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The Franklin Institute

DEVOTED TO

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AMATEUR CINEMATOGRAPHY AND THE KODACOLOR PROCESS.*

BY

C. E. KENNETH MEES, D.Sc.[1]

Director, Research Laboratories, Eastman Kodak Company;
Member of the Institute, Associate Editor.

IN August, 1923, a paper describing a new process of amateur cinematography which had just been introduced was published in the JOURNAL OF THE FRANKLIN INSTITUTE. The process depended upon the use of a narrow film, 16 mm. wide, instead of the 35 mm. of the standard motion picture film. This film, after exposure, is not developed to a negative and printed, as is customary with 35-mm. film, but is developed by what is known as the *reversal* process. In this reversal process, the exposed image is first developed, and then the developed silver is dissolved in a *bleaching* bath, as it is called, which oxidizes the silver. This leaves behind the undeveloped silver bromide which was not affected by the developer, because it was not exposed to light. After a fresh exposure to light, this remaining silver bromide is developed in its turn and gives a positive. This is illustrated in Fig. 1, which is a drawing made from pictures taken through the microscope. In Section 1, we see the grains of the light-sensitive silver bromide in the emulsion, and in Section 2

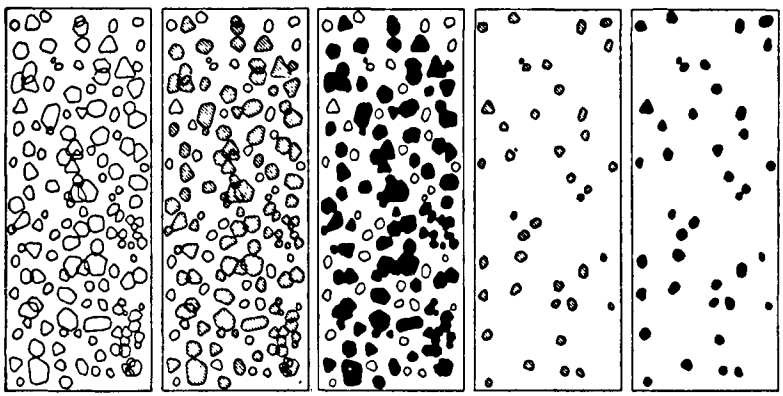
* Presented at a meeting held Thursday, October 25, 1928.

[1] Communication No. 378 from the Kodak Research Laboratories.

(Note.—The Franklin Institute is not responsible for the statements and opinions advanced by contributors to the JOURNAL.)

we have marked, by crosshatching them, those which have been affected by light during exposure in the camera. They would not show any change to the eye, of course, because the change by light is not visible. The grains form what is called the *latent image*. Now, after development, these exposed

FIG. 1.



grains turn into black metallic silver, and this is shown in Section 3 of the diagram. Then the bleaching bath removes all the silver, and it leaves behind the silver bromide grains which were not exposed, as shown in Section 2. Now, these are re-exposed and developed and make the final positive, as we see in Section 5.

In this reversal process, control of the results is obtained by regulating the second exposure in proportion to the amount of exposure given in the camera. Thus, if in the camera a heavy exposure is given, much of the silver will be developed at the beginning, and when this is removed, there will be only a small amount of silver bromide left to be used for the production of the positive. It is necessary, therefore, to give a very heavy exposure in order to make all this developable and so get enough density in the image. On the other hand, if the camera exposure is light, there will be a great deal of undeveloped silver bromide available to form the final image, and it is necessary to give only a short exposure, as otherwise too dense an image will be produced. This reversal process

enables pictures of excellent quality, and with a very small amount of grain, to be obtained at low cost upon the narrow film so that the tiny pictures will stand the high magnification necessary for projection upon the screen.

Since 1923 this process has developed until it is now a considerable industry. The early apparatus described in my previous paper has been supplemented by other forms of cameras and projectors, so that at the present time a wide range of extremely convenient apparatus made by several different manufacturers is available for the public. As a general rule, at the present time, the cameras are driven by spring motors which are wound up between exposures, and the trend of design is in the direction of cameras as small and compact as possible, so that an amateur motion picture camera is little more bulky than the very small and compact cameras used for still photography.

The processing is now done on automatic machines consisting of a number of tanks which carry racks with rollers, by means of which the film is fed continuously through the solutions. The machines are entirely automatic, the films being fed in as they come from the customer and taken out of the drying cupboard as positives ready for projection. As the film travels down the machine, it is first developed to a negative, the developed silver is then removed in a bleaching bath, the film is cleared of the bleach and resensitized, and it is then exposed to an extent dependent upon the original exposure and controlled by the optical density of the film itself. The control is effected by the passage of the film between a source of red light and a thermopile. The current from the thermopile, produced by the heating effect of the red light passing through the film, controls a galvanometer vane, which is interposed in an optical system by means of which a beam of white light is projected upon the film. Thus, the second exposure is dependent upon the current in the thermopile and, therefore, upon the transmission of the red light by the film.

After this second exposure, the new image is developed as a positive, the film is fixed, washed, and dried; all these operations going on as the film travels forward through the machine. In a little more than an hour from the time the

film enters the machine, it is ready for projection. Each machine takes a new film every five minutes.

The processing stations are available all over the world, these containing more than a hundred of the automatic machines that are described above. At the present time, the traveler can get his film processed in Rio de Janeiro or in Japan or in China as easily as he can in Florida. In a few days, the pictures ready for projection are returned to him from the station.

From these original films, duplicates are now made by the same reversal process. A print is made in a small contact printer upon a fine-grained positive film prepared for the purpose, and this is then developed by the reversal process in an automatic machine, so that, from the original positive, any number of prints can be made in the same way and with the same ease as prints are made from a negative.

In photography it is, of course, more rational to use a positive than a negative as an original since the negative is only a means to an end and is of no value in itself. The earliest photographic processes gave positives, and when a process was first introduced in which a negative was obtained which had to be printed in order to get a final result, it was regarded as a defect in the process that it was unable to give a direct photograph; a defect, of course, which was more than compensated by the ease with which copies could be multiplied. Provided, as is the case by the use of the reversal process, that prints can be obtained from positives as easily as they can from negatives, the use of positives has the additional advantage that the original is of value as well as the prints made from it.

There have been many other developments in the field of amateur cinematography. Thus, at first, titles were made by setting up the original material with a printing press and photographing it in a camera of the ordinary type. By the use of typewritten titles and special cameras enabling the pictures to be made at high speed, the cost of titles has been very greatly reduced and their use enormously increased, so that at the present time it is very easy for an amateur to insert appropriate titles in his pictures and thus give them a more permanent value for projection in his absence. The

greatest development in the whole field of amateur cinematography lies in the increasing use of the 16-mm. for the projection of pictures originally made in the standard size. Very soon after the small size was introduced on the market, a library was formed in which reduction prints from standard professional negatives were available for renting and thus could be shown in the home. This was later supplemented by the supply from standard short-length negatives of prints which could be bought by the owners of projectors, and such prints, made by many different producers, are now available on the market.

The availability of the small film has made it possible to plan the use of films for teaching purposes on a much larger scale than could be considered if the expensive standard width film were the only one available, and complete programs of class room films for the instruction of children are now in course of preparation. These films have been designed by teachers collaborating with those still engaged in scholastic work throughout the country and are made to fit into the curriculum of the elementary schools and high schools. An experiment in this field has been conducted by the Eastman Kodak Company. This consisted in the production of a number of special films adapted to class room instruction. These films were then shown to groups of children in a number of schools in twelve cities in different parts of the country, groups in other schools in the same city being taught by equally competent teachers without the films.

The results obtained in this experiment show that the children who received instruction with the use of film made much greater progress in a series of objective tests than the children covering the same area of instruction but without the use of the film. Moreover, these results show that those pupils who had instruction with the film manifested greater interest in the work of the class room; that this interest was a sustained one extending through the school term; that these children did more reading of a finer quality; that their self-activities were stimulated; that they discussed and wrote more freely on the topics studied; that they correlated features of the film with their own personal experiences and with community life. The results achieved, therefore, through the

use of films, were the outstanding objectives of class room work.

As an instance of one of the many ramifications which the development of the amateur standard film has made possible, there may be mentioned an interesting application invented by Mr. George L. McCarthy and placed on the market, by the Eastman Kodak Company, under the name of the *Recordak*. Mr. McCarthy conceived the idea that instead of listing the details of the checks passing through a bank every day, it would be feasible to photograph them, the checks

FIG. 2.

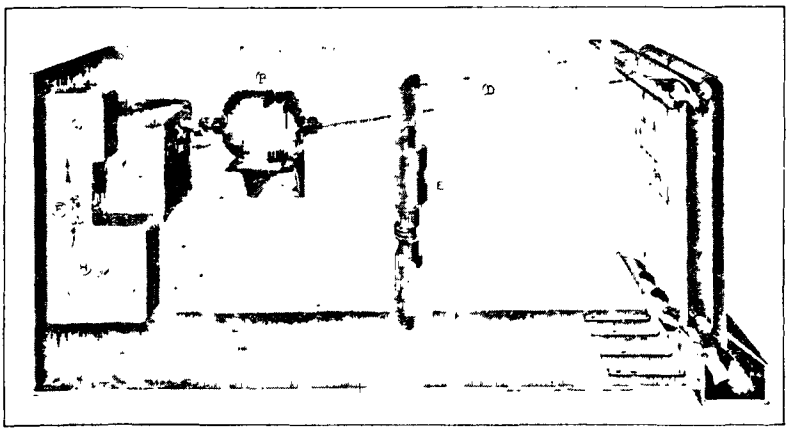


Diagram of Recordak.

being carried on an automatic belt in front of a camera which would record two checks on every frame of the amateur standard film. A hundred feet of film have four thousand frames and can thus record eight thousand checks. Another advantage of doing this is to provide a bank with a definite proof of the existence of each check drawn by a depositor before it is returned to the depositor, so that, should it be claimed later that the bank had made an error and had entered against a customer's account a check which did not exist, the photograph of the check could be produced as proof of the accuracy of the record. The Recordak instrument designed for this purpose is a very ingenious machine (Fig. 2). The checks are simply fed into a hopper, and as the amount

of one is tabulated on an adding machine connected to the Recordak, its predecessor passes into the machine and is photographed, being held on a vertical belt by wires passing over the face. The camera is double and will make either one or two records of the check, as may be desired, since in most cases it is necessary for the bank to send one set of records to another bank and retain one set for itself. The great economy of time and labor which is obtained by this new introduction is causing Recordaks to be installed in the banks as rapidly as they can be made available, and it is probable that the Recordak will play as important a part in the machinery of the banking business as the adding machine, thus introducing what may truly be called *accounting by photography.*

From the very beginning of photography, it has been the dream of every worker to produce pictures in natural colors instead of in monochrome, and in this field the Kodak Company has been experimenting for more than twenty-five years. Every process that appeared to be promising has been studied very carefully, but no process was considered to fulfill the requirements which were held by the officials of the company to be necessary for a process of amateur color photography. These requirements arose from the fundamental idea on which the Kodak Company is based. When Mr. Eastman started his work on photography, he was impressed by the difficulty that photography presented to the ordinary unskilled person; and the idea that photography should be accessible to anybody who was interested in obtaining photographs, whether he wished to acquire expert knowledge or not, led to the design of portable, flexible film and and the film cameras which have become so widely known as *Kodaks.* The same ideas to which the origin of the company itself was due still held in 1923, when it was made possible for anybody without professional skill to take motion pictures as easily as he had previously been able to take still pictures. In the search for a process of color photography, the same conception prevailed, and in the summer of 1928 it was felt that a process had been worked out which would meet those requirements.

The processes of color photography depend upon the fact first demonstrated in 1861 by Clerk Maxwell in a lecture at

the Royal Institution that colors can be duplicated in photography by taking advantage of the fact that any color can be matched by a mixture of three primary colors—red, green, and blue-violet. Maxwell took three photographs of a colored ribbon—one through a red solution, another through a green solution, and a third through a blue—and from the three negatives he made three lantern slides and projected them on the same screen in register from three lanterns, each one projecting its slide through the solution which had originally been used for taking the negative. From these three primary colors—red, green, and blue-violet—in the correct proportions, any color can be obtained. Thus, if red light is thrown on the screen and green light added to it, we shall get first orange-red and then orange, which, as the intensity of the green grows in proportion to the red, finally becomes yellow. Similarly, the addition of green to blue-violet produces shades of blue-green and finally of pure bluish green exactly opposite to the red sensation in color, while mixtures of red and blue produce all possible shades of violet, ranging through magenta into purple. Since the three colors together generate white when they are mixed in equal proportion, we can dilute any of the strong colors—red, yellow, orange, green, blue-green, or blue-violet—by the addition of one or two other colors.

In 1869, a little book in French by Louis Ducos du Hauron was published under the title *Les Couleurs en Photographie.* In this, du Hauron explained the analysis of light into three primary colors and proposed the obtaining of three negatives through three color filters, but, instead of proposing that the pictures should be projected on top of one another, he suggested two other methods of combining the colors to obtain a finished picture. One of these processes has since become of very considerable practical importance in color photography and is now known as the *screen plate* process. Du Hauron suggested that the surface of a glass plate or film might be covered with tiny unit filters and a sensitive emulsion coated on top of this. Exposures are made through the color filters, so that the image is cut up into a lot of little sections—so small that they are invisible to the eye, each section being taken through one of the filters. If the emulsion is so treated

in development that a positive is obtained, this will give a direct photograph in colors. This process was realized on a commercial scale in the Lumière Autochrome plate, which was first put on the market in 1907.

Another process which was discussed at greater length by du Háuron is that which is now known as the *subtractive* process. When a color picture is projected on the screen by means of a three-color projection lantern, the pictures made in this way, by adding three different lights, are said to be made by the *additive* process. Suppose that over a sheet of yellow gelatin, which will absorb blue light, is put a sheet of magenta gelatin, which will absorb green light; then, since the yellow absorbs the blue light, and the magenta, the green light, only red light is transmitted. A red image can thus be obtained either by projecting it through a red filter or by putting a magenta image on the top of a yellow one. In the same way, a green image can be obtained by putting a blue-green one on top of a yellow one, when the yellow will cut the blue out of the blue-green and leave only green.

In working the subtractive process, the three negatives are taken just as for the additive process, and then positives are printed in some way which enables them to be made of a colored material, the commonest being to make them by printing in bichromated gelatin. By this process, the three negatives can be printed in colored dye, the picture taken through the red filter being printed on gelatin dyed blue-green; the one taken through the green filter, on gelatin dyed magenta; and the one taken through the blue filter, on gelatin dyed yellow; and, now, if the three are cemented together in register, the resulting transparent color picture will reproduce the colors of the original subject.

We thus see that there are three distinct processes of color photography which must be borne in mind in any consideration of the subject:

(1) The additive processes, in which the three positives are projected through their corresponding filters on to a screen.

(2) The screen plate processes, in which the unit filters are so small that they are invisible but in which the process is still essentially an additive one.

(3) The subtractive processes, in which the filter negatives are printed in their complementary colors, and these prints are then superimposed.

For motion picture work, the additive processes would appear to present great advantages, but they have one serious disadvantage: they require a modification of the projection apparatus, and this has hindered their introduction into the motion picture theater field. The very beautiful Gaumont process of color motion picture photography, for instance, involved the taking of three pictures for every frame. These were taken one above the other through three lenses fitted with three filters and were then projected through a triple lens also fitted with filters, the three pictures being superposed on the screen to give a picture in color.

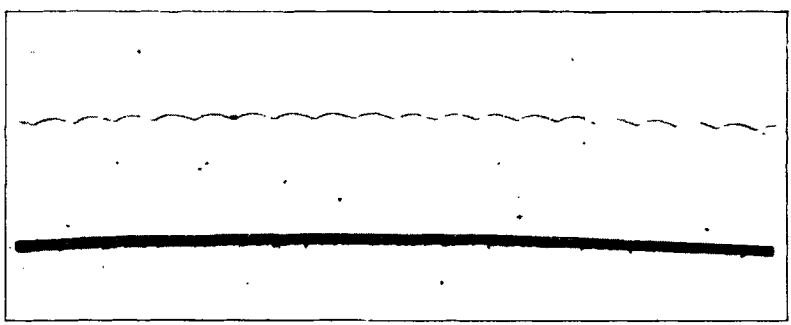
The processes which are in use at the present time for the production of colored motion pictures for the theaters are subtractive processes, and they usually take advantage of the fact that a film has two sides, so that one picture can be put on one side, and another, in register, on the other side of the film. We ourselves experimented with processes of this kind and have obtained some excellent results from a process which we term Kodachrome. While these two-color processes represent a practical compromise which may be useful in connection with the motion picture theater, the quality of the results is not satisfactory for outdoor scenes, and both the taking and the printing is too complicated for use by amateurs.

A most satisfactory process for amateur cinematography in colors would be the use of a screen film in which the film would carry filter units preferably dispersed regularly in dots or lines. There are, however, great practical difficulties in the preparation of such a screen and in securing the adherence of the emulsion to it, which have hitherto prevented such a process from attaining practical realization.

In 1908, R. Berthon patented a process which realized most of the advantages of a screen film process without involving too great difficulty in making the film. In this process, the color filters are placed in the lens, while the film is embossed on the support side with a number of small lenses. The film

therefore looks as is shown in Fig. 3. In this figure, the corrugated edge represents the surface of the film base, in which the small lenses shown have been embossed. We see below this the thickness of the base itself, and then a thick black line which represents the thickness of the emulsion. When this film is put into the camera, the embossed lenses will form miniature images of the filters on the film emulsion. Suppose, for instance, that the lens contains a stop with three holes in it—one covered by a red filter; a second, by a green; and a third, by a blue filter. Then, behind each lens there would be formed on the emulsion a group of three dots—one

FIG. 3.



Section of Kodacolor film.

dot corresponding to the red filter; one, to the green; and the third, to the blue filter. In this way, the effect produced by the embossed film and the filters in the lens is exactly similar to that which would be obtained by the use of a screen film in which the filter units were in contact with the emulsion, but the process has the advantage that the film is much easier to make and to use. The lenses can be either spherical or cylindrical. If they are cylindrical, the filters used in the lens can be strips parallel to the embossing on the film, and we shall then get line images of them on the film.

Some years after this invention, Berthon became associated with Anthon Keller-Dorian, the owner of an engraving plant in Alsace, and the Société Keller-Dorian-Berthon was formed to develop the process for commercial purposes. In 1925 this process was offered to the Eastman Kodak Company

and was thought to offer possibilities of success, especially as it was closely akin in its requirements to the method of amateur cinematography which the company had already developed. Rights in the process were therefore purchased, and development was started with a view to perfecting the process for use with the amateur 16-mm. film.

A great deal of study was involved. It was necessary to standardize the methods of making the lenses on the film; to design and make a suitable emulsion strongly sensitive to green and red light and yet with sufficiently fine grain for the minute structure of the separate color elements to be resolved; and, especially, to work out suitable methods by which the film could be developed and reversed while the rendering of color was retained. It was the successful accomplishment of all this work which made the Kodacolor process a commercial possibility.

In order to make color pictures by this process, a photographer using a standard Ciné-Kodak fitted with an f/1.9 lens has only to insert a color filter into the lens and thread the Kodacolor film in the camera. After the film has been exposed, it is sent away for processing and comes back to the photographer as a roll of black and white film which can be projected in an ordinary projector and will give a black and white picture on the screen. But if the projector is fitted with a special color filter like that which is used in the camera, then a colored picture will be obtained on the screen.

The color filter is composed of three separate areas—red, green, and blue—and it slips into the lens in the place of the ordinary hood, which is removable.

But the secret of the Kodacolor process is in the film. The film surface is embossed by running it through steel rollers with tiny cylindrical lenses composed of the film base material and extending lengthwise of the film. The lenses on the film are about four times narrower than the dots making up the illustrations in this journal and they are therefore invisible except under a microscope. They cover completely the surface of the side of the film opposite from the sensitive emulsion. That surface faces the camera lens, and the emulsion is away from the lens (see Fig. 3).

When the trigger of the camera is pressed, light reflected

from the subject passes selectively through the three-color filter, on through the camera lens, and thence through the tiny embossed lenses on the film to the sensitive emulsion coating on the opposite side, where it is recorded (Fig. 4).

FIG. 4.

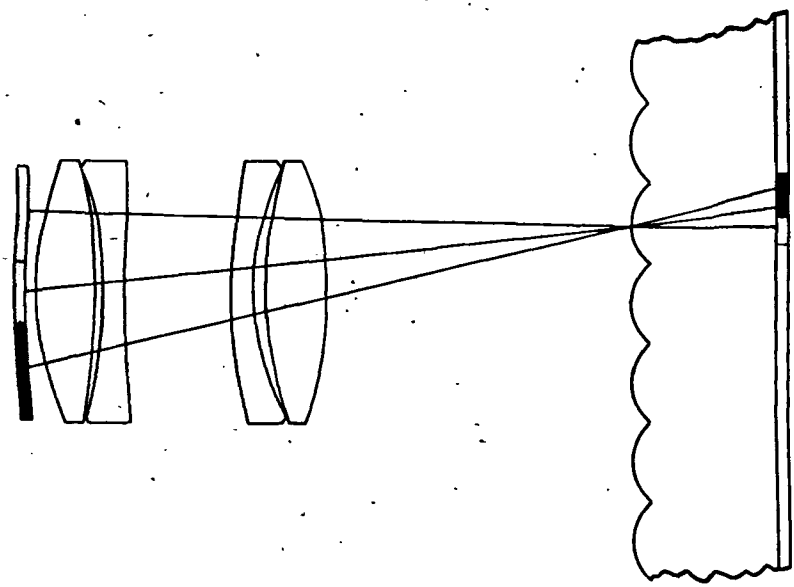
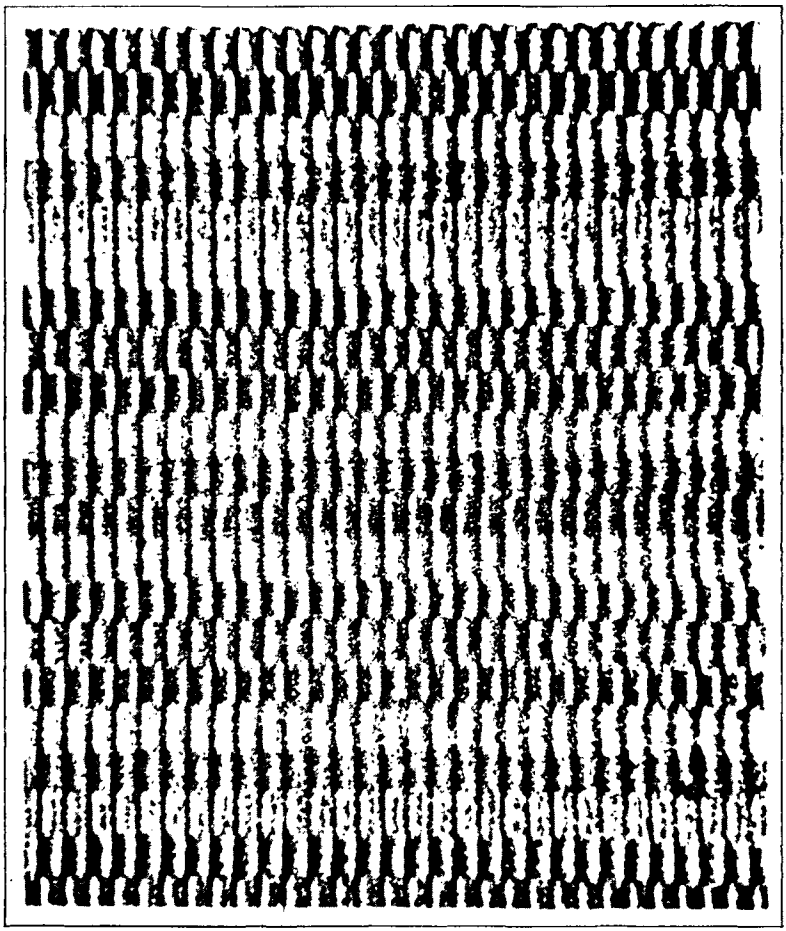


Diagram of Kodacolor process.

The function of the lenses embossed on the film is to guide the rays of light falling upon each tiny area and lay them on the sensitive emulsion as three distinct impressions corresponding to the three filter areas, so that the three colors covering the lenses are imaged behind each tiny cylindrical lens as three parallel vertical strips, because the tiny cylindrical lenses are parallel to the strips of color on the filter. Thus, the width of each of the minute areas of emulsion is subdivided into three parts related to the three filter areas and affected by light that is able to pass through the different colors. The sum of these invisibly small affected areas of film constitutes the whole photographic image. In Fig. 5 is shown a photomicrograph of a picture showing a number of colored strips running horizontally across the image. The vertical strips are produced by the embossed lenses.

A red ray from an object in front of the camera, for instance, reaches the sensitive material of the film at a spot related to the red area of the filter. The *reversal process*

FIG. 5.



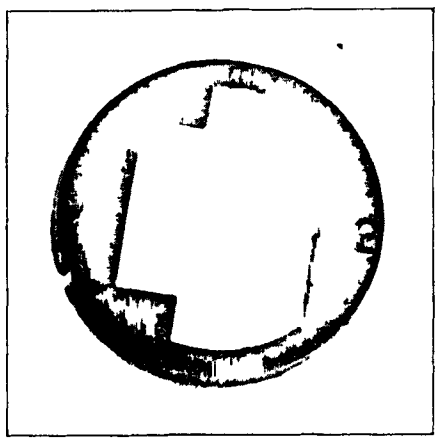
Photomicrograph of Kodacolor picture of narrow colored strips. The strips are horizontal, the embossed lenses running vertically.

turns this affected spot into a transparent area, leaving opaque the adjoining, unaffected areas related to the green and blue segments of the filter.

So, also, with the green and blue and with combinations of colors. The sum of the points on the scene containing red makes a photograph from red light on the emulsion areas related to the red filter area; the sum of the blue also makes a separate photograph; and, similarly, the green.

It is necessary to have some method of controlling the exposure for the separate colors. The film may vary somewhat in its sensitiveness. Even if we make one batch of film

FIG. 6.



. Ratio diaphragm cap.

which has equal sensitiveness to the three different filters, it does not follow that this can be duplicated exactly, and on the exactness of adjustment depends the color rendering. This adjustment for the different sensitiveness of the emulsions is accomplished by cutting down the length of the color filters used in the camera. This is done by what is termed a *ratio diaphragm cap* (Fig. 6). These caps are chosen, after the emulsion has been tested, to correspond to the color sensitiveness of that particular batch of emulsion, and the cap is then placed on the end of the film in such a position that it cannot be missed by the user. All that is necessary is for him to take out his color filter from the camera when placing a new film in the camera, throw away the old cap, and replace it by a new one from the end of the fresh film.

In this way, he can always be sure of getting the correct ratio of exposures.

Now, in order to project the pictures, the developed film is put in the projector, which contains exactly the same optical system reversed. Behind the film are the condenser and the source of light. The color filter consists of the same three primary colors—red, green, and blue. The reverse of the fact that white light divides up into the colors of the spectrum is that light coming evenly out from the three colors of the filter on a projector and superimposed on a screen appears white. But cover up the green and blue segments of the filter, and the screen will turn red. Cover up the red and blue, and the result will be green. Varying the areas of each color through which the light may shine will give infinite shadings between these colors. Black is the total elimination of light.

Now, when we have a picture on the film, the opaque areas of the film cover up, in effect, certain of the filter areas; they prevent the light from going through where it is not needed by cutting off, at the film, rays which would otherwise pass through the embossed lenses, through the projecting lens, and through the filter area in question to the screen.

From any point on the scene, the only colors which are permitted to be projected are those which, on the screen, blend into the corresponding original colors of the scene photographed. The pattern of these rays, from all the cylindrical lenses on each frame, projects a picture on the screen, with each ray contributing its speck of light to the color or blend of colors at one point.

Owing to the absorption of the light by the color filter used in projection, it is necessary to be content with a small picture on the screen, and for this reason small screens are used, so that the illumination can be sufficient.

As will be seen, the projector system is optically an exact inversion of the camera system, but there is in general one difference between the camera and the projector: the focal length of the projection lens is usually considerably greater than that of the camera lens. The relative apertures must be the same; that is, the cone of light coming through the projector lens must have exactly the same angle as the cone

of light coming through the camera lens, since otherwise the optical conditions will not be reproduced correctly and the colors will not be correct. A difference in focal length can, however, be compensated by the use of a supplementary lens, which places images of the color filters at the right point in the optical system; and this lens, which is termed a *compensator*, is fitted on to the lens of the projector. The *compensator* makes no difference when black and white pictures are being projected except to change slightly the focal length of the projection lens, but is required to obtain an even screen when Kodacolor pictures are being projected.

Although the Kodacolor process has been on the market for only a short time, the results which are being obtained by the users show quite definitely that it is successful. The results are, on the whole, excellent, and provided that the photographer exercises reasonable care in the choice of his subjects and in the operation of the camera, there is no more difficulty at the present time in obtaining motion pictures in color than in the making of still photographs.

ROCHESTER, NEW YORK,
October 19, 1928.

Botanical Orders and Chemical Composition. THOMAS PERCY HILDITCH of the University of Liverpool (*Proc. Royal Soc.*, B, 1928, *103*, 111–117) notes a marked tendency toward the production of quite specific fatty acids in the seed fats of certain orders of plants. Thus the *Palmae* or palms produce lauric acid; the *Myristiceae* or nutmeg family produce myristic acid; the *Cruciferae* or cress or mustard family produce erucic acid; the *Umbelliferae* or parsley family and the closely related *Ariliaceae* or English ivy family produce petroselinic acid, which is a metamer of oleic acid and has its double bond between the sixth and the seventh carbon atoms from the carboxyl group. Oleic, linoleic, palmitic, arachidic, lignoceric, and other acids may also be present as minor constituents of the fats.

J. S. H.

Preservation of Peaches. CHARLES W. CULPEPPER, JOSEPH S. CALDWELL, and R. C. WRIGHT of the Bureau of Plant Industry (*U. S. Dep. Agric. Technical Bull.* No. 84, 1–13, 1928) have published the results of a research of three years' duration on the preservation of peaches with the retention of their flavor so that the product could be used in the manufacture of ice cream.

Peaches of suitable varieties and of the proper degree of ripeness were either crushed or pulped. The resulting material was divided into three lots, to two of which were added 10 per cent. and 40 per cent., respectively, by weight of sugar, while the third lot contained no added sugar. Each lot was divided into two portions, one of which was canned, the other frozen. The process of canning was conducted in the usual manner, but enamel cans were found most suitable. For preservation by cold storage, the uncooked material was placed in hermetically sealed cans which were then kept at a temperature of 15° F. until their contents had frozen; they were then kept at a temperature of 15° to 25° F. until used. Cold storage at a temperature of 32° F. was not satisfactory. The chief advantage in the addition of sugar was the prevention of fermentation of the raw fruit prior to complete freezing of the contents of the cans; addition of 40 per cent. by weight of sugar was found preferable. Peaches preserved by each of these processes kept in good condition for periods as long as 21 months. They retained their value as a flavor for ice cream. Freezing or cold storage gave a slightly better preservation of the fresh fruit flavor, while canning had the advantage of a lower cost of production and distribution.

J. S. H.

THE SCATTERING OF A STANDING ELECTRO-MAGNETIC WAVE.

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Communication No. 31.

Introduction. During consideration of the pure Compton effect, the question arose as to what would happen if instead of the running electromagnetic wave the electron were to encounter a standing electromagnetic wave. Would the scattered radiation be the sum of that expected from the two component waves or would the effect be more complicated? An answer to this question has been attempted on the basis of the Schroedinger wave mechanics; it is dealt with essentially from the point of view put forth by Gordon,[1] but in addition the simple perturbation theory given by Klein [2] is shown to yield the same solution, if carried out to its ultimate conclusion. The Schroedinger theory has not yet been developed to the point where it is utterly consistent in telling the whole story of the Compton effect, but in carrying out the computations in connection with the standing electromagnetic wave we shall try to make the procedure as consistent as possible and still retain the theory in its present state.

The procedure for the simpler case of a running wave may be sketched as follows. (1) The Schroedinger wave equation for an electron in an electromagnetic field is derived by inserting the appropriate Hamiltonian operator in the diffusion equation. (2) A solution ψ_s is found for this differential equation for the case in which the vector and scalar potentials involved are those of a plane X-ray wave and where the amplitudes of these potentials are assumed so small that terms involving their squares are to be neglected. The constants appearing in this solution are correlated with

[1] Gordon, W.: *ZS. f. Physik*, 40, 117, 1926.

[2] Klein, O.: *ZS. f. Physik*, 41, 407, 1927.

the energy and momentum of the electron. For each value of the momentum $\mathbf{p}$ of the electron the simple solution is to be multiplied by a normalizing factor $C(p)$. We must now find in terms of these individual solutions one solution ψ which at $t = 0$ satisfies given initial conditions. Since the momentum of the electron may vary continuously from minus infinity to plus infinity, such a ψ will take the form of the integral

$$\psi = \int_{-\infty}^{\infty} \psi_s C(p) z(p) dp, \tag{1}$$

where $dp = dp_x dp_y dp_z$ and where ψ_s corresponds to the solution for constant p_x, p_y, p_z. This integral thus represents an infinite sum in these individual solutions with coefficients $z(p)$ which are to be determined to satisfy these initial conditions. (3) We then build up from this value of ψ and from its conjugate $\overline{\psi}$ the expression for the effective current density which in terms of ψ and $\overline{\psi}$ has been given by Gordon and Klein as

$$\rho \mathbf{u} = \frac{e}{2m_0} \frac{h}{2\pi i} \left(\overline{\psi} \operatorname{grad} \psi - \psi \operatorname{grad} \overline{\psi} - \frac{4\pi i}{h} \frac{e}{c} \mathbf{U} \psi \overline{\psi} \right). \tag{2}$$

(4) From this value of the current density we construct the vector potential $\mathbf{U}'$ which appears as an integral with respect to p_x, p_y, p_z, each element of the integral being characterized by the particular values of p_x, p_y, p_z, by $h\nu$, by the unit vector $\mathbf{n}$ in the direction of the incident radiation, and by the unit vector $\boldsymbol{\xi}$ in the direction of observation. The coefficient of the time in the exponent is of the form $h\nu_{1P}$ where

$$h\nu_{1P} = E - E_{1P} + h\nu, \tag{3}$$

and E_{1P} is a certain definite function of the p's, $\mathbf{n}$ and $\boldsymbol{\xi}$. If now we consider a problem in which an electron is permitted to have only two stationary states characterized by p_x, p_y, p_z, E and by p_{x1}, p_{y1}, p_{z1}, E_1, the Schroedinger theory for the problem gives for the vector potential of the emitted radiation an expression in which the coefficient of t is equal to $h\nu_1$ where

$$h\nu_1 = E - E_1 + h\nu. \tag{4}$$

It gives only one direction in which there is any intensity for

this radiation, the direction being characterized by the unit vector ξ_1 given by the vector equation

$$\mathbf{p}_1 - \mathbf{p} = \frac{h\nu}{c}\mathbf{n} - \frac{h\nu_1}{c}\xi_1. \tag{5}$$

Now in this problem the quantities $\mathbf{p}_1$, E_1, ξ_1 occur in the expression for $\mathbf{U}'$ in identically the same positions as do certain quantities E_{1P}, $\mathbf{p}_{1P}$ and ξ_1 (which are related by an equation of the type (5) and by the relation $|\mathbf{p}_{1P}| = \sqrt{\frac{E_{1P}^2}{c^2} - m_0^2c^2}$) in the expression for $\mathbf{U}'$ for the general problem. Our hypothesis of correlation is therefore to say that in that problem a quantum of energy of frequency ν_{1P} given by

$$h\nu_{1P} = E - E_{1P} + h\nu \tag{6}$$

may be radiated in a direction given by ξ_1 satisfying the vector equation

$$\mathbf{p}_{1P} - \mathbf{p} = \frac{h\nu}{c}\mathbf{n} - \frac{h\nu_{1P}}{C}\xi_1, \tag{7}$$

that the probability of such an event taking place is dependent upon the coefficient of $e^{2\pi i\nu_{1P}(t-R/c)}$ in the expression for $\mathbf{U}'$, and that whenever the quantum is radiated it is accompanied by the emission of an electron of energy equal to the quantity E_{1P} and of momentum equal to the quantity $\mathbf{p}_{1P}$. The correlation procures conservation of energy as between the initial and final states of the electron and radiation, and it procures conservation of momentum provided that the momentum of the quantum is regarded as $\frac{h\nu}{c}$.

In applying this procedure to the case of a standing electromagnetic wave, the vector and scalar potentials for the sum of two equal and opposite plane X-ray waves must be used in the original differential equation for ψ. That part of the emitted vector potential which is dependent on the time then turns out to consist to a first approximation of two additive parts, one resulting from the component

X-ray wave travelling in one direction and the second from the other component wave travelling in the opposite direction. The function multiplying the frequency factor in each of these terms governs the probability that the X-ray concerned will be scattered by the given electron. The radiation observed in any direction then consists of two parts, differing in frequency and depending in intensity on the probability that the component wave will be scattered in that direction. The correlation with the motion of the recoil electron for each of these terms is carried out separately as indicated above in the more simple case.

1. *Derivation of the Schroedinger equation.* The fundamental equation of the Schroedinger wave mechanics, as it is accepted to-day, is the diffusion equation [3]

$$-\frac{h}{2\pi i}\frac{\partial \psi}{\partial t} = H\left(\frac{h}{2\pi i}\frac{\partial}{\partial q_r}, q_r\right)\psi. \tag{8}$$

H is here the Hamiltonian operator, which for the purposes in hand may be regarded as being derived from the form

$$H = \sum_r \dot{q}_r p_r - L, \tag{9}$$

where L is the quantity $L(\dot{q}_1, \dot{q}_2, \cdots q_1, q_2, \cdots t)$ which makes $\delta \int L dt = 0$ give the classical equations of motion. For an electron in an electromagnetic field having vector potential $\mathbf{U}$ and scalar potential φ, L is known to be given by the following equation:

$$L = -m_0 c^2 \sqrt{1 - \dot{q}^2/c^2} - e\varphi + \frac{e}{c}(\mathbf{U}\cdot\dot{\mathbf{q}}), \tag{10}$$

where the dot represents the scalar product of the two adjoining vectors. Since p_r may be defined as $\partial L/\partial \dot{q}_r$, we have for $\mathbf{p}$ the vector equation

$$\mathbf{p} = \frac{m_0 \dot{\mathbf{q}}}{\sqrt{1 - \dot{q}^2/c^2}} + \frac{e}{c}\mathbf{U} \tag{11}$$

and the important Hamiltonian function (9) may be expressed as

[3] Darwin, C. G.: *Roy. Soc. Proc.*, 117, 258, 1927.

$$H = \mathbf{p}\cdot\dot{\mathbf{q}} + m_0c^2\sqrt{1 - \dot{q}^2/c^2} + e\varphi - \frac{e}{c}(\mathbf{U}\cdot\dot{\mathbf{q}}) \qquad (12)$$
$$= \frac{m_0c^2}{\sqrt{1 - \dot{q}^2/c^2}} + e\varphi,$$

which shows H to be the sum of the kinetic and potential energies which it is so often assumed to be. H must, however, for use in (8) be expressed in terms of the momenta p_r. Equation (11) squared gives

$$\left(p_x - \frac{e}{c}U_x\right)^2 + \left(p_y - \frac{e}{c}U_y\right)^2 + \left(p_z - \frac{e}{c}U_z\right)^2$$
$$= \frac{m_0^2\dot{q}^2}{1 - \dot{q}^2/c^2} = m_0^2c^2\left(-1 + \frac{1}{1 - \dot{q}^2/c^2}\right), \qquad (13)$$

which solving for $1/\sqrt{1 - \dot{q}^2/c^2}$ gives the relation

$$\frac{m_0c^2}{\sqrt{1 - \dot{q}^2/c^2}} = c\left[\left(\mathbf{p} - \frac{e}{c}\mathbf{U}\right)^2 + m_0^2c^2\right]^{\frac{1}{2}}. \qquad (14)$$

When (14) is inserted in (12), H is expressed as a function of the p's and q's as required. For the H operator appearing in the diffusion equation (8) p_r is to be replaced by $(h/2\pi i)(\partial/\partial q_r)$ so that that equation becomes

$$-\frac{h}{2\pi i}\frac{\partial\psi}{\partial t} = c\left[\left(\frac{h}{2\pi i}\text{grad} - \frac{e}{c}\mathbf{U}\right)^2 + m_0^2c^2\right]^{\frac{1}{2}}\psi + e\varphi\psi, \qquad (15)$$

and this is the fundamental equation for the ψ associated with an electron in an electromagnetic field. Expanding, (15) becomes

$$-\frac{h}{2\pi i}\frac{1}{c}\frac{\partial\psi}{\partial t} = \left[-\frac{h^2}{4\pi^2}\nabla\cdot\nabla - \frac{h}{2\pi i}\frac{e}{c}(\mathbf{U}\cdot\text{grad} + \text{div }\mathbf{U})\right.$$
$$\left. + \frac{e^2}{c^2}\mathbf{U}^2 + m_0^2c^2\right]\psi + \frac{e}{c}\varphi\psi. \qquad (16)$$

The operator acting on ψ may be applied twice, giving

$$\left(-\frac{h}{2\pi i}\frac{1}{c}\frac{\partial}{\partial t} - \frac{e}{c}\varphi\right)^2\psi = \left[-\frac{h^2}{4\pi^2}\nabla\cdot\nabla - \frac{h}{2\pi i}\frac{e}{c}(\mathbf{U}\cdot\text{grad}\right.$$
$$\left. + \text{div }\mathbf{U}) + \frac{e^2}{c^2}\mathbf{U}^2 + m_0^2c^2\right]\psi \qquad (17)$$

In view of the fact that

$$\text{div } \mathbf{U}\psi = \mathbf{U}\cdot\text{grad } \psi + \psi \text{ div } \mathbf{U} \tag{18}$$

and of the equation of continuity

$$\text{div } \mathbf{U} + \frac{1}{c}\frac{\partial \varphi}{\partial t} = 0 \tag{19}$$

equation (17) reduces to

$$\left[-\frac{h^2}{4\pi^2}\left(\nabla\cdot\nabla - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) - 2\frac{h}{2\pi i}\frac{e}{c}\left(\mathbf{U}\cdot\text{grad} + \frac{\varphi}{c}\frac{\partial}{\partial t}\right) + m_0^2c^2 + \frac{e^2}{c^2}(\mathbf{U}^2 - \varphi^2)\right]\psi = 0. \tag{20}$$

The diffusion equation now appears for the case of an electron in an electromagnetic field as the older and better known wave equation of Schroedinger. This may be seen to be identical with that found by Gordon (l.c. p. 119 equation 9).

2. Solution of the Schroedinger equation. The solution of this Schroedinger equation requires that we make assumptions as regards the form of the vector and scalar potentials. In the case of the ordinary Compton effect, we assume simple harmonic potentials with the frequency of the X-ray, i.e.

$$\begin{aligned} \mathbf{U} &= \mathbf{a}\cos\frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{r} - ct), \\ \varphi &= a\cos\frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{r} - ct), \end{aligned} \tag{21}$$

where the amplitudes are so small that their squares may be neglected. Then $\mathbf{U}^2$ and φ^2 are immediately to be neglected. For the case in which we are interested, that of a standing electromagnetic wave, we shall assume potentials of the form

$$\begin{aligned} \mathbf{U} = \mathbf{U}_1 + \mathbf{U}_2 &= \mathbf{a}\left[\cos\frac{2\pi\nu}{t}(\mathbf{n}\cdot\mathbf{r} - ct) + \cos\left\{\frac{2\pi\nu}{c}(-\mathbf{n}\cdot\mathbf{r} - ct) + \delta\right\}\right], \\ \varphi = \varphi_1 + \varphi_2 &= a\left[\cos\frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{r} - ct) + \cos\left\{\frac{2\pi\nu}{c}(-\mathbf{n}\cdot\mathbf{r} - ct) + \delta\right\}\right], \end{aligned} \tag{22}$$

where ν is the frequency of the X-ray, $\mathbf{n}$ a unit vector in the direction of this incident radiation, and δ an arbitrary constant phase angle. It is of interest to note that for the case of a plane electromagnetic wave, if $\mathbf{a}$ is taken in the direction of and proportional to the electric vector,[4] the scalar potential is necessarily zero, owing to the fact that the electric vector lies in the plane of the wave front and so is perpendicular to the direction of propagation $\mathbf{n}$. This is easily shown, for

$$\operatorname{div} \mathbf{U} = -\frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{a}) \sin\frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{r} - ct)$$

which vanishes for $\mathbf{a}$ perpendicular to $\mathbf{n}$, in which case $(\partial\varphi/\partial t) = 0$. The electric vector is then $-(1/c)(\partial\mathbf{U}/\partial t)$ and has the direction of $\mathbf{U}$. Hence, since standing waves are necessarily made up of plane waves, φ in (22) may vanish for a proper choice of $\mathbf{a}$. But, however $\mathbf{a}$ may be chosen, terms involving squares of the amplitudes are small enough to be neglected.

To solve the differential equation (20) we may follow either Gordon or Klein. First, following Gordon, let us assume a solution of the form

$$\psi = e^{(2\pi i/h)(-Et+\mathbf{p}\cdot\mathbf{r})} e^{(2\pi i/h)\lambda} \tag{23}$$

where λ is a function of x, y, z and t, and p_x, p_y, p_z and E are constants satisfying the relation

$$p_x^2 + p_y^2 + p_z^2 - \frac{E^2}{c^2} + m_0^2c^2 = 0. \tag{24}$$

This gives us an equation in λ

$$\frac{h}{2\pi i}\left(\nabla\cdot\nabla\lambda - \frac{1}{c^2}\frac{\partial^2\lambda}{\partial t^2}\right) - 2\frac{e}{c}\left[(\mathbf{p} + \operatorname{grad}\lambda)\cdot\mathbf{U} + \left(-E + \frac{\partial\lambda}{\partial t}\right)\frac{\varphi}{c}\right] + 2\left[\mathbf{p}\cdot\operatorname{grad}\lambda + \frac{E}{c^2}\frac{\partial\lambda}{\partial t}\right] + \left[(\operatorname{grad}\lambda)^2 - \frac{1}{c^2}\left(\frac{\partial\lambda}{\partial t}\right)^2\right] = 0. \tag{25}$$

[4] Waller, I.: *Phil. Mag.*, 4, 1228, 1927.

As suggested by Gordon's results, let us assume λ to be of the form

$$\lambda = \lambda_1 + \lambda_2 = A_1 \sin \frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{r} - ct) + A_2 \sin\left\{\frac{2\pi\nu}{c}(-\mathbf{n}\cdot\mathbf{r} - ct) + \delta\right\} \quad (26)$$

where A_1 and A_2 are coefficients the values of which are to be determined by the conditions laid upon λ by the differential equation.

A complete theory would assume such an expression for λ that $e^{(2\pi i/h)\lambda}$ would be equal to unity at $t = 0$, at which time the vector and scalar potentials would not yet be acting and so would be zero. Owing to the customary incompleteness of expression in problems of this kind λ is periodic and never disappears. Our best hope is to say that ψ is like (23), but with a slowly varying coefficient of λ which makes $\lambda = 0$ at $t = 0$ and then at $t = 0$

$$\psi = e^{(2\pi i/h)(-Et + \mathbf{p}\cdot\mathbf{r})}. \quad (27)$$

This is the value of ψ which the fundamental hypothesis of this theory associates with an electron with energy E and momentum $\mathbf{p}$, the association being rendered possible through the relation (24) which was required by the differential equation (20) and also demanded as the relation between the momentum and energy for the relativistic dynamics of the electron.

We must now show two things about our assumed solution (23): prove that it does satisfy the differential equation (20), and find the appropriate values of the constants A_1 and A_2. Because of the wave form assumed for λ, the d'Alembertian vanishes and our equation (20) becomes

$$2\frac{e}{c}\left[\mathbf{p}\cdot\mathbf{U}_1 + \mathbf{p}\cdot\mathbf{U}_2 + \operatorname{grad}(\lambda_1 + \lambda_2)\cdot(\mathbf{U}_1 + \mathbf{U}_2) - \frac{E}{c}(\varphi_1 + \varphi_2) + \left(\frac{\partial\lambda_1}{\partial t} + \frac{\partial\lambda_2}{\partial t}\right)\frac{1}{c}(\varphi_1 + \varphi_2)\right]$$
$$- 2\left[\mathbf{p}\cdot\operatorname{grad}\lambda_1 + \frac{E}{c^2}\frac{\partial\lambda_1}{\partial t}\right] - 2\left[\mathbf{p}\cdot\operatorname{grad}\lambda_2 + \frac{E}{c^2}\frac{\partial\lambda_2}{\partial t}\right]$$

$$-\left[(\text{grad}\,\lambda_1)^2-\frac{1}{c^2}\left(\frac{\partial\lambda_1}{\partial t}\right)^2\right]-\left[(\text{grad}\,\lambda_2)^2-\frac{1}{c^2}\left(\frac{\partial\lambda_2}{\partial t}\right)^2\right]$$
$$-2\left[\text{grad}\,\lambda_1\cdot\text{grad}\,\lambda_2-\frac{1}{c^2}\frac{\partial\lambda_1}{\partial t}\frac{\partial\lambda_2}{\partial t}\right]=0. \quad (28)$$

In view of the fact that the sum of the squares of the direction cosines equals unity, the following expressions vanish:

$$(\text{grad}\,\lambda_1)^2-\frac{1}{c^2}\left(\frac{\partial\lambda_1}{\partial t}\right)^2=0,$$
$$(\text{grad}\,\lambda_2)^2-\frac{1}{c^2}\left(\frac{\partial\lambda_2}{\partial t}\right)^2=0. \quad (29)$$

The equation may now be expressed, inserting the values of $\mathbf{U}$ and φ from (22), as the sum of terms in $\cos(2\pi\nu/c)(\mathbf{n}\cdot\mathbf{r}-ct)$, of terms in $\cos\{(2\pi\nu/c)(\mathbf{n}\cdot\mathbf{r}-ct)+\delta\}$ and of other terms:

$$\cos\frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{r}-ct)\left[\frac{e}{c}\left(\mathbf{p}\cdot\mathbf{a}-\frac{E}{c}a\right)-A_1\frac{2\pi\nu}{c}\left(\mathbf{p}\cdot\mathbf{n}-\frac{E}{c}\right)\right]$$
$$+\cos\left\{\frac{2\pi\nu}{c}(-\mathbf{n}\cdot\mathbf{r}-ct)+\delta\right\}\left[\frac{e}{c}\left(\mathbf{p}\cdot\mathbf{a}-\frac{E}{c}a\right)\right.$$
$$\left.+A_2\frac{2\pi\nu}{c}\left(\mathbf{p}\cdot\mathbf{n}+\frac{E}{c}\right)\right]+\frac{e}{c}\left[\text{grad}\,(\lambda_1+\lambda_2)\cdot(\mathbf{U}_1+\mathbf{U}_2)\right.$$
$$\left.+\frac{1}{c}\left(\frac{\partial\lambda_1}{\partial t}+\frac{\partial\lambda_2}{\partial t}\right)(\varphi_1+\varphi_2)\right]$$
$$-\left[\text{grad}\,\lambda_1\cdot\text{grad}\,\lambda_2-\frac{1}{c^2}\frac{\partial\lambda_1}{\partial t}\frac{\partial\lambda_2}{\partial t}\right]=0. \quad (30)$$

In order that the coefficient of $\cos(2\pi\nu/c)(\mathbf{n}\cdot\mathbf{r}-ct)$ should vanish, the coefficient A_1 must have the value

$$A_1=\frac{e\left(\mathbf{p}\cdot\mathbf{a}-\frac{E}{c}a\right)}{2\pi\nu\left(\mathbf{p}\cdot\mathbf{n}-\frac{E}{c}\right)}. \quad (31)$$

Similarly, so that the term in $\cos\{(2\pi\nu/c)(-\mathbf{n}\cdot\mathbf{r}-ct)+\delta\}$ may vanish, we have

$$A_2 = -\frac{e}{2\pi\nu}\frac{\left(\mathbf{p}\cdot\mathbf{a} - \frac{E}{c}a\right)}{\left(\mathbf{p}\cdot\mathbf{n} + \frac{E}{c}\right)}. \tag{32}$$

The remaining terms all contain as a factor the product of a λ derivative with a vector or scalar potential or with another λ derivative, and in view of the fact that A_1 and A_2 both consist of terms proportional to the amplitudes of the electromagnetic potentials, are all terms of the order already neglected in deriving (25). Consequently we have for the solution of the diffusion equation for an electron hit by a standing X-ray wave, the wave being defined by the vector and scalar potentials (22), the expression

$$\psi = e^{(2\pi i/h)(-Et+\mathbf{p}\cdot\mathbf{r})}e^{(2\pi i/h)\lambda} \tag{33}$$

subject to the condition

$$\mathbf{p}^2 - \frac{E^2}{c^2} + m_0^2c^2 = 0, \tag{24}$$

where

$$\lambda = \frac{e}{2\pi\nu}\frac{\left(\mathbf{p}\cdot\mathbf{a} - \frac{E}{c}a\right)}{\left(\mathbf{p}\cdot\mathbf{n} - \frac{E}{c}\right)}\sin\frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{r} - ct)$$

$$-\frac{e}{2\pi\nu}\frac{\left(\mathbf{p}\cdot\mathbf{a} - \frac{E}{c}a\right)}{\left(\mathbf{p}\cdot\mathbf{n} + \frac{E}{c}\right)}\sin\left\{\frac{2\pi\nu}{c}(-\mathbf{n}\cdot\mathbf{r} - ct + \delta\right\}. \tag{33a}$$

This may be written more simply as

$$\psi = e^{(2\pi i/h)(-Et+\mathbf{p}\cdot\mathbf{r})}e^{(2\pi i/h)(A_1\sin\theta_1 + A_2\sin\theta_2)}, \tag{34}$$

where

$$\theta_1 = \frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{r} - ct) \quad \text{and} \quad \theta_2 = \frac{2\pi\nu}{c}(-\mathbf{n}\cdot\mathbf{r} - ct) + \delta.$$

Alternative solution. This simple solution of (20) may also be arrived at by a direct application of perturbation theory

as indicated by Klein. In that case to emphasize the smallness of the amplitudes and to have $\mathbf{U}$ and φ in the more convenient exponential form, we shall express (22) as

$$\begin{aligned} \mathbf{U} &= \sigma \mathbf{a}'(e^{i\theta_1} + e^{-i\theta_1} + e^{i\theta_2} + e^{-i\theta_2}), \\ \varphi &= \sigma a'(e^{i\theta_1} + e^{-i\theta_1} + e^{i\theta_2} + e^{-i\theta_2}), \end{aligned} \tag{22a}$$

where θ_1 and θ_2 have the same significance as before, and σ is a small parameter such that terms containing its square are to be neglected. a' is seen to be $a/2\sigma$ and a similar relation holds for $\mathbf{a}'$. In solving an equation from the point of view of perturbation theory, we seek a solution of the perturbed equation in terms of the unperturbed solutions, i.e. in this case in terms of solutions of the equation in which $\mathbf{U} = \varphi = 0$, which we have seen to be satisfied by

$$\psi = e^{(2\pi i/h)(-Et + \mathbf{p}\cdot\mathbf{r})}. \tag{27}$$

Our problem is then to get a solution of (20) which is not far different from (27). Let us assume a solution of the form $\psi + \sigma f$ where ψ is given by (27). Making this substitution and neglecting terms in σ^2 the equation (20) becomes

$$\sigma\left[-\frac{h^2}{4\pi^2}\left(\nabla\cdot\nabla f - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}\right) + m_0^2 c^2 f\right] = 2\frac{h}{2\pi i}\frac{e}{c}\left(\mathbf{U}\cdot\operatorname{grad}\psi + \frac{\varphi}{c}\frac{\partial\psi}{\partial t}\right), \tag{35}$$

or inserting the values of the potentials

$$-\frac{h^2}{4\pi^2}\left(\nabla\cdot\nabla f - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}\right) + m_0^2 c^2 f = 2\frac{e}{c}\left(\mathbf{a}'\cdot\mathbf{p} - \frac{E}{c}a'\right)e^{(2\pi i/h)(-Et+\mathbf{p}\cdot\mathbf{r})}\left[e^{i\theta_1} + e^{-i\theta_1} + e^{i\theta_2} + e^{-i\theta_2}\right].$$

As indicated by Schroedinger,[5] we shall assume f to be of the form

$$f = w_+(x, y, z)e^{-(2\pi i/h)(E+h\nu)t} + w_-(x, y, z)e^{-(2\pi i/h)(E-h\nu)t} \tag{36}$$

and equate the resulting coefficients of $e^{-(2\pi i/h)(E+h\nu)t}$ and $e^{-(2\pi i/h)(E-h\nu)t}$ on the two sides of the equation. This leads to two separate equations

[5] Schroedinger: *Ann. d. Physik*, 81, 115, 1926.

$$-\frac{h^2}{4\pi^2}\nabla\cdot\nabla w_+ - \frac{(E+h\nu)^2}{c^2}w_+ + m_0^2c^2w_+$$
$$= \frac{2e}{c}\left(\mathbf{a}'\cdot\mathbf{p} - \frac{E}{c}a'\right)e^{(2\pi i/h)\mathbf{n}\cdot\mathbf{r}}\left(e^{(2\pi i/h)(h\nu/c)\mathbf{n}\cdot\mathbf{r}} + e^{-(2\pi i/h)(h\nu/c)\mathbf{n}\cdot\mathbf{r}+i\delta}\right)$$
$$-\frac{h^2}{4\pi^2}\nabla\cdot\nabla w_- - \frac{(E-h\nu)^2}{c^2}w_- + m_0^2c^2w_-$$
$$= \frac{2e}{c}\left(\mathbf{a}'\cdot\mathbf{p} - \frac{E}{c}a'\right)e^{(2\pi i/h)\mathbf{p}\cdot\mathbf{r}}\left(e^{-(2\pi i/h)(h\nu/c)\mathbf{n}\cdot\mathbf{r}} + e^{(2\pi i/h)(h\nu/c)\mathbf{n}\cdot\mathbf{r}-i\delta}\right). \quad (37)$$

To eliminate the remaining exponentials, let

$$\begin{aligned} w_+ &= B_1e^{(2\pi i/h)[\mathbf{p}+(h\nu/c)\mathbf{n}]\cdot\mathbf{r}} + B_2e^{(2\pi i/h)[\mathbf{p}-(h\nu/c)\mathbf{n}]\cdot\mathbf{r}+i\delta}, \\ w_- &= C_1e^{(2\pi i/h)[\mathbf{p}-(h\nu/c)\mathbf{n}]\cdot\mathbf{r}} + C_2e^{(2\pi i/h)(\mathbf{p}+(h\nu/c)\mathbf{n}]\cdot\mathbf{r}-i\delta}. \end{aligned} \quad (38)$$

Since the relation (24) holds between the p's and E this gives for the values of the constants

$$\begin{aligned} B_1 &= -C_1 = \frac{e}{h\nu}\frac{\left(\mathbf{p}\cdot\mathbf{a}' - \frac{E}{c}a'\right)}{\left(\mathbf{p}\cdot\mathbf{n} - \frac{E}{c}\right)}, \\ C_2 &= -B_2 = \frac{e}{h\nu}\frac{\left(\mathbf{p}\cdot\mathbf{a}' - \frac{E}{c}a'\right)}{\left(\mathbf{p}\cdot\mathbf{n} + \frac{E}{c}\right)}. \end{aligned} \quad (39)$$

The solution of the perturbation equation is consequently

$$\begin{aligned} f = \frac{e}{h\nu}&\frac{\left(\mathbf{p}\cdot\mathbf{a}' - \frac{E}{c}a'\right)}{\left(\mathbf{p}\cdot\mathbf{n} - \frac{E}{e}\right)}\left[e^{(2\pi i/h)\{[\mathbf{p}+(h\nu/c)\mathbf{n}]\cdot\mathbf{r}-(E+h\nu)t\}}\right. \\ &\left. - e^{(2\pi i/h)\{[\mathbf{p}-(h\nu/c)\mathbf{n}]\cdot\mathbf{r}-(E-h\nu)t\}}\right] \\ + \frac{e}{h\nu}&\frac{\left(\mathbf{p}\cdot\mathbf{a}' - \frac{E}{c}a'\right)}{\left(\mathbf{p}\cdot\mathbf{n} + \frac{E}{c}\right)}\left[e^{(2\pi i/h)\{[\mathbf{p}+(h\nu/c)\mathbf{n}]\cdot\mathbf{r}-(E-h\nu)t\}-i\delta}\right. \\ &\left. - e^{(2\pi i/h)\{[\mathbf{p}-(h\nu/c)\mathbf{n}]\cdot\mathbf{r}-(E+h\nu)t\}+i\delta}\right]. \end{aligned} \quad (40)$$

The complete solution $\psi + \sigma f$ may then be written

$$e^{(2\pi i/h)(\mathbf{p}\cdot\mathbf{r}-Et)}\left[1+\sigma\frac{e}{h\nu}\cdot\frac{1}{2\sigma}\frac{\left(\mathbf{p}\cdot\mathbf{a}-\frac{E}{c}x\right)}{\left(\mathbf{p}\cdot\mathbf{n}-\frac{E}{c}\right)}\left\{e^{(2\pi i/h)[(h\nu/c)\mathbf{n}\cdot\mathbf{r}-h\nu t]}\right.\right.$$

$$\left.-e^{-(2\pi i/h)[(h\nu/c)\mathbf{n}\cdot\mathbf{r}-h\nu t]}\right\}+\sigma\frac{e}{h\nu}\cdot\frac{1}{2\sigma}\frac{\left(\mathbf{p}\cdot\mathbf{a}-\frac{E}{c}a\right)}{\left(\mathbf{p}\cdot\mathbf{n}+\frac{E}{c}\right)}$$

$$\left.\times\left\{e^{(2\pi i/h)[(h\nu/c)\mathbf{n}\cdot\mathbf{r}+h\nu t]-i\delta}-e^{-(2\pi i/h)[(h\nu/c)\mathbf{n}\cdot\mathbf{r}+h\nu t]+i\delta}\right\}\right]$$

$$=e^{(2\pi i/h)(\mathbf{p}\cdot\mathbf{r}-Et)}\left[1+\frac{e}{2\pi\nu}\frac{\left(\mathbf{p}\cdot\mathbf{a}-\frac{E}{c}a\right)}{\left(\mathbf{p}\cdot\mathbf{n}-\frac{E}{c}\right)}\frac{2\pi i}{h}\sin\frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{r}-ct)\right.$$

$$\left.-\frac{e}{2\pi\nu}\frac{\left(\mathbf{p}\cdot\mathbf{a}-\frac{E}{c}a\right)}{\left(\mathbf{p}\cdot\mathbf{n}+\frac{E}{c}\right)}\frac{2\pi i}{h}\sin\left\{\frac{2\pi\nu}{c}(-\mathbf{n}\cdot\mathbf{r}-ct)+\delta\right\}\right]. \quad (41)$$

For the sake of comparison solution (34) may be written

$$e^{(2\pi i/h)(\mathbf{p}\cdot\mathbf{r}-Et)}\left[1+\frac{2\pi i}{h}A_1\sin\theta_1+\frac{2\pi i}{h}A_2\sin\theta_2\right]. \quad (34a)$$

since the A's are small, and this, in view of the known values of A_1 and A_2 (31 and 32) is seen to be identical with (41). This establishes the essential identity of the two processes worked out by Gordon and Klein, the erroneous sign appearing in Klein's equation (88) being obviously an oversight.

This simple solution corresponds to but one of the many possible solutions of (20); the appropriate solution for a given problem is to be developed as an infinite sum in terms of these separate simple solutions, each of which before summation is multiplied by a normalizing coefficient $C(p)$

and by an arbitrary function $z(p)$. This arbitrary function $z(p)$ is determined from the initial conditions of the problem; i.e. at $t = 0$, the coefficients $z(p)$ must be adjusted to fit the given conditions. In this case, inasmuch as the electron may have any momentum whatsoever, adding all the corresponding values of ψ must throw the sum into the form of an integral

$$\psi = \int_{-\infty}^{\infty} e^{(2\pi i/h)(\mathbf{p}\cdot\mathbf{r}-Et)} e^{(2\pi i/h)(A_1 \sin\theta_1 + A_2 \sin\theta_2)} C(p) z(p) dp, \quad (42)$$

where $dp = dp_x dp_y dp_z$.

3. *Derivation of the current density expression.* According to classical theory the vector and scalar potentials of the emitted radiation are to be expressed as the retarded potentials of the current and charge densities. Hence our next problem is to derive expressions for these densities. Following Gordon and Klein we may express these densities in terms of the ψ functions according to the following relations:

$$\begin{aligned} \rho\mathbf{u} &= \frac{e}{2m_0}\frac{h}{2\pi i}\left(\bar{\psi}\,\mathrm{grad}\,\psi - \psi\,\mathrm{grad}\,\bar{\psi} - \frac{4\pi i}{h}\frac{e}{c}\mathbf{U}\psi\bar{\psi}\right) \\ \rho &= -\frac{e}{2m_0c^2}\frac{h}{2\pi i}\left(\bar{\psi}\frac{\partial\psi}{\partial t} - \psi\frac{\partial\bar{\psi}}{\partial t} + \frac{4\pi i}{h}e\varphi\psi\bar{\psi}\right). \end{aligned} \quad (43)$$

These can be shown to qualify as current and charge densities because they satisfy the equation of continuity. We shall content ourselves with finding the expression for the current density, since the same relations as regards frequency and momentum conditions would grow out of the charge density and the resulting scalar potential. The evaluation of the separate terms of $\rho\mathbf{u}$ involves the multiplication of such integrals as $\bar{\psi}$ and grad ψ, and $\rho\mathbf{u}$ becomes

$$\begin{aligned} \rho\dot{\mathbf{u}} = \frac{e}{2m_0}\int\int\Big[&(\mathbf{p}+\mathbf{p}') + \left\{\frac{2\pi\nu}{c}\mathbf{n}(A_1 + A_1') - \frac{2e}{c}\mathbf{a}\right\}\cos\theta_1 \\ &+ \left\{\frac{2\pi\nu}{c}\mathbf{n}(-A_2 - A_2') - \frac{2e}{c}\mathbf{a}\right\}\cos\theta_2\Big] \\ &\times e^{(2\pi i/h)\delta W} z(p) z(p') C(p) C(p') dp\, dp', \end{aligned} \quad (44)$$

where

$$\begin{aligned} \delta W = &-(E - E')t + (\mathbf{p} - \mathbf{p}')\cdot\mathbf{r} + (A_1 - A_1')\sin\theta_1 \\ &+ (A_2 - A_2')\sin\theta_2, \end{aligned}$$

where p' is an arbitrary variable of integration, where E' is defined by the relation $(E'^2/c^2) - m_1^2c^2 = \mathbf{p}'^2$, and where the A''s are the same function of the p''s that the A's are of the p's. The range of integration of the p''s is from minus infinity to plus infinity as for the p's. The factor $e^{(2\pi i/h)\delta W}$ may be written

$$e^{(2\pi i/h)[(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}-(E-E')t]}\left[1+\frac{2\pi i}{h}(A_1-A_1')\sin\theta_1+\frac{2\pi i}{h}(A_2-A_2')\sin\theta_2\right], \quad (45)$$

since the terms in the A's are small; the next term in the Taylor's expansion would involve terms of the order of those which we are neglecting. We may use this value of $e^{(2\pi i/h)\delta W}$ in equation (44), giving

$$\rho\mathbf{u}=\frac{e}{2m_0}\int\int\left[(\mathbf{p}+\mathbf{p}')+\left\{\frac{2\pi\nu}{c}\mathbf{n}(A_1+A_1')-\frac{2e}{c}\mathbf{a}\right\}\cos\theta_1+\frac{2\pi i}{h}(\mathbf{p}+\mathbf{p}')(A_1-A_1')\sin\theta_1+\left\{\frac{2\pi\nu}{c}\mathbf{n}(-A_2-A_2')-\frac{2e}{c}\mathbf{a}\right\}\cos\theta_2+\frac{2\pi i}{h}(\mathbf{p}+\mathbf{p}')(A_2-A_2')\sin\theta_2\right]$$
$$e^{(2\pi i/h)[(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}-(E-E')t]}z(p)z(p')C(p)C(p')dpdp'. \quad (46)$$

This expression may be simplified by expressing the cosine as $\frac{1}{2}(e^{i\theta}+e^{-i\theta})$ and $i\sin\theta$ as $\frac{1}{2}(e^{i\theta}-e^{-i\theta})$. This will give rise to terms with coefficients of the t exponent $-(E-E'+h\nu)$ and $-(E-E'-h\nu)$, and upon interchanging p and p' (which values will occur since we are integrating both p and p' throughout all possible values) to terms in $(E-E'-h\nu)$ and $(E-E'+h\nu)$. The second and third of these expressions correspond to an increase in the frequency of the emitted radiation which at least in the realm of X-rays has not been experimentally shown; hence we will neglect the terms involving these exponents. Terms of a similar character have, however, been observed in the visible region in the Raman effect. The terms involving the two remaining exponents of t may both be considered by using the real part only of the integral

$$\rho\mathbf{u} = \int\int[(\mathbf{p}+\mathbf{p}') + \mathbf{T}_1 e^{i\theta_1} + \mathbf{T}_2 e^{i\theta_2}]e^{(2\pi i/h)[(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}-(E-E')t]} z(p)z(p')C(p)C(p')dpdp', \quad (47)$$

where

$$\mathbf{T}_1 = \left\{\frac{2\pi\nu}{c}\mathbf{n}(A_1+A_1') - \frac{2e}{c}\mathbf{a}\right\} + \frac{2\pi}{h}(\mathbf{p}+\mathbf{p}')(A_1-A_1'),$$
$$\mathbf{T}_2 = \left\{\frac{2\pi\nu}{c}\mathbf{n}(-A_2-A_2') - \frac{2e}{c}\mathbf{a}\right\} + \frac{2\pi}{h}(\mathbf{p}+\mathbf{p}')(A_2-A_2'). \quad (47a)$$

4. *Derivation and significance of the vector potential of the emitted radiation.* This value of the current density must now be inserted in the usual expression for the retarded potential to give the vector potential of the emitted radiation,

$$\mathbf{U}' = \frac{1}{c}\int\frac{[\rho\mathbf{u}]}{R}d\tau, \quad (48)$$

where $d\tau = dx\,dy\,dz$. This gives the sum of three nine-fold integrals

$$\mathbf{U}' = \mathbf{U}_0' + \mathbf{U}_1' + \mathbf{U}_2', \quad (49)$$

where

$$\mathbf{U}_0' = \frac{e}{2m_0Rc}\int\int\int(\mathbf{p}+\mathbf{p}')e^{(2\pi i/h)\{(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}-(E-E')[t-(R/c)+(\xi\cdot\mathbf{r}/c)]\}} \times z(p)z(p')C(p)C(p')dp\,dp'd\tau, \quad (49a)$$

$$\mathbf{U}_1' = \frac{e}{2m_0Rc}\int\int\int\mathbf{T}_1 e^{(2\pi i/h)\{(h\nu/c)\mathbf{n}\cdot\mathbf{r}-h\nu[t-(R/c)+(\xi\cdot\mathbf{r}/c)]\}} \times e^{(2\pi i/h)\{(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}-(E-E')[t-(R/c)+(\xi\cdot\mathbf{r}/c)]\}} \times z(p)z(p')C(p)C(p')dp\,dp'd\tau, \quad (49b)$$

$$\mathbf{U}_2' = \frac{e}{2m_0Rc}\int\int\int\mathbf{T}_2 e^{(2\pi i/h)\{-(h\nu/c)\mathbf{n}\cdot\mathbf{r}-h\nu[t-(R/c)+(\xi\cdot\mathbf{r}/c)]\}+i\delta} \times e^{(2\pi i/h)\{(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}-(E-E')[t-(R/c)+(\xi\cdot\mathbf{r}/c)]\}} \times z(p)z(p')C(p)C(p')dp\,dp'd\tau, \quad (49c)$$

and where for t has been substituted its retarded value $\left(t-\frac{R}{c}+\frac{\xi\cdot\mathbf{r}}{c}\right)$. R is here the distance from the point of observation to the center of the electron and ξ is the unit vector in the direction of observation.

The integration of these nine-fold integrals with respect to six of the variables, p' and τ, may be easily effected by throwing that part of the integral into the form of a Fourier integral, as is frequently done in Schroedinger's theory. The characteristics of such a Fourier integral are indicated by the following relation:

$$h^3F(P, P) = \int\int F(P, P')e^{(2\pi i/h)(\mathbf{P}-\mathbf{P}')\cdot\mathbf{r}}dP'd\tau \qquad (50)$$

where $F(P, P')$ symbolizes some function of P_x, P_y, P_z, P'_x, P'_y, P'_z, and we see that the integrated function is exactly h^3 times the function that appears in the integrand multiplying the required exponential term but with $\mathbf{P}'$ replaced by $\mathbf{P}$.

Let us first apply this method of integration to $\mathbf{U}_0'$. That will require a change of variable such that

$$\mathbf{P} = \mathbf{p} - \frac{E}{c}\boldsymbol{\xi} \quad \text{and} \quad \mathbf{P}' = \mathbf{p}' - \frac{E'}{c}\boldsymbol{\xi}.$$

The function $F(P, P')$ is

$$(\mathbf{p} + \mathbf{p}')e^{(2\pi i/h)(E-E')[t-(R/c)]}z(p)z(p')C(p)C(p')JJ',$$

where J is the Jacobian of P and J' of P'. The resulting integral is

$$\mathbf{U}_0' = \frac{eh^3}{2m_0Rc}\int [e^{(2\pi i/h)(E-E')[t-(R/c)]} \times (\mathbf{p} + \mathbf{p}')JJ'z(p)z(p')C(p)C(p')]_P dP, \qquad (51)$$

where the subscript P indicates that all the p's involved are to be expressed as functions of the P's and the p''s of the P''s and then $\mathbf{P}'$ is to be put equal to $\mathbf{P}$, making the resulting expression a function of P alone. Putting $\mathbf{P}'$ equal to $\mathbf{P}$ gives us the relation

$$\mathbf{p} - \mathbf{p}' - \frac{E - E'}{c}\boldsymbol{\xi} = 0 \qquad (52)$$

and that, since $\boldsymbol{\xi}$ is a unit vector, may be expressed as

$$(\mathbf{p} - \mathbf{p}')^2 = \frac{(E - E')^2}{c^2}. \qquad (53)$$

This condition leads necessarily, because of (24), to the relations $E = E'$ and $\mathbf{p} = \mathbf{p}'$. The t exponent is thus made to vanish causing this part of the vector potential to be independent of the time. Consequently $\mathbf{U}_0'$ will emit no radiation and that fact alone causes us immediately to lose interest in it.

The second and third integrals, $\mathbf{U}_1'$ and $\mathbf{U}_2'$, will however give a contribution to the emitted electromagnetic radiation. Let us consider first $\mathbf{U}_1'$. The exponent here may be written as

$$-\frac{2\pi i}{h}(E - E' + h\nu)\left(t - \frac{R}{c}\right) + \frac{2\pi i}{h}\left\{(\mathbf{p} - \mathbf{p}')\cdot\mathbf{r} - \frac{E - E' + h\nu}{c}\boldsymbol{\xi}_1\cdot\mathbf{r} + \frac{h\nu}{c}\mathbf{n}\cdot\mathbf{r}\right\}. \quad (54)$$

We shall write $\boldsymbol{\xi}_1$ instead of $\boldsymbol{\xi}$ for the direction of observation merely to emphasize the fact that it belongs to this part of the integral $\mathbf{U}_1'$. The first parenthesis depending on t and the distance R is independent of r, and hence forms part of the function $F(P, P')$ in the Fourier integral (50); the second parenthesis must be expressed as $(\mathbf{P} - \mathbf{P}')\cdot\mathbf{r}$. This may be accomplished by changing the variables to

$$\begin{aligned}\mathbf{P} &= \mathbf{p} - \frac{E + h\nu}{c}\boldsymbol{\xi}_1 + \frac{h\nu}{c}\mathbf{n},\\ \mathbf{P}' &= \mathbf{p}' - \frac{E'}{c}\boldsymbol{\xi}_1.\end{aligned} \quad (55)$$

To integrate with respect to these new variables one must multiply the integrand by the Jacobian of P, J, and by the Jacobian of P', J'. These factors will both form part of the function $F(P, P')$. Equations (55) together with the relation (24) enable us to express the p's and E as functions of the P's, and also the p''s and E' as functions the P''s. Replacing $\mathbf{P}'$ by $\mathbf{P}$ is equivalent to replacing $\mathbf{p}' - (E'/c)\boldsymbol{\xi}_1$ by $\mathbf{p} - \boldsymbol{\xi}_1(E + h\nu)/c + (h\nu/c)\mathbf{n}$, i.e. to replacing $\mathbf{p} - \mathbf{p}' - \boldsymbol{\xi}_1(E - E' + h\nu)/c + (h\nu/c)\mathbf{n}$ by zero. In other words, remembering that $|\mathbf{p}'|$ is equal to $\sqrt{(E'^2/c^2) - m_0^2c^2}$, we see that replacing $\mathbf{P}'$ by $\mathbf{P}$ is equivalent to replacing E' by a quantity E_{1P} which is a solution of the equation

$$\mathbf{j}_1 \sqrt{\frac{E_{1P}^2}{c^2} - m_0^2 c^2} - \frac{E_{1P}}{c}\boldsymbol{\xi}_1 = \mathbf{p} - \frac{E + h\nu}{c}\boldsymbol{\xi}_1 + \frac{h\nu}{c}\mathbf{n}, \quad (56)$$

where $\mathbf{j}_1$ is a unit vector the direction of which is determined by this relation. We have then for the resulting expression for $\mathbf{U}_1'$

$$\mathbf{U}_1' = \frac{eh^3}{2m_0 cR}\int e^{-(2\pi i/h)(E - E_{1P} + h\nu)[t - (R/c)]} \times [\mathbf{T}_1 C(p)C(p')z(p)z(p')JJ']_P dP, \quad (57)$$

where the subscript P again indicates the same thing that it did in (51).

If we should now attempt to complete the integration of the vector potential (57) which we have caused to be a function of the initial electron momentum $\mathbf{p}$, and of ν, $\mathbf{n}$ and $\boldsymbol{\xi}$, we should be attempting to consider simultaneously all vector potentials which might exist and all emitted radiations which occur when an X-ray wave of a known frequency encounters electrons of all possible velocities. We shall consequently proceed as we do in the case of an atomic transition and pick out one term corresponding to a given transition or more precisely to one given initial velocity of the electron. To do this we must separate the integrand of (57) into two parts, one determining the intensity of the component waves and the other the number of terms in the infinite sum lying between p and $p + dp$. This requires us to give a more definite meaning to the series coefficients $z(p)$, for we must find for the new system of coördinates P a function $Z(P)$ which will have as nearly as possible the same characteristics as $z(p)$ in the old. We shall place upon our solution (42) for the initial state of the electron, in which λ is to be taken as zero, the condition that the integral of the expression for the density (43) using this value of ψ taken over all space shall give the electronic charge e. The total charge then takes the form of the integral

$$e = \frac{eh^3}{m_0 c^2}\int EC^2(p)z^2(p)dp, \quad (58)$$

where the integration with respect to $d\tau$ and dp' has already

been carried out using the Fourier integral theorem. Before we can give a definite meaning to $z(p)$ we must evaluate $C(p)$, the normalization constant. For this we shall use the physical criterion of normalization stated by Klein which holds for the relativistic as well as for the non-relativistic case, that for each state the density integrated over all space must be equal to the electronic charge e, but since this integral is here divergent we shall in accord with the procedure adopted by Born [6] use the mean value of this integral over the integration range instead of its actual value. Thus we have for the simple solution $\psi = C_s e^{(2\pi i/h)(\mathbf{p}_s \cdot \mathbf{r} - E_s t)}$ the condition on C_s that

$$\lim_{\tau \to \infty} \frac{1}{\tau} \int C_s^2 \frac{eE_s}{m_0 c^2} d\tau = e. \tag{59}$$

In the limit this gives for C_s the value

$$C_s^2 = \frac{m_0 c^2}{E_s}. \tag{60}$$

For the non-relativistic case C_s is then unity and the average value of $\int \psi \bar{\psi} d\tau$ is also unity in accord with the usual condition for normalization. This value substituted in (58) gives the condition

$$1 = h^3 \int z^2(p) dp \tag{61}$$

which differs by the factor h^3 from that given by Gordon, a difference which is immaterial since it is a constant factor. Moreover, by comparing (59) which is the case for an electron in a single state with (58) which has been derived from ψ expressed as a sum of terms, the two are seen to be consistent if we arbitrarily take $h^3 z^2(p) dp$ as the number of terms involved between p and $p + dp$ which are to be taken in order to give the initial distribution of charge. In changing to another set of coördinates P, the number of terms to be taken in a corresponding range dP to give the same initial distribution must be the same and we can define a new function $Z(P)$ such that

$$z^2(p) dp = Z^2(P) dP, \tag{62}$$

[6] Born, M.: *ZS. f. Physik*, 38, 803, 1926.

so that if J is the Jacobian of the transformation, for which $J\,dP = dp$, we have

$$[z(p)]_P\sqrt{J} = Z(P). \tag{63}$$

We have then for $\mathbf{U}_1'$, on substituting the value of $C(p)$ already found, the expression

$$\mathbf{U}_1' = \frac{ec}{2R}h^3\int e^{-(2\pi i/h)(E-E_{1P}+h\nu)[t-(R'/c)]}\frac{\mathbf{T}_{1P}}{\sqrt{EE_{1P}}} \times\left[Z(P)Z'(P)\sqrt{\frac{J'}{J}}\right]_P dp, \tag{57a}$$

in which it is to be observed we have changed the variable of integration back to p. $Z'(P)$ here is defined analogously to $Z(P)$ as $[z(p)]_P\sqrt{J'}$ and thus obviously is a different function from $Z(P)$. If we now work out the vector potential corresponding to the transition between two states of energy designated by E and by E_{1P}, we find

$$\mathbf{U}_1' = \frac{ec}{2R}\frac{\mathbf{T}_1}{\sqrt{EE_{1P}}}e^{-(2\pi i/h)(E-E_{1P}+h\nu)[t-(R'/c)]}. \tag{64}$$

We can now set up a correspondence between this expression and the expression represented in equation (57a) by saying that the integral is the equivalent of having $h^3Z(P)Z'(P)\sqrt{J'/J}$ contributions per unit range of p in the vector potential of amount equal to that given by equation (64), each of these terms representing a transition from the energy state E to the energy state E_{1P}.

It may be noted that we here differ slightly from the procedure adopted by Gordon, who retains the variable P in the expression (57a), so that the states are symbolized by P instead of by p, and the number of them per unit range of P is taken as $h^3Z(P)Z'(P)$. This results in the assignment of an amplitude $\frac{ec}{2R}\frac{\mathbf{T}_{1P}\sqrt{JJ'}}{\sqrt{EE_{1P}}}e^{-2\pi i\nu_{1P}[t-(R'/c)]}$ to each state, a form of expression differing from that we have chosen by the factor $\sqrt{JJ'}$. It would certainly seem that the number of states should be specified in terms of p rather than of P.

However, this operation of procedure only plays a minor part. For example, it does not affect the fact that the intensity of the emitted radiation is proportional to the cube of the frequency.

Carrying out a similar process for the integral $\mathbf{U}_2'$, we get for the final result

$$\mathbf{U}_2' = \frac{ec}{2R} e^{i\delta} \frac{\mathbf{T}_{2P}}{\sqrt{EE_{2P}}} e^{-(2\pi i/h)(E-E_{2P}+h\nu)[t-(R/c)]}, \tag{65}$$

The replacement of $\mathbf{P}'$ by $\mathbf{P}$ indicated by the subscript P in (65) is the equivalent of replacing

$$\mathbf{p}' - \frac{E'}{c}\boldsymbol{\xi}_2 \quad \text{by} \quad \mathbf{p} - \frac{E + h\nu}{c}\boldsymbol{\xi}_2 - \frac{h\nu}{c}\mathbf{n}. \tag{66}$$

The phase angle δ merely introduces a constant factor $e^{2\pi i/k}(k \geqq 1)$ which has unity as its maximum value. For certain values of δ this constant will have negative values. It thus affects the intensity and phase of the electromagnetic wave emitted due to this part of the integral, but the frequency in which we are particularly interested is not affected.

It remains for us to correlate the constants appearing in this solution with the motion of the recoil electron. Obviously the coefficient of the exponential in t determines the frequency emitted and the rest of the function determines the intensity of the radiation in the given direction. Hence we will define ν_{1P}, the frequency of the emitted radiation, by the relation

$$h\nu_{1P} = E - E_{1P} + h\nu \tag{67}$$

restricting ourselves for the time being to the consideration of $\mathbf{U}_1'$.[7] The terms $h\nu$ and $h\nu_{1P}$ represent the energy of the

[7] The term is supposed to govern the rate of radiation of electromagnetic energy in the following steps. We are first to carry out a calculation analogous to the above for the scalar potential. It is then necessary to calculate the electric and magnetic fields in terms of these potentials, and to calculate the electromagnetic Poynting flux per unit solid angle with origin at the point designated by R. This quantity will of course be proportional to the square of the amplitude referred to. We are to divide this rate of radiation of electromagnetic energy by $h\nu_{1P}$, and the result is to be taken as proportional to the probability of the tran-

incident and emitted X-rays, and E was shown to be the initial energy of the electron. Hence, if we are to provide for a situation in which conservation of energy is to hold, E_{1P} must be associated with the energy of the recoil electron. In this connection it is to be noted that there is no other quantity in our theory which has a prior claim to this title. It is also appropriate to associate with E_{1P} a quantity $\mathbf{p}_{1P}$, defined by $|\mathbf{p}_{1P}| = \sqrt{(E_{1P}^2/c^2) - m_0^2c^2}$, which is the momentum of the recoil electron. This quantity E_{1P} is given by equation (56) which since it represents three equations serves to determine both E_{1P} and also j_{1x}, j_{1y}, j_{1z} in view of the relation $j_{1x}^2 + j_{1y}^2 + j_{1z}^2 = 1$. The determination of E_{1P} of course fixes for us the magnitude of the recoil momentum of the electron and of $\mathbf{j}_1$ the direction of emission of this electron in terms of the foregoing hypotheses. Thus (56) may be written, in view of the definitional relation (67), as

$$\mathbf{p}_{1P} + \frac{h\nu_{1P}}{c}\boldsymbol{\xi}_1 = \mathbf{p} + \frac{h\nu}{c}\mathbf{n}. \tag{68}$$

(56) now appears as identical with the Compton-Debye condition for the conservation of momentum for the system formed by the electron and the quantum, if we christen $(h\nu/c)\mathbf{n}$ and $(h\nu_{1P}/c)\boldsymbol{\xi}_1$ the momenta of the incident and emitted quanta as is the custom. Looked at from another point of view, this procedure gives us the right so to christen them. A similar correlation may be made for the condition (66) accompanying the integration of $\mathbf{U}_2'$, which leads to the final relation

$$\mathbf{p}_{2P} + \frac{h\nu_{2P}}{c}\boldsymbol{\xi}_2 = \mathbf{p} - \frac{h\nu}{c}\mathbf{n}, \tag{69}$$

where $\mathbf{p}_{2P}$ is the momentum of the recoil electron in that case,

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sition associated with the amplitude in question. In this connection it is to be noted that associated with any one transition there is radiation of electromagnetic energy in one and only one direction, the situation being analogous to what we have in the case of reflection of an electromagnetic wave from a grating. The method of specifying the probability above outlined results in the conclusion that when the probability of the transition is re-multiplied by $h\nu_{1P}$ it yields for a solid angle a rate of radiation of energy equal to that calculable on classical electromagnetic theory.

and ν_{2P} is defined by

$$h\nu_{2P} = E - E_{2P} + h\nu, \tag{70}$$

E_{2P} being the energy of the recoil electron. There is, however, no necessity that either the direction of observation nor the momentum of the recoil electron should be the same in the two integrals and hence I have used a different notation in expression $\mathbf{U}_2'$.

Conclusion. To interpret these results, the first obvious thing is that the vector potential of the emitted radiation is the sum of two separate potentials due to the separate effects of the two component waves. This was not immediately obvious without carrying through the computation. The conditions for each of these, (68) and (69), and the definitions, (67) and (70), are the usual Compton-Debye equations for such encounters, for the two waves one moving diametrically opposite to the other so that $(-\mathbf{n})$ appears in one equation and $(+\mathbf{n})$ in the other. There is a certain probability governed by the functions $\mathbf{T}_1$ and $\mathbf{T}_2$ that the electron will be affected by one or the other of the component waves so that its motion and the emitted radiation will then be determined by these energy-momentum conditions. According to the old electromagnetic theory, the picture presented to our minds would be one of an electromagnetic field composed of two oppositely directed trains of waves operating as a whole on the electron. There would be nothing to differentiate the action of the two separate trains of waves; whenever one of them did anything the other would do something. For even the faint potentialities inherent in the fact that a unidirectional wave would possess a Poynting flux would now be lost to this combined wave, at any rate in positions of zero Poynting flux. Now, however, the analysis has of its own accord seen a reason for splitting this stationary wave up into two oppositely directed parts. It has done this by realizing that at any rate to an approximation and only to an approximation, the solution of the whole equation leads to an expression for the vector potential for the emitted radiation consisting of the sum of two parts, one depending on one of these waves and the other depending on the other, and we may say that it would

not be content with any other method of splitting up this composite wave which we might choose for ourselves. It then results from the interpretation of our results in terms of probability considerations that each of these two trains of waves are able to act in the sense indicated, that is, that the scattering of the two component waves is to be considered quite independently one of the other.

If, for two electrons moving in the same direction with the same velocity, it should happen that each responded to one of the component running waves and scattered an X-ray quantum, one could compare the radiation scattered by the two electrons in a given direction of observation. Assuming that the radiation is scattered by both electrons in the same direction gives us the condition $\xi_1 = \xi_2$. Equations (68) and (69) would determine $\mathbf{p}_{1P}$ and $\mathbf{p}_{2P}$ since they each furnish three equations between the three unknown components of the recoil momentum. ν_{1P} and ν_{2P} are known functions of $\mathbf{p}$, $\mathbf{p}_{1P}$ and $\mathbf{p}_{2P}$, and so would not prevent this determination. Thus the momenta of the recoil electrons would be different both as regards direction and magnitude. The frequency being a function of the recoil momentum as well as of the initial momentum of the electron would also be different. This means that an observer would with one setting of his instrument find two scattered lines both of frequencies differing from that of the incident X-ray. Of course, there is nothing remarkable in this, as will be immediately evident to the experimentalist who would naturally expect that if he sent a beam of X-rays in one direction he would get a line in one position and that if he sent it in the opposite direction he would get a line in another position; he would therefore not be very much surprised if he sent the two beams together to get these two lines.

We may discuss from a slightly different point of view the fact that giving the velocity of the electron before impact in magnitude and direction, the frequency of the incident X-ray, and the directions of both the incident and emitted radiation determines uniquely the frequency of the emitted quantum and the vector momentum of the recoil electron. According to Schroedinger's [8] conception of the reflection of an electro-

[8] Schroedinger: *Ann. d. Physik*, 82, 257, 1927.

magnetic wave from the charge density wave, formed by the sum of the ψ waves associated with the initial and recoil electrons, as taking place according to Bragg's law for the first order, for a given incident X-ray wave and a specified direction of the scattered ray, *only* one charge density wave will qualify as a grating to be used for reflection. If the initial motion of the electron is given, there is consequently but one value of the velocity of the recoil electron which will produce this grating. In other words, for one assigned initial state there is one and only one other final state of the electron which will produce a grating appropriate to the reflection of our X-ray in the given assigned direction. The change in frequency can be computed as a Doppler effect due to reflection from this moving mirror the velocity of which is known. In this way we can explain more completely the thought underlying the analysis carried out in this paper. In forming our expression for $\rho\mathbf{u}$ we summed over all possible values of $\mathbf{p}$ and $\mathbf{p}'$, which may be identified with the momentum of the electron before and after impact—that is, physically we built a double infinity of these Schroedinger gratings. At the end we picked out one value of $\mathbf{p}$, the initial momentum of the electron, and that corresponded to cutting down the number of our gratings to a single infinity, but even with this restriction since $\mathbf{p}'$ could have any value we had a set of gratings with all possible velocities, directions and wave-lengths. However, owing to the mutual interference of the radiation from the various parts of these gratings it results that any one of them sends electromagnetic radiation off in one and only one direction. Moreover, if we specify the frequency of an X-ray wave and its incident and reflected directions there would be but one of these many gratings which could reflect for this case. The radiation travelling in the opposite direction would pick out another grating for its purpose. As the direction of observation is changed, two different gratings would be chosen automatically. Hence the infinite set of gratings would include all the individual gratings which are necessary to tell the story for all possible directions of observation or, for that matter, for all directions and frequencies of the incident radiation. The selective character of this grating reflection is indicated in the analysis in that the

integrals have no value unless their space exponents vanish and the condition that they vanish is equivalent to the conservation of energy and of momentum which is the same condition that Schroedinger derives from his grating analogy. In fact the final expression for $\mathbf{U}'$, the vector potential, in integral form represents exactly what we would have if we considered a number of gratings of the above kind, each one operative in respect of an initial beam of X-rays for one and only one direction of reflection.[9]

In conclusion it is a pleasure to thank Prof. W. F. G. Swann for the inspiration and help which he has so generously given during long discussions of the subject considered in this paper.

[9] It is satisfying to note that Bothe (*ZS. f. Physik*, 41, 332, 1927) has shown *experimentally* that the component waves of a standing X-ray wave are scattered separately, thus verifying this theory.

Denicotinized Tobacco. Several processes may be used in the preparation of denicotinized tobacco. Nicotine may be removed by the use of suitable solvents; or the tobacco may be subjected to a resweating either by treatment with superheated steam or by heating in a vacuum chamber. E. M. BAILEY, O. L. NOLAN, and W. T. MATHIS (*Conn. Agric. Exp. Station Bull.* 295, 338–351, 1928) have analyzed 17 samples of denicotinized tobacco as found on the market in cigars, cigarettes, and smoking tobacco. For comparison they also analyzed 18 samples of ordinary tobacco as found on the market, and collected from the literature 40 additional analyses of ordinary tobacco as found in cigars, cigarettes, and smoking and chewing tobacco. The total nicotine content, calculated on the moisture-free basis, ranged between 0.47 and 3.63 with an average of 1.96 per cent. in the 58 analyses of ordinary tobacco, and between 0.74 and 2.73 with an average of 1.41 per cent. in the 17 analyses of denicotinized tobacco. When tobaccos of corresponding leaf types were compared, the denicotinized products, as a rule, contained but little less nicotine than the ordinary products. While considerable reductions in nicotine content were indicated in certain instances, yet it was not difficult to find brands of ordinary tobacco with a nicotine content not greatly in excess of that present in the most thoroughly processed of the denicotinized products. Nicotine is an important, if not the chief, factor in the production of the satisfying effects derived from smoking. Use of denicotinized products may lead to increased indulgence; and the actual nicotine intake may equal or exceed that when ordinary tobacco is used. None of the denicotinized tobaccos analyzed were sufficiently low in nicotine content to warrant unrestricted indulgence by persons who suffer ill effects from this alkaloid.

Free nicotine occurs in tobacco, and is due, apparently, to the dissociation of the nicotine salts of organic acids. The harsh and irritating effects on smoking may be due to free nicotine. Denicotinized tobacco did not differ markedly from ordinary tobacco with respect to the free nicotine content. The distribution of the nitrogen as nitrate nitrogen and as ammoniacal nitrogen was practically identical in the two classes of tobacco.

J. S. H.

THE QUANTUM MECHANICS OF AN ELECTRON OR OTHER PARTICLE.

BY

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THE last three years have witnessed the performance of several physical experiments which bid fair to rival the most famous experiments of the last half century in revolutionary importance. Unlike most revolutionary experiments, however, their results have offered to the theoretical physicist a problem of explanation peculiarly baffling in character. The experiments deal with two entities which had, as it seemed, been demonstrated to be of radically different natures, namely, light and electrons, and in consequence of them we now see that the difference between these entities is not nearly so large as had been supposed. It is the purpose of this article to give an account in simple terms of the resulting changes in physical conceptions and of the manner in which the new phenomena and problems of a similar sort have been handled in the new theory of Quantum Mechanics.

The revolutionary experiments upon light were performed in 1925 by Bothe and Geiger in Germany and by Compton and Simon in this country. They are well described in A. H. Compton's article [1] in a recent number of this journal, also in his book on X-rays. The experiments actually deal with X-rays, but so many facts point to the conclusion that X-rays are merely light of extremely short wave-length that it seems quite safe to extend the general conclusions to ordinary light or to electromagnetic radiation of any other kind.

From these experiments we are led to the view that when a beam of X-rays passes through a piece of matter, nothing else happens at all during most of the time; the X-rays are not even scattered. Only now and then, here and there, does something cataclysmic happen in an atom, and, when

[1] A. H. Compton, JOUR. FRANK. INST., 205, 155, 1928.

it does, several things happen together: either a photo-electron is ejected from the atom and simultaneously a quantum of the incident X-rays disappears, or else both a slower recoil-electron is ejected and also a quantum or photon of X-ray energy starts off in a direction different from that of the incident X-rays to form part of the familiar scattered radiation. Such a scattered quantum does not, however, in accord with the wave-theory and like the waves of sound from a pistol-shot, spread out in all directions from the atom in the form of a spherical disturbance; instead of this, the entire quantum moves off in a fairly definite direction which can be calculated by simple rules from the direction in which the associated recoil-electron was ejected from the atom. The method of making this calculation is very similar to that employed in treating the collision of two billiard balls; it is just as if the incident X-ray photon had struck the electron, given up part of its energy to it and bounced off in a new direction, all in accord with the laws of elastic impact. It might seem at first sight as if we had been led back to the old corpuscular theory of light.

And yet there is every reason to believe that with these same X-rays interference experiments can be performed just as well as with ordinary light. For instance, we might allow a stream of these billiard-ball photons to fall upon a screen with a small hole in it and receive those which passed through the hole upon a second screen. We should then find that the streamlet which came through the hole did not continue quite straight ahead but spread sidewise somewhat, producing the phenomenon of diffraction. Now this effect we might easily explain, at least qualitatively, on the corpuscular hypothesis: perhaps those photons which pass near the edge of the hole are attracted by the material of the screen and are thereby deviated from their paths. But if we now experimented further and opened a second small hole near the first, thereby opening a path for a second streamlet to come through, we should find that the number of photons striking certain parts of the second screen was actually less than when only one hole was open, these parts forming the dark regions in the interference pattern. To judge from the results of experiments on very faint light, this effect

would occur even if only one photon per minute were coming through the holes; the blackening after many hours on a photographic plate placed behind the holes would still be less in certain places with both holes open than with only one open. The interference can therefore not be ascribed to interaction between different photons which are simultaneously present in the region.

Now such a result is practically impossible to explain on a corpuscular theory of the ordinary sort. It would seem that each photon, after passing through one hole, must understand how to lay down the entire interference pattern; it must be guided in its subsequent motion by the shape of all parts of the screen. Perhaps a set of laws could be set up which would guide the photons under all conditions to the right place,—possibly something like the combination of waves and corpuscles once tentatively suggested by Newton; but no one has yet elaborated any such explanation, and it seems to be the general conviction of theorists that, if it can be worked out at all, the result would be a theory at least inconveniently complicated. But before considering what the quantum theorists have done about the problem, let us turn to the other side of the experimental picture.

From 1900 to 1925 it was generally accepted that electrons, unlike light rays, were corpuscular in nature—little specks of electricity which could be thought of as moving inside a vacuum tube in a continuous manner along definite paths. There can hardly be any doubt that their energy and electric charge really are thus distributed discontinuously at least at the instant when they arrive at an electrode, for under certain circumstances they can be counted as they come (for instance by causing each electron to produce a visible "kick" of an electrometer fiber). And yet within the past two years[2] Davisson and Germer in America and Thompson in England have obtained definite interference effects with a beam of electrons! It now seems probable that even the simple interference experiment described above can be performed with such a beam, although it will be a difficult one because of the extremely short "wave-length" of electrons moving with appreciable energy.

[2] Cf., e.g., Davisson and Germer, *Nat. Acad. Sci., Proc.*, 14, 619, 1928; Thompson, *Nature*, 122, 279, 1928.

The chief difference in the mechanics of electrons and photons seems, in the light of these discoveries, to lie in the fact that electrons influence each other's motions (by means of their electromagnetic fields) whereas photons move independently of each other. With this exception the same type of law should be found to hold for both; and it must be something that is neither wave-theory nor corpuscular theory. Fortunately it happens that a new form of Quantum Mechanics has been emerging just during the past three years, arising out of the impasse into which the old quantum theory had gotten, and this new theory is equal to the task of describing the new facts adequately. The new theory is really not so much a fusion of the wave and the corpuscular theories as a remarkable combination of elements taken from both of them. It was initiated by Heisenberg in 1925, developed independently a little later in quite a different direction by deBroglie [3] and Schrödinger,[4] and finally completed a year ago for the purposes of "particle mechanics" by Heisenberg.[5] The theory is not yet complete, for the proper treatment of radiation processes is still partly obscure, as is also the mechanics of high-speed electrons. We shall accordingly limit the discussion in what follows to the case of particles—electrons or protons or atomic or molecular groups of these—moving with speeds small as compared to the speed of light. We shall also omit all mention of the earlier "matrix" form of the theory, which is mathematically equivalent to the other but is much more abstract and to many physicists repellent.

THE INDETERMINATION PRINCIPLE.

The great guiding idea of Heisenberg has been that in constructing our theories we ought to be very clear in our minds as to what quantities can really be made the object of physical observation; only observable quantities and such others as can be calculated from them by definite rules can be said to possess physical significance. The principle itself it as old as physics, of course; it is only Newton's "hypotheses non fingo" in a new dress. The novelty in Heisenberg's

[3] L. deBroglie, *Ann. de Physique*, 3, 22, 1925.

[4] E. Schrödinger, *Ann. d. Physik*, 79–81, several articles, 1926.

[5] W. Heisenberg, *Zeits. f. Phys.*, 43, 172, 1927.

position was his suggestion that the principle had not been sufficiently observed in the old quantum theory. Thus, atomic energy and spectral frequency are observable quantities, for, although they are pretty abstract conceptions and are far removed from the sights and sounds actually noted and recorded in a physical laboratory, yet they are calculable from the latter according to well-defined rules. On the other hand, orbital frequency and orbital size were never in the same class; from certain assumptions as to these, energy and spectral frequency could be calculated, but the process could not be reversed, as is well illustrated in the long and fruitless struggle on the part of the quantum theorists to discover the motions of the electrons in the helium atom.

Now the application of this idea to the concept, "motion of a particle," leads in the light of recent experimental discoveries to a remarkable conclusion. It appears that we must simply give up the idea of an electron as a particle occupying at successive instants a *continuous* series of successive positions; at least we must regard this idea as one which is purely imaginary and has no meaning in terms of physical observation. We must introduce into the conception of a moving particle a certain degree of haziness, a certain element of indeterminateness and mere probability entering fundamentally into the groundwork of natural processes.

It is not however, space-time itself that requires to be modified; space-time remains in the new theory as continuous and as infinitely divisible into elements as it ever was. It appears to be possible, for instance, at least in principle, to locate an electron at a certain exact point and at a certain exact moment, and the point and the moment may be anywhere on a continuous scale of spatial and temporal magnitudes. From the theoretical standpoint, the simplest method of so locating an electron is probably one similar to the method used by surveyors in finding the location of an inaccessible mountain-top, namely, to train two microscopes along converging lines upon a point in space and to contrive to "see" the electron in both instruments as it passes the point at a certain instant. Such an observation is scarcely possible as yet in practice, but this circumstance seems to inhere in difficulties of instrumental construction and manipulation

rather than in the nature of the laws governing natural processes. The method has the advantage of being pretty certainly capable of refinement to yield any desired degree of precision; we need only shorten sufficiently the wave-length of the light employed in order to attain the necessary resolving power in the microscopes. For instance, to attain a precision of one per cent. of an atomic diameter in the observed position, we should need to use light of about $\lambda = .01$ Ångstrom, i.e., soft γ-rays; but this seems to present no difficulty in principle, for it is now known that X-rays can be refracted like light and there is every reason to think that the same thing is true of all shorter waves. By employing first X-rays, then γ-rays, then cosmic rays and so on we might in principle attain any degree of precision in the observation of position.

Difficulties of another sort arise, however, when we pass from observations of position to the consideration of observations of motion. It is instructive to consider how we might attempt to observe the motion of an electron in a Bohr orbit.* We might take our cue from the astronomers and try to see it by means of our microscopes at successive points in the orbit. But then we should have to take account of one of the newly-discovered peculiarities of radiation, namely, the Compton effect; because of this, every time a pair of photons rebounds from the electron into our microscopes, the electron will receive an impulse which changes its motion. Thus we could never follow it round in a smooth Kepler ellipse; the observed points could at best only fall roughly on the ellipse. It seems a significant fact that this blurring of the observed path is at least enough to prevent the distinction by observation of any one quantum orbit from the next one. For instance, in a hydrogen-like atom, the radius of the nth circular orbit being $r = n^2h^2/4\pi^2me^2$, the difference in radius between the nth and $(n + 1)$st orbits is only $\Delta r = (2n + 1)h^2/4\pi^2me^2$ (h = Planck's constant, m and e = mass and charge of the electron); to distinguish such a small difference in radial position we should have to employ light of $\lambda \leqq \Delta r/2$ (roughly speaking), which would impart a Compton impulse of magnitude

* Cf. Heisenberg, loc. cit.

$$\Delta p = \frac{h\nu}{c} = \frac{h}{\lambda} \geqq \frac{8\pi^2 me^2}{(2n+1)h},$$

more or less, and this would change the angular momentum by an amount of the order of $r\Delta p = 2n^2h/(2n+1)$; this expression has the value $2h/3$ for the first orbit, $8h/5$ for the second, and so on, whereas the difference in angular momentum between successive orbits of similar shape is only h.

It thus appears that we must cease to regard the Bohr orbits as possessing any physical reality. Similar considerations are believed to apply to the motion of any portion of matter that can be treated as a "particle." If this is correct, then orbital motion is a conception which has a relatively precise meaning only for phenomena on a certain scale of magnitude, such as planetary motion, where the effect even of billions of Compton kicks is negligible, whereas the conception becomes hazy when we turn to small-scale phenomena and loses all meaning if we go to the extreme of trying to distinguish adjacent "quantum" paths.

Closer analysis shows that the same difficulty inheres in any measurement of *velocity*. The fundamental mechanical definition of velocity is, of course, the distance between two positions occupied by the moving entity at two different times, divided by the difference in time. According to quantum mechanics, however, there will be an indefiniteness in the positions, if measured by the method described, of the order of λ, and there will be a Compton impulse upon the particle of the order of $h\nu/c$ or h/λ, so that at the instant when the position of the particle is determined its momentum (in any one direction) is undefined by an amount of order h/λ. The product of the degrees of indefiniteness in position and in momentum is thus of the order of h.

We have here an instance of an aspect of natural processes which evidently lies very near the root of those features which find their expression in the quantum theory. It is an instance of what has been called the Indetermination Principle of Heisenberg, the exact formulation of which is contained in equation (13) below. The principle is believed to hold quite generally, but its application often requires very painstaking

and clear reasoning and we shall refer for further discussion of it to the original literature.[5, 6, 7]

In consequence of the Indetermination Principle the fundamental concepts of the new mechanics differ radically from those of the old, and especially in two respects. In the first place, as we have seen, there is in the new theory a characteristic incompatibility between certain pairs of magnitudes when they are made objects of observation. This is a very important feature of the theory and neglect of it in dealing with special cases is very apt to lead to paradox. Thus, we have seen that when position is measured precisely, momentum must remain largely indefinite; whereas if we arrange a situation where momentum is accurately known (of which an example is given below), then the position is indefinite. In a similar way energy and motion are incompatible: when we follow a particle along a path, its energy remains somewhat indefinite; if on the other hand we select an atom in a certain quantum state characterized by a definite value of the energy, then, as we shall see presently, the process of selection has created a situation in which nothing referring to electronic motion appears in the observations.

In the second place, it follows from the Principle that an element of probability must necessarily enter into the statements that we make about physical systems and into our statements of physical laws. We can no longer say that a system of particles is at a time, t, definitely in a certain configuration and moving in a certain manner; we can only assign a probability function for the results that would be obtained if positions or momenta of the particles were to be measured with precision at that time. For example, when we have by suitable means isolated an atom in a definite quantum state, we cannot specify the position or motion of the electrons within it, we can only assign a probability function for the positions in which they would be found if they were to be located.

Accordingly it is to be expected that the fundamental laws of mechanics will have to do with this probability func-

[6] E. H. Kennard, *Phys. Rev.*, 31, 344, 1928.

[7] C. G. Darwin, *Roy. Soc. Proc.*, 117, 258, 1927.

tion or with some quantity closely related to it. This is the form in which the theory has been given by Schrödinger and Heisenberg, and we shall now turn to a more detailed consideration of this theory.

THE NEW PARTICLE DYNAMICS.

The principles of Quantum Mechanics are most easily grasped in direct comparison with those of classical theory.

In classical mechanics it was customary to write down each coördinate of a particle, Cartesian or generalized, as a function of the time, e.g. $q_n = q_n(t)$. These equations were tacitly interpreted as meaning that, if at a given time all of the particles were located experimentally, the values thus found for the coördinates would be the values of these functions for the given value of the time. The variation of the coordinates with the time was then assumed to occur in accordance with Newton's laws of motion, which might be dressed up in the form of the Hamiltonian equations, the Hamiltonian function being obtained from the forces that were assumed to act between the particles.

In the new mechanics, in accordance with what has been said above, we have in place of the coördinates as functions of the time a probability function for their values. The theory is most easily expressed, however, not in terms of the probability itself but in terms of what Pauli called the "probability amplitude"; if this is denoted by $\psi(q_1, q_2 \cdots q_n, t)$, or $\psi(q, t)$ for short, where $q_1 q_2 \cdots q_n$ are the generalized coördinates of the system, then the probability function is assumed to be $\psi\bar{\psi}$, where the bar over the second ψ means that the conjugate value is to be taken if ψ is a complex number. Thus the probability amplitude has, according to Heisenberg,[5] the fundamental significance that, if all of the n coördinates were to be determined with precision at a given moment, the chance of finding values for them lying between q_1 and $q_1 + dq_1$, q_2 and $q_2 + dq_2$, etc., would be

$$Pdq = \psi\bar{\psi}dq, \tag{1}$$

where dq stands for $dq_1 dq_2 \cdots dq_n$. This relationship between the amplitude and the probability is very similar to

the relation between amplitude and energy in any of the familiar types of wave-motion. It requires that always

$$\int \psi \bar{\psi} dq = 1, \tag{2}$$

the integral being extended over all physically admissible values of the coördinates.

In lieu of Newton's laws of motion or their equivalent it is then assumed as a fundamental law, following Schrödinger,[4] that ψ obeys the wave-equation*

$$\frac{h}{2\pi i}\frac{\partial \psi}{\partial t} = H\left(-\frac{h}{2\pi i}\frac{\partial}{\partial q}, q\right)\psi, \tag{3}$$

where $H\left(-\frac{h}{2\pi i}\frac{\partial}{\partial q}, q\right)$ stands for a differential operator formed from the Hamiltonian function by replacing in it each power of a momentum by the same power of $\left(-\frac{h}{2\pi i}\right)$ multiplied by a partial derivative of an order equal to the power, taken with respect to the corresponding variable. For example, in the case of one particle in a conservative field of force, if we employ Cartesian variables x, y, z and if p_x, p_y, p_z denote the corresponding components of momentum, then

$$H = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) + V,$$

where V is the potential energy and m the mass of the particle, and (3) becomes

$$\frac{h}{2\pi i}\frac{\partial \psi}{\partial t} = -\frac{h^2}{8\pi^2 m}\Delta\psi + V\psi, \tag{4}$$

in which Δ stands for $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$. The determination of the Hamiltonian function is, of course, in all cases an important fundamental problem. Fixing this function corresponds in

* The train of ideas that led to this rather mysterious-looking postulate is to be found in the paper of deBroglie, loc. cit.

the new theory to fixing the constitution of the system in the old theory, from which the Hamiltonian was then deduced; its determination is thus a matter of hypothesis to be tested by agreement of deductions from it with experiment. Fortunately, carrying over the classical Hamiltonian for the Rutherford atom into the new theory has resulted in so many successes that the classical form of this function seems to be at least very nearly the correct one for atomic systems. The occurrence of the universal natural constant h in this equation constitutes, at least so far as mechanics is concerned, its fundamental point of entry into the theory of physical phenomena.

A knowledge of ψ covers in the new theory all that can be known about the state of the system. This may at first sight seem surprising in view of the fact that ψ contains only the coördinates whereas in classical theory one had to assign the values of the velocities or the momenta as well in order to describe the state of the system completely. The explanation lies in the fact that ψ, being complex, is, so to speak, a more powerful number than the real numbers of ordinary theory; if we write $\psi = Re^{i\theta}$, where R and θ are real functions of the q's and t, then $\psi\bar{\psi} = R^2$, so that the modulus R alone determines the probability while the argument θ is left free to express additional features of the situation corresponding to the velocities of the particles.

One of the most important types of problem in classical theory was one in which the initial positions and velocities of the particles were assumed to be known and their positions and velocities at future times were calculated. The corresponding problem in the new theory is obviously one in which the values of ψ at a certain initial time are assumed to be known; the future history of the system is then represented by that solution of the wave-equation which reduces to the given function of the coördinates at the initial moment. Such an initial value of ψ would be determined in the usual way by previous knowledge, by the conditions of an experiment, etc. The values assumed would be subject (at least in principle) to experimental check, only it is characteristic of the new theory that such a check would have to be made by a statistical method: we should have to repeat the experi-

ment or to reproduce the given situation a great many times and each time determine the position of the particles at the same moment relative to the time of beginning the repetition. The results would then vary from one repetition to the next, and by treating them statistically we should obtain the probability function $\psi\bar{\psi}$. A statistical study of this sort has actually been made in certain cases, as in Rutherford's observations upon the scattering of α-particles. From these observations we should then know the absolute value of ψ for all values of the coördinates; by repeating the determination at a slightly different time and introducing the assumption that ψ obeys the wave-equation, we could then arrive at the values of ψ itself, but we shall not discuss the mathematical method of doing this.

Such statistical studies are, however, comparatively rare in practice; usually an experiment is so arranged that the inevitable statistical variations are too small to be of importance, and experimental conditions are usually known in gross terms, such as size of an electrode or applied potential. The process of passing rigorously from such conditions to the complete value of ψ, which must contain the coördinates of all of the electrons or atoms forming the material under observation, is very complicated and has been replaced in all applications of the theory to date by a shrewd surmise as to the form which the result must take, to be checked by agreement of the calculated consequences with experiment; illustrations of this will be given below in the discussions of electrons having definite speeds and of atomic energy levels.

THE CLASSICAL THEORY AS AN APPROXIMATION.

Any new theory of mechanics must, of course, pass into the classical theory under those conditions where the classical theory constitutes a well-established first approximation to the truth; otherwise it could not be correct. The simplest connection of Quantum Mechanics with classical theory lies in those cases in which a particle is confined within a certain region of space, ψ being zero (or practically so) outside of that region. Such a distribution of ψ has been called a "wave-packet" (Schrödinger) or "probability packet" (Heisenberg). As time passes, such a packet will move about in accordance

with the wave-equation. This motion can often be resolved more or less into a translatory motion of the packet as a whole and a change in size; if the change in size is small relatively to the distance covered in the translatory motion, then the packet represents a particle whose position is definite to a certain extent and which can with a certain degree of definiteness be said to be moving with a certain velocity. If we suppose in addition that the residual indefiniteness in these magnitudes is small as compared to the scale of magnitude upon which the phenomena are being viewed, then we have the case in which the classical theory is approximately valid.

There are even some cases in which the translatory motion of a packet is exactly the classical motion of the particle itself; these are, in fact, the simplest cases of all, the necessary condition being that the space derivatives of the force shall be either zero or at least constant. Thus one finds [8,7] by solving (3) or (4) that the packet representing a particle in a uniform gravitational field "falls" with the classical acceleration, the packet for an electron in a uniform magnetic field moves round in a circle at the classical rate, and in a harmonic field the packet vibrates back and forth with the classical frequency. It is only in more complicated fields, like the Coulomb field around a nucleus, that novel results are obtained. In the case of a harmonic field the amplitude of the "center" of the packet may even have any value, there being no trace of quantisation in its motion. One cannot, however, quite assert that the particle moves in violation of the old quantum rule, for the size of the packet is always necessarily great enough to obliterate the difference between the motions in a successive pair of the old quantum states. Quantisation is a phenomenon, not of motion, but of energy or angular momentum, as we shall see below.

Usually, however, a packet not only moves but changes "size" as well; in the last two cases just mentioned, the size merely pulsates, but in the cases of free motion and of a uniform gravitational field the packet continually enlarges and, if it is very small to begin with, it spreads very rapidly. The latter behavior is intimately connected with the In-

[8] E. H. Kennard, *Zeits. f. Phys.*, 44, 326, 1927.

determination Principle: if the packet is initially small, its position is then fixed rather closely and there must be a correspondingly large indefiniteness in its velocity, which requires in turn a rapidly increasing indefiniteness in its subsequent positions and so a rapid enlargement of the packet with time. It is an interesting fact that in all of the simple cases just mentioned correct results are obtained also if we assume that the particle moves under classical laws but that its initial position and velocity are not precisely known, only a probability curve being assignable for each. In the case of other force fields, however, such as the Coulomb field, an interpretation of this kind is almost certainly impossible (no rigorous proof of this fact is known to the writer).

A ONE-DIMENSIONAL EXAMPLE.

A better idea of the theory can perhaps be got by considering a concrete example. Let us take the case of a particle moving in one direction in a uniform gravitational field. Its wave-equation can be written (cf. (4)).

$$\frac{h}{2\pi i}\frac{\partial\psi}{\partial t} = -\frac{h^2}{8\pi^2 m}\frac{\partial^2\psi}{\partial x^2} - mgx\psi, \tag{5}$$

and a possible packet is given by:

$$\psi = \frac{C}{\sqrt{1 + \frac{ht}{2\pi i m x_i^2}}} \exp\left[-\frac{\left(x - x_m - \frac{\xi_m}{m}t - \frac{1}{2}gt^2\right)^2}{2\left(x_i^2 + \frac{t^2}{m^2}\xi_i^2\right)} \right.$$
$$- i\left\{\frac{ht}{2\pi m x_i^2}\left(x - x_m - \frac{1}{2}gt^2\right)^2 + \frac{4\pi}{h}\xi_m x_i^2 \right.$$
$$\left.\left.\left(x - x_m - \frac{1}{2}\frac{\xi_m}{m}t\right) + \frac{8\pi^3}{h^3}gm^2\xi_m x_i^6\right\} \div 2\left(x_i^2 + \frac{t^2}{m^2}\xi_i^2\right)\right.$$
$$\left. - i\frac{2\pi}{h}gmt(x - x_m) + i\frac{\pi}{3h}g^2mt^3\right] \tag{6}$$

where $i = \sqrt{-1}$, t = time, ξ_i is given by

$$x_i\xi_i = \frac{h}{2\pi}, \tag{7}$$

and C, x_i, x_m, ξ_m, and g are constants whose meaning we shall presently discover (exp y means, of course, e^y). The reader can verify if he wishes that this rather complicated expression for ψ really is a solution of (5); the method of obtaining it is an indirect one.[7, 9] The corresponding probability function for position is:

$$\psi\bar{\psi} = C\bar{C}\left(1 + \frac{h^2t^2}{4\pi^2m^2x_i^4}\right)^{-1} \exp\left\{-\frac{\left(x - \frac{t}{m}\xi_m - \frac{1}{2}gt^2 - x_m\right)^2}{x_i^2 + \frac{t^2}{m^2}\xi_i^2}\right\}; \tag{8}$$

$\psi\bar{\psi}dx$ taken for given values of x and t denotes the probability that, if the position were precisely determined at time t, a value lying between x and $x + dx$ would be obtained. Of the constants, C must have such a value that $\int Pdx = \int \psi\bar{\psi}dx = 1$. The constant x_m is simply the "mean expectation" of value for x at the time $t = 0$, for if we put $t = 0$ in (6) we find $\int_{-\infty}^{\infty} x\psi\bar{\psi}dx = x_m$; whereas x_i is the "measure of definiteness" at $t = 0$ as defined by the equation $x_i^2 = 2[(x - x_m)^2]_m = \int_{-\infty}^{\infty} 2(x - x_m)^2\psi\bar{\psi}dx$. The measure of definiteness at time t is similarly

$$x_{it} = \sqrt{x_i^2 + \frac{t^2\xi_i^2}{m^2}} = x_i\sqrt{1 + \frac{h^2t^2}{4\pi^2m^2x_i^4}}. \tag{9}$$

If in (6) and (8) we substitute

$$x' - x_m = \frac{x_i}{x_{it}}\left(x - \frac{t}{m}\xi_m - \frac{1}{2}gt^2 - x_m\right)$$

we find

$$\psi\bar{\psi} = C\bar{C}\left(1 + \frac{h^2t^2}{4\pi^2m^2x_i^4}\right)e^{-(x'-x_m)^2/x_i^2},$$

[9] E. H. Kennard, *Phys. Rev.*, 31, 876, 1928.

in which the factor containing x' is independent of t. From this result it follows easily that the probability curve at any time t can be obtained from that at time $t = 0$, when $x = x'$, merely by stretching all abscissas in the ratio x_{it}/x_i while holding the ordinate at $x' = x_m$ fixed, then displacing the whole curve a distance $t\xi_m/m + \frac{1}{2}gt^2$ toward $+x$, and finally decreasing ordinates in the proper ratio to keep the area under the curve equal to unity. We can express this in another way by saying that the packet spreads out with time, (its diameter ultimately increasing at the uniform rate $h/2\pi m x_i$ or ξ_i/m) while at the same time the whole packet moves with a velocity consisting of a uniform component $u_m = \xi_m/m$ upon which is superposed a component with uniform acceleration g. We have thus an example of a concentrated probability packet which both moves as a whole and changes size as it goes.

THE AMPLITUDE FOR MOMENTUM.

In terms of this example we can bring out more clearly the status of velocity and momentum in the new theory. In accordance with the general discussion above, it is not immediately obvious what should be meant by "the velocity" of the particle whose position in space is described by the amplitude ψ; were we to locate the particle in order to begin an observation of velocity, we should seriously disturb it through the Compton effect and should alter completely the value of ψ. Apparently we can do nothing better than the following. Let us suppose that, as in the above example, at time $t = 0$, ψ differs appreciably from 0 only over a finite range D (in the example, say $D = 10x_i$). Let us wait a long time t and then locate the particle with precision, say at a point x_1. Then we may regard $u = (x_1 - x_0)/t$, where x_0 is any point within the region occupied initially by the packet, as the resulting observed value of the velocity, or mu as the observed value of its momentum. The value of u so found is not affected by the observational disturbance of ψ, which occurs only after we are through with the particle, and the uncertainty as to the value that ought to be taken for x_0 is only of order D/t and so can be made as small as we please by sufficiently increasing t.

In the case of free motion ($g = 0$ in the example) this

definition of velocity as an observable magnitude is quite satisfactory, for it is easily shown that the result of such an experiment is independent of the times chosen as the initial and final ones so long as the interval between them is long. But in any other case than free motion, as in our example if $g \neq 0$, the result of such an observation does depend upon the times; of course this is really only what one would expect in the case of any "accelerated" motion, but in Quantum Mechanics it leads to consequences disastrous for the concept of velocity, because the relatively long time that is necessarily required to make an observation precludes us from saying with precision to just what instant the measurement refers. The most natural view, in the author's opinion, is that velocity and momentum are conceptions which really possess a simple immediate significance as physically observable magnitudes only in the specially simple case of free motion.

Even in the case of free motion, moreover, the peculiar statistical foundation of the new theory manifests itself in the circumstance that the result of such an observation of velocity or momentum can only be predicted statistically (because of the spreading out of the packet with time). Thus in the example considered above it is readily shown that with $g = 0$ we can take as the probability amplitude for the momentum, $M(\xi)$, where $\xi = mu$ and

$$M(\xi) = \frac{1}{\sqrt{h}} \int_{-\infty}^{\infty} \psi(x) e^{2\pi i h^{-1} \xi x} dx$$

$$= C \sqrt{\frac{x_i}{\xi_i}} \exp \left\{ - \frac{(\xi - \xi_m)^2}{2 \xi_i^2} + \frac{2\pi i}{h} x_m \xi \right\}, \qquad (10)$$

the probability that an experimenter would obtain, by the method described, a value of ξ lying within a range $d\xi$ being

$$P_\xi d\xi = M \bar{M} d\xi = C \bar{C} \frac{x_i}{\xi_i} e^{-(\xi - \xi_m)^2 / \xi_i^2} d\xi. \qquad (11)$$

From (11) we find that $\int_{-\infty}^{\infty} \xi P_\xi d\xi = \xi_m$, $\int_{-\infty}^{\infty} 2(\xi - \xi_m)^2 P_\xi d\xi = \xi_i^2$, so that the constants ξ_m and ξ_i appearing in (6) represent

respectively the mean expectation of value and the measure of definiteness for the momentum. The constancy of ξ_m in time is, of course, due to the absence of external forces; if $g \neq 0$, ξ_m as defined by means of (10) and (11) is found to increase at the rate of mg units per second. The mean expectation of velocity, $u_m = \xi_m/m$, is simply the velocity of advance of the whole packet, as is reasonable.

The extension of the idea of momentum as an observable quantity to cases of accelerated motion, as we have seen, presents difficulties. For theoretical purposes, however, it is often convenient to generalize the mathematical formulas and to define as the probability amplitude for any momentum p corresponding to a generalized coördinate q:

$$M(p) = \frac{1}{\sqrt{h}} \int \psi(q) e^{2\pi i h^{-1} pq} dq, \tag{12}$$

with a suitable generalization for the case of many degrees of freedom. It can then be shown [8] that, if q_i and p_i are the "measures of definiteness" of q and p respectively, in general

$$q_i p_i \geqq \frac{h}{2\pi}. \tag{13}$$

This equation represents a very general case of Heisenberg's Indetermination Principle.

MAGNITUDE OF THE QUANTUM INDEFINITENESS.

At this juncture it may be illuminating to turn aside for a moment from the general discussion and to give some illustrations of the actual magnitude of the quantum indetermination. This feature of the theory only makes itself felt, of course, in very small-scale phenomena.

For example, a bacterium weighing 10^{-12} g. might be represented by a packet like that expressed by eq. (6) with $x_i = .03\ \mu$, so that its position is fixed initially within about $0.1\ \mu$; then ξ_i equals 3.5×10^{-22}, so as to make $x_i \xi_i = h/2\pi = 1.04 \times 10^{-27}$, and the indefiniteness in velocity is under $10^{-5}\ \mu$ per second. To double in size, such a packet will require a time t that may be found by putting $x_{it} = 2x_i$ in

(9), from which $t = \sqrt{3}mx_i/\xi_i$ or more than four hours. Similarly, for the smallest water drop visible to the naked eye the doubling might easily take a million years. Thus such large bodies as these can be handled very well by the approximate methods of classical mechanics. On the other hand, if a molecule of air were fixed in position within the length of its own diameter (say $x_i = 6 \times 10^{-9}$ cm.), its velocity would necessarily be indeterminate by at least 3000 cm./sec. and its wave-packet would enlarge by at least one molecular diameter every 10^{-11} seconds; whereas the packet for an electron defined in position within the radius of the smallest Bohr orbit for the hydrogen atom (say, $x_i = 3 \times 10^{-9}$ cm.) would at least double in size in 1.5×10^{-17} sec. or in less than a tenth of the time required for one revolution around its orbit in the old quantum theory. Thus, according to the new theory, even if we could locate a particle of atomic dimensions within a distance equal to its own diameter we should lose track of it again very quickly indeed.

MONOCHROMATIC WAVES OF ψ.

Returning now to our general train of thought, let us consider the very important case in which we have to deal, not merely with a "wave-disturbance," but with actual "waves" analogous to the monochromatic wave-trains of optics. In eq. (6) let us put $g = 0$ so as to have a case of free motion, and let us make x_i very large; ψ can then be written

$$\psi = \gamma(x, t)e^{2\pi ih^{-1}(Wt - \xi x)}, \qquad (14)$$

where $W = \xi^2/2m$ (and represents the energy), ξ is written for ξ_m, and γ is a function that is almost constant in space and in time but nevertheless manages to vanish at infinity and to make $\int \psi\bar{\psi}dx = 1$. The momentum is now confined practically to the value ξ_m (for which we have written ξ), for in (10) $\xi_i = h/2\pi x_i$ and is small. The packet will still move along with a speed $u_m = \xi_m/m$ or $u = \xi/m$; and its motion is in this case not much blurred by spreading, for by (9) the increase in its diameter in time t is only of the order of

$$x_{it} - x_i = \frac{h^2t^2}{4\pi^2m^2x_i^2}\frac{1}{x_{it} + x_i} < \frac{h^2t^2}{8\pi^2m^2x_i^3},$$

whereas the time required for the packet to traverse a distance nx_i is $t = nx_i/u$, so that with both n and x_i large the quantity $(x_{it} - x_i)$ may still be small and the packet can therefore move over many times its own diameter (n large) before spreading appreciably. Such a packet represents a particle whose velocity is very closely defined, while its position is largely indefinite and yet confined within sufficiently close limits so that it can be regarded, on the relatively large scale of physical experimentation, as located at a moving geometrical point.

The fact that such a packet moves approximately as a unit suggests that whenever we have a stream of cathode rays or of β-particles of approximately uniform speed we can suppose each one to be accompanied by a finite train of waves of this type (if it were necessary to take account of interaction between the particles we should have to replace all of these separate packets by a single ψ containing the coördinates of all of the particles). A complete treatment of the situation would involve a quantum-mechanical analysis of the system producing the cathode-rays, but this is too complicated to handle, so we are compelled to proceed as in optics and infer the character of the wave-disturbance from what it is found to do. The length of the train will depend upon how closely the speed of the particle is determined by the experimental conditions. When the particle strikes an obstacle, like the electrons falling upon the crystal in the experiments of Davisson and Germer, its wave-train is scattered to the four winds and produces diffraction and interference patterns along the path of its flight. Since, however, the waves are only probability waves, this scattering implies no division of the electronic charge and energy; it is only that the position of the charge and energy becomes increasingly indefinite.

The monochromatic waves that can thus be supposed to accompany electrons of definite speed are usually extremely short. The general formula for their length is, from (14), $\lambda = h/\xi = h/mu = h/\sqrt{2mW}$, W being the kinetic energy of an electron; this gives $\lambda = 1.3 \times 10^{-7}\sqrt{1/V}$ where V is the energy in equivalent volts and λ is in centimeters. Thus even millivolt electrons have a wave-length of only about 4×10^{-6} cm. or the same as very short ultra-violet light and

the electrons of a hundred volts or so with which Davisson and Germer worked have wave-lengths of X-ray magnitude. The study of electron diffraction and interference thus requires a technique similar to that employed for X-rays and we can understand very readily why these phenomena were only discovered very recently (and under the stimulus of the new theory, as a matter of fact).

Probably a theory for photons, similar to the above for electrons, will eventually be worked into satisfactory shape, the field vectors being interpreted as, or replaced by, a probability amplitude for the photons. The explanation of an interference experiment like that described at the beginning of this article, whether performed with electrons or with photons, will then be somewhat as follows. Each wave-packet representing a particle passes through the apparatus according to the wave-equation, exhibiting such phenomena as refraction and interference. The wave amplitude at any point then determines the probability of finding each particle at one place or another, or of each particle's producing at each point some physical effect such as the blackening of a photographic plate. Thus the probability amplitude is propagated according to the wave-equation, but charge and energy, when they put in an appearance, do it in a single unitary act confined to a highly restricted region. Of course it will always be possible to believe, if one will, that electrons and photons are really particles of the old sort moving along definite paths and that the new theory with its essential element of probability merely expresses a hopeless limitation in our knowledge of their exact motion. So long, however, as it is impossible either to eliminate this element of probability, or to imagine exact laws according to which their true motion might be taking place, it seems likely that physicists generally will prefer the simpler view according to which electrons and photons are entities to which the concept of continuous motion can be applied only with an essential degree of vagueness.

Before leaving the subject of electron streams it should be mentioned that theorists have usually worked with a wave-function simpler even than (14), viz.:

$$Ce^{2\pi i h^{-1}(Wt-\xi x)}, \tag{15}$$

where C is an arbitrary constant. Such a function cannot, of course, constitute the amplitude for a particle, since the condition that $\int \psi\bar{\psi}dx = 1$ would require that $C = 0$; it represents only an ideal mathematical case, analogous to the exactly harmonic wave-train of optics, which is of necessity of infinite length. The general solution of (4), the wave-equation for a free particle in one dimension, can be written subject to certain restrictions in the form

$$\psi = \int_{-\infty}^{\infty} M(\xi) e^{2\pi i h^{-1}(Wt - \xi x)} d\xi,$$

where $M(\xi)$ is the amplitude for momentum as defined above, just as the wave function in optics can be written as a Fourier integral in which the square of the coefficient in the integrand represents the intensity at a given point in the spectrum. The chief application of the simple form (15), originated by Born, is to problems in which the velocity of a particle is pretty definite while its position is (on the molecular scale) indefinite, as in calculating the ionization produced by a stream of cathode particles in a gas. A somewhat different interpretation from the meaning usually given to ψ is employed in such cases, however; the quantity

$$S_x = \frac{h}{4\pi m i}\left(\psi \frac{\partial \bar{\psi}}{\partial x} - \bar{\psi}\frac{\partial \psi}{\partial x}\right) = C\bar{C}\xi/m \qquad (15a)$$

is taken to represent the stream density of the particles or the number of them that cross unit area per second in the direction of the x-axis. This interpretation is closely analogous to the use of the conception of the intensity of a beam of radiation in optics, and the vector S is the exact analogue of Poynting's vector. The interpretation is justified by the fact that in the case of one particle the integral of the normal component of S over any closed surface can be shown to equal the rate of decrease in the integral of $\psi\bar{\psi}$ throughout the enclosed volume; if we are dealing with a large number N of particles moving independently, they can all be represented by the same wave-packet with ψ multiplied by $\sqrt{N}$, so that $\int \psi\bar{\psi}d\tau = N$, and the integral of the normal component of the vector S over any closed surface will then equal the de-

crease in the probable number of particles to be found within the surface and so may be regarded as the rate at which they are crossing it in an outward direction.

The method of handling ionization problems is therefore as follows.[10] One assumes a wave-train of the type (15), representing a stream of electrons moving in a definite direction with kinetic energy W. When these fall upon an atom the waves are scattered in all directions and this scattering is calculated by solving the wave-equation for the combined system of electron and atom. The scattered waves moving out in any chosen direction from the atom are then found to consist of one or more terms like that in (15), and the vector S for each term, divided by its value for the incident wave-train, is taken to represent the fraction of the electrons falling upon unit area in the incident stream which are scattered per unit area in the chosen direction and with energy corresponding to that term.

Experiments upon the simple elements of Quantum Mechanics, which we have been considering so far, are, it must be confessed, very scarce as yet because of their difficulty. We must conclude, therefore, with a brief consideration from the mechanical standpoint of the fields in which the new theory has scored its greatest triumphs. This is, first of all, the field of atomic energy levels, and secondarily that of radiation intensities.

ATOMIC ENERGY LEVELS AND QUANTUM STATES.

In approaching the subject of atomic energy from the mechanical side, we have first to raise the question, what is to be meant by energy as an observational quantity. In classical theory the energy was given as a function of the positions and velocities of the particles, but if we employ this definition in the new theory a precise measurement of energy becomes impossible because positions and velocity cannot be measured with precision simultaneously. Energy thus exhibits quantum-incompatibility with position and velocity (or momentum) taken together, and we are thrown back upon a consideration of direct experimental methods for its measurement. Now it must be admitted that no such

[10] M. Born, *Nachrichten der Ges. der Wiss. zu Göttingen*, 146, 1926.

method has ever actually been used, but an excellent clue is furnished by the Stern-Gerlach experiment, in which the deflection of an atom moving through a non-homogeneous magnetic field is made to measure the magnetic moment of the atom and so also its angular momentum, which is a mechanical quantity in the same general quantum status as the energy. It seems reasonable to assume that an atom might similarly be subjected to forces proportional to its energy, and its resulting displacement, or some indirect effect of this, might then be taken as measuring this magnitude; for instance, if gravitational mass and energy really are proportional to each other, we might "weigh" the atom.

The analysis of such an experiment, in which an atom is assumed to move under a force proportional to its internal energy, shows [11] that the result is a splitting up of the probability packet. We can write

$$\psi' = \psi_c(x, y, z, t)\psi(q, t) \tag{16}$$

where x, y, and z are the coördinates of the center of mass of the atom and q stands for all of its internal coördinates; for convenience we have written ψ' where ψ would ordinarily stand so as to reserve the symbol ψ for the internal factor alone. $\psi(q, t)$ is a solution of Schrödinger's equation for the atom relative to its center of mass, viz.:

$$\frac{h}{2\pi i}\frac{\partial \psi}{\partial t} = H\left(-\frac{h}{2\pi i}\frac{\partial}{\partial q}, q\right)\psi, \tag{17}$$

where H is the Hamiltonian with the part referring to the center of mass omitted. Now the general solution of this last equation can be written

$$\psi = \sum_n c_n e^{2\pi i h^{-1} W_n t} u_n(q, W_n) + \int \gamma(W) e^{2\pi i h^{-1} W t} u(q, W) dW \tag{18}$$

where $u_n(q, W_n)$ or $u(q, W)$ is one of those solutions of the Schrödinger equation for the atom,

$$H\left(-\frac{h}{2\pi i}\frac{\partial}{\partial q}, q\right)u = Wu, \tag{19}$$

[11] E. H. Kennard, *Phys. Rev.*, 31, 876, 1926.

which is single-valued and finite everywhere over the physically admissible range for the q's; only values of W for which this special type of solution exists ("Eigenwerte" or characteristic numbers) and the solutions corresponding to them (characteristic functions) are to be employed, and the sum in (18) extends over such of these values of W as form a "discrete spectrum" while the integral extends over any continuous ranges of such values of W forming a "continuous spectrum." The u's have the property that $\int u_n \bar{u}_m dq = 0$ if $n \neq m$, and the arbitrary constant which they contain as a factor is to be so chosen that $\int u_n \bar{u}_n dq = 1$ (with a suitable modification for the continuous spectrum). The coefficients c_n or $\gamma(W)$ are arbitrary; if ψ is given at a certain time t_0, these constants can be adjusted to make ψ reduce to the given function of the q's at time t_0. Now what would happen in the measurement of the energy is that the various components of the packet, as represented by the separate terms in (18) corresponding to different values of W_n, or by the parts of the integral corresponding to different elements dW, would undergo different displacements in the x, y, z space of the center of mass, with the result that the whole packet would be spread out into a spectrum in this space according to the values of W. The probability of finding the system in the place belonging to a given W_n or dW is easily shown to be $C_n \bar{C}_n$ or $\gamma \bar{\gamma} dW$, respectively. Now in a special case it might happen that the whole packet came to a single spot in the spectrum; in such a case we should naturally regard the corresponding value of W as an observed value of the energy. Otherwise we should have a probability curve for the energy; by analogy with the meaning of ψ and $M(\xi)$ we can call c_n or $\gamma(W)$ the probability amplitude for energy.

We have thus obtained both a realization of atomic energy as an observable physical magnitude and a method of calculating its possible values from the Hamiltonian function. The possible values that the energy can have are simply the characteristic values of W for the ordinary Schrödinger equation, (19), in which H denotes the Hamiltonian for the atom with its mass-motion omitted. This is the famous method of calculating energy levels which has

succeeded so wonderfully and is too well known to require further comment.

Any one term or part of the integral in (18), with the coefficient omitted so as to preserve the relation $\int \psi\bar{\psi}dq = 1$, such as

$$\psi_n = e^{2\pi i h^{-1} W_n t} u_n(q, W_n) \tag{20}$$

or

$$\psi \Delta W = \int_{\Delta W} e^{2\pi i h^{-1} W t} u(q, W) dW,$$

thus characterizes the atom in a state in which its energy is definitely equal to a certain value W_n, or lies within the range ΔW. Such a ψ_n can be regarded as a "standing wave" of probability amplitude, albeit one of a novel sort, since the amplitude does not pulsate in magnitude but merely revolves steadily in the complex plane. The corresponding probability function for the positions of the electrons,

$$\psi\bar{\psi} = u_n(q, W_n)\bar{u}_n(q, W_n),$$

is entirely independent of time. We have thus in this case a wholly static atom. Perhaps such a result may seem paradoxical in view of the fact that under many circumstances the electrons could certainly be followed in orbital motion around an attracting center like the nucleus. The solution of the paradox lies in the fact that a single characteristic solution such as that in eq. (20) refers to an atom that has been carefully selected so as to have a perfectly definite value of the energy, and any such process of selection requires a certain lapse of time; for instance, to select an atom in the normal state, we might simply wait until the atom had almost certainly jumped spontaneously into that state. During this lapse of time any definiteness as to the electronic motions within the atom which may have existed initially gradually disappears and by the time the energy has become definitely equal to W_n all trace of definite motion is gone.

To give a simple example, for the normal state of the hydrogen atom eq. (20) takes the form:

$$\psi_1 = (\pi a^3)^{-1/2} e^{2\pi i h^{-1} W 1 t - r/a}, \qquad \psi\bar{\psi} = \frac{1}{\pi a^3} e^{-2r/a},$$

where $a = h^2/(4\pi^2 me^2)$ (a thus equals the radius of the smallest Bohr orbit), $W_1 = -2\pi^2 me^4/h^2$, and r = distance from the nucleus to the point at which ψ_1 has the value given (e in the equation stands as usual, of course, for the Napierian base!). The probability of finding the electron in a given element of volume is thus in this case greatest at the nucleus (but we must remember that the theory here given is non-relativistic and treats both nucleus and electron as a point charge, so that the true value of ψ may well be quite different just at the nucleus). Yet we should not really be likely to find the electron so close in, for the probability of finding it at a distance away between r and $r + dr$ is $4\pi r^2 \psi\bar{\psi} dr$, which has a maximum for $r = a$. The radius of the Bohr orbit thus represents in this case the most probable distance of the electron from the nucleus. The electron is, however, more likely to be found still farther away rather than nearer, it has a chance of one fifth of being found more than twice as far out, five per cent. for three times as far, and so on. In the higher states of excitation the probability distribution broadens out still more just as did the old Bohr orbits.

The picture of the atom which we thus obtain from the new Quantum Mechanics is decidedly different from the one furnished by the old quantum theory. It has something of resemblance to the static atom imagined at one time by various physicists although the indefiniteness in the location of the electrons makes the resemblance rather a superficial one. Probably the new picture will seem somewhat repellent to many thinkers, who find a more concrete model a great help in fixing their ideas; and it may be that the future will lead us again to a conception of the older sort, although it seems unlikely to be the one of Bohr, at any rate.

INTERACTION BETWEEN ATOMS AND RADIATION.

For the sake of completeness we shall now consider briefly the problem of the interaction of atoms with radiation, which embraces all the phenomena of refraction, scattering, absorption and emission. The correct method of handling such problems seems to have been discovered, but its true interpretation probably remains to be found; speaking more generally, the complete quantum theory of the electro-

magnetic field remains to be elaborated. There seems to be no reason for abandoning the hope that when this has been found the theory of radiation will turn out to be as simple in conception as is the new theory of mechanical systems, the abstruse processes of the generalized matrix theory being no longer needed. What we have at present is rather like snatches of a rocky coast seen through breaks in the fog. For simplicity we shall confine the discussion to atoms with a single electron. In one of his original papers Schrödinger takes account of the effect of radiation upon an atom, of a frequency lying outside of its absorption band, by treating the radiation simply as a force field and introducing appropriate terms into the Hamiltonian function. It then turns out that the simplest solution of the wave-equation, (17), is not a single term ψ_n but a series somewhat like (18); if we suppose for simplicity that the characteristic functions u are real, the corresponding probability function for an atom in the nth quantum state can be written, to the first order in the amplitude b of the radiation:

$$\psi\bar{\psi} = u_n^2 + 2bu_n \cos 2\pi\nu t \left\{ \sum_s \frac{\beta_{ns} u_s}{(W_n - W_s)^2 - h^2\nu^2} + \int \frac{\beta_n(W) u(q, W) dW}{(W_n - W)^2 - h^2\nu^2} \right\}.$$

the β's denoting constants characteristic of the atom and ν the frequency of the radiation. Now Schrödinger interpreted $\psi\bar{\psi}$ as actual density of electricity, normalizing his ψ so that $\int \psi\bar{\psi} dq = e$ (electronic charge) instead of unity. The harmonic term in the expression just given for $\psi\bar{\psi}$ would then mean a vibrating distribution of charge, and this vibrating electricity he assumed to radiate according to the classical laws. This assumption gives, as a matter of fact, so far as tests have yet been made, a satisfactory account of the phenomena of dispersion and scattering, so that the theory must be at least mathematically correct.

Such an interpretation of $\psi\bar{\psi}$ is obviously at variance with the standpoint taken in this paper and it does not seem to have been accepted generally by theoretical physicists except as a means of visualizing a successful rule for calculat-

ing the optical behavior of atoms. On the other hand, we have at present nothing to replace it.

A still more baffling case from the standpoint of interpretation is presented by the theory of "spontaneous emission." When an atom is definitely in a certain excited state, ψ is supposed to have a form like that in eq. (20) and $\psi\bar{\psi}$ is constant in time; according to Schrödinger the electricity should therefore be at rest and there should be no emission. Actually, however, we know that emission may occur, accompanied by a quantum jump; and it is believed, with considerable support from experimental checks, that the following method of calculation leads to correct values of the emission, or jump, probabilities for a one-electron atom. Let the amplitude for the actual state of the atom be ψ_n as given in (20), normalized as before, so that $\int \psi_n \bar{\psi}_n dq = e$, and let ψ_m be the corresponding amplitude for any other state of lower energy, W_m. Then we find that (the u's being real)

$$\psi_n \bar{\psi}_m + \bar{\psi}_n \psi_m = 2u_n u_m \cos 2\pi \frac{W_n - W_m}{h} \cdot t. \quad (26)$$

Now assume that there is a fluctuating distribution of electricity with density $\rho = (\psi_n \bar{\psi}_m + \bar{\psi}_n \psi_m)$ and calculate the classical radiation from it, divide the rate of emission of energy in this classical radiation by $h\nu_{nm}$ where $\nu_{nm} = (W_n - W_m)/h$; and the result is the probability per second that a photon of frequency ν_{nm} will be emitted and that the atom will simultaneously jump into the state m.

Such a method of calculating emission probabilities obviously has no footing at all within the frame of the Heisenberg-Schrödinger mechanics as we have sketched it. We are required to combine two amplitudes each of which in our theory represents the atom only when it has been specially selected so as to be in a definite quantum state, and it cannot possibly be selected so as to be definitely in each of two states at once. Nor does this method of treating spontaneous emission agree with Schrödinger's treatment of scattering and dispersion, for there we worked with a single ψ, consisting of a series of terms added together and normalized so that $\int \psi \bar{\psi} dq = e$, whereas here we are required to take two ψ's belonging to different states, each one normalized separately

by the same rule, and then to combine them without adding them!

The only possible conclusion to be drawn from this situation seems to be that there is a link still missing in the theory. We can see in a general way what sort of element remains to be supplied. For one thing, it will probably be necessary to work with a probability amplitude for the photon, and to deduce the Schrödinger rule for calculating radiation from a law connecting the time rate of change of the amplitude for the photon with the atomic ψ; this law must, however, be rather different from the Schrödinger wave-equation for atoms because photons can (apparently, at least) be created and destroyed. Again it may be that the series that we obtain for ψ when the atom is under the influence of radiation should be interpreted to mean that the state of the atom under these conditions is not fully defined, there being only a certain probability for each of the quantum states. Probably the most radical innovation, however, will be the one to take account of the spontaneous quantum jumps.

Several physicists have attempted to account for spontaneous changes in atomic energy (ignoring for the moment the accompanying emission of energy) by introducing into the wave-equation a term corresponding to radiation damping in classical theory, the effect of which would be to make the coefficients c and γ in (18) change continuously with time until eventually the whole expression becomes reduced to the single term referring to the normal state. Such an attempt seems, however, foredoomed to failure because there seems to be something essentially irregular about the transitions between states.

Consider, for instance, an atom in a metastable state of energy W_2, from which there is a small but finite transition-probability to the normal state of energy W_1, corresponding, let us say, to a mean life of an hour in the higher state. The transition, when it occurs, will be accompanied by the emission of a photon, and by observing this we can fix the time of the jump. Then we have this situation: before the photon comes out, the atom must surely be represented by a single term like (20), consisting of the function proper to the metastable state, while after the jump ψ must have the form that

represents the normal state. Thus we must suppose that, at a time which is itself controlled only by a probability law, ψ begins to change and quickly passes over into a different form, which it thereafter retains.

Spontaneous "jumps," therefore, are real physical occurrences and are not merely an illusion. To give an account of them it would seem as if a statistical element must be introduced into the very wave-equation itself.

ELECTRICAL AND MAGNETIC PROPERTIES OF THE ATOM.

The interpretation of $e\psi\bar{\psi}$ (with ψ normalized to unity) as density of electricity has applications even beyond the theory of radiation. The electrical field calculated from such a distribution, using the value of ψ proper to the quantum state in which the atom is assumed to be, can be shown to give a good approximation to the effect of the atom upon rather distant bodies, in so far as that effect arises in the correct quantum-mechanical theory from terms in the Hamiltonian representing the mutual potential energy associated with electrostatic forces between the constituents of the atom and of the distant body. That is to say, in order to calculate the interaction between two atoms close together one must employ the special methods of quantum mechanics, whereas if the atoms are rather far apart as compared with their diameters either one of them may be replaced approximately, as regards its electrostatic effect upon the other, by a distribution of charge $e\psi\bar{\psi}$ (with a suitable generalization for atoms containing more than one electron). The distant magnetic effects also can be obtained, it is generally believed, in a similar manner. In cases in which the density $e\psi\bar{\psi}$ varies with time, there must be a set of associated currents to move the charges about, and it is easily seen that the density of these currents is eS where S is the vector defined by the first equation in (15a) above. Now in many cases S is not zero even when $e\psi\bar{\psi}$ is constant in time; and in such cases the magnetic moment due to these currents is believed to represent the magnetic moment of the atom. In this way the diamagnetic and paramagnetic properties of bodies are treated on the new theory; [12] but it has not been possible as yet to secure very extensive comparison of results with experiment.

[12] Cf., e.g., J. H. Van Vleck, *Phys. Rev.*, 30, 31, 1927.

Such a use of $e\psi\bar{\psi}$ and S is, of course, entirely in harmony with the probability interpretation of $\psi\bar{\psi}$; the Schrödinger distributions of charges and currents are employed simply as an auxiliary idea to enable us to visualize rules of calculation which find their ultimate theoretical justification in an application of the more abstract fundamental principles of the theory. It seems rather probable that a similar view will eventually come to be taken of the role of the vibrating charge in connection with radiation phenomena.

CONCLUSION.

The new Quantum Mechanics, as we have seen, cannot yet be considered as a coherent and completed theory; its condition at the present time has been aptly compared to that of Maxwell's theory soon after its publication, only that the confusion in Quantum Mechanics is much greater because it has been the product of several minds. The parts of the theory which rest upon the firmest experimental basis are probably the part that deals with atomic and molecular energies and the treatment of electron and photon beams by means of a probability amplitude propagated according to a wave-equation. According to the theory, protons also should exhibit wave-properties in the same way as do electrons, but as yet experimental proof of this is lacking. Perhaps we must still regard it as possible that protons are fundamentally different from electrons in their mechanical properties; an experimental test of this point is urgently needed. The new Fermi-Dirac statistics finds a basis in Quantum Mechanics, but the only application of it which has met with convincing success is again one to electrons, namely, the new form given to the theory of metallic conduction by Sommerfeld.

On the whole, however, it seems certain that the Quantum Mechanics of Schrödinger and Heisenberg will form an important part of the completed theory that is to come. Whether the student of the future will also be required to learn the matrix form of the theory seems more open to question, although this form is still in favor in many quarters. Perhaps the decision on this point will come when the proper treatment of electromagnetics shall have been discovered.

SOURCES OF VISIBLE AND INFRA-RED RADIATIONS FOR DEEP THERAPY.

BY

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PROGRESS in the scientific development of radiation therapy has been seriously retarded by insufficient consideration and understanding of physical science on the part of investigators and practitioners. Radiant energy or radiations of all kinds—infra-red, visible, ultra-violet, Roentgen, radium, and others—are tools which can be adequately described only by physical units such as wave-length and intensity. The chief factor in this description is wave-length or its reciprocal —frequency. In the scientific literature of researches and practices, the spectral aspect has not been adequately treated and ovbiously in many cases has not been understood. For example, ultra-violet radiation to many persons is merely a specific thing when, as a matter of fact, it is the name applied to a great range of the spectrum including many wave-lengths, or frequencies. Its properties vary enormously with wave-length and, therefore, researches which do not definitely specify wave-lengths are of little value. This is equally true of radiations of other spectral ranges. In previous publications the writer has presented many physical data pertaining to ultra-violet [1] and visible [2] radiation. These are only a part of an adequate literature to which the medical practitioners and investigators in physiology may turn for an understanding of the physical aspects of radiation. In the present paper some of the fundamental physics of infra-red are presented with the hope that the present confusion may be partially clarified at least.

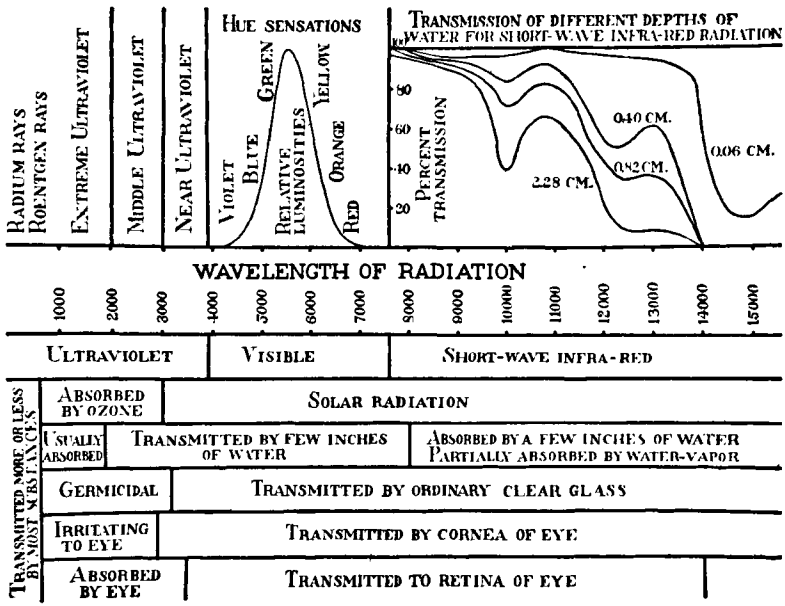
For those who are not familiar with the spectrum of radiation, Fig. 1 is inserted. The spectral range includes all wave-

[1] M. Luckiesh, "Ultra-violet Radiation," D. Van Nostrand Company, 1922,

[2] M. Luckiesh, "Color and its Applications," D. Van Nostrand Company. 1915 and 1921.

lengths from zero, through the ultra-violet and visible regions and the near infra-red. Certain properties of radiation and characteristics of the eye and of various media are also presented. In this paper wave-lengths are expressed in Ängstrom units. For example, λ7600 is the wave-length approximating the boundary line between the visible and near infra-red. Inasmuch as water plays a major rôle in the present discussion, spectral transmission curves of four different thicknesses are presented in Fig. 1 and also in other illustrations. Coblentz [3]

FIG. 1.



A diagrammatic view of various data pertaining to radiant energy. The wave-lengths are in Ängstrom units. The spectral range presented consists of the entire ultra-violet and visible radiations and the short-wave or "near" infra-red. This diagram is taken from "Light and Work," by M. Luckiesh, D. Van Nostrand Company, 1924.

has made an extensive study of the spectral transmission of water and of other media, which may interest the reader who wishes to go deeply into the subject.

In this paper the term "deep therapy" applies only to the use of visible and infra-red radiations for heating at a depth.

[3] W. W. Coblentz, "Radiometric Investigation of Water of Crystallization, Light Filters, and Standard Absorption Bands," Bul. Bureau Standards, 7, 1911, p. 619.

Deep therapy is now practiced with a large assortment of devices emitting radiations of vastly different wave-lengths covering a wide range of the spectrum. All of them cannot be most suitable for heating at a depth in the human body. Among these are many incandescent radiators ranging from relatively low-temperature carbon filaments to high-temperature special tungsten filaments. These are enclosed in bulbs of various colors or spectral-transmission characteristics. Among these are red, yellow, blue, violet and purple bulbs. Besides these there are various non-luminous radiators in use —devices heated electrically to temperatures lower than those of incandescence. It is obvious to anyone familiar with the physical aspects that most of these devices are made, recommended, and used without much knowledge of the wave-lengths of radiation involved and in some cases without even a vague conception of the physics of the subject.

Here we often find a good example of the correctness of the old adage to the effect that a little knowledge is dangerous. Vaguely it is known by many that radiation consists of "vibrations." Therefore, mysterious and even metaphysical ideas are entertained in regard to the effect of these vibrations upon human ailments. Even these vague ideas are not monopolized by the questionable practitioners but, owing to lack of knowledge, are entertained by some medical men of unquestionable character.

The one known fact of therapy is that of heating at a depth. No other fundamental cause of beneficial results is known. No direct influence of wave-length upon curative value has been found as in the case of ultra-violet radiation. Therefore, at least the physics of the subject must deal with that of the absorption of radiation by the body. Knowledge of this aspect clarifies much of the confusion now existing in regard to infra-red therapy.

Heating can be done by radiation, convection, and conduction. When a hot towel is placed upon the skin, heating at a depth is accomplished almost solely by conduction. The skin in contact with the towel may be the hottest part of the body. Some heat from the towel is lost by radiation and conduction outward. If air comes into contact with the towel, it is heated and convection currents of air carry this heat

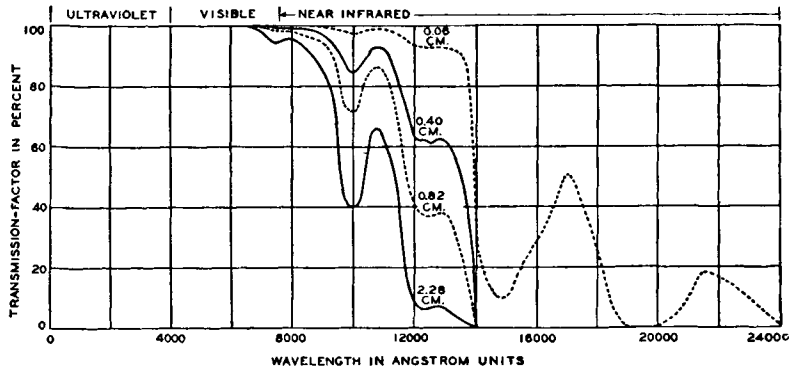
away. The liquids of the body carry some heat away by their movement, but this is not true convection and at best is perhaps a small part of the effect of the hot application.

If an electric filament lamp is held close to the body, some radiant energey enters the body. On being absorbed it is converted into heat and heating at a depth is achieved without direct contact with the hot bulb as in the case of a hot towel. Not only is this more comfortable for the patient, but greater heating at a depth can be achieved by the absorption of the radiant energy than by direct conduction. In a consideration of the physics of heating by absorption of radiant energy or radiation, it is only necessary to know the spectral transmission of the body. This is not known in actual detail but enough is known to bring order out of the present chaos. In fact, it is practicable to consider the spectral transmission of the human body to be similar to that of water, of which it chiefly consists. The present state of the art of infra-red therapy and the uncertainty of measuring the results of it do not require any greater accuracy of assumption than the foregoing. However, we may be guided further by the fact that blood transmits infra-red somewhat as red-tinted water does. The conclusions drawn in this paper would not be altered in general, and only relatively slightly in detail, if exact spectral transmission-factors of the elements of the human body were available. These conclusions will show how inconsistent the claims for the various radiators are and that many must be ruled out of consideration if the greatest heating at a depth is desired and if the greatest efficiency is a consideration.

That the human body is somewhat transparent to visible radiation is readily demonstrated by holding the hand over a light-source such as a flashlight. In the dark the hand is seen to transmit considerable light or visible radiation. It is seen that the color of the light is only a tint of red. This indicates that much of the radiation throughout the spectrum—including violet, blue, green, and yellow rays—passes through the hand. Therefore, radiation of all wave-lengths in the visible region may be considered to be more or less penetrating and useful in deep therapy. If we replaced the eyes with an instrument sensitive to the near infra-red, some of these invisible radiations would be found to pass through the hand.

Years ago in connection with a study of infra-red and the eye,[4] certain computations were made which are applicable to the present problem. In addition to these, certain modern sources have been considered, so that a view may be obtained of present radiators of a great range in temperature. Water transmits radiant energy of all wave-lengths throughout the visible and only partially in the near infra-red region. Pure water is transparent to ultra-violet energy throughout most of its spectral extent but inasmuch as sources used for deep therapy do not emit much ultra-violet, this region is not considered herewith for heating at a depth. Besides, the salts and colored compounds in the body absorb most or all of it.

FIG. 2.



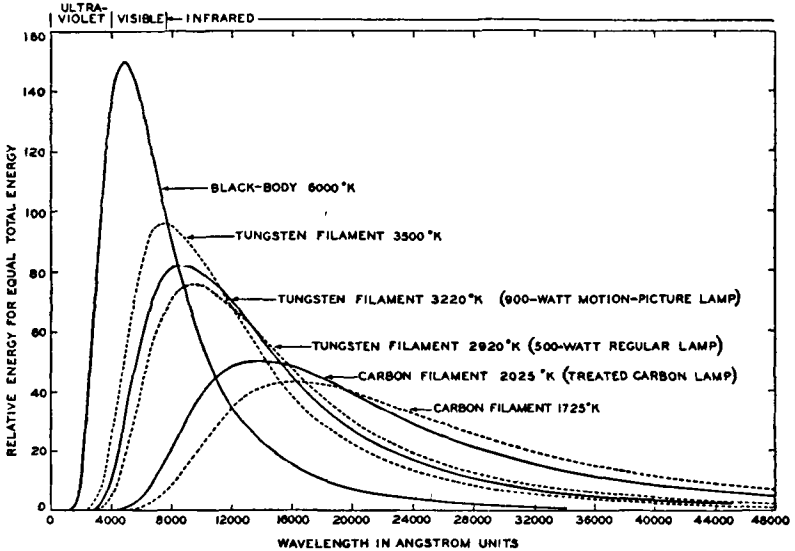
The transmission-factors of radiant energy of various wave-lengths—throughout the visible and near infra-red regions—are presented for water of four different thicknesses.

In Fig. 2 the transmission factors of four different thicknesses of water are given for radiant energy of various wave-lengths throughout the visible and near infra-red regions. It will be noted that a thickness of 2.28 cm. of water (nearly an inch) is (1) transparent to radiations of wave-lengths corresponding to the visible spectral region, (2) only partially transparent to radiations in the near infra-red, and (3) is totally opaque to radiations of longer wave-lengths than 14,000 Ängstrom units. The visible spectrum lies between approximately λ4000 (wave-length 4000 Ängstrom units) and λ7600.

[4] M. Luckiesh, "Infra-red Radiant Energy and the Eye," *Amer. Jour. Physiol.*, 50, December, 1919.

The ultra-violet extends from λ4000 toward shorter wave-lengths. The infra-red extends from λ7600 toward longer wave-lengths. Although a layer of water 2.28 cm. in thickness is completely opaque to infra-red energy greater than λ14,000, a layer 0.06 cm. in thickness transmits partially out to λ25,000 but is opaque to wave-lengths longer than this until λ80,000 is reached. This latter region is discussed later.

FIG. 3.



The spectral distributions of energy from seven incandescent solids of different absolute temperatures (Kelvin scale). The total radiant energy emitted is the same for all these radiators. As the temperature increases the amount of energy emitted in the visible and near infra-red regions increases.

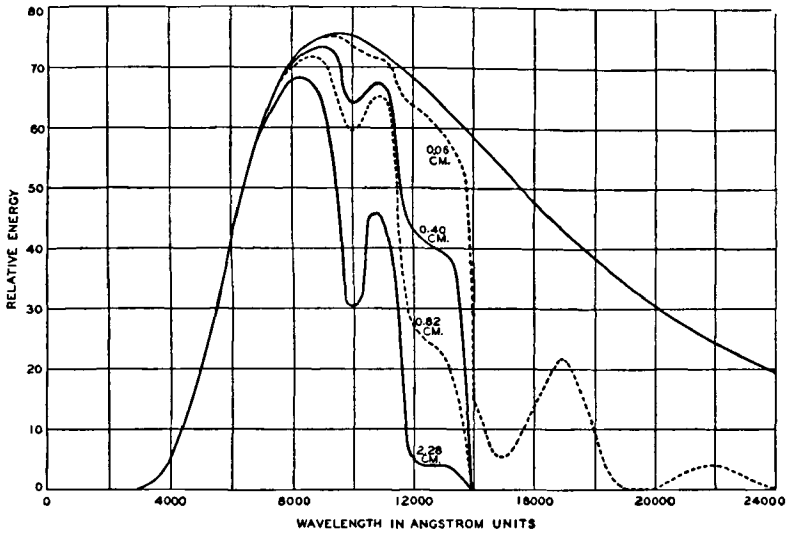
Such data as the foregoing prepare us to consider the relative value of radiators of various temperatures. In Fig. 3 are shown the spectral distributions of energy as emitted by seven incandescent solids. The black-body is a theoretical source emitting radiant energy in accordance with fundamental physical laws. Incandescent carbon and tungsten do this approximately. It is seen that as the temperature increases a relatively greater percentage of radiation is emitted in the visible and near infra-red. The areas contained within the curves and the common base-line represent relative amounts of total energy emitted by the sources at the temperatures indi-

cated. The ratio of the area between any two wave-length vertical lines, to the total area, is the percentage of the total energy in any case emitted within this range of wave-lengths. It is seen that, as the temperature increases. a greater percentage of energy is emitted by an incandescent solid between λ4000 and λ10,000 (the region of almost complete transparency of an inch of water). Thus we are ready to eliminate the low-temperature incandescent solids in favor of the high-temperature ones.

Let us pause to emphasize this. In obtaining heating at a depth, it is desirable to have as little absorption as possible in the first or outer layer of the body. This leaves more radiant energy to penetrate deeper, where by absorption it is converted into heat, thereby heating this deeper layer. From Figs. 2 and 3 it is seen that the maximum of the curve for a tungsten filament at 3500° K. is near λ7600 or the border line between the visible and near infra-red. Therefore, much more of the energy of the wave-lengths to which water is transparent, or fairly so, is emitted by this source than by incandescent solids of much lower temperature. Therefore, of all the actual sources considered in Fig. 3, the tungsten-filament lamps appear to be best suited for deep therapy. Any modification by means of colored bulbs decreases their efficacy by reducing the amount of radiant energy available. Why use a blue bulb to eliminate much of the yellow and red radiation which should be used in heating at a depth just as the blue radiations are? Still many blue bulbs are in use. Why use a red bulb to eliminate blue and green radiations when these are likewise valuable in penetrating the body? Still many of these are advocated.

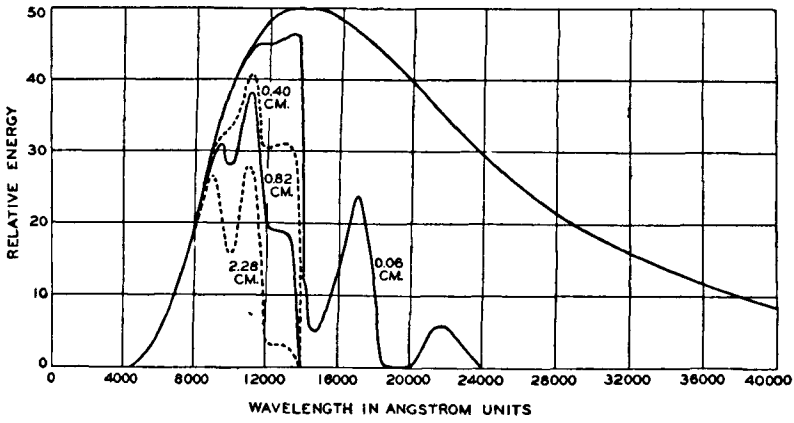
Figs. 4 and 5 make this point clearer and at the same time indicate that an ordinary high-temperature tungsten-filament lamp is more desirable for heating at a depth in the human body than the old 4 watt-per-candle carbon-filament lamp of much lower filament-temperature. In Fig. 4 the major portion of the spectral energy distribution of the present 500 watt tungsten-filament lamp is presented. The relative amount of energy of each wave-length transmitted by 2.28 cm. of water is shown in the lowest curve. The ratio of the area under this lowest irregular curve to the total area under the outer regular

FIG. 4.



The spectral distribution of energy emitted by an ordinary 500-watt tungsten lamp is shown in the upper curve. The amounts of this energy of various wave-lengths which pass through four different layers of water respectively are shown by the four irregular curves. Inasmuch as the areas represent amounts of energy, the ratio of the area under the irregular curve (marked 2.28 cm.) to the total area under the uppermost regular curve indicates the percentage of the total energy which remains unabsorbed after a depth of 2.28 cm. of water is reached.

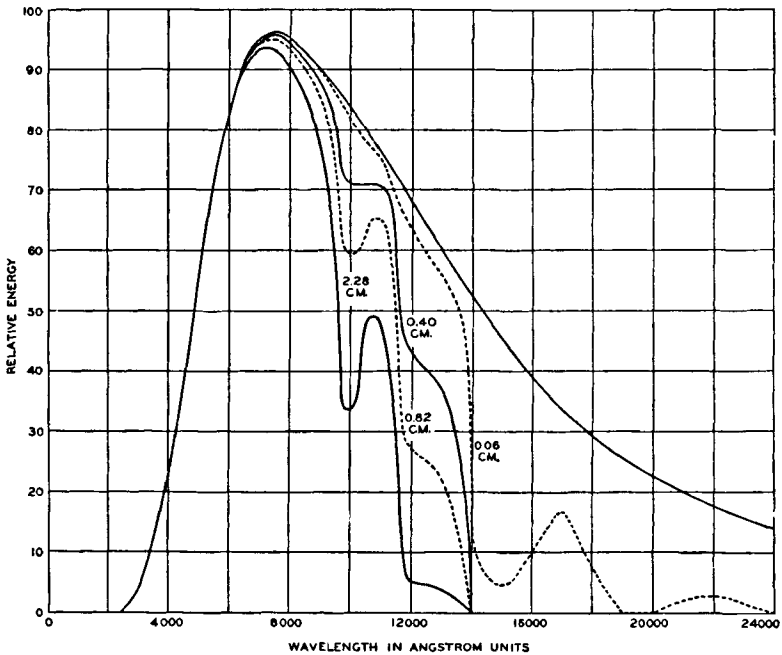
FIG. 5.



The uppermost regular curve is that of spectral distribution of energy emitted by the old 4 watt-per-candle carbon-filament lamp. The irregular curves indicate the relative amounts of energy transmitted respectively by the four layers of water.

curve is the percentage of total energy transmitted by the 2.28 cm. of water. The relative amounts of energy of various wave-lengths transmitted by the other three thicknesses of water are also shown and the same process applies to them.

FIG. 6.



Showing the spectral distribution of energy emitted by solid tungsten at 3500° K. (near its melting point) and the relative amounts of energy of various wave-lengths transmitted by the four layers of water respectively.

In Fig. 5 the same physical basis is given for the old carbon lamp—still unjustifiedly used in deep therapy. The ratio of the area under the irregular curve to the total area under the outside regular curve (spectral energy distribution for a 4 watt-per-candle carbon filament) is seen to be much smaller than in the case of the 500-watt tungsten-filament lamp. This means that in the case of this tungsten-filament lamp more energy remains unabsorbed after 2.28 cm. of water have been penetrated than in the case of the carbon lamp. Therefore, the former is the better radiator for heating at a depth.

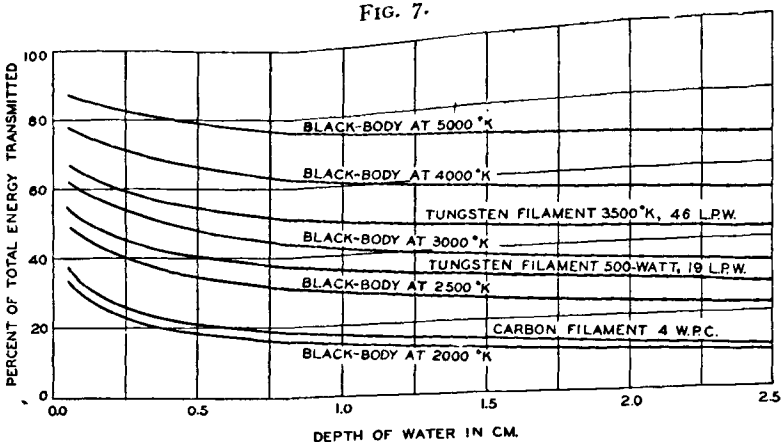
The foregoing point is further emphasized by Fig. 6, in

which the source of radiation is tungsten at 3500° K.—near the melting point. Here it is seen that the area under the lowest irregular curve (which shows the spectral transmission of 2.28 cm. of water for the radiation from this source) is a still larger percentage of the total energy emitted by the source than in the case of Fig. 4. This indicates that for deep therapy the best available source of radiant energy is a solid of highest practicable temperature up to the neighborhood of 5000° K. Of the electric filament lamps, the high-efficiency tungsten lamps with colorless glass bulbs are the best for deep therapy in the light of present knowledge. Furthermore, there is nothing unknown which leads to the belief that this will have to be altered when all details of heating the depths of the body are known.

In Table I the amounts of energy absorbed in layers of water of four thicknesses are presented for theoretical black-body radiators of various temperatures and for four practical sources. The percentages of total radiant energy absorbed in four layers of water respectively are given in the first four columns of figures. In the last column are the percentages of total energy remaining unabsorbed from various sources of radiation after passing through 2.28 cm. (0.9 inch) of water. It is seen that as the temperature of the source increases, the percentage of unabsorbed energy increases. Inasmuch as the source of lowest temperature is incandescent, non-luminous radiators are not under consideration in this comparison. Furthermore, it is seen that, of the practicable incandescent sources available, the regular high-efficiency gas-filled tungsten lamps are best. If any bulb besides a colorless one were used, the value of the lamp for deep therapy would be reduced whether the bulb were blue, violet, purple, red or any other color.

Inasmuch as the body consists largely of water, these values and conclusions are indicative of the actual and tenable ones. Any salts in solution cannot appreciably alter the general conclusions or the order of rank of these sources. Finally, the general reddish tint of the bodily tissue cannot possibly reverse the order of rank.

FIG. 7.



Showing the relation between the transmission-factor, or total energy transmitted, and the thickness of water for the nine sources considered in Table I.

TABLE I.

Percentage of Radiant Energy, from Various Sources, Absorbed in Various Layers of Water.

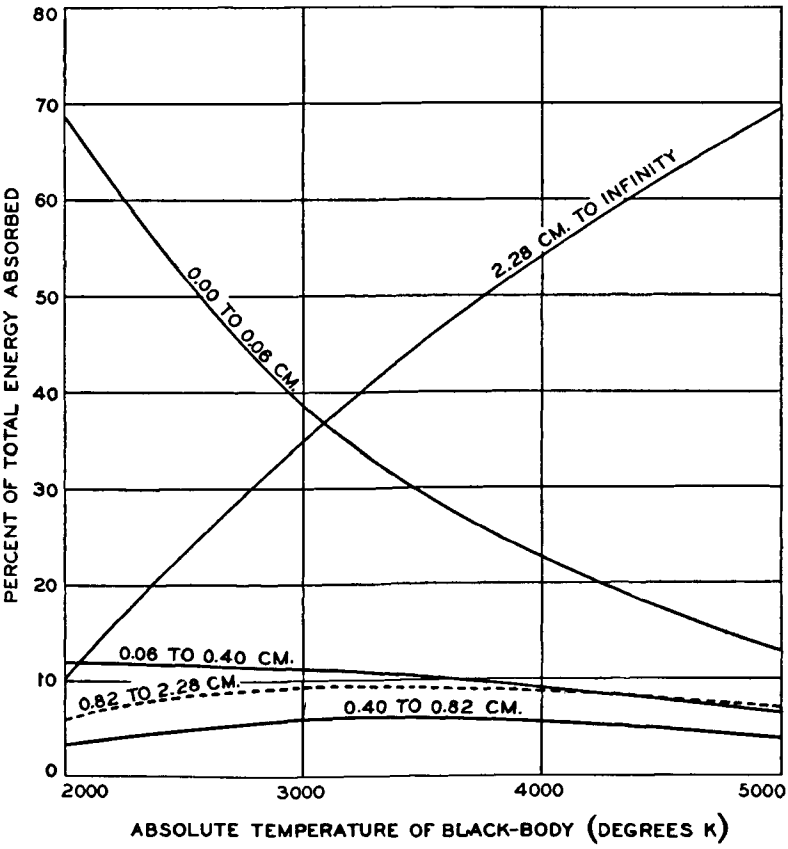
Source.	Percentage of Total Energy Absorbed in Water of Depth.				Remaining Unabsorbed at Depth of 2.28 cm.
	0.06 cm.	0.40 cm.	0.82 cm.	2.28 cm.	
Black-body 2000° K.	68.8	80.6	83.8	89.7	10.3
" " 2500° K.	51.7	63.3	68.3	76.7	23.3
" " 3000° K.	38.5	49.8	55.7	65.1	34.9
" " 4000° K.	22.8	31.7	37.2	45.9	54.1
" " 5000° K.	13.0	19.6	23.4	30.4	69.6
(*a*) Carbon filament, 2090° K.	64.1	77.3	81.0	87.9	12.1
(*b*) Tungsten filament, 2500° K.	50.4	64.5	70.5	80.0	20.0
(*c*) " " , 2920° K.	46.5	58.0	62.2	70.5	29.5
(*d*) " " , 3500° K.	33.6	43.6	48.4	56.4	43.6

(*a*) Old treated carbon-filament; 4 watts per candle.
(*b*) Approximating regular vacuum tungsten lamps; 9 lumens per watt.
(*c*) Actually a 500 watt-gas-filled tungsten lamp; 19 lumens per watt.
(*d*) Near the melting point of tungsten; 46 lumens per watt.

Fig. 7 presents a graphic view of the data found in Table I. This further emphasizes the superiority of high-temperature incandescent solids over low-temperature ones for penetrating water, or on the reasonable assumption upon which this discussion is based, for deep therapy. Here the horizontal scale

is thickness of water in centimeters and the vertical scale is in percentage of energy transmitted. For any given source the energy transmitted decreases with thickness—rapidly at first, then more slowly. It is seen that as the temperature

FIG. 8.



The percentage of total energy absorbed by water of various depths depends upon the temperature of the source of the radiant energy.

increases the curve is higher in the diagram, indicating that water is more transparent to radiations from high-temperature sources than from low-temperature ones.

In Fig. 8 are presented the percentages of total energy absorbed by layers of water at various depths for incandescent

solids of different temperatures from 2000° K. to 5000° K. The practicable incandescent-solid sources are within this range as seen in Figs 2 and 7. It will be noted that the first layer, even though it is only 0.06 cm. in thickness, absorbs considerable of the total energy. The next layer (from depth 0.06 cm. to depth 0.40 cm.) absorbs about twice as much of the total energy as the next layer (from depth 0.40 cm. to 0.82 cm.) which is slightly thicker. By consulting the other layers it is seen that the absorption of successive layers of equal thickness diminishes rapidly. In fact, this absorption follows a well-known law.

The intensity of radiation of any given wave-length after traversing any given depth d can be computed from the following equation:

$$I = I_0 e^{-xd},$$

where I_0 and I are the original and the final intensities respectively, e is the base of Naperian logarithms, and x is the extinction-coefficient. This can be further simplified for the purposes of computation into,

$$\log T = -xd \log e,$$

where T is the transmission-factor, e equals 2.7183, and $\log e$ equals 0.4343. If I_0 is taken as unity, I is numerically equal to the transmission-factor for the thickness d of the substance, for radiant energy of a given wave-length. In this manner many computations can be made throughout the spectrum and for many thicknesses of media. Years ago in connection with spectral work the writer devised a short-cut graphic method of making one set of computations or measurements answer for an infinite range of thicknesses.[2] This method is now widely used and may be of interest to those who would pursue this subject further.

In Fig. 7 it is interesting to note that as the temperature of the source increases to 5000° K., the percentage of total energy remaining to be absorbed in the layer from depth 2.28 cm. to depths beyond increases. This emphasizes again that among the incandescent solids a high-temperature source is desirable. Inasmuch as temperatures of incandescent solids of 5000° K. or more are not practicable at present, the data are not presented for higher temperatures.

Non-luminous bodies, heated electrically or otherwise to temperatures much less than that of incandescent solids, may also be judged from the basis of physics. It is known that water is transparent to radiant energy between λ80,000 and λ500,000—a range in wave-length represented in microns as from 8μ to 50μ. In Fig. 2 it is seen that the wave-length of maximum radiation shifts toward shorter wave-lengths as the temperature of an incandescent solid is increased. Conversely the shift is toward longer wave-lengths if the temperature is decreased.

The position of the maximum of the curve of spectral distribution of energy may be readily computed for the theoretical black-body from the well-known law:

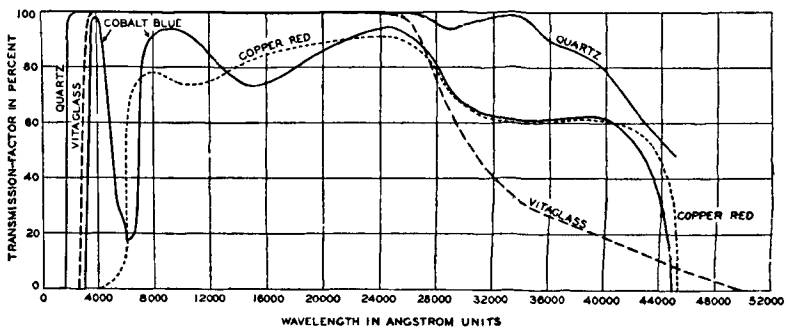
$$\lambda_m T = C,$$

where λ_m is the wave-length of maximum radiation, C is a constant, and T is the absolute temperature. When the wave-length is expressed in Ängstrom units and the temperature is expressed on the Kelvin scale, the value of the constant C is 28,900,000. This is known as Wein's displacement law and the accuracy of the law may readily be tested by the reader by multiplying λ_m and T in each case in Fig. 2. When the wave-length is expressed in any other unit the constant C is varied accordingly. For this purpose it will be recalled that one micron (μ) equals 1000 millimicrons ($m\mu$) or 10,000 Ängstrom units. The Ängstrom unit is convenient in the ultra-violet and visible, but becomes unwieldly in the infra-red where the micron is more satisfactory. However, to avoid confusion the Ängstrom unit has been used throughout this paper.

Obviously, if non-luminous radiators are to be effective, they must radiate appreciable energy between λ80,000 and λ500,000 (8μ and 50μ). If they are to be efficient, much of the total energy emitted must be of wave-lengths within this range. Simple computations of the wave-length of maximum radiation by means of Wein's displacement law reveal the temperatures which non-luminous radiators must have if they are efficiently to supply radiation which will penetrate the body as in the case of that emitted by incandescent solids of high temperature. The wave-length of maximum radiation must

be between λ80,000 and λ500,000. These simple computations reveal the temperature range for such sources to be from 360° K. to 60° K. The upper limit is 87° C. or about the temperature of a hot-water bag. The lower limit of temperature is −213° C. or 213 degrees below zero on the Centigrade scale. Hot-water bags have often been effectively used, but a source having a temperature 213 Centigrade degrees below the freezing-point of water has not been seriously pro-

FIG. 9.



The transmission-factors of representative glasses for radiant energy of various wave-lengths.

posed by anyone, notwithstanding the fact that such a recommendation has as much scientific foundation as many of the therapeutic devices sold at a considerable price and in appreciable numbers. Therefore, it is seen that possibilities among non-luminous sources are confined to temperatures in the neighborhood of hot-water bottles. This brings us back to the hot towels, water bottles, etc., with all their discomfort, inconvenience, and lack of accurate control which modern deep therapy has largely eliminated.

It is seen that the foregoing considerations exclude non-luminous hot radiators from the class of incandescent solids as efficient and effective sources for deep therapy. Still, there are many costly devices of this kind in use. Whatever beneficial results may be obtained by them must result from heating the surface of the body just as a hot towel does. Actually heating at a depth then is achieved by the conduction of heat into deeper layers of the body. There can be little or no direct penetration of the radiant energy as in the case of

radiation from high-temperature incandescent solids such as the tungsten-filament lamp.

Owing to the questions which so often arise in regard to the spectral transmission of glasses in the infra-red regions, a few examples are presented in Fig. 9. It is seen that they are fairly transparent to the near infra-red, but that their transparency decreases in general beyond λ30,000 (or 3μ). Even colored glasses have fairly high transmission-factors in the near infra-red. The clear glasses, including quartz, are almost perfectly transparent out to λ28,000 and quite effectively transmit energy as far as λ40,000.

In conclusion, it may be emphasized again from these considerations of the physical basis of deep therapy that, of the incandescent solids, the high-temperature sources supply the most penetrating (visible and near infra-red) for deep therapy. The high-efficiency gas-filled tungsten-filament lamps with regular bulbs are the most efficient and the most effective, of all the sources considered, for heating at a considerable depth in the human body. Furthermore, it may be emphasized that special bulbs—colored or colorless—are unnecessary and even unjustifiable and that colored bulbs reduce the efficiency and efficacy of incandescent solids for deep therapy because they absorb some of the penetrating radiation. These conclusions are drawn from a consideration of heating at a depth as being the only direct cause of beneficial results of such therapeutic practice as yet known.

VOLTAGE-INTENSITY RELATIONS OF 29 LINES OF THE MERCURY SPECTRUM.

BY

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Member of the Institute.

HERETOFORE the principal data of the spectroscopic theorist have been wave-lengths. With the growth of the quantum theory, increasing use is being made of line intensities. The measurements here reported on the variation with voltage of the intensities of most of the bright lines in the photographic spectrum of the mercury arc show a number of interesting relations of the sort that modern theory proposes to treat. No exact explanation of these relations is here attempted, as the conditions of operation of the commercial arc which was used were not sufficiently simple to warrant it.

The variation of intensity of lines of the mercury spectrum with the voltage across the arc has been studied by Athanasiu [1] and by Harrison and Forbes [2] both using radiometric methods. Valasek [3] studied the mercury spectrum using the photographic method. This last method has been used in the present investigation. It has the advantage that greater resolution of lines may be secured and faint lines (unless masked) may be measured as easily as strong lines. The lines on which measurements are here recorded are the 29 most prominent lines in the arc spectrum between 2378 and 4108, with the exception of the region about 2537 where the absorption of 2537 and the attendant bands complicate measurement. All but three of the lines belong to known series.

The source of radiation was a "Uviarc." This is a mercury arc in a straight vertical quartz tube 1 cm. in diameter, 16.0 cm. between electrodes. It was operated on rectified A.C. from a mercury arc rectifier. The current was maintained at 4.0 amperes by cooling the arc uniformly over its

[1] Athanasiu, *Compt. Rend.*, 178, 2071, 1924.

[2] Harrison and Forbes, *J. Opt. Soc. of Amer. and R. S. I.*, 10, 1, 1925.

[3] Valasek, *Phys. Rev.*, 29, 817, 1297.

entire length with a blast of air. The voltage across the arc was varied from 23 to 118 volts, 23 volts being the lowest voltage at which the arc could be maintained steadily. A thermocouple bound to the tube and shielded from the air blast gave a rough measure of the temperature of the arc. The arc was always brought to a steady state (current and temperature steady) and allowed to remain so for several minutes before making an exposure.

Light from the center of the arc was focussed by a quartz collecting lens on the slit of a Hilgar type E quartz spectrograph of 60 cm. focal length, a camera shutter being placed at the collecting lens. The shutter was electrically operated by a pendulum, the exposures being uniformly of one second duration. The intensity of exposure was regulated by a series of stops placed in the spectrograph close to the prism. The apertures in these stops varied from 6.73 sq.cm. to .0647 sq.cm. The width of the apertures was kept at 2.59 cm. in order to preserve the resolving power of the spectrograph (the slit was vertical).

In order to make the densitometer readings as reliable as possible, the spectrometer slit was kept fairly wide, .3 mm. This produced a line on the plate varying from about 10 A width at 4000 A to 1 A at 2300 A. With this width triplets were not always completely resolved. However, the results show that all the lines of a triplet maintain fairly closely the same relative intensity and, since the strongest line is usually considerably more intense than the others, it is believed that the lack of resolution has not introduced errors. Measurements on multiplets too close to be resolved are attributed to the strongest line.

All the spectra used in this series of measurements were obtained on three Eastman No. 36 plates backed with black paper. They were developed together in 1 : 100 Rodinol 17 minutes at 21° C.

The photographic densities of the lines were measured on a densitometer designed by Barnes and Fulweiler.[4] In it the plate moves slowly past a slit somewhat narrower than the lines to be measured. The density of a line was taken as the logarithm of the ratio of the galvanometer deflection

[4] Barnes and Fulweiler, *J. O. S. A. and R. S. O.*, 15, 331, 1927.

produced by an unexposed portion of the plate to that produced by the line. In some cases the unexposed portion was taken immediately above or below the line, but in most cases it was permissible to use a portion at the side of the line, for, in general, the continuous spectrum and scattered light were too faint to produce appreciable blackening. The deflection for clear plate was about 30 cm.; the densest lines gave deflections of only a few millimeters. Readings of small deflections were made to a tenth of a millimeter.

Intensity-density curves were first prepared from a series of spectra taken at a constant voltage, 23 volts, but with different stops, the logarithm of the stop area being taken as the intensity. A separate intensity-density curve was drawn for each line. It was found that the curves of faint lines were not exactly parallel to those of neighboring strong lines. This was perhaps due to the tendency of strong lines to exhaust the developer about them in development. Below a density of .2 the curves are very flat (1 to 3 slope) and so cannot be used with great precision to find intensities. Above a density of 1.5 precision again falls off as the densitometer deflections are so small.

For a line in a spectrum excited at other than 23 volts, the desity of the negative is measured and the intensity corresponding found on the I-D curve of that line. This intensity less the logarithm of the area of the stop used to photograph the spectrum gives the logarithm of the ratio of the brightness of the line at the exciting voltage to its brightness at 23 volts, and will be referred to simply as the intensity (or logarithmic intensity) of the line at that voltage. On this basis the logarithmic intensity of all lines is zero at 23 volts.

The over-all precision of the measurements can best be inferred by observing how closely the observed points lie on a smooth curve in the accompanying voltage-intensity graphs. Under the most favorable conditions of photographic density much greater precision was obtained than is here indicated in general, but these conditions could not always be met, for the range of densities in a spectrum is very great. Two density determinations of the same plate and the same spectrum showed an average difference of .007 for densities

TABLE I.

First Subordinate Series (Triplet).

Volts per cm.	.75	1.16	1.25	1.31	1.72	1.81	2.76	3.41	4.87	5.09	5.16	6.69
Temperature (° C.)	63	150	165	165	185	224	240	350	300	400	400	450
Vapor Pressure (atm.)	.00004	.004	.007	.007	.014	.07	.07	.90	1.00	2.01	2.01	4.25

Log Intensity.

Designation.	Wave-Length.	Photographic Density at .75 volts/cm.											
p_1d_1—$3d_1$.....	3650	2.15	0	.23	.28		.40	.59	.73	1.09		1.61	1.49	
p_1d_1—$4d_1$.....	3022	1.298	0	.54	.52		.86	1.17	1.26	1.51		2.07	1.93	
p_1d_1—$5d_1$.....	2803	.675	0	.74	.75	.82	1.04	1.06	1.50	1.85	2.15	2.17	2.16	2.38
p_1d_3—$3d_3$.....	3663	2.06	0	.05	.11		.32	.40	.73	1.00		1.45	1.38	
p_1d_3—$4d_3$.....	3026	.565	0	−.02			.27	.38	.78	.99				
p_2d_2—$3d_2$.....	3126	1.96	0	.16	.25		.45	.51	.87	1.10		1.46	1.46	
p_2d_2—$4d_2$.....	2652	1.233	0	.26	.27	.38	.55	.60	1.05	1.28	1.65	1.72	1.73	1.96
p_2d_2—$5d_2$.....	2482	.614	0	.52	.54	.92	.81	1.35	1.61			2.03		
p_2d_3—$3d_3$.....	3132	2.11	0	.10	.20		.36	.36	.76			1.40	1.36	
p_2d_3—$4d_3$.....	2654	1.086	0	.10	.20	.21	.40	.48	.92	1.15	1.60	1.61	1.62	1.60
p_3d_3—$3d_3$.....	2967	1.76	0	?.25	.25	.32	.46	.51	.87	1.01		1.40	1.43	1.56
p_3d_3—$5d_3$.....	2378	.310	0	.51	.58		.80	.86	1.31	1.53		1.99	2.00	

TABLE II.

Second Subordinate Series (Triplet).

Volts per cm.			.75	1.16	1.25	1.31	1.72	1.81	2.76	3.41	4.87	5.09	5.16	6.69
			Log Intensity.											
Designation.	Wave-Length.	Photographic Density at .75 volts/cm.												
p_1S—$3S$	3341	1.52	0	.03	.09		.28	.37	.77	.98		1.43	1.43	
p_1S—$4S$	2925	.556	0	.08	.17	.24	.38	.55	.90	1.14	1.69	1.64	1.67	1.93
p_1S—$5S$	2760	.145	0	.40		.42	.69		1.20	1.39	1.85	1.95	1.92	2.12
p_1S—$6S$	2675	.055	0							1.36	1.81	?1.97	1.88	1.99
p_2S—$3S$	2894	1.391	0	−.06	.02	.06	.20	.27	.65	.86	1.28	1.29	1.29	1.49
p_2S—$4S$	2576	.486	0	.09	.16	.20	.39	.48	.91	1.14		1.64	1.64	
p_2S—$5S$	2447	.107	0	.43			.80		1.28	1.50		1.99	2.00	
p_3S—$2S$	4047	2.41	0		−.11			.10	?.07			.75	?.87	
p_3S—$3S$	2753	1.007	0	−.11	.02	−.02	.15		.62	.83	1.31	1.29	1.28	1.53
p_3S—$4S$	2464	.292	0	.03			.33		.86	1.08		1.56	1.60	

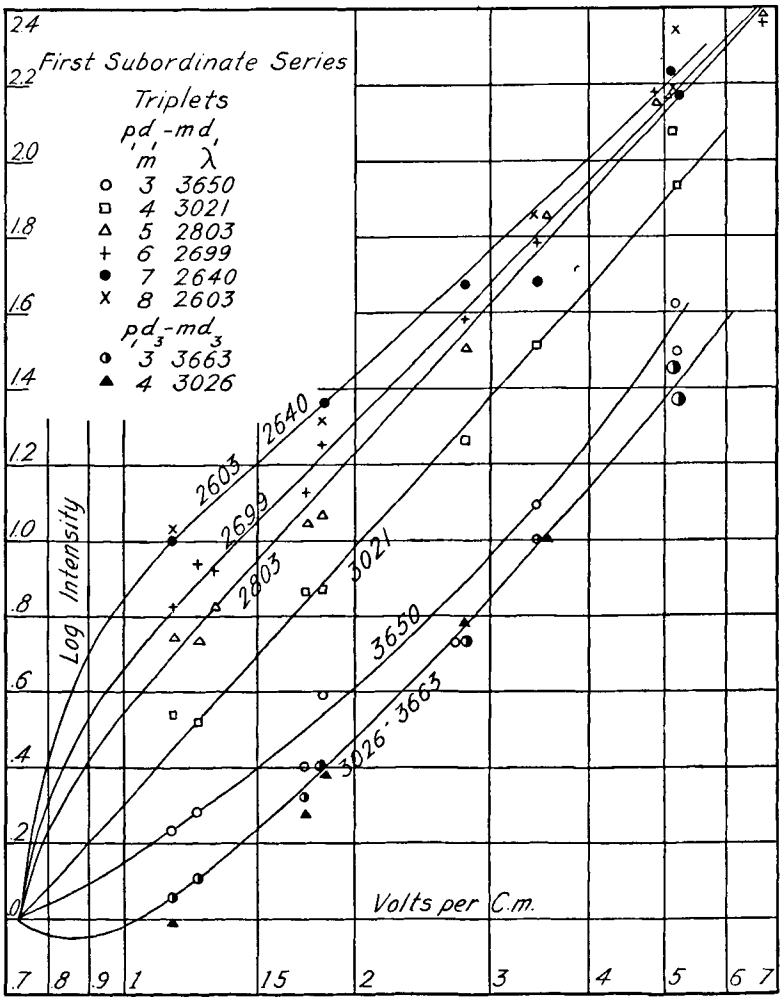
TABLE III.

Miscellaneous Lines.

Volts per cm.			.75	1.16	1.25	1.31	1.72	1.81	2.75	3.41	4.87	5.09	5.16	6.69
							Log Intensity.							
Designation.	Wave-Length.	Photographic Density at .75 volts cm.												
							First Subordinate Series (Singlet).							
$2P—5D$.....	3906	.479	0	.30			.81	.70		1.32		1.82	1.87	
$2P—6D$.....	3704	.150	0	.43			.74			1.46		1.78	1.90	
							Second Subordinate Series (Singlet).							
$2P—4S$.....	4108	.162	0						.67	.92		1.48	1.46	
							Combinations between Triplet and Singlet Levels.							
$2p_2—2S$.....	4078	1.94	0	−.29	−.18		−.10	−.03	.21	.42		.86	.82	
$2p_2—3S$.....	2857	.271	0				.13		.49	.69		1.22	1.23	1.51
							Unclassified Lines.							
	2478	.02	0						1.20	1.42			2.20	
	3790	.09	0							1.45			1.84	
	3984	.142	0	.42								?1.63		

below 1 and an average difference of .05 for densities between 1 and 2. Two different spectra on the same plate of the arc

FIG. 1.

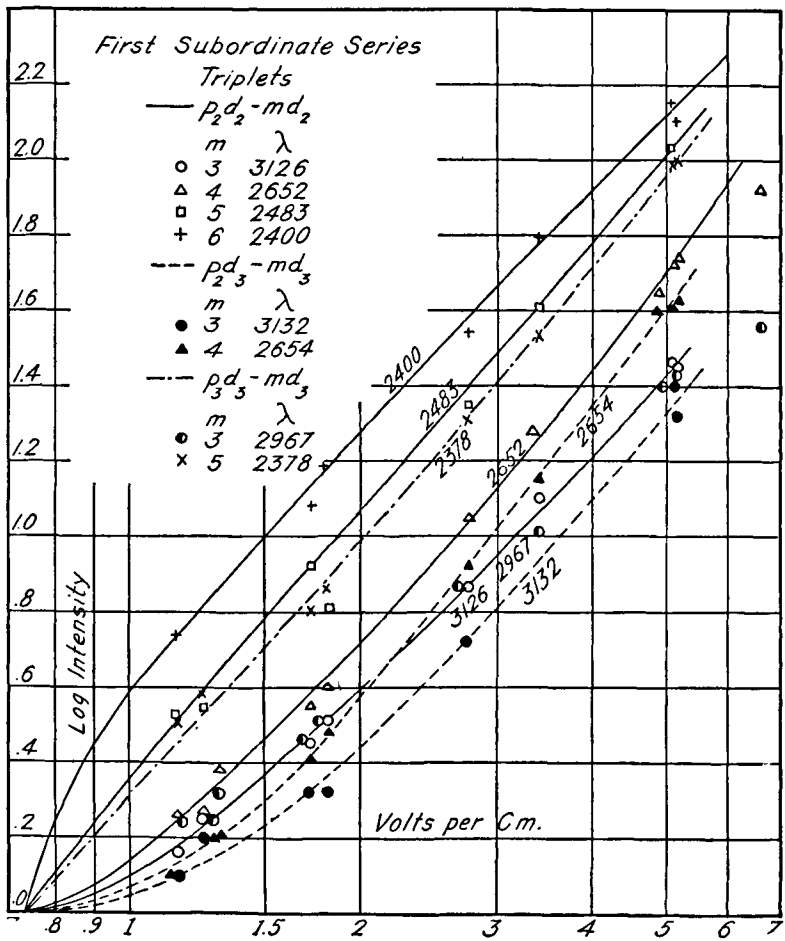


Variations of intensity of lines of the first subordinate series (triplet) of mercury with exciting potential gradient in arc, p_1d_1, p_1d_3 levels.

at the same voltage taken with different stops gave an average difference of intensity of .02 (5 per cent.). Two different spectra on different plates taken of the arc at the same voltage

and with the same stop gave an average difference of intensity of .03.

FIG. 2.

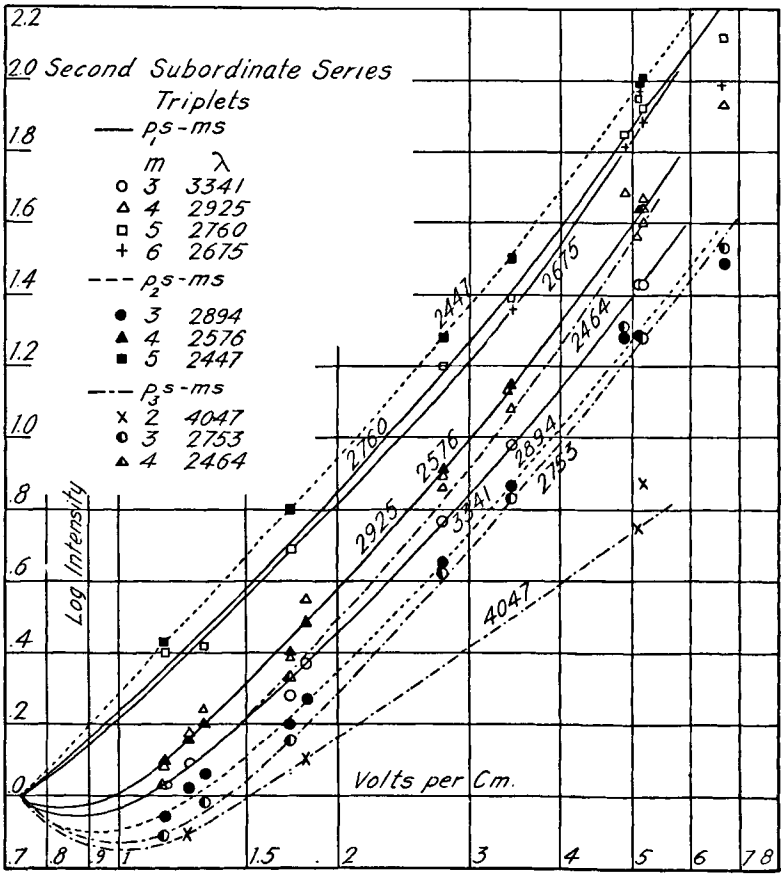


Variations of intensity of lines of the first subordinate series (triplet) of mercury with exciting potentials gradient in arc, p_2d_2, p_2d_3, p_3d_3 levels.

The data for the 29 lines measured are collected in the accompanying tables grouped according to series. In computing voltage gradients, 11 volts have been subtracted from the over-all voltage drop to allow for cathode drop. The gradient of .75 volt per cm. corresponds to an over-all voltage drop of 23 volts, the lowest voltage at which the arc

could be maintained. The logarithmic intensity at this gradient is taken arbitrarily as zero for every line. The temperatures given are those recorded by the thermocouple outside the tube and are only rough approximations to the

FIG. 3.

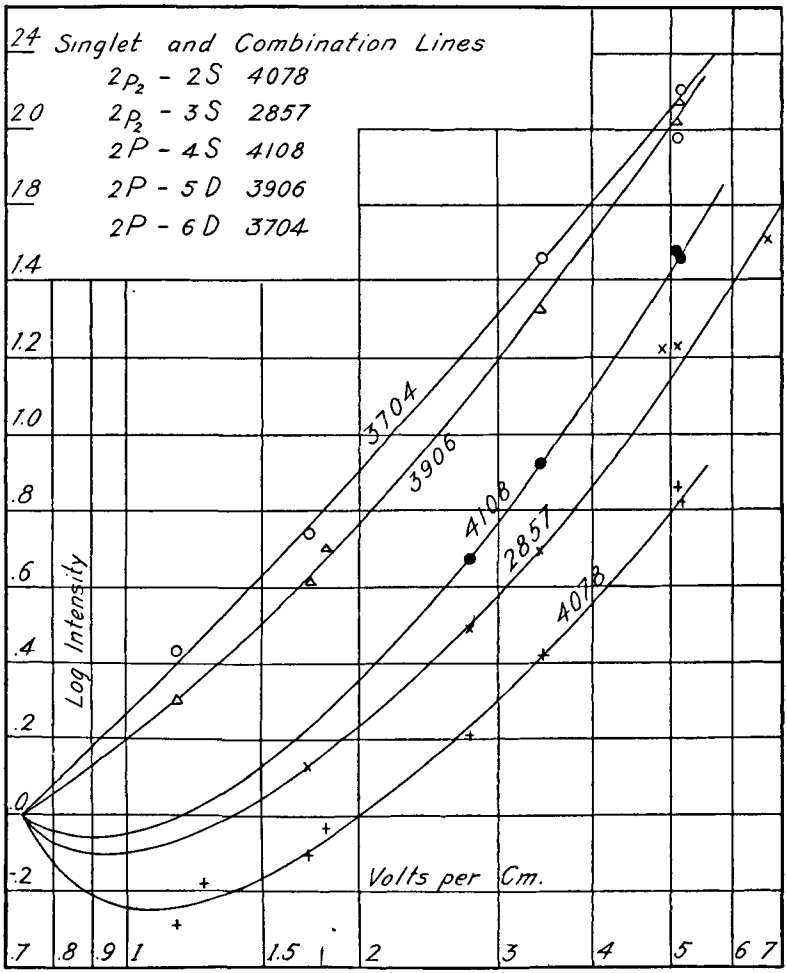


Variation of intensity of lines of the second subordinate series (triplet) of mercury with exciting potential gradient in arc.

temperature of the vapor. The pressures are those of saturated mercury vapor at the recorded temperatures. In order to indicate the comparative brightness of the lines, densities have been given for the spectrum taken with the stop of 5.26 sq.cm. area at .75 volt per cm.

In the graphs of these data logarithmic intensities have been plotted on a linear scale against voltage gradients on a logarithmic scale, so that a straight line graph indicates an

FIG. 4.



Variation of intensity of miscellaneous lines of the mercury spectrum with exciting potentila gradient in arc.

energy emission proportional to some power of the voltage gradient.

From the tables, and more obviously from the graphs, several generalizations may be drawn. Above a gradient of

1.5 volts per cm. all the curves tend to become parallel. This is the graphical indication that the brightnesses of the lines relative to one another vary but little at exciting voltages above 1.5 volts per cm., but that all increase in the same ratio. This behavior is apparently due to practically all of the vapor being completely excited or ionized above this voltage. The increase in brightness should then be proportional to the amount of vapor in the tube. This is roughly verified if the vapor pressure is assumed to be saturated. The energy radiated by each line in this region is proportional to about the 2.3 power of the voltage gradient.

Below 1.5 volts per cm. there is to be observed very considerable variation in the relative brightnesses of the lines. Considering the lines of a series there is seen to be a systematic variation in the voltage intensity curves as the term number, m, is varied. The lower members increase in brightness very slowly and may even decrease to a minimum of intensity below 1.5 volts per cm. As the term number rises the lines increase in brightness with voltage more and more rapidly, that is, the curves are more and more steep. (The apparent exception of the line 2675 to this generalization may be accounted for as due to error of observation. A small change in the data no greater than the probable error would bring it into concordance.) The separation between the curves of one member and that of the succeeding member grows steadily less with increasing term number, so that we may presume that the curves approach a limiting curve which would correspond to the limiting member of the series. In the second subordinate series and in the group of $2p$ and $2P$ lines, the curve of the last lines observed has throughout the constant slope which all the lines have above 1.5 volts per cm. But in the first subordinate series the curves of the higher members rise more rapidly at the lower voltages than at the higher.

Curves of the same triplet lie quite close together; in some cases they are identical within the precision of measurement, notably 3126 and 2967, 2894 and 2753, 2925 and 2576; but, on the other hand, others, though close, are yet distinctly different, as 3341 and 2894, 2803 and 2482. Athanasiu reported that 2652, 2967, and 3026 all maintained the same

relative brightness under conditions quite similar to these maintained in the present experiments. This is in disagreement with the present results.

This work was proposed by Professor C. B. Bazzoni and the author is indebted to him for assistance in its prosecution. Acknowledgment is also due to Professor Jas. Barnes and the authorities of Bryn Mawr College for the use of the densitometer.

UNIVERSITY OF PENNSYLVANIA,
PHILADELPHIA, PA.,
November 1928.

COMMITTEE ON SCIENCE AND THE ARTS.

ABSTRACTS OF REPORTS.

PERMALLOY.

HALL OF THE INSTITUTE,
PHILADELPHIA, December 7, 1927.

No. 2876.

The Franklin Institute of the State of Pennsylvania, acting through its Committee on Science and the Arts, investigating the invention of Permalloy, by Mr. Gustaf W. Elmen, of New York, reports as follows:

The name Permalloy is the trade name applied to certain nickel-iron alloys that, when properly heat-treated, possess certain magnetic properties to an unusual degree. In order that these properties may be developed these alloys must contain more than thirty per cent. of nickel and have the crystal structure of nickel instead of that of iron.

While the desired qualities of high permeability, ease of magnetization under weak magnetic fields and small hysteresis loss, are found whenever the nickel is over 30 per cent., these are found in the highest degree in permalloy containing about 80 per cent. of nickel. The following composition was found by analysis in a bar made by the inventor:

Ni.....	78.23 per cent.	Cu.....	0.10 per cent.
Fe.....	21.35 per cent.	C.....	0.04 per cent.
Co.....	0.37 per cent.	S.....	0.035 per cent.
Mn.....	0.22 per cent.	Si.....	0.03 per cent.
		P.....	Trace

Carbon is the most harmful of the elements present but the heat treatment given to the alloy to secure the highest permeability overcomes its effect.

This material was developed in the laboratories of the Bell Telephone Company, through the investigations of Gustaf W. Elmen to whom a United States Patent No.

1,586,884 was granted on June 1, 1926 on an application filed May 31, 1921.

One of the objects of the invention as stated in the patent is to provide a suitable loading material for signaling conductors to increase their range and speed of operation. Another object is so to apply this material to a conductive core as to produce a highly efficient transmission line for long range, high speed signaling.

In order to prevent the development of eddy currents due to the rapid changes in the magnetizing forces to which the material is subjected, it is used in a laminated form and its composition is such as to provide high resistivity.

In telegraphic and telephonic transmission it has been the practice to increase the inductance of the circuits by winding the conductors in series turns over an iron core in what are called loading coils.

Many years ago, Heaviside investigated the transmission of signals and acertained that the best results, i.e. highest speed and clearest signals, were obtained when the conductor approached, in its characteristics, as closely as possible a pure resistance.

In most conductor systems this requires the neutralization of the capacity by the addition of inductance.

Pupin developed this in the form of lumped inductance units, or loading coils, but sinte the capacity to be neutralized is distributed it is desirable that the inductance should be distributed also.

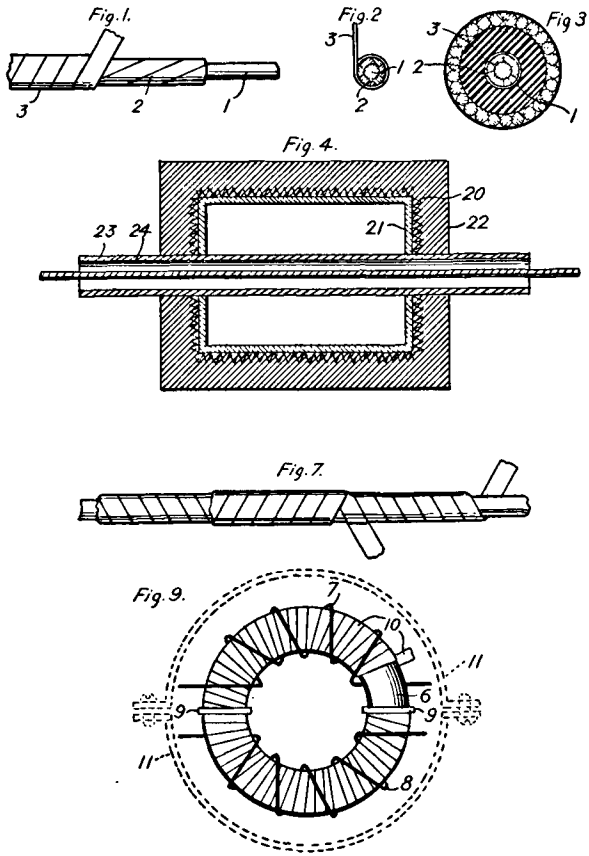
The main difficulty in applying these methods of improvement has been that there was no material known having sufficiently high permeability to make the methods effective.

In the manufacture of permalloy, iron and nickel of a high degree of purity in the proportion of about 21½ per cent. of iron and 78½ per cent. of nickel are fused together in an induction furnace. It is then poured into a mold and cooled. The resulting bar is then repeatedly swaged until it is changed into a rod. This is then drawn through a die until it becomes a wire of size about B. and S. No. 20. This wire is passed through several pairs of rolls and flattened into a tape about 0.006 inch thick and 0.125 inch wide.

It is then passed between cutting rolls that trim its edges squarely and give to the tape an exact width.

Figure 1 of Plate I shows how this permalloy tape is applied to the conductor. This conductor is composed of a bundle of copper wires, *1* and *2*, the combined area being equivalent to size No. 5 on the B. and S. gauge. Six special shaped strands, *2*, are wound helically around a central con-

PLATE I

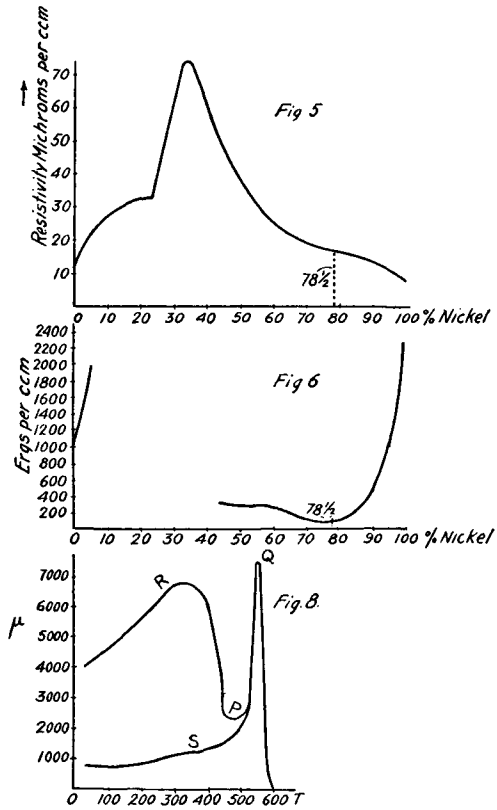


ductor, *1*, fitting it closely and leaving a smooth surface. Around this is wound the permalloy tape, *3*, as shown, care being taken to fit the tapes without overlapping.

The taped conductor is then drawn through an electric muffle furnace, Figure 4, having the heating elements, *20*, between the fire clay muffle, *21*, and the fire brick *22*. The

conductor passes through a copper lined iron tube, *24*, which extends through the middle of the furnace and projects several inches beyond the furnace walls. The conductor is drawn out from the furnace in a straight line to such a distance that it is cooled to nearly room temperature before it is

PLATE II



wound on a reel which should be of large diameter to prevent injuring the conductor, thus affecting its permeability.

After the heat treatment the loaded conductor, if designed for a submarine cable, is insulated, armored and mechanically reinforced by being wrapped with a layer of gutta percha, jute and finally a layer of helically wound steel wires as shown in a cross section, Figure 3.

Figure 9 shows the method of constructing a loading coil. Small permalloy wire is insulated and wound into a core *6*, and the windings *7* and *8* are mounted on this core after it has been wound with insulating tape, *10*. In order to secure constant permeability the core is sometimes provided with gaps filled with non-magnetic material as at *9*.

Experimentation has shown that with varying proportions of nickel in this alloy there is a variation in its magnetic characteristics and these are shown by the curves in Plate II. Figure 5 of this Plate shows the relation between the percentage of nickel in the alloy and the resulting resistivity. An alloy of iron and nickel has a higher resistivity than that of either metal alone and the resistivity of Permalloy, having 78½ per cent. of nickel, is about one third greater than that of pure iron.

It is evident, from this curve, that an alloy of about 34 per cent. has the maximum of resistivity and this alloy may be found of value for some uses in which the permeability is not so important.

Figure 6 shows the hysteresis loss for different percentages of nickel in the alloy. The ordinates of this curve give the work in ergs per cubic centimeter represented by the hysteresis loop for a maximum induction of 5000 c.g.s. units per square centimeter of cross section. The minimum hysteresis loss is at 78½ per cent. nickel and it is at this percentage that the alloy has its maximum permeability.

The curves shown in Figure 8, show the relation between temperature and permeability when a specimen having 70 per cent. of nickel is subjected to a magnetizing force of $H = 0.03$.

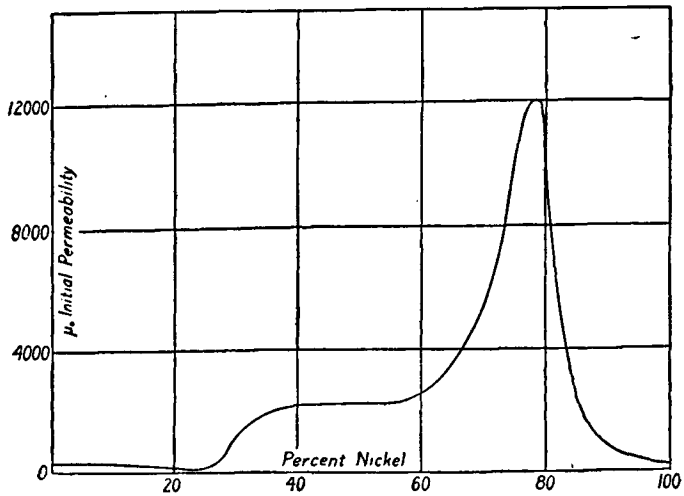
If the cooling down from its critical temperature, that is from the temperature at which the permeability suddenly falls from a maximum, was at the rate for giving the best results, the curve will be of the form *Q*, *P*, *R*, showing a rapid fall from *Q* to *P* and then rising to *R*. If, however, the cooling was too fast or too slow, the curve may be like *Q*, *P*, *S*, with no recovery of permeability.

Thirty-three claims are allowed in this patent of which the eighth and tenth are as follows:

"A magnetic material characterized by a higher permeability than that of iron at magnetizing forces of 2/10 of a gauss or less and lower hysteresis loss than iron, comprising at least nickel and iron of the magnetic group, the nickel being within a few per cent. of 78½ per cent. of the nickel-iron content."

"The method of developing high permeability at low magnetizing forces in a magnetic material which comprises annealing said material, bringing it to a temperature approximating its magnetic transition temperature, and then cooling at a rate intermediate an annealing rate and a rate at which undue stresses and strains would be set up therein."

PLATE III



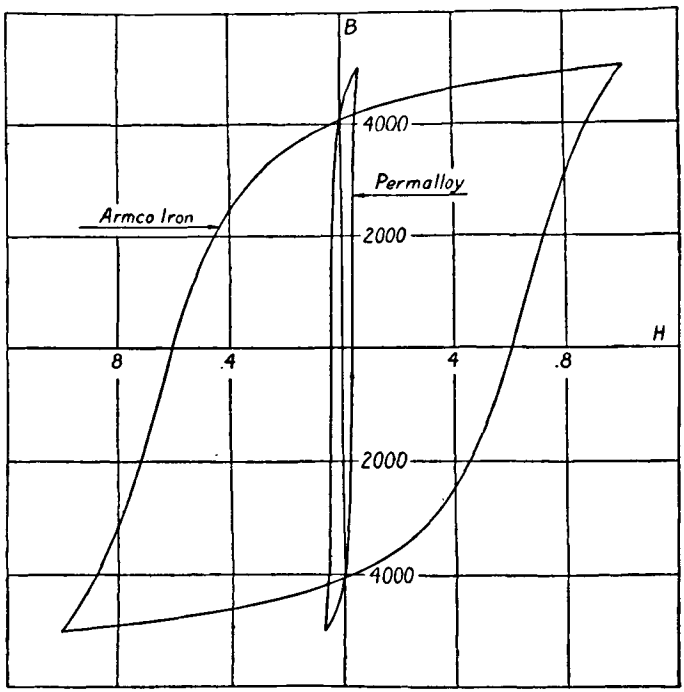
A comparison of the curve in Plate III with Figure 6 of Plate II shows the correctness of the statement that the maximum initial permeability occurs at the same percentage of nickel as the minimum hysteresis loss, at 78½ per cent. This plate is from a figure given in a paper on "Permalloy" by Mr. Elmen, printed in the JOURNAL OF THE FRANKLIN INSTITUTE for May, 1923.

Another figure in this paper, Plate IV, shows two hysteresis curves in which the magnetization is carried to a maximum of 5,000 gausses. In the loop shown, the magnetizing force necessary to obtain a certain magnetic density is less in permalloy than in iron and the hysteresis loss is much less than that in the iron.

One of the most successful applications of Permalloy has been in the loading of submarine telegraph cables for the purpose of increasing the speed of transmission.

An early attempt to obtain increased speed was made with the Key West–Havana telephone cable which is loaded with soft iron wire having a diameter of 0.008 inch, the permeability of which was the greatest of any material that could be obtained at the time this line was built.

PLATE IV



The New York–Azores cable, a Permalloy loaded line, has twenty times the permeability of the Key West–Havana line and has an inductance of about 54 millihenries per nautical mile.

This cable has been in operation for about three years and with the improvements that have been made in the terminal apparatus has attained a speed of 1900 letters per minute, about four times the speed of an unloaded cable of the same size and length.

The increasing use of devices and instruments in which magnetism was applied led to an extended study of magnetic

materials. The methods employed in these studies and the results obtained were fully discussed in a book on "Magnetic Induction in Iron and Other Metals" by J. A. Ewing, published in 1894. This book contained essentially all that was known regarding the prior art at the time of its publication.

Subsequent to the publication of this book many experiments were carried on by Barrett, Brown and Hadfield in a research on the Electrical Conductivity and Magnetic Properties of upwards of one hundred different alloys of iron.

A paper giving the results of these experiments was published in the Journal of the Institution of Electrical Engineers of February, 1902. Thirteen alloys of the nickel-iron series were investigated containing from less than one per cent. to a maximum of 31.4 per cent. of nickel.

In the table of results the greatest emphasis appears to be put upon the conductivity and specific resistance of these alloys.

The magnetic qualities of iron-nickel alloys were also investigated and alloys of higher permeability than iron were found, but since the highest proportion of nickel was only 31.4 per cent., the increased permeability of higher percentages of nickel in the alloy was not then discovered.

In the *Journal for Metallurgical and Chemical Engineering* for January, 1910, there is published an article on "The Magnetic and Electrical Properties of the Iron-Nickel Alloys" by Burgess and Aston. Eighteen percentages of nickel were combined with iron in these tests, the maximum being 75.06 per cent. Curves of the magnetization of these alloys are shown both when the alloy is unannealed and when annealed at 675° and 1000° C. Hysteresis loops and electrical resistance curves are also shown. This paper gives quite full results of the investigation over the range covered but it does not carry the research far enough to compare with the 78½ per cent. of nickel in the permalloy composition.

The use of Permalloy in the field of electrical communication is constantly increasing.

It is now being used as a core in loading coils. For this purpose the Permalloy is reduced to a finely divided state by grinding and after a suitable heat treatment it is subjected to a pressure of 200,000 pounds per square inch between steel

dies resulting in the present powdered-permalloy core. This core has only one third the volume of the iron core previously used thus requiring much less space in the loading vaults. These loading coils have greater magnetic stability and permeability than with the iron cores previously used and are being placed in the telephone plants of the Bell Telephone Company at the rate of about 700,000 each year.

Another use for Permalloy is in the audio frequency transformers of radio receiving sets in which the highest quality of reproduction is desired.

It is also used in telephone and telegraph relays which are now being manufactured at the rate of about 20,000 each year.

The inventor submitted a piece of apparatus for the demonstration of the relative permeability of iron and permalloy.

This consists of two small transformers, each supplied with a primary winding of eight turns and a source of electric current that can be sent through either of these coils as desired, the core of one coil being iron and that of the other permalloy.

The secondary windings, consisting of many turns, may each be connected to a galvanometer, the throw of the needle indicating the amount of the induced voltage when the current in the primary coil is reversed. Although there are many more turns on the secondary of the iron core than on that of the permalloy core, a reversal of the current on that core shows only a very small throw of the galvanometer needle while a reversal on the coil with the permalloy core gives a throw of more than half the entire scale reading.

In consideration of his extended researches in the magnetic characteristics of nickel-iron alloys resulting in the invention of Permalloy, an alloy having high permeability and low hysteresis loss, and of its successful uses for the loading of submarine telegraph cables to secure high speed transmission, for the cores of loading coils and audio frequency transformers, THE FRANKLIN INSTITUTE awards its ELLIOTT CRESSON MEDAL to MR. GUSTAF W. ELMEN, of the Bell Telephone Laboratories, New York.

PRECISION REFRACTOMETER.

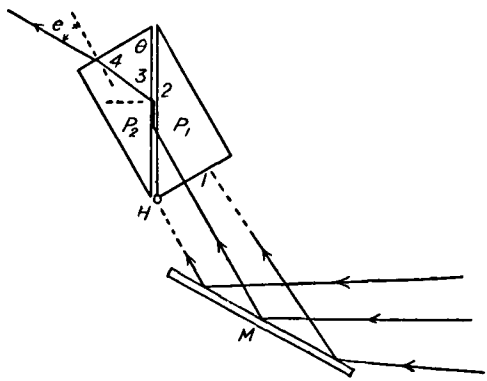
HALL OF THE INSTITUTE,
PHILADELPHIA, April 4, 1928.

No. 2877.

The Franklin Institute of the State of Pennsylvania acting through its Committee on Science and the Arts, investigating the Precision Refractometer invented by Mr. Warren P. Valentine, of Philadelphia, Pennsylvania, reports as follows:

When rays of light fall upon a prism of a transparent substance and are transmitted by it their paths are deviated

FIG. 1.



by an amount depending on the angle of the prism and on the value of its refractive index. This deviation varies also with the wave-length of the light used.

The most accurate determination of indices of refraction is made by the careful observation of the angle of deviation produced in various wave-lengths of light by an accurately worked prism of fairly large dimensions of the material, the index of which is to be determined. This method takes a considerable amount of time to carry out and is even more troublesome when applied to liquids as the liquid has to be contained in a hollow prism built up of accurately plane parallel pieces of glass.

There is a large amount of refractometry which has to be done in connection with various industries for which the highest accuracy is not demanded and for which the time

and trouble involved in such determinations would not be justified by the expense. For such purposes a number of instruments have been devised which are very rapid in use and require a minimum of material and labor. One of these is the Abbe refractometer. The essential parts of this instrument are shown in Figure 1.

P_1 and P_2 are two prisms of dense glass. Surfaces 1, 3 and 4 are polished, surface 2 is mat. The prism P_1 is hinged at H so that it can be swung away from P_2. In determining the index of a liquid, a drop is placed on surface 2 which is closed up until it is nearly in contact with surface 3, the liquid drop being squeezed out into a thin film. Monochromatic light from a suitable source is directed towards the prism system by a mirror M, it strikes the mat surface 2 and is scattered into the liquid film and the prism P_2. Since no ray can enter P_2 with a greater angle of refraction than that of the ray at grazing incidence, the emergent rays when collected by a telescope will all converge to points on one side of a line in the focal plane. The field will therefore be divided into a dark and a light portion with a sharp edge, upon which the cross-hairs in the telescope are set. The rotation of the telescope is registered on a circular arc which is graduated in units to suit the problem under investigation.

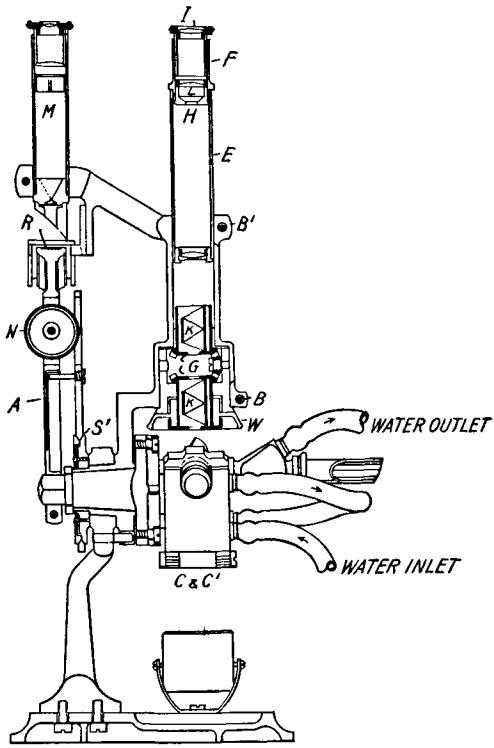
The instrument can also be used with white light by using two direct vision prisms in the telescope to compensate for the dispersion of the liquid and the prism P_2.

The Valentine refractometer which is the subject of this report, is the result of many improvements on the Abbe refractometer as manufactured by Zeiss. These improvements resulted from a study of this instrument when Mr. Valentine decided in 1917 to construct a duplicate of it. On account of the fact that mechanicians who were competent to do instrumental construction of high grade, were engaged in work for World War requirements and on account of the difficulty of obtaining the necessary materials, especially optical glass, this instrument was not finished until January, 1919. Since no improvements had been made in the Zeiss-Abbe instrument since it was originally designed, several desirable changes that would add to its stability and accuracy were considered and Mr. Valentine constructed a second

refractometer in which he incorporated changes in its mechanical and optical design that would secure these results.

Two models are referred to in the following description: the Precision Refractometer, with sector scale of refractive indices divided to the third decimal by estimating tenths of the smallest division; and the Improved Precision Refractometer, Figure 2, with sector scale divided to the fourth

FIG. 2.



decimal and reading to the fifth decimal by estimating tenths of the smallest division.

In order to avoid an error due to parallax, Mr. Valentine redesigned telescope *E*. The field lens *L* and the cross-hairs *H* are fixed relatively to each other. The eye-lens, *I*, being mounted in a screwed cell, may be adjusted to the sight of the particular individual observer and the entire ocular

focussed upon the border line. The entire telescope unit is self-contained and held rigidly in place by means of the clamp screw B' and thus may easily be removed for inspection or cleaning. For reading the extremely small divisions on the sector scale a small compound microscope is provided to which a drum scale is fitted for splitting the divisions into tenths.

Every possible means is employed to secure the highest accuracy. The refractive index of the glass of which the refractory prism is made is determined to the sixth decimal. Errors of flatness in the refracting and transmitting surfaces are held within one third of a wave-length.

The Valentine Refractometers have come into general use. They are used extensively in the paint, oil and drug industries, in chemical and physical laboratories, in dispensaries to measure the quantity of albumin or protein in blood serum and urine. They are also used to determine the oil content of flaxseed, cotton and other oil-bearing seeds, to control the melting point in the hardening of vegetable and mineral waxes and the hydrogenation of soap stock, to measure the total solids in tomato juice and catsup and the concentration of sugar syrups as in sugar manufacture and the sugar content of fruit jellies and preserves. Mr. Valentine submitted with his application a list of almost a hundred users of his refractometers.

In consideration of the meritorious work shown in the improvement of the mechanical and optical parts of the Abbe refractometer, thereby increasing its accuracy, THE FRANKLIN INSTITUTE awards its EDWARD LONGSTRETH MEDAL to MR. WARREN P. VALENTINE, of Philadelphia, Pennsylvania.

THE WORK OF ARTHUR GRAHAM GLASGOW.

HALL OF THE INSTITUTE,
PHILADELPHIA, March 7, 1928.

No. 2880.

The Franklin Institute of the State of Pennsylvania, acting through its Committee on Science and the Arts, investigating the work of Mr. Arthur Graham Glasgow in the Gas Industry, reports as follows:

Arthur Graham Glasgow was born in Buchanan, Virginia, May 30, 1865. He was graduated at Stevens Institute of Technology in 1885, with the degree of M.E. He was in the service of the United Gas Improvement Company from 1885 to 1891 in various capacities, afterwards becoming Engineer and General Manager of the Standard Gas Light Company of the City of New York. Since May, 1892, he has been managing partner in Europe of the firm of Humphreys and Glasgow, Ltd., contracting and constructing gas engineers. Under his direction, this firm has designed, constructed and brought into successful operation, carburetted gas water works throughout the United Kingdom, the Continent of Europe, and in Australia, New Zealand, China and other countries. He was President of the official delegation representing the United States Government at the World's Gas Congress at Paris in 1900. During the late war, Mr. Glasgow was Fixed Nitrogen Administrator in the United States War Department.

Mr. Glasgow is the author of numerous papers presented before various organizations in this country and in England. For his paper on "Carburetted Water Gas as a Coal-Gas Auxiliary," presented before the American Gas Light Association in 1899, Mr. Glasgow received the Beal Medal. The committee awarding this medal, said: "Mr. Glasgow treated his subject in a masterly manner. The paper is broad in its scope and of great and general usefulness to the gas engineer. The deductions are reached by the author after a thorough and analytical examination of all the underlying bases of gas manufacture."

Mr. Glasgow is a member of the Institution of Civil Engineers of Great Britain, Institution of Mechanical Engineers of Great Britain, The American Society of Mechanical Engineers, American Society of Civil Engineers, American Gas Association, and the Western Gas Association.

In its investigation of the work of Mr. Glasgow the Institute has been in communication with many leading authorities and executives in the gas industry who were practically unanimous in their appraisal of the high value of Mr. Glasgow's achievements and of his standing as an engineer and executive.

Mr. Glasgow's work may be summarized as follows:

1. As an outstanding authority on water gas manufacture, he has been associated with nearly all of the important improvements and developments that have been brought about in water gas technology and practice.

2. As the author of some thirty-five papers on various phases of gas manufacture, he has made important and lasting contributions to the literature of the general subject.

3. In the commercial development of water gas manufacture and utilization, he has played an important part, as shown by the fact that he has been granted twenty-six patents for various inventions of equipment and methods, and that to date about 1,885 water gas sets have been installed by his firm in various countries, with a total daily capacity of 1,870,900,000 cubic feet.

In consideration of his outstanding and valuable work in the technology and development of the manufactured gas industry, THE FRANKLIN INSTITUTE awards its WALTON CLARK MEDAL to MR. ARTHUR GRAHAM GLASGOW, of London, England.

Action of Light upon Ferric Citrate. According to JOSEPH B. BURT (*Jour. Am. Pharm. Asso.*, 1928, *17*, 650–658) the magma of ferric hydroxide does not dissolve completely in a solution of citric acid when the two compounds are present in exactly molecular proportions; an excess of the acid is required to produce complete solution of the hydroxide. Study was made of the action of sunlight upon the resulting solution, (1) as prepared, (2) after complete neutralization with ammonia water, and (3) after half neutralization with that base. At the end of two days, the solutions containing free citric acid had become colorless, and carbon dioxide gas had been evolved from them. Acetone was also formed; but formaldehyde was absent. One molecule of citric acid was oxidized with the formation of one molecule of acetone, three molecules of carbon dioxide, and one molecule of water. The acid solution as prepared yielded more gas than the half-neutralized solution. The completely neutralized solution became lighter in color, but did not become colorless; and no gas was evolved from it. The end point of the reaction apparently depended upon the complete reduction of the iron to the ferrous state, rather than on the exhaustion of an excess of citric acid. Use of quartz vessels instead of vessels of ordinary glass did not increase the yield of carbon dioxide, other conditions being constant; therefore, the light need not necessarily be rich in ultra-violet rays. The end point was not very definite, for the iron in the reduced solution was partially re-oxidized when this solution was permitted to stand in diffused light. It is recommended that solutions of this type, for example, the elixir of iron, quinine and strychnine, and the elixir of iron, quinine and strychnine phosphates, be rendered as nearly neutral as possible, and then be kept in containers of amber glass.

J. S. H.

Essential Oils as Antiferments. ELLERY H. HARVEY (*Am. J. Pharmacy*, 1928, *100*, 524–529) has determined the value of thirty-two typical essential oils by means of their influence on the hydrolysis of sugar by yeast. The essential oils of clove, cinnamon, thyme, and wintergreen were found to rank high in effectiveness. Absorption of ultra-violet radiation affected both the color and the antiferment value of essential oils. Such oils should be kept in brown bottles, and not in blue or clear glass bottles. Acetic acid, in concentrations above 1.5 per cent., was found to be an efficient antiferment.

J. S. H.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

DISPOSAL OF WASTE CRANK CASE OILS.

A REPORT, containing recommendations for controlling the disposal of waste automobile crank case oils so as to avoid contamination of public sewers, was issued under date of November 15, 1928, by J. B. Gordon, Sanitary Engineer of the District of Columbia.

This report contains excerpts from replies received to a questionnaire sent by Mr. Gordon to many cities in the United States and foreign countries. These replies show that a large percentage of municipalities recognize the danger involved in allowing great quantities of oil, often containing considerable amounts of gasoline, to enter the sewers. Many cities have adopted regulations covering the disposal of waste oils, but often such regulations are not successful in accomplishing the desired object, because, while the regulations prohibit the dumping of oils into the sewers, no recommendation is made as to what is to be done with them.

The report gives briefly the results of an investigation conducted by the Bureau of Standards, in which it was shown that by the use of a special rerefining process oils suitable for use in automobile engines can be recovered from waste crank case oil. A table gives the properties of the new, used, and rerefined oils, based on tests of samples obtained by the bureau from the Post Office Department and the Office of the Superintendent of Public Buildings and Parks. In a second table values are given for used and rerefined oils collected from a public filling station.

Mr. Gordon concludes his report as follows:

"With all the information as cited herein at hand, including the reports from the United States Bureau of Standards showing the value of the product, the Plumbing Regulations of the District of Columbia were recommended to be revised in this respect and amended to require the main-

* Communicated by the Director.

tenance by garages, filling and service stations, automobile repair shops, etc., of approved containers or underground tanks, separate and apart from garage traps, for the retention of all waste crank case oils, the contents of the same to be removed by the occupant or operator at regular intervals or upon notice from the District authorities. Regulations for the safe transportation of this commodity are also included in the proposed amendments.

"The adoption of these regulations will necessarily require provision to be made for the safe and systematic disposition of the waste oil so retained. The dumping grounds in the District and vicinity have prohibited the receiving of such refuse (waste oils or contents of garage traps) and in order that the public may be fully protected in the matter of disposition of these accumulations of waste oil retained under such regulations, particularly as to the more or less isolated sections being properly provided with disposal service, a proposal is being submitted for approval whereby a contract may be made under competitive bidding for the collection, transportation and disposition of such waste oil as may become available. Such services as may be contracted for will be guaranteed by proper bond, regulated by direct supervision of the District Government as to scope of service, maintenance of facilities for collection, transportation and disposition and all other conditions which will make for a thorough and systematic service.

"In view of the fact that the rerefined product has been determined usable and valuable, it is expected that this contract service will be without cost to either the District or the public, the contractor realizing his earnings from the ownership of the available oils so collected. No monopoly is to be granted the contractor except that requests made by the public on the District for the removal of oil will become an order on the contractor for the rendering of this service. In this way, the outlying sections of the District will be supplied with disposal service on a par with the more densely populated part of the city. The occupant or operator of any premises may dispose of the oil so retained in any manner he may choose the only stipulation being that its disposition must be in accordance with the Regulations.

"Under these conditions it is estimated that more than 200,000 gallons of waste oil should be available for collection annually. It is believed that the public will be quick to realize the importance attaching to the exclusion of this waste from the public sewers as well as appreciate the advantage to be gained from its free and regular removal, especially the latter as it costs the operator some 2¢ to 3¢ per gallon to have it removed when deposited with silt in garage intercepting traps. A decided improvement of the condition of the interior of public sewers may confidently be expected to result from the adoption of these measures, the danger from fires and explosions in sewers minimized, thousands of gallons of oil which has heretofore been a source of danger and expense conserved for reuse and a commercial enterprise of appreciable moment encouraged and developed."

MELTING POINT OF NICKEL.

THE bureau has practically completed a determination of the melting point of nickel, based upon two samples prepared in its own laboratories. These samples, known as N19 and N23 were from different melts, and each was found to contain about 0.06 per cent. of impurities, chiefly iron, cobalt, and copper. The main difference between the two samples lies in the fact that N19 contains these impurities in about equal parts, while N23 contains more iron and is nearly free of copper.

Nine freezes were observed on N19 by two observers. The average value obtained was 1454.9° C., the maximum deviation of any one freeze being 0.5° C. In addition, the readings of each observer, taken alone, average 1454.9° C. The previous value which has been generally accepted as the best is 1452° C.

Eight freezes have been taken on the second sample, N23, the average being likewise 1454.9° C., and the average of each observer's readings, taken alone, again being 1454.9° C., the maximum deviation of any reading of either observer being 0.4° C.

The bureau believes that enough work has been done to show conclusively that the two samples of nickel are so

nearly alike that all the observations on the 17 freezes may be treated as observations of the same sample. Considered from this point of view, 17 readings have been secured with a maximum deviation of 0.5° C., and a probable error of 0.05° C. It is believed that the result secured is not in error by more than 1° C. on the International Scale. This conclusion is based on the following estimates of the errors that may be present:

Source of Error.	Amount. Degrees C.
Purity of nickel	− 0.5
Transmission of sector	± 0.1
Effective wave-length	± 0.5
Setting pyrometer	± 0.2
Maximum	1.3

THE PRODUCTION OF XYLOSE ON A SEMI-COMMERCIAL SCALE.

THE Alabama Polytechnic Institute and the University of Alabama, coöperating through the Alabama Industrial Development Board, have arranged to work with the Bureau of Standards in the erection and operation of a small factory in which experiments can be made on the recovery of xylose from cotton-seed bran and peanut shells, on a semi-commercial scale. The plant is now under construction by the Federal Phosphorus Company at Anniston, Alabama.

Xylose at present is a rare sugar, selling for about $100.00 per pound, but possessing properties which ought to make it a desirable material for use in the food, textile, and leather industries. Its investigation is part of an extensive research program which the bureau is conducting with the object of discovering new uses for the waste products of agriculture. This work does not stop with the production of a marketable material in the laboratory, but includes the discovery of profitable ways to dispose of this material. As a matter of fact, it is not so difficult to find how to make things from these waste products. The difficulty lies in finding how to make money out of them. The chief difficulties are economic rather than chemical.

Therefore, the bureau has found it necessary to build small factories in which the laboratory processes can be tried out

on a production basis, thus making it possible to figure manufacturing costs with some degree of assurance. These small plants are useful in another way as well. They make possible the production of enough material to be actually used by likely consumers in their own plants, so that markets for the product may be explored. Thus, in the case of xylose, it seems probable that it will find a ready market in the industries mentioned, but there is no way to answer this question definitely while it is a laboratory curiosity. The erection of the plant at Anniston, with a capacity of 100 pounds of xylose per day with a manufacturing cost probably not exceeding fifty cents per pound, should soon give definite information on the subject.

USE OF ANTIOXIDANTS IN PRESERVING RUBBER.

DURING the past two or three years several substances have been placed on the market which are designed to be incorporated in rubber compounds to prevent or at least retard deterioration through aging. These substances are designated by the general name of antioxidants. In the case of one or two of these, it has been claimed that it is beneficial to dip rubber articles already vulcanized into solutions of the antioxidants. In order to obtain data on the influence of this dipping process on the aging of rubber articles, tests have been made with five different antioxidants. Inasmuch as the penetration of antioxidants during the dipping process is probably not very great, the logical products on which to test the effect are thin rubber products such as surgeons' gloves, rubber bandages, rubber bands, etc. Numerous samples of these articles were dipped in benzol-alcohol solutions of antioxidants for five minutes and then allowed to dry in the air. These were then exposed to aging under two conditions; first, indirect light such as in an ordinary room; and second, stored in the dark.

The results of tests of samples exposed to light show that in all cases the antioxidants have had a decided beneficial effect. Aging in storage is a much slower process than when rubber is exposed to light and to date no difference is apparent between dipped and undipped articles.

PROTECTIVE COATINGS FOR PAPER CURRENCY.

THROUGH the use of paper having increased fiber strength, the proportion of the United States paper money that is redeemed because of its being broken or torn has been reduced until it is almost negligible. Most of the paper currency that is now withdrawn from circulation on account of being considered unserviceable is declared unfit because of oil or grease stains, dirt, creasing, and other evidences of surface wear. On account of this condition, the coöperative investigation of the Bureau of Standards and the Bureau of Engraving and Printing has been extended to include a study of various materials for protecting the surfaces of the notes for the purpose of further prolonging the working life of paper currency.

At present, sheets of notes are resized with animal glue, but the demands of large scale production have prevented the employment of the low temperature drying methods that are essential for preserving in full the improvements contributed by the glue. The addition of tanning materials such as formaldehyde and of softening agents such as glycerine to the freshly sized paper has been studied, and the use of such reagents seems to offer considerable promise of improving the resistance of the resized notes to surface wear. The necessity of removing quickly the relative amount of water that is retained by papers resized with glue or other proteins such as casein or albumen, has caused attention to be centered quite largely on the use of protective materials that may be applied with organic solvents. It is planned, however, to study the present resizing equipment in order to determine if air-drying methods may be applied in order to enhance the effects of the glue sizing.

Various lacquers and varnishes have also been included in the investigation. Both cellulose nitrate and cellulose acetate lacquers have been tested. The work with varnishes has been limited in scope, but the materials used tended to make the paper surfaces somewhat sticky when warm and to increase the brittleness of the treated notes. Several nitrocellulose lacquers were tried but these were generally considered to be impractical for large scale application because of the strong and persistent odors of the solvents which were of the type of

amyl or butyl acetate, fusel oil, or similar materials. Objection was also raised to the excessively glossy surfaces imparted to the bills by the nitrocellulose lacquers used in the small-scale work.

With the coöperation of manufacturers of the material, the possibility of using cellulose acetate as the protective material has been studied quite extensively. Many combinations of different cellulose acetates, plasticizers, and solvents were tested until there was developed a solution capable of imparting to printed surfaces improved resistance to oil, water, and wetrubbing, without noticeably decreasing the high folding strength of the paper. The cellulose acetate forms a closely attached, invisible film that is free from gloss or stickiness and that does not bloom or discolor when dried at room temperature. Practical use of such a material would require special equipment for its application and for the removal of the organic solvents employed. In addition, as is the case with other surface protective agents, the question of the adherence of the overprinted sealing and numbering inks to such treated surfaces would require further careful study. These factors are now being considered.

Most of this work has been on a laboratory scale. It is now planned to undertake full-scale work with especial references to the drying of paper currency that has been resized with glue.

TESTS OF TERRA COTTA FACED WALLS.

THE Bureau of Standards in coöperation with the National Terra Cotta Society has just completed a second series of tests on terra cotta faced walls. The purpose of this work is to determine the best method of setting terra cotta ashlar on buildings. Owing largely to lack of information on which to base requirements, most of the building codes do not give specific details covering this subject.

The test walls were $4\frac{1}{2}$ feet wide and $7\frac{1}{2}$ feet high, and included the different types of construction now in vogue in different parts of the country. In some walls the terra cotta voids were filled with brick and mortar; in others the voids were partially filled with concrete grout; and in still others the voids were left unfilled. The strengths of these walls

were determined under different types of loading. On some walls, the load was applied on the backing wall behind the terra cotta; on others it was applied on the terra cotta facing only; and on still others it was applied uniformly over the terra cotta facing and the backing wall. The vertical deformations of the terra cotta facing and the backing walls were measured at different loads. The separation of the terra cotta facing from the backing wall was also measured during loading.

It appears from the results that the filled terra cotta facing adds considerably to the strength of the walls. These would apparently justify reckoning filled terra cotta as a portion of the thickness of walls. Such a possibility has decided commercial advantages in that the structural framework would not have to carry so great a load and, therefore, could be constructed more economically. The adoption of thinner walls would also permit the use of a larger floor area for renting purposes.

It was also the purpose of this work to compare the strength of walls containing terra cotta ashlar with two vertical partitions and six cells, with those walls containing ashlar having three vertical partitions and eight cells. Although it is possible to build brick filled walls with six cell ashlar which are equally as strong as those with eight cell ashlar, it was found that the concrete walls containing eight cell ashlar were stronger than those containing six cell ashlar.

DRYING BEHAVIOR OF SHALES AND CLAYS.

ONE of the investigations being conducted at the Columbus branch of the bureau is concerned with the drying behavior of several shales and clays of the glacial and alluvial type. A humidity dryer of semi-commercial size is used in this work. Water loss, shrinkage, and surface and center temperatures are determined on three green brick of commercial dimensions, the brick being made with a small combination pug mill and brick machine.

Although the data are as yet not sufficient to warrant drawing positive conclusions, a number of interesting facts have developed. It was found that a commercial sized brick

made from the Rutland (Ohio) alluvial clay could be made to lose about 76 per cent. of the added tempering water when dried at an average temperature of 26° C. and an average humidity of 94 per cent. for 244 hours. At this point an equilibrium appeared to be established. At no time was the temperature above 27° C. or the humidity below 92 per cent. The brick lost weight steadily at a temperature of 25.6° C. and a humidity of 97 per cent., even after it had already lost about 33 per cent. of the added tempering water.

The above is of interest in view of the fact that there is a rather prevalent opinion that clay ware will not dry out at such high humidities at this temperature range but on the contrary will absorb water.

Maintaining the same temperature, the humidity was lowered to 85 per cent. and the brick again lost weight. After it had lost about one ounce, the humidity was raised to 94 per cent. and the brick gained 0.5 ounce in weight and expanded 0.003 inch in 10 hours. The brick were dried at 105° C. for 15 hours and again subjected to a humidity of 94 per cent. at a temperature of 26.7° C. Under this treatment the brick absorbed water to the extent of 3 ounces (about 4 per cent. of the dry weight) and expanded 0.011 inch in 69 hours. Thus, after about 75 per cent. of the tempering water has been evolved, the clay can be made to gain or lose water at will by varying the humidity or temperature. However, the clay only regained about 50 per cent. as much water at 94 per cent. humidity after being dried at 105° C. as it held when equilibrium had been attained at this humidity after the drying process had been carried on for about 244 hours.

Two facts of importance are to be noted here: first, the reabsorption of water on proper change of condition, and secondly, the expansion which takes place coincidently. It is entirely possible that considerable dryer trouble might be traced to such change of conditions.

Shrinkage had practically ceased when about 75 per cent. of the tempering water had been eliminated, this being the time that the equilibrium had been attained apparently at 94 per cent. humidity. That is, this percentage possibly represents water present in a purely mechanical fashion. But, inasmuch as the remaining 25 per cent. of water apparently

exhibits equilibrium conditions at various humidities (temperature constant at about 26° C.), it is possible that this portion of the water is present in a definite adsorbed or hydrate condition, each of the hydrates having its characteristic vapor pressure.

A further interesting fact is that the shrinkage varies with the length of time required to dry the brick to the end of the shrinkage period. In the table below, it is to be noted that the shrinkage of the Rutland alluvial clay is in every case greater, the greater the length of the drying time.

Drying Time, Hours.	Total Shrinkage, Inches.	Shrinkage, Per Cent.
4	0.435	5.15
6	.456	5.42
8	.458	5.44
10	.469	5.56
12	.485	5.75
220	.522	6.23

From this it would appear that to state shrinkage correctly, the conditions under which the drying takes place should also be stated.

REACTION OF WATER ON THE CALCIUM ALUMINATES.

AN item on this subject was published in Technical News Bulletin No. 124 (August, 1927). A full report, giving in detail the results of this investigation, will appear in the December, 1928 issue of the Bureau of Standards Journal of Research, as Research Paper No. 34 under the title. "Reactions of Water on the Calcium Aluminates."

In this investigation the four calcium aluminates ($CaO.Al_2O_3$, $3CaO.5Al_2O_3$, $5CaO.3Al_2O_3$, $3CaO.Al_2O_3$) were made, and the mechanism of their reaction with water was studied. Tricalcium aluminate, which appears to be the only aluminate present in Portland cement of normal composition and normal properties, reacts so rapidly with water that the changes in composition of resulting solutions could not be followed. The monocalcium aluminate and the 3 : 5 calcium aluminate occur in cements characterized by a high alumina content. These two aluminates together with the 5 : 3 calcium aluminate and a high alumina cement react with

water to form metastable and supersaturated monocalcium aluminate solutions in the early periods. The metastable solutions decompose as the reaction proceeds with the precipitation of varying amounts of hydrated alumina and crystalline hydrated tricalcium aluminates with attendant increases in both the pH and molar ratio CaO/Al_2O_3, in the resultant solutions. If the metastable solutions are filtered from the reaction mixtures, they also decompose with the precipitation of the above phases. The resultant solutions, which are in equilibrium with these two phases, apparently attain the same composition as those left in contact with the calcium aluminates or high alumina cement for long periods.

A study of the subsequent changes in the aluminate solutions attended by increasing concentrations of calcium hydroxide, a product of the hydrolysis of the calcium silicates, shows that the alumina is almost completely precipitated from solution at a pH value of 12.0 or above.

The constitution of the aluminate solutions is discussed. Calculations based upon electrometric measurements and chemical analyses indicate that the alumina in the aluminate solutions is in combination with lime as the calcium salt of monobasic aluminic acid.

Tricalcium silicate and beta dicalcium silicate appear to hydrolize less in an aluminate solution than in water.

An electrometric titration study of aluminum chloride and calcium hydroxide is described. The conditions under which the aluminum precipitates as the hydroxide and as the calcium chloraluminate were studied. The formation of calcium chloraluminates from the reaction of calcium chloride with aluminate solutions is also described.

The optical properties of the various hydration products are given.

TEST OF SLATER'S CEMENT.

FREQUENT calls for tests on slater's cements have led to an improvised method for determining the adhesive and cohesive properties of such materials under conditions simulating those of service. The intended purpose of slater's cement is to produce and maintain a waterproof joint between slate shingles on the roof. The material is called upon to withstand

rigorous weather conditions and remain sufficiently plastic to accommodate itself to structural movements.

Previous tests have been confined mainly to determining the effects of high and low temperatures on the physical state of the cement. The new test was designed to determine if structural movements of a given magnitude will break the joint and permit leakage.

The test is modeled on a procedure now in use for testing "elastic pointing materials." A joint is made of two blocks of stone, one of which is fixed and the other so arranged that it can be moved toward or away from the fixed stone by means of a thread and nut. The movements can be accurately controlled and measured by a gauge between the two blocks. A cavity is made in the ends of the two blocks in such a way that it will hold water when the joint is sealed with the material to be tested. In testing a slater's cement, strips of slate shingles are sealed to the ends of the stone with sealing wax and then the joint is made between the slates with the cement in question. After the joint has set for one month the cavity is filled with water. If no leaks are noted the joint is stretched a small amount to simulate the effect of a structural movement.

Experiments so far completed indicate that a joint of slater's cement is usually capable of withstanding a movement of 0.01 inch at 70° F. after a drying period of one month, and some tests have indicated considerably higher values. This procedure seems to offer a satisfactory means of comparing different cements of this nature and to afford a basis for improvement of the product.

EFFECT OF WORKING GYPSUM MIXES AFTER STIFFENING.

CALCINED gypsum for certain purposes, such as for use in construction, must have a fairly quick set when mixed with water. However, sufficient time must be allowed to permit thorough mixing of the material and pouring into the forms before it starts to set. It has been generally accepted that the gypsum is useless if set has started before it is in place, inasmuch as little strength can be expected from the gypsum when worked through the set. Preceding the set a stiffening of the calcined gypsum putty occurs, and the question arose

as to whether the material could be used after the stiffening of the putty had started. Some tests have been made to determine for what period the putty can be worked without materially reducing the strength of the set gypsum.

A large batch of calcined gypsum was mixed with sufficient water to bring it to standard consistency, and three 3 by 6 inch cylinders were cast two minutes after mixing, and at one minute intervals thereafter until the paste was too stiff to be forced into the mold, even by using considerable pressure. A typical set of results is given below.

Time after Adding Water, Minutes.	Compressive Strength, lbs. /in.^{2}
2	1,070
3	1,060
4	1,100
5	1,070
6	910
7	710

The material for the specimens at six and seven minutes had to be pressed into the mold. Similar results have been obtained using retarded mixes with longer time intervals between the casting of the specimens, and using mixes containing various proportions of wood chips. It seems evident, therefore, that no weakening of the set gypsum occurs when the paste may be molded or formed without exerting undue pressure.

LIGHT-COLORED GROUND COATS FOR SHEET IRON.

A TREATMENT of the metal used in this work has been described in Technical News Bulletin No. 133 (May, 1928). The metal is coated with a film of cobalt nitrate. This film decomposes at 300° C. (about 570° F.) and is thus rendered insoluble. The light-colored enamel is then applied to the treated metal.

Sixty-three light colored ground coat enamels have been prepared and tested, the majority of them being clear glasses, although several of the frits [1] were very white and opaque. These were divided into four groups as follows:

[1] A term used to denote the glass which forms the basis of a vitreous enamel.

Ingredient.	Group.			
	G-2.	G-3.	G-4.	G-5.
	Per cent.	Per cent.	Per cent.	Per cent.
Feldspar	—	—	—	42
Silica	65	65	60	16.7
Boric acid	5 to 17	5	5	11.3
Sodium oxide	12 to 27	10 to 19	12 to 21	13.5
Lead oxide	0 to 18	6 to 15	6 to 18	—
Fluorspar	—	0 to 9	5 to 17	1.5 to 11.5
Cryolite	—	—	—	0 to 10
Sodium antimonate	—	—	—	0 to 10

When applied to treated metal, the composition of the enamel does not seem to be the most important factor in its adherence. The strength of the test specimens and the type of fracture obtained depend more upon the thermal expansivity of the enamel and the firing temperature. It appears, therefore, that any ground coat enamel now in use may be converted into a light-colored one by simply eliminating the colored oxides and giving the metal a suitable treatment such as described above.

Of the four groups in the above table, G–2 contained the best enamels as these did not have as much tendency to crawl or blister, and had a lighter color than any of the others.

If the Group G–2, containing 65 per cent. silica, be plotted as the ternary diagram $PbO–Na_2O–B_2O_3$, the enamels just described will lie within a lozenge-shaped field bounded by the following limits: boric oxide, 8 to 11 per cent.; lead oxide, 0 to 6 per cent.; and sodium oxide, 21 to 24 per cent. These enamels are fairly free from the ordinary enamel defects, as compared with the rest of the group or the other groups. They have good resistance to solution as tested with cold 20 per cent. hydrochloric acid and fire down smoothly at 850 to 900° C. (about 1560 to 1650° F.). They are, however, rather difficult to melt and to grind.

NOTES FROM THE RESEARCH LABORATORY, EASTMAN KODAK COMPANY.*

GRAIN GROWTH IN SILVER HALIDE PRECIPITATES.[1]

S. E. Sheppard and R. H. Lambert.

PARTICULARS are given of an experimental study of the grain growth of silver bromide precipitates which shows that the process is regulated by definite quantitative laws. Two principal periods are distinguished, the first consisting in nucleation of the new solid phase; the second, in the growth of grains.

Precipitation was accomplished by mixing silver nitrate and potassium bromide, each containing gelatin, at the juncture of a Y-tube; the mixed material was then ripened at constant temperature by digestion in a solution containing excess of potassium bromide.

Samples were removed from time to time and grain sizes were measured photomicrographically. Following a statistical analysis, examination now indicates that the grains in the first classes (small grains) decrease proportionately to the number present at any given time. A break in the curve, however, occurs at a time independent of excess soluble bromide but depends on the size of grains. Growth before the break is attributed to Ostwald ripening. After the break, the growth is attributed to collision and recrystallization.

A theory of grain growth is suggested involving a double layer about individual grains through hydration of imperfectly oriented ions of silver bromide and of the solubilizing potassium bromide. This explains the effective dispersion of grains during Ostwald ripening and subsequent coalescence, orientation, and recrystallization.

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* Communicated by the Director.

[1] Communication No. 357 from the Kodak Research Laboratories and published in the *Colloid Symposium Monograph*, 6: 265. 1928.

THE FOGGING BY ACIDS AND OXIDIZING AGENTS AND THE INTENSIFICATION OF PHOTOGRAPHIC LATENT IMAGE.[2]

E. P. Wightman.

FURTHER arguments are given in support of the viewpoint of Wightman (*Brit. J. Phot.*, **74**, 447 (1927)) that the cause of latent fog formation from the action of certain oxidizing agents and acids is probably caused by the formation of bromine from the soluble bromide in the plate, with which hypothesis Lüppo-Cramer is in disagreement (*Z. wiss. Phot.*, **25**, 121 (1928); *Phot. Korr.*, **64**, 49 (1928)). The hypothesis was originally based on the fact, discovered by Lüppo-Cramer, that very dilute bromine solution causes fog and on a previously proposed mechanism of latent image intensification and fog formation by hydrogen peroxide, further evidence for which is now presented.

[2] Communication No. 366 from the Kodak Research Laboratories and published in *J. Amer. Chem. Soc.*, **50**: 2923. 1928.

NOTES FROM U. S. BUREAU OF MINES.*

WORLD'S PRODUCTION OF COPPER INCREASING AT RAPID RATE.

THE world's production of copper during 127 years, since the beginning of the nineteenth century, amounts to more than 40,000,000 tons, according to the United States Bureau of Mines, Department of Commerce. In comparison, all previous production since the beginning of man's utilization of copper—estimated by some as less than a million tons—is relatively insignificant.

The important part played by the United States in supplying the world with copper is interesting and significant, it is pointed out in an economic study of the production of copper during the nineteenth and twentieth centuries, which has just been completed by C. E. Julihn and the Common Metals Division of the Bureau of Mines. The United States has produced more than 19,500,000 tons of copper, or 48 per cent. of the world's output since 1800, although the production of this country was negligible prior to 1850. The increase in rate of United States production from 1845 to 1917 was even greater than the increase in world production during the same period. During the six decades ending with 1920 the average increase per decade in world production was 60.1 per cent. During the same period the average increase in the United States production was 145.8 per cent.

The United States began to produce copper in the decade ending with 1850; the quantity produced was less than 1 per cent. of the world total. Following that decade, however, the increase was rapid up to the end of the nineteenth century, during the last decade of which the United States produced 52 per cent. of the world total. Since then the relation of the United States production to that of the world has remained fairly constant.

Of the world's output of copper since 1800, about 1¼ per cent. was produced in the 25-year period between 1801 and

* Published by permission of Director of the Bureau of Mines.

1825; 2½ per cent. between 1826 and 1850; 5¾ per cent. between 1851 and 1875; 18¾ per cent. between 1876 and 1900; and 63½ per cent. between 1901 and 1925. About 8¼ per cent. was produced during the years 1926 and 1927.

Compared with the first decade of the nineteenth century, world production of copper for the first decade of the twentieth century had increased forty fold, and sixty-seven fold for the succeeding 10 years. For more than 100 years there has been an average increase of 53 per cent. per decade in the rate of production. Only in the period 1919 to 1925 was acceleration of world production interrupted.

The past 127 years have witnessed a geographical shift in the production of copper. During the first half of the nineteenth century Europe produced 63 per cent. of the world's output; South America furnished 17 per cent., Asia 13 per cent., and North America 5 per cent. During the next 50 years the North American output was 37 per cent. of the world total; European production amounted to less than 30 per cent., South American 21 per cent., and the remainder was produced by Australasia, Asia, and Africa. In the first 25 years of the present century North America produced 66 per cent.; South America 11 per cent.; Europe 10 per cent.; and Africa, Asia and Australasia the remainder. Statistics for 1926–27 indicate that North America still retains first place in world copper production with 61 per cent., South America second with 18 per cent., Europe third with 8 per cent., Africa fourth with 7 per cent., and Asia fifth with 5 per cent.

The first important copper production in the United States came from the Lake Superior district in Michigan. This State continued to be the most important source until 1887, when it was surpassed by Montana. In 1907 Arizona became the leading producer and with the exception of one year has held that position to date. Utah became the fourth largest producer of copper in 1903 and in 1926 ranked second.

The results of the Bureau's study are given in Economic Paper No. 1, "Summarized Data of Copper Production." While the Bureau publishes annual reports of the production of the various industrial metals, detailed long-time records of production have not been available. For such studies the facts of production from at least the early years of the nine-

teenth century are desirable, as they serve to link the steadily expanding production of that century with the relatively small production of all previous centuries; but for most of the metals no such record is available. Details of world production of most metals previous to the middle of the last century are decidedly sketchy, and even subsequent to that time the record leaves much to be desired.

The present report on copper is the first of a proposed series of economic papers to be prepared by the Bureau's Common Metals Division, which will present similar analyses of long-time production of other metals such as lead, zinc, iron, chromium, manganese, nickel, aluminum, tin, gold, and silver. Another group of papers on resources is in contemplation, and eventually the Bureau hopes to present papers dealing with distribution of industrial flow and ultimate consumption. Economic papers discussing the opportunities for American citizens or corporations to exploit foreign mineral deposits will also be prepared.

DEVELOPMENT OF AN AMERICAN MINERAL PAINTS INDUSTRY.

A PROGRAM of investigation which promises to assist in the development of a substantial American industry in a field hitherto largely neglected is the study of the ochers and mineral pigments of this country being conducted by the United States Bureau of Mines, Department of Commerce, at its Northwest Experiment Station, Seattle, Washington. This study, which was first concerned with the ochers and mineral pigments of the Pacific Northwest States, is being extended to cover all the known ocher deposits in the country. Coöperation has been obtained from most of the state geological departments, and they are collecting samples and studying the occurrence of the pigments in their respective states. The value of an ocher depends principally on its color and the closer it approaches the golden yellow hue of the French ocher, the greater will be its value. When the original sand and impurities have been removed by washing in water, the ocher should have an impalpable fineness and an iron oxide content of 17 per cent. or more, and should be able to retain a large portion of its color strength when diluted with zinc oxide.

A report on "The Ochers and Mineral Pigments of the Pacific Northwest" is now in press. The report covers the occurrence; methods of purification; and tests of color, oil absorption, and dilution with zinc oxide. Yellow ocher from near Spokane, Washington, and red and brown pigments from other parts of the Northwest were studied. Other colors were produced by heating the ochers and siennas alone or after intimate mixing with manganese ore from the Olympic peninsula of Washington. Some of the ochers studied possess light yellow hues similar to the imported French ocher.

All interested parties are invited to send one-pound samples of ochers or other pigment materials to the Northwest Experiment Station, United States Bureau of Mines, Seattle, Washington, for preliminary classification tests.

EXTRACTION OF POTASH AND OTHER VALUABLE PRODUCTS FROM GREENSANDS.

AN investigation of some of the various processes proposed for the utilization of greensands, of which there are very large deposits along the Atlantic seaboard, is in progress at the Nonmetallic Minerals Station, of the United States Bureau of Mines, at New Brunswick, New Jersey. A bibliography of the literature on greensands has been completed and will be published in the near future. An economic survey of some of the proposed processes is also in course of preparation.

In the laboratory, experiments have been in progress for about two months on the so-called Charlton-Shreve process, involving the digestion of greensands with slaked lime at 200° C. The results of these experiments may be briefly summarized as follows:

Digestion for two hours of 1 part greensand with 0.9 parts of lime, and 5.0 parts of water at 200° C. and 225 pounds pressure in an autoclave resulted in 55 per cent. of the potash being rendered soluble. Digestion for four hours rendered soluble 67 per cent. of the potash. The greensand used contained 6.5 per cent. potash, and was ground as fine as is possible in a small ball-mill. These results agree reasonably well with those of previous investigators.

PATH OF THE STOCK IN BLAST FURNACES.

THE proper preparation each day of some 1,600 to 1,800 tons of iron ore, 500 to 600 tons of limestone, and 600 to 800 tons of coke in the upper 60 feet of a modern blast furnace depends upon contact with about 100,000,000 cubic feet of gas which during 24 hours moves counter-current to the foregoing quantities of descending stock. Equal descent of the stock and uniform gas flow would be ideal for producing maximum gas-solid contact.

For several years the United States Bureau of Mines has been accumulating at its North Central Experiment Station and in the field information on the path of materials in blast furnaces. Gas samples from the tuyeres of 13 furnaces furnished the first definite information that the stock was likely moving faster along the furnace periphery than in the center. Faster movement of stock along the walls seemed a logical result of localized combustion, disclosed by gas samples removed through the tuyeres. Measurements of gas velocities at several levels of an industrial furnace, together with fundamental laboratory data on the flow of gas through beds of broken solids, made it possible to calculate the size distribution of stock across several planes. Such calculations show that the particle size is more uniform across lower levels than at the stock line. As the materials are charged they segregate according to particle size. Mixing of sizes takes place as the stock descends. Particles in the center of the furnace move laterally toward the line of fastest movement, just as a piece of driftwood moves toward the position of a stream where the current is moving most rapidly.

Faster movement of stock along the periphery of the furnace has been further substantiated by stock flow measurements in a furnace. The average rate for 3 points next to the wall was 3 inches per minute, while the average rate near the center was 2.1 inches per minute. Other data have been obtained on conditions inside of the furnace. Such information is needed to point the way toward greater smelting efficiency.

Cod Liver Meal. Cod liver meal is a by-product of the manufacture of cod liver oil, and is used as a feedstuff. R. M. BETHKE, G. ZINGALIAN, D. C. KENNARD, and H. L. SASSAMAN of the Ohio Agricultural Experiment Station (*Jour. Agric. Research*, 1928, *36*, 747–753) have made a series of biological tests of cod liver meals prepared from fresh cod livers after extraction of the oil. They used chickens and rats as experimental animals, and studied particularly the antirachitic value of the meals; this is a measure of the content of fat-soluble D vitamin.

Several samples of the meal showed differences in chemical composition and in antirachitic value. This value was not proportional to the fat content of the meal; and the ether extract or fat obtained from the meal was less potent than ordinary cod liver oil. The conclusion is drawn that cod liver meal should not be used as a substitute for cod liver oil as an antirachitic in the production of poultry and livestock.

J. S. H.

Benzene Poisoning as a Possible Hazard in Chemical Laboratories. J. J. BLOOMFIELD of the U. S. Public Health Service (*Pub. Health Rep.*, 1928, *43*, 1895–1897) suggests that benzene poisoning may occur in laboratories where benzene (benzol) is used as a solvent in the analysis of such materials as rubber, paint, varnish and oil products, and in the cleaning of apparatus, or the removal of stains from the skin. In industrial plants, where the exposure to benzene vapor is continuous, a concentration of benzene as low as 100 parts per million of air is a substantial hazard. However, in laboratories, the use of benzene usually is intermittent. In chronic benzene poisoning, the leucocyte or white cell count of the blood is reduced to a considerable extent, the count falling to values below 5,000 per cubic millimeter. The differential leucocyte count, or the relative numbers of the various types of white cells, also undergoes changes; the per cent. of lymphocytes shows a relative increase, and the per cent. of polymorphonuclear cells a marked decrease. It is recommended that benzene be used only as a solvent, never as a cleansing agent. The further recommendation is made that it be replaced as a solvent by less toxic compounds, such as toluene, xylene, or high-flash naphtha. The health of the laboratory workers should be checked by means of blood counts made at intervals of one or two months.

J. S. H.

THE FRANKLIN INSTITUTE

STATED MONTHLY MEETING, DECEMBER 19, 1928

THE stated monthly meeting of the Institute was called to order at eight-twenty-five by the Secretary, Dr. Howard McClenahan.

The Chairman announced that this was the regular monthly meeting and stated that the minutes of the preceding meeting had been printed in full in the December number of the Journal of the Institute. He requested that these minutes be approved as printed. Upon motion the minutes were so approved.

The Chairman announced that since the last meeting, four Non-Resident members had been elected to the Institute and announced also, with regret, the death of three members.

The Chairman stated that nominations for the election of officers for the ensuing year were in order at this meeting, and called for such nominations. He quoted the By-law of the Institute governing such nominations, and then presented the following names:

Office	Nominee	Term	Nominated by
President	Mr. Nathan Hayward	One year	Mr. Henry Howson Mr. S. T. Wagner Mr. J. S. Rogers Mr. C. A. Hall
Vice-President	Mr. C. C. Tutwiler	Three years	Mr. E. H. Sanborn Mr. W. G. Littleton
Treasurer	Mr. Benjamin Franklin	One year	Dr. J. S. Hepburn Mr. R. W. Lesley
Board of Managers	Dr. James Barnes Mr. G. H. Benzon, Jr. Mr. G. H. Clamer Mr. Theobald F. Clark Dr. A. W. Goodspeed Mr. Charles Penrose Dr. G. D. Rosengarten Mr. W. C. Wetherill	Three years	Dr. G. A. Hoadley Mr. L. F. Levy Mr. S. Shoemaker

The Chairman then presented the resignation of Mr. Nathan Hayward from membership in the Board of Managers and requested that it be accepted. It was then moved and seconded that this action be taken and the motion was unanimously carried.

The Chairman stated that a member of the Board of Managers should be nominated at this time, to serve for the unexpired term of Mr. Hayward, and presented the name of Mr. Malcolm Lloyd of Philadelphia, he was nominated by Mr. Morris L. Clothier and Mr. James L. Whitaker.

The Chairman called for other nominations for any of the offices or for membership in the Board of Managers. No other nominations being offered, the Chairman declared the nominations closed and stated that a ballot would be prepared in accordance with the nominations which had been offered, for submission to the membership of the Institute, in advance of the January monthly meeting.

The Chairman stated that two John Scott Medals and Awards were to be presented at this meeting, to Dr. H. D. Arnold of the Bell Telephone Laboratories, Inc., and to Mr. James C. McBride of the Worthington Steam Pump Company, members of The Franklin Institute, and to this end introduced Mr. Louis Heiland, Secretary of the Board of Directors of City Trusts, to take charge of the meeting during these presentations.

Mr. Heiland called upon Mr. McBride and presented him with the Certificate of Award, the John Scott Medal, and a Premium of one thousand dollars, for the development of the locomotive feed water heater.

He then called upon Dr. H. D. Arnold and presented him with the Certificate of Award, the John Scott Medal, and a Premium of one thousand dollars, for the development of the three electrode high vacuum thermionic tube.

Proper acknowledgments of the presentations were made by the two recipients of them. Both Dr. Arnold and Mr. McBride are members of The Franklin Institute and had requested that the presentations be made before the Institute.

The Chairman, Dr. McClenahan, then resumed the Chair, and presented as the lecturer of the evening Dr. Hermann J. Muller, of the University of Texas, who spoke on "The Enigma of the Gene and of Its Mutations." Dr. Muller spoke delightfully concerning his subject, telling of his own important researches in this field of thought. The lecture was followed by an animated discussion for about a half hour, after which the meeting adjourned with a vote of hearty thanks to Dr. Muller for his interesting talk.

HOWARD MCCLENAHAN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, December 5, 1928.)

HALL OF THE INSTITUTE,
PHILADELPHIA, December 12, 1928.

DOCTOR GEORGE S. CRAMPTON *in the Chair.*

The following reports were presented for final action:

No. 2895: Masonite.

The device considered in this report is for the production of disintegrated wood fibers and their utilization in the manufacture of useful materials. The fibers are made by running the waste wood from sawmills through a machine that reduces it to small chips and by inclosing them in a hollow cylinder capable of withstanding high pressure.

Superheated steam is introduced into the cylinder and after a sufficient time for the steam to penetrate and soften the chips a gate is opened in the base of the cylinder and this is of such a shape that when the steam pushes out the chips they are compressed and the pressure being removed suddenly, the absorbed steam explodes the wood into its fibers—the result being a fibrous mass.

This is made into felted sheets from a water bath and the excess water is removed by passing these sheets between rollers or by drying. They are then cut into convenient lengths and placed between the steam heated upper and lower platens of a press.

The resulting product has a smooth surface, the thickness and density of the inner portion depending upon the temperature of the platens, the pressure employed and the length of the treatment.

The material obtained by this process is either "Masonite," an insulation board about a half inch in thickness having dense surfaces and fibrous interior, or "Presdwood," a much thinner board of uniform density and having surfaces that can be finished by standard methods. Both the products are in extensive use for construction and industrial application.

The John Price Wetherill Medal was recommended to Mr. William H. Mason, of Laurel, Mississippi, "in consideration of his process for the successful use of wood waste."

No. 2897: The Work of Mr. W. H. Gartley in the Gas Industry.

This report gives, in detail, a statement of Mr. Gartley's forty-five years devoted to the gas industry, calling attention to his inventions of devices that have been of direct application in the industry, to his work as chief engineer of the Philadelphia Gas Works and to his papers presented before the American Gas Institute.

He was awarded the Beal Medal by this Institute for his paper on the "Delivery of Uniform Candle Power to the Consumer," this award being for "the best paper at the American Gas Institute Meeting of 1906."

During the World War he was a member of the Committee on Coal Tar By-Products for the Chemical Committee of the Council of National Defense. His expert knowledge of the manufacture of toluol was of great service to the War Industries Board as was also the production of toluol by the Gas Works of Philadelphia under his engineering operation.

The award of the Walton Clark Medal was recommended to Mr. W. H. Gartley "In consideration of his engineering contributions to the advancement of the gas industry, to the development and practical operation of large city gas works, of his ingenuity, sound judgment, and service to the public in his field of work."

The following report was presented for first reading:

No. 2879: Temperature Control in Gas Furnaces.

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, December 12, 1928.)

NON-RESIDENT.

MR. HOWARD W. AMBRUSTER, Consultant and Importer, 261 Broadway, New York City. For mailing: 597 Highland Avenue, Westfield, N. J.

MR. CARL T. HOLMQUIST, Vacuum Tube Engineer, 1850 Otto Street, Chicago, Illinois. For mailing: 529 North Leclaire Street, Chicago, Illinois.

MR. MALCOLM M. HUBBARD, Student—Massachusetts Institute Technology, Cambridge, Mass. For mailing: 8 Cottage Street, Newport, R. I.

DR. J. C. MCLENNAN, Director of Physical Laboratory, Professor of Physics University of Toronto, Toronto, Canada.

CONTRIBUTING.

Representing American Engineering Corporation.

MR. H. WELLS, Llanerch, Pa.

Representing Bell Telephone Company.

MR. D. S. HILBORN, Engineering Department, Bell Telephone Company, 1835 Arch Street, Philadelphia, Pa.

CHANGES OF ADDRESS.

MR. MILTON W. DEISLEY, 1829 West Venango Street, Philadelphia, Pa.

MR. J. E. EGLESON, Old Chester Road, Essex Fells, N. J.

MR. CHARLES HART, Wrought Iron Company of America, 732 Commercial Trust Building, Philadelphia, Pa.

MR. ERNEST L. HOPPING, S. W. corner Ninth and Sansom Streets, Philadelphia, Pa.

DR. CHARLES H. HERTY, 101 Park Avenue, Room 1834, New York City.

MR. ROBERT JOB, 980 St. Antoine Street, Montreal, Quebec, Canada.

MR. MICHAEL MELODY, 506 East Durham Street, Mt. Airy, Philadelphia, Pa.

MR. JAMES WARNOCK, JR., 511 Chestnut Street, Philadelphia, Pa.

NECROLOGY.

Mr. Strickland L. Kneass, Philadelphia.

Mr. T. P. Chandler, Radnor, Pa.

Mr. J. Snowden Bell, New York City.

LIBRARY NOTES.

RECENT ADDITIONS.

American Mathematical Society. Bulletin. Volumes 1–33. Thirty-three volumes. 1895–1927.

American Mathematical Society. Transactions. Volumes 1–29. Twenty-nine volumes. 1900–1927.

ARMITAGE, F. P. A History of Chemistry. 1912.
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BRAGG, SIR WILLIAM H. The Structure of an Organic Crystal. Fison Memorial Lecture, 1928.
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CHATFIELD, CHARLES HUGH, AND CHARLES FAYETTE TAYLOR. The Airplane and Its Engine. First edition. 1928.
COHEN, LOUIS. Heaviside's Electrical Circuit Theory. 1928.
Colloid Symposium Monograph. Volume 6. 1928.
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GEIGER, H., AND K. SCHEEL, editors. Handbuch der Physik. Band 6. 1928.
GIBSON, CHARLES R. Modern Conceptions of Electricity. 1928.
HACKH, INGO W. D. Chemical Reactions and Their Equations. Second edition, revised. 1928.
Handbook Encyclopedia of Engineering. First edition. 1928.
HOOVER, HERBERT C. Principles of Mining. First edition. 1909.
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INGALLS, ALBERT G., editor. Amateur Telescope Making. 1928.
LIBBY, WALTER. An Introduction to the History of Science. 1917.
LIDDELL, DONALD M., editor. Handbook of Non-Ferrous Metallurgy. Prepared by a staff of specialists. Two volumes. First edition. 1926.
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LODGE, SIR OLIVER. Ether and Reality. 1925.
MOYER, JAMES A., AND RAYMOND U. FITTZ. Refrigeration. Including Household Automatic Refrigerating Machines. First edition. 1928.
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PURDAY, H. F. P. Diesel Engine Design. Third edition. 1928.
RICHTMYER, F. K. Introduction to Modern Physics. 1928.
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SEYMOUR, HENRY. The Reproduction of Sound. No date.
SOTHERN, J. W. M. Marine Diesel Oil Engines. Third edition. 1928.
STUDLEY, BARRETT. Practical Flight Training. 1928.
SUMNER, P. H. The Science of Flight. Two volumes. 1926–1929.
THOMSON, SIR J. J. Beyond the Electron. 1928.
WHYTE, CHARLES. Our Solar System and the Stellar Universe. 1923.
Wien-Harms Handbuch der Experimentalphysik. Volume 7, Pt. 1; Volume 13, Pt. 2; Volume 15; Volume 18; Volume 23, Pt. 2. Five volumes. 1928.
WILES, ROBERT. Cuban Cane Sugar. 1916.

BOOK REVIEWS.

MOLECULAR REARRANGEMENTS. By C. W. Porter, Professor of Chemistry in the University of California. 167 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1928. Price, $4.

Research is steadily penetrating into the intimate structure of matter. The ultra-microscope with its interesting revelation of colloidal phenomena is now very familiar. In another direction we have the study of atomic arrangements by means of x-rays, these investigations having brought about many changes in our views as to the molecular structure of compounds. A still more intimate study is as to the transformations that occur within a given molecule by rearrangement of its atoms. Often such investigations proceed irregularly and encounter scornful treatment by those who should consider the matter carefully and impartially. The history of science shows a good many such arbitrary condemnations. When Newlands presented to the Chemical Society his suggestions as to the significant relations of the atomic weights of certain groups of elements he was criticised severely, and his communication was refused publication in the Society's Journal. Yet to-day the arrangement of the elements according to atomic weights is one of the most prominent features in chemistry. In the preface of this book we are told that certain suggestions in regard to internal rearrangements were derisively disapproved as the "flip-flop theory."

Wöhler's experiment showing the conversion of ammonium cyanate into urea was the first observed case of molecular rearrangement but it was more particularly famous as the first production from inorganic sources of a compound then known only as derived from living organisms. The text of this work shows a very large number of transformations of this type, but of course many far more complicated than Wöhler's. The phenomena are discussed in seven chapters and all the changes are amply illustrated by structural formulas. The work constitutes therefore, a very comprehensive presentation of inter-molecular transformations. There are, of course, border-lands which may be disputed areas, but the author has been careful to keep within certain specific lines and has presented a very interesting and valuable summary of some of the special developments of organic chemistry.

HENRY LEFFMANN.

AMATEUR TELESCOPE MAKING. Albert G. Ingalls, Editor, with contributions by Russell W. Porter, Prof. Charles S. Hastings, Ph.D., Rev. William F. A. Ellison, F.R.A.S., Dr. George Ellery Hale, Sc.D., Dr. Elihu Thomson, Ph.D., Clarendon Ions, John M. Pierce, and A. W. Everest. With a foreword by Dr. Harlow Shapley. 285 pages, illustrations, 8vo. New York, Scientific American Publishing Company, 1928. Price, $3.

The productions of amateurs in a diversity of fields are often of a quality that might easily earn their originators recognition in professional ranks, and the value of amateur work, not alone as a diversion of absorbing interest but as a real aid in the choice of a professional career has long been widely appreciated. The Scientific American which may rightfully be termed the dean of popular scientific journalism in this country, for several generations has been at the forefront in fostering the interest of amateur devotees of science as well as providing a valuable

fund of information on progress in every branch of science. One highly interesting activity of this enterprising journal in recent years is the issue of a number of articles which deal with the construction of astronomical reflecting telescopes of small size adapted in design to the requirements and capacity in craftsmanship of amateurs. In harmony with the interest aroused in the subject by these articles, the Editor was induced several years ago to seek the coöperation of the noted authors whose names appear on the title-page of this volume, in the contribution of articles covering every phase of the construction of reflecting telescope for compilation into book form. The present treatise, now in its second edition is the result.

Consisting of separately written sections, the book necessarily lacks the continuity of a formal text. There is also a certain amount of repetition which, however, is helpful rather than detrimental in emphasizing critical processes by discussion from more than one point of view. Part I by Russell W. Porter and Part II by the Rev. William F. A. Ellison are veritable condensed treatises on the subject. Detailed directions are given for the process of grinding the concave mirror by means of the pitch lap and the foucault-knife-edge test for determining its accuracy. Equally minute directions are to be found for the production of prisms, flats and making the lenses of eye-pieces as well as the mounting for these elements. Methods of silvering glass receive the full treatment which its importance deserves. The material in Part II is reprinted from Doctor Ellison's The Amateur Telescope, an English publication. Among the special contributions of the different authors may be mentioned: silvering, teloscopic oculars, grinding and polishing machines, special designs of telescope, solar phenomena and the spectrohelioscope and an extensive collection of miscellaneous data. The book as a whole is a unique collection of useable information not available elsewhere in a single work.

LUCIEN E. PICOLET.

ANNUAL SURVEY OF AMERICAN CHEMISTRY, Volume 3. Prepared under the auspices of the Division of Chemistry and Chemical Technology National Research Council. Edited by Clarence J. West, Director, Research Information Service National Research Council. 395 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1928. Price, $3.

The good work still goes on actively. The development of American chemical research in the years immediately following the war was an encouraging feature, but there was a fear that as the necessity for such research became less as far as regards the general public, there would be a cessation of activity. This seems so far not to have occurred. Three volumes embodying the principal results of American investigation have now appeared, and it would seem from the present treatise that there is no abatement in the zeal in which the work is being carried on. Any one who knows the value of systematic research will be gratified by this fact. The arrangement of the present volume is similar to that of the preceding ones. The editorial board has adopted the policy of varying somewhat the subjects included, by which certain less active fields will be only occasionally covered. The authors of the essays also will be different in different years, thus giving greater variety to the method of treatment and to the topics presented. When the first volume was issued there were some unfavorable comments, probably mostly by

those who are strongly impressed with the belief that science should be international, and that a summary limited strictly to one nation is inadvisable. The support with which the first two volumes have been met indicates that the system is much favored. It is to be hoped that the volumes will continue, as they will serve not only for the comprehensive report of what is being done in chemistry in its many departments in this country, but as a stimulation to further extension of such research.

The volume is excellently printed and it contains a very large amount of information.

HENRY LEFFMANN.

INTRODUCTION TO MODERN PHYSICS. By F. K. Richtmyer, Professor of Physics at Cornell University. First Edition XV–596 pages, 6 plates, 169 figures, 8vo. cloth. New York, McGraw-Hill Book Company, Inc., 1928. Price, $5.

In this Introduction to Modern Physics the author strives to acquaint us with the prevailing theories in much the same manner as we employ in becoming acquainted with our best friends. Meeting a theory or a person for the first time, we are filled with respect and admiration for their ability and accomplishments. On the other hand, a feeling of intimacy and deep-seated interest towards such entities comes only from a knowledge of their background. Through a study of their past we can follow the slow but steady development from almost nothing at all and the overcoming of this and that difficulty thus acquiring a deeper appreciation of the person or a more complete recognition of the significance of some theory.

The author announces that the book was written for a pedagogica purpose. As such he has employed true pedagogic methods in presenting the subject matter to its readers. Here and there a personal touch may be found taking the form of an epigrammatic expression or an italicized word furnishing a glimpse of the writer's dry humor. All these serve to whet the interest of the student and promote a friendly and receptive attitude toward such phases of scientific knowledge.

In this particular case the term "modern physics" includes those discoveries and theories which have come into being since 1890. In the earlier part of this present period, progress in the delevopment of physics was punctuated by the photoelectric effect, the x-rays, the electron, radio-activity and evolution of the quantum theory. The author quotes: "Indeed, the reconciliation of these two theories, (electromagnetic and quantum) each based on incontrovertible experimental evidence, may be said to be one of the two great problems of modern physics; the other problem being that of the structure of matter."

In keeping with plan of presentation the first three chapters respectively are: Historical Sketches of the First Period: Earliest Times to 1550 A.D.; Second Period (1550–1800 A.D.); The Rise of the Experimental Method and Third Period (1800–1890 A.D.); The Rise of Classical Physics. The chapter on the electromagnetic theory of light should be of particular interest since the author presents in a logical manner the development of the corresponding theoretical equations from the elementary laws of mechanics and integral calculus The first element of doubt concerning the universal applicability of the electromagnetic theory of light is presented in the chapter on the photoelectric effect. A chapter

on the origin of the quantum theory follows which is outstanding for its clear, direct and logical treatment of the subject. With the advent of the quantum theory the way is now open for an interpretation of various observed phenomena in the light of this theory. Among those considered are specific heats, series relations in line spectra, the origin of spectral lines, the nuclear atom, arrangement of electrons in atoms and X-rays. The last chapter is given over to consideration of the second great problem of modern physics, i.e. the structure of matter. Appendix I supplies complete and valuable information concerning the elements, giving the atomic numbers, atomic weights as well as number and weight of the isotopes. Appendix II gives the arrangement of electrons in orbits according to Foote while III contains a list of some important constants.

Of any book in physics written for the purpose of interesting the student in the subject this one should accomplish its purpose with ease. Fortunately Dr. Richtmyer did not turn a deaf ear toward his importune former students and other friends.

T. K. CLEVELAND.

CHEMICAL REACTIONS AND THEIR EQUATIONS. A guide for students of chemistry. By Igno W. D. Hackh, Professor of chemistry, College of Physicians and Surgeons, San Francisco. Second edition, revised. 145 pages, illustrations. 8vo. Philadelphia, P. Blakiston's Sons & Company, 1928. Price, $2.

All who have had experience in teaching chemistry will heartily agree with the statement in the preface of the first edition to this book that students are particularly deficient in ability to balance equations, even simple equations are often turned in unbalanced. A further difficulty is a lack of care in calculations and a general unwillingness to check up work or to examine the calculation carefully to see whether the equation is rational. One is not surprised that the author of this book has found even graduate students at a great university unable to work out a moderately complicated equation. Students of the first and second year are generally very poor in this field. The present work is a summary of the principles of chemistry especially directed to the treatment of chemical reactions. There is no broad discussion of the great principles of chemistry or physical chemistry but there is a very large amount of illustrating material which will serve very well to help students in what is found to be by experience a difficult part of the study of chemistry.

HENRY LEFFMANN.

SOLUBLE SILICATES IN INDUSTRY. By James G. Vail. American Chemical Society Monograph Series. 443 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1928. Price, $9.50.

This is an American Chemical Society Monograph issued by the Chemical Catalog Company, constituting No. 46 of that series, which has already covered in a most excellent manner a wide field of applied chemistry and brought much credit to American science. A Philadelphia reviewer may be pardoned for reacting pleasantly to the fact that the author is of that city, for recent publications in the field of American chemistry have shown few instances in which it has appeared in conspicuous position in either the pure or applied department of the science.

Silicon is one of the most abundant elements of the earth's crust and its oxide

is presented to us in a great variety of forms from the brilliant quartz crystals, and amorphous forms so prized by collectors, to common sand. In one state, the typical crystalloid, the substance that gave the word to our language,—in the other state a typical colloid it finds many applications in pure and applied chemistry.

The volume in hand relates only to the commercial silicates, of which, of course, sodium silicate is the most prominent, and it is still further limited to the soluble salts, so that many products such as glass and porcelain are not mentioned. The author's long and active association with a company manufacturing soluble silicates has given him extensive and accurate knowledge of the manufacturing processes and the many uses of the products. The book is liberally provided with illustrations of apparatus, graphs, and tables. Turning over its pages one is much impressed with the high development to which the manufacture of these silicates has been brought and also the very large number of applications that they have obtained. The comprehensiveness of the work is shown by the fact that space is devoted to considerations of the relations of the compounds to the public health, including a brief note of a case of non-fatal poisoning by sodium silicate.

Typography, press-work and general make-up are in accordnace with the excellent standard of the publishing company and the text is clearly and concisely written.

HENRY LEFFMANN.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 292. Characteristics of Five Propellers in Flight, by J. W. Crowley, Jr., and R. E. Mixson. 23 pages, illustrations, quarto. Washington, Government Printing Office, 1928. Price, fifteen cents.

This investigation was made by the Committee at Langley Field for the purpose of determining the characteristics of five full-scale propellers in flight. The equipment consisted of five propellers in conjunction with a VE 7 airplane and a Wright E-2 engine. The propellers were of the same diameter and aspect ratio. Four of them differed uniformly in thickness and pitch and the fifth propeller was identical with one of the other four with the exception of a change of the airfoil section. The propeller efficiencies measured in flight are found to be consistently lower than those obtained in model tests. It is probable that this is mainly a result of the higher tip speeds used in the full-scale tests. The results show also that because of differences in propeller deflections it is difficult to obtain accurate comparisons of propeller characteristics. From this it is concluded that for accurate comparisons it is necessary to know the propeller pitch angles under actual operating conditions.

R.

PUBLICATIONS RECEIVED.

Industrial Chemistry, An Introduction. An elementary treatise for the student and general reader, by Emil Raymond Riegel, Ph.D. 649 pages, illustrations, portrait, 8vo. New York, The Chemical Catalog Company, Inc., 1928, price $9.00.

A Survey of Nitrocellulose Lacquer, by Bruce K. Brown and Francis M. Crawford. 368 pages, 8vo. New York, The Chemical Catalog Company, Inc., 1928, price $7.50.

Outlines of Paint Technology, by Noël Heaton. 400 pages, illustrations, table, 8vo. London, Charles Griffin & Company, Limiced; Philadelphia, J. B. Lippincott Company, 1928, price $8.00.

Enamels: Their Manufacture and Application to Iron and Steel Ware, by Laurence R. Mernagh. 234 pages, illustrations, plates, diagrams, tables, 8vo. London, Charles Griffin & Company, Limited; Philadelphia, J. B. Lippincott Company, 1928, price $7.00.

Amateur Telescope Making, by Albert G. Ingalls, Editor. 285 pages, illustrations, 12mo. New York, Scientific American Publishing Co., 1928, price $3.00.

Colloid Symposium Monograph Papers presented at the Sixth Symposium on Colloid Chemistry, University of Toronto, June 1928, edited by Harry Boyer Weiser. 346 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1928, price $6.50.

Molecular Rearrangements, by C. W. Porter. American Chemical Society Monograph Series. 167 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1928, price $4.00.

Electrical Engineering Laboratory Practice, by Oskar E. Edison, E.E., and Ferris W. Norris, E.E. 276 pages, illustrations, 8vo. Boston, Ginn and Company, 1928, price $2.80.

Annual Survey of American Chemistry, Vol. III, July 1, 1927, to July 1, 1928, prepared under the auspices of the Division of Chemistry and Chemical Technology, National Research Council. Edited by Clarence J. West, Director Research Information Service, National Research Council. 395 pages, 12mo. New York, The Chemical Catalog Company, 1928, price $3.00.

La Physique de la Gravitation et la Dynamique de L'Univers, par Thomas Tommasina. 300 pages, potrait, quarto. Paris, Gauthier-Villars et Cie., 1928.

Soluble Silicates in Industry, by James G. Vail. American Chemical Society Monograph Series. 443 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1928, price $9.50.

Lichtelektrische Erscheinungen, von Bernhard Gudden. 325 pages, illustrations, 8vo. Berlin, Julius Springer, 1928, price in paper, 24 marks.

National Advisory Committee for Aeronautics. Technical Notes, No. 296, Bearing Strength of Wood under Steel Aircraft Bolts and Washers and Other Factors Influencing Fitting Design, by G. W. Trayer. 25 pages, tables, diagrams, quarto. No. 299, The Effect of Fillets between Wings and Fuselage on the Drag and Propulsive Efficiency of an Airplane, by Melvin N. Gough. 4 pages, plate, diagrams, quarto. No. 300, The Variation in Pressures in the Cockpit of an Airplane in Flight, by Thomas Carroll and William H. McAvoy. 6 pages, illustrations, quarto. No. 301, Drag and Cooling with Various Forms of Cowling for a "Whirlwind" Engine in a Cabin Fuselage, by Fred E. Weick. 25 pages, plates, quarto. Washington, Committee, 1928.

CURRENT TOPICS.

Airplane Sign Posts. The Journal has been asked to coöperate with The Daniel Guggenheim Fund for the Promotion of Aëronautics in a nation-wide campaign for roof-markings which will identify the country's towns to the aviator. Such identification is a first requisite for an air transportation system, which will heighten the efficiency of American business by swifter means of communication.

In asking that industrial corporations should assist in roof-marking their buildings, where practical, Harry F. Guggenheim states: "Sign posts of this kind for the airplane are an absolutely essential item in the safety of air transportation. The need for them has been repeatedly stressed to the Fund by Colonel Charles A. Lindbergh as a result of his experience during his United States tour a year ago, and in subsequent cross-country flying. In the opinion of Colonel Lindbergh, who is a technical adviser to the Fund, this identification represents one of the most worth while steps that can be taken for the advancement of civil aviation. His opinion is shared by other pilots from coast to coast and has the emphatic endorsement of all those interested in aëronautics."

As a basis for the work, the Fund, supported by the Postmaster General, has asked some 8,000 postmasters in towns with a population between 1,000 and 50,000 to take the initiative in seeing that their communities are identified by roof-markings. The postmaster is urged to enlist the support of civic organizations such as the American Legion, the service clubs, or the Chamber of Commerce, or individual corporations. To the organization or person responsible for the identification the Fund has asked Colonel Lindbergh to send a certificate of appreciation. The record of roof-markings will be kept by the Fund and made available to the Department of Commerce, the Post Office Department and other government agencies. The movement has the backing of the Departments of War, Navy and Commerce.

Many corporations throughout the country, a number of them in this industry, have already been active in marking the roofs of their own buildings for purposes of aërial navigation. This is not only a civic act helpful to the community in which the corporation is located; it is also of direct benefit to business as a whole, since it is the latter which will benefit particularly from a commercial air transportation system. The Fund asks that a corporation with an available roof in an unmarked town of between 1,000 and 50,000 population should communicate directly with the postmaster.

In a printed bulletin the Fund describes the type of roof most suitable for marking and urges the use of block letters in chrome yellow with a black background. It is suggested that it should be from 10 to 20 feet in height and on roofs preferably of tiled shingles, tin or other metal, or slate where the visibility is not reduced by smoke. Besides the name of the town or city, the marking should include an arrow pointing due north with the letter "N" over it and a smaller letter indicating the airport if there is one.

Effect of Magnetic Fields upon Thermal Conductivity of Iron, Gold, Copper, Silver and Zinc. (*Physical Review*, Sept., 1928.) It is for the ferro-magnetic metals, iron and nickel, that the effect of magnetic fields has been chiefly studied, though much work has been done on bismuth and tellurium. For these the application of a field decreases the thermal conductivity.

A "bar method" was used, the thermal conductivity being given by the temperatures of two points on the bar. The bars of metal, slightly longer than 10 cm. and of cross-sectional area ranging from .051 to .086 sq. cm., were heated by an electric current. This was alternating for all except iron for which a direct current was used on account of the skin effect. Both ends of the bars as well as the walls of the vessel surrounding the bars were maintained at the temperature of melting ice. Thermo-couples, spot-welded to the metal of the bars, gave the temperatures. The current was adjusted so that a parabolic distribution of temperature was attained. The electro-magnet had a 10.7 cm. gap and, when supplemented by a solenoid about the gap, produced a field of 10,000 gauss. The solenoid carried 800 amperes and was oil-cooled. This was for the longitudinal field and could be used with no time limit. The transverse field of 8,000 gauss was obtained by using different pole tips without the solenoid.

Not only the thermal but also the electrical conductivity was measured. The application of a transverse field of 8,000 gauss causes no change in either the thermal or the electrical conductivity of iron, copper, silver, gold or zinc. A transverse field half as strong reduces the thermal conductivity of iron by .4%. No effect upon either conductivity due to the longitudinal field of 10,000 was found for silver, gold and zinc. This field caused an increase in the electrical conductivity of iron and copper amounting to .2 per cent. and .04 per cent. respectively. Both of these metals by the same field suffered losses of thermal conductivity amounting to 1.14 and .23 per cent. respectively. A single crystal of zinc showed no behavior different from ordinary zinc.

G. F. S.

The Physics of the Universe. SIR JAMES JEANS. (*Nature*, Supplement, Nov. 3, 1928.) This was the first Henry Herbert Wills lecture and was delivered at the University of Bristol on Oct. 30, 1928. Quite apart from its scientific value it is an unusual production by reason of its felicitous phrases and pungent illustrations.

Even after geologists came to study the earth as a changing structure and astronomers learned to speculate on the evolution of the stars man still continued to believe the atoms to be immutable. This view persisted till almost the close of the nineteenth century. "Like a child's box of wooden bricks, the atoms made many buildings in turn. Then Crookes, Lenard, and, above all, Sir. J J. Thomson, began to break up the atom." In 1895 Sir J. J. Thomson showed that the fragments chipped off from no matter what atoms, the electrons, are identical, possessing the same mass and equal negative electrical charge. Two years later Lorentz, in explaining the Zeeman effect, made it clear that the moving parts in atomic structures are the same electrons. To Rutherford is due the credit of furnishing a model of the atom that has served for decades to coordinate the facts. A central nucleus with a positive charge is surrounded by negatively charged electrons moving in distant orbits. "The volume of space kept clear by the electrons is enormously greater than the total volume of the electrons; roughly, the ratio of the volumes is that of the battlefield to the bullets. The atom, with a radius of about 2×10^{-8} cm., has about 100,000 times the dimensions, and so about 10^{15} times the volume, of a single electron, of which the radius is about 2×10^{-13} cm. In all probability the 'nucleus' is even smaller than the electrons. The number of orbital electrons in an atom is called the 'atomic number' of the atom." Then the investigations of radioactivity initiated by Becquerel led to the conclusion that the nucleus itself was not an unchanging thing. Given adequate time the nucleus of a uranium atom spontaneously changes into the nucleus of a lead atom and eight alpha particles each of which is the nucleus of a helium atom. With the exceptions of potassium and rubidium radioactivity resides only in the heavy elements, those having atomic numbers larger than 83. Though the lighter elements do not spontaneously disintegrate their atoms can be broken up as Rutherford showed in 1920 by bombarding oxygen and nitrogen with rapid alpha particles. In consequence a vigorous hypothesis exists to the effect that "the whole universe is built up of only two kinds of ultimate bricks, namely, electrons and protons. Each proton carries a positive charge which is exactly equal in amount to the negative charge carried by an electron. The protons are supposed to be identical with the nucleus of the hydrogen atom; all other nuclei are supposed to consist of closely packed

structures of protons and electrons." The atom, however, contains, besides electrons and protons, a third ingredient, electromagnetic energy. "All radiation carries mass about with it, one gram of mass being associated with 9×10^{20} ergs or 2.15×10^{13} calories of radiation. As a necessary consequence, any substance which is emitting radiation must also be losing mass. The ultimate fate of a gram of uranium may be expressed by the equation.

$$1 \text{ gram uranium} = \begin{cases} 0.8653 \text{ gm. lead.} \\ 0.1345 \text{ gm. helium.} \\ 0.0002 \text{ gm. radiation."} \end{cases}$$

The resulting lead and helium contain just the same protons and electrons as the original uranium. On the earth at least no transformation of radiation into mass is ever found by way of compensation for the change of mass into radiation. The development of heat in the earth due to radioactive processes causes our planet to radiate at a rate that means a loss of mass from it equal to about an ounce a minute.

The Origin of Terrestrial Radium and Uranium.—The average life of radioactive atoms varies from billions of years down to thousandths of a sec. "If a whole earth were built of pure radium only a single atom would be left after a quarter of a million years. Since the earth is many millions of years old, we may be confident that every atom of radium now on earth was born on earth." Radium traces its ancestry back to uranium, which disintegrates slowly. It takes 5,000,000,000 years for this element to diminish to half its original mass. Less than half of this number of years ago the earth was formed from the sun so that most of the earth's stock of uranium at the beginning is now in existence as uranium on the earth. An initial store of about 10^{19} gm. would suffice."

The Ages of the Sun and Stars.—Maxwell showed how to investigate by theory the behavior of a gas whose molecules repelled one another with a force inversely as the fifth power of the distance. If molecules of different masses were present their velocities tended to assume values for which the kinetic energy was the same for all. This adjustment was due to the continuous action of their mutual repulsions. A similar mathematical treatment has been applied to the universe of stars which attract one another with a force inversely as the square of the distance. Here too a time will come when the kinetic energies of the stars will be the same no matter what the mass be and this equipartition of energy will be due to the action of gravitation. However millions of millions of years must elapse before such a time can arrive. A comparison of the kinetic

energies of the stars furnishes ground for the belief that they have about the same value and that in consequence the millions of millions of years have passed already. Such must then be the order of magnitude of the age of the stars. Other considerations based on binary stars lead to about the same age. "The general age of the stars is probably between five and ten million million years."

The Origin of Solar Uranium.—If the entire mass of the sun had been composed of uranium the process of spontaneous disintegration would have reduced the quantity of this element to a single atom within a million million years. If we substitute thorium, the most longlived of the radioactive elements, the corresponding time is three times as great. "When the earth was born the sun's age was greater than either of these times, so that the earth's portion of radioactive matter must have been generated during the sun's life in the sun itself." The author now reaches the question of the manner in which this kind of matter was generated in the sun. Two processes occurring on the earth are considered, viz., change of atomic make-up by rearrangement of electrons and change of nuclear make-up by spontaneous disintegration. They seem at first very different. Nothing helps or hinders the disintegration. Conditions of temperature and pressure make no difference. "On the other hand, the change produced in ordinary atoms by electronic rearrangement is extremely susceptible to external physical conditions. Every spectroscopist knows how to chip off one, two, or even three electrons from the atom at will. Nevertheless, as was first made clear in a remarkable paper which Einstein published in 1917, the difference is merely one of degree and not of kind." An electron can revolve about the central nucleus in any one of a series of orbits of greater and greater dimensions. To cause it to remove from the innermost orbit where its energy is least to the next orbit a definite impartation of energy is requisite. "If it is left entirely undisturbed the electron must, after a certain time, lapse back spontaneously to its original smaller orbit. If it were not so, Planck's well-established law of black-body radiation could not be true. Thus a collection of hydrogen atoms in which the electrons describe orbits larger than the smallest possible is similar to a collection of uranium atoms in that the atoms spontaneously lapse back to their states of lower energy as a result merely of the passage of time, losing mass and emitting radiation in the process." To cause an electron to move from one orbit to another located at a greater distance from the nucleus it is not sufficient to flood the atom with radiation of miscellaneous wave-lengths. On the contrary the atom resembles a penny-in-the-slot machine. It will not work unless the incentive is presented in the right form. A definite wave-length of radiation

causes the electron to move to another orbit and, when it resumes its former orbit, radiation of the same wave-length is emitted. Both the absorption and the emission takes place by quanta which are equal to the frequency of the radiation multiplied by Planck's constant.

The author deduces an interesting geometrical relation "that any structure will only be disturbed by radiation whose wave-length is less than 860 times the dimensions of the structure." Since the nucleus, like the entire atom, is composed of positive and negative charges it ought to behave like an atom toward radiation incident upon it, but the radiation either absorbed or emitted by the nucleus is of much shorter wave-length than that absorbed or emitted by the atom, the atom being 100,000 as large as the nucleus. The quanta effective in disintegrating the nucleus are accordingly by theory 100,000 times as great as those causing rearrangements of electrons. "If we compare the hydrogen atom to a penny-in-the-slot machine, nothing less than five-hundred-pound notes will work the radioactive nuclei."

In passing the author pays his respects to modern alchemists. "In its widest form, their ambition is to combine the electrons and protons of base metals with the third atomic ingredient, namely, electromagnetic energy, so as to form atoms of gold. It would need at least an appreciable fraction of an ounce of energy to produce an ounce of gold, and with electric power at even a farthing per Board of Trade Unit, energy and radiation cost eleven million pounds per ounce. Whatever the gold standard may have to fear on the political side, it would appear to be thoroughly impregnable on the side of physics and chemistry."

It is possible to calculate the temperatures at which bodies will emit predominatingly the wave-lengths of radiation that should be effective in changing atomic and nuclear configurations. "Atoms, for example, whose general dimensions are of the order of 10^{-8} cm., begin to be broken up when the temperature approaches 30,000 degrees; nuclei, whose general dimensions are of the order of 10^{-13} cm., must remain unaffected until the temperature approaches 3,000,000,000 degrees. The shortest wave-length of radiation emitted in the transformation of uranium is about 0.5×10^{-10} cm., and this corresponds to a temperature of 5,800,000,000 degrees." Nowhere in the universe is this temperature attained except possibly in the centers of 'white-dwarf' stars and of spiral nebulæ. Since the sun is far below this temperature it seems impossible that its uranium could have been fabricated from lighter elements. It must have resulted from the disintegration of unknown heavier elements which at one time existed in the sun, and indeed, may still exist there.

"Every square centimeter of the earth's surface discharges radiation out into space at the rate of about 1,500 calories a second, from which we can calculate that the sun's total mass is diminishing at about 250,000,000 tons a minute. Whereas the flow of mass from the earth's surface, a total loss of about an ounce a minute, is about equal to the flow of water from a dripping tap, the flow of mass from the sun's surface is about 150 times the flow of water over Niagara. Many stars lose mass even more rapidly. S. Doradus loses mass at the rate of about 45,000,000 Niagaras." The earth more than recoups its loss by the influx of meteors, but the sun has continued to lose through its long life of 7×10^{12} years, so that it now possesses only a few dozen of each thousand atoms that it originally had. "This transformation of atoms into radiation, although unknown to terrestrial physics, must clearly be one of the fundamental physical processes of the universe. Neither total energy nor total mass is any longer constant; the conservation both of mass and of energy has disappeared from physics, and only a kind of sum of the two is conserved."

The fate of the universe is an interesting question, and on it thermodynamics sheds some light. Two competing schemes of enlarged thermodynamics in general are not in agreement but fortunately they offer concordant results for one limiting case which is that of space almost free from matter. This happens to be the average condition of the universe. "Hubble has estimated that if all the matter within about 100 million light-years of the sun were uniformly spread out, it would have a mean density of the order of only about 10^{-31} gm. per cubic centimeter, so that even the very 'empty' atoms would be at several thousand million times their diameters apart." "Maximum entropy would be attained when every atom has turned into radiation that is capable of doing so. Furthermore there is no place where radiation congeals into matter." There can be no creation of matter out of radiation, and no reconstruction of radioactive atoms which have once broken up. The fabric of the universe weathers, crumbles, and dissolves with age, and no restoration or reconstruction is possible. It will be noted that this conclusion is quite at variance with one recently announced by Millikan.

A few decades ago temperatures of 10,000° seemed rather high but in this paper the radiation accompanying the annihilation or creation of the proton and associated electrons is stated to be characteristic of the temperature of 22×10^{11} degrees.

Elsewhere the author speaks seriously of a temperature of 2×10^{62} degrees.

G. F. S.

Two addresses by Vladimer Karapetoff, Professor of Electrical Engineering, Cornell University, before a conference of teachers in electrical engineering in Pittsburgh, Pa., July 25th, 1928. I. On Professional Development of Teachers in Advanced Branches of Electrical Engineering. II. On Electric Transients. In his first discourse, Professor Karapetoff in relation to electrical engineering deals with a condition of American education which is arousing marked concern among many of the leaders of scholarship of the country. In the course of a recent article on the mass-production of culture in the United States, Doctor Paul Shorey in this connection says: "It is a common indictment of American education that the average college graduate of ten years standing cannot be distinguished from the now-collegiate travelling man in the smoking car by his grammar, his vocabulary or the range of his information." He concludes: "In general and cultural education, my personal impression is that there has been a distinct decline since my own college days and early years of teaching. Many thoughtful teachers and writers seem to share the same opinion."

The discussion is limited to those teachers who guide seniors and graduate students, instructors in general and elementary branches being specifically excluded. Doctor Karapetoff specifies the theme of his first discourse in two divisions: (*a*) The total contribution to the substantial progress of electrical science and art by professors of electrical engineering, year after year, remains small in quantity and unimportant in quality. (*b*) Work done by graduate students in most colleges of engineering is small in quantity and inconsequential in quality and in results.

It is possible that powerful natural causes beyond anyone's control govern this situation. As such, may be mentioned: isolation from the rest of the world, privileged economic position; rapid commercial expansion; intensely practical ideas; material prosperity; ignorance of foreign tongues, unwillingness to sacrifice some of the standard of living and to exert a sustained mental effort in a single direction; interest in elementary teaching; general shiftlessness, with a pronounced tendency towards administrative and commercial jobs; side issues and social obligations; poor preparation in mathematics, physics, and methods of attack and analysis; over-use of the self suggestion about being underpaid and not having the necessary facilities; employment of high-grade experts by manufacturing and operating companies and a large amount of research done by them; etc.

The speaker's views of the proper functions of professors in advancing the electrical art are defined, his estimate of the superiority of German, and to some extent the British professors are pointed

out, and suggestions are made whereby professors of electrical engineering may hope to attain the leadership and vision such as exists in Germany and England.

The second address specifically deals with the teaching of electric transients under which term are included the phenomena and the theory of operation of electric circuits, apparatus, and machinery under variable conditions, especially during periods of starting and stopping, on initial short-circuit, on sudden open-circuit, rapid changes in load or voltage, arcover, etc. More and more are engineers being called upon to deal with transient phenomena. The speaker discusses why transient phenomena should be taught, what should be taught among transient phenomena, how they should be taught and where to find time in the curriculum necessary for such a course. The required preparation on the part of the student and the qualifications of teachers conclude the address.

L. E. P.

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Vol. 207 FEBRUARY, 1929 No. 2

SOME LONG DISTANCE TRANSMISSION PROBLEMS.*

BY

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The Bell Telephone Company of Pennsylvania.

I HAVE chosen for the subject of my paper "Some Long Distance Transmission Problems." The last four or five years have been of stirring interest to all students of transmission. There has been a great deal of progress during this period in both the transmission of power and that of speech. The power engineers are developing a more complete understanding of the practical possibilities of the art of transmitting power to a distance. Judging from the experimental trials of interconnected lines for distances such as that between New York and Chicago, it seems clear that their interest in this subject is very great.

The actual progress made in the communication field, including the extension of service across the Atlantic has been outstanding in its significance and perhaps unparalleled in any other similar period of time. I thought it would be interesting to outline briefly the trends of development in the two arts, as they now appear, to establish some comparisons, and to indicate some possibilities of development.

At the Midwinter Convention of the American Institute of Electrical Engineers, held in Philadelphia, which some of you

* Presented at a meeting held Thursday, April 12, 1928.

(Note.—The Franklin Institute is not responsible for the statements and opinions advanced by contributors to the JOURNAL.)

may have attended, there were a number of very interesting and important papers read on the subject of transmission of power to comparatively long distances.

The characteristics of the line considered in this important series of papers were as follows:

Potential	220,000 volts
Size of Conductor	750,000 circular mils
Spacing of Conductors	19 feet
Resistance per mile	.0782 ohms
Reactance per mile	.80 ohms
Susceptance per mile	.00000542 ohms

The characteristic impedance and the propagation constant, as computed, are equal to—

Characteristic Impedance	383.7 − j18.7 ohms
Propagation Constant	0.000102 + jo.002085

The impedance of a typical 220-kilovolt high tension line is therefore lower than that of the open wire circuits used in the communication art, the impedance of such circuits usually being about 600 ohms.

A line of this type, if used between New York and San Francisco, would have an equivalent loss of 2.7 TU. It would be an excellent talking circuit, even taking into account echo effects. Using the information as given by Mr. Murray in "Superpower System for the Region between Boston and Washington," it may be estimated that the cost of a line of this type would be about $100,000,000. If use were made of these twin circuits of the high tension type for communication purposes, three physical and one phantom channel could be established over these circuits. The carrying capacity of these four circuits would be 70,000 messages annually and the annual cost of the conductors alone would be $200 per message. The establishment of carriers over these same circuits would decrease somewhat the above very large and uncommercial cost per message.

In discussing the papers on power transmission and particularly the one read by Mr. R. P. Thomas, Dr. Kennelly brought out the fact that for lines of great length of the order of 500 miles, the deviation between the nominal π of the line and the equivalent π was quite important.

To quote Dr. Kennelly—

"In the case of Mr. Thomas's line, 500 miles long, the nominal π of any one of the three line conductors has an appreciable error at 60 cycles per second, and has to be corrected by some process which takes the uniform distribution of capacitance into account. This is done in the paper by means of the equation at the foot of its third page, which has been shown by Mr. Thomas in an earlier page to be suitable for the purpose. An alternative method, preferred by the writer, is to form the equivalent π of the lines, such that the electrical behavior of this π according to Ohm's law, gives the same voltage, current and power at its terminals under the assigned conditions of load, as the actual lines with its distributed constants."

I thought that it might be worth while to carry further the suggestion made by Dr. Kennelly and determine for a 220-kilovolt line of variable length, having the linear characteristics indicated above, the constants of the equivalent diagrams. Some interesting conclusions are brought out in this way. Before showing you these diagrams, I should like to show the "power limits," so-called, of 220-K.V. lines of various lengths. Figure 1*a* (Figure 5 of the paper by Evans and Sels) [1] shows how this power limit decreases rapidly, reaching a value of about 125,000 kilowatts at 500 miles.

Figure 1 shows the illustration furnished by Dr. Kennelly in his discussion, showing the deviation between the nominal π and the so-called actual π of a transmission line 500 miles in length.

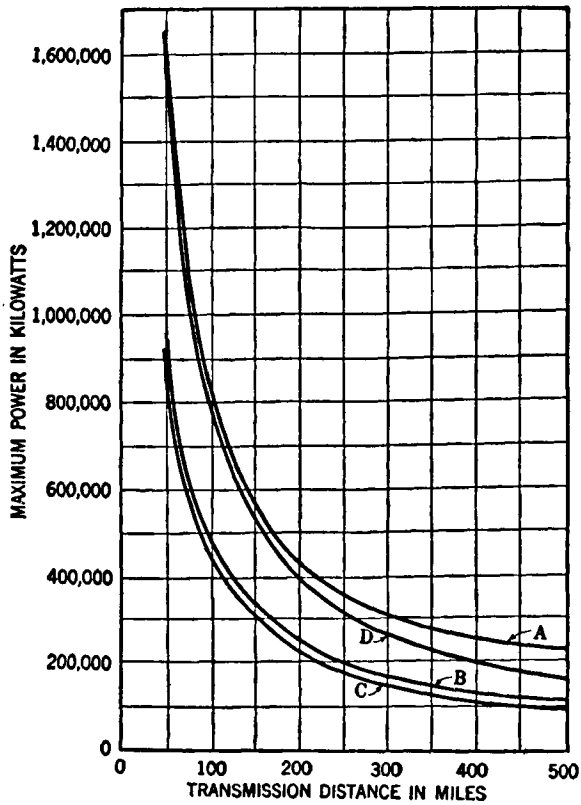
Figures 2 and 3 show the results of the computations made to determine the constants of the equivalent π diagrams, the first figure showing the constants of the architrave impedance and the second showing the constants of the pillar impedance.

Some of the results are quite striking and I am sure will prove of interest. Referring first to Figure 2 it will be noted that the resistance component of the equivalent π diagram reaches a maximum at about 500 miles, becomes zero for a line of 976 miles and reverses its sign, reaches a second maximum and becomes zero in value for a line of 2,280 miles

[1] "Power Limitations of Transmission Systems," *Trans. A. I. E. E.*, February 1924.

in length, approximately. The reactance component follows a similar course. It reaches a maximum for a line of, roughly, 800 miles in length, slightly over one-quarter wave-length,

FIG. 1*a*.



1*a*. Power Limits—220,000-Volt Line.

RELATION OF POWER LIMITS TO LENGTH OF TRANSMISSION LINE.

These diagrams are based on a transmission circuit employing 636,000 cir. mil. aluminum conductor with 21 ft. equivalent spacing, and with 220 k.v. maintained at the supply end. Curve "A" shows the maximum power limit with 60-cycle supply with unrestricted receiver voltage and condenser capacity. Curve "B" shows the maximum power limit with 60-cycle supply and 220 k.v. maintained at the receiver. Curve "C" shows the power limit at the envelope with 60-cycle supply and 220 k.v. maintained at the receiver. Curve "D" corresponds to Curve "B," except that the power supply is 25 cycles.

reaches zero for a line slightly longer than one-half wave-length, becomes a capacity reactance in changing sign, reaches a second maximum and crossing zero changes sign again for a length slightly in excess of one full wave-length.

The computations were carried out using the following relations:

(1) $$\text{Architrave impedance} = Z_0 \sinh Pl,$$

(2) $$\text{Pillar admittance} = \frac{\tanh \frac{Pl}{2}}{Z_0},$$

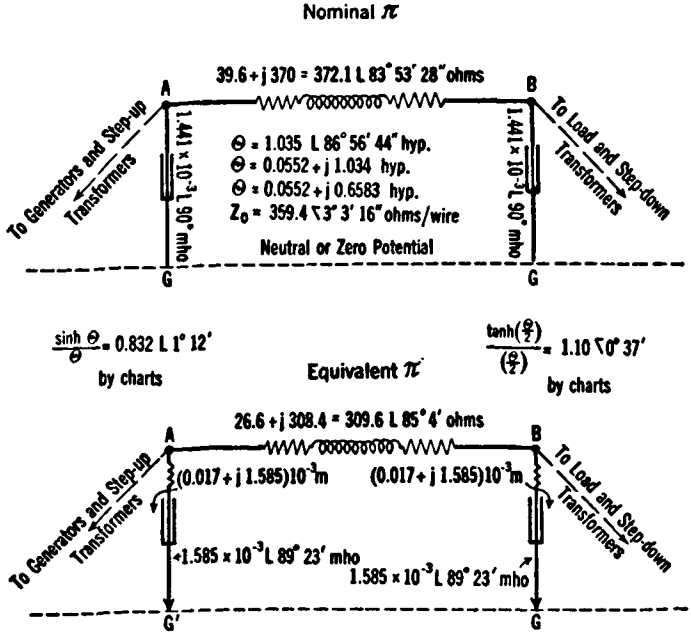
where

Z_0 = characteristic impedance of transmission line,

P = propagation constant of transmission line per unit length,

l = length of line.

FIG. 1.


Nominal π
39.6 + j 370 = 372.1 L 83° 53' 28" ohms
A
B
To Generators and Step-up Transformers
To Load and Step-down Transformers
1.441 × 10⁻³ L 90° mho
Θ = 1.035 L 86° 56' 44" hyp.
Θ = 0.0552 + j 1.034 hyp.
Θ = 0.0552 + j 0.6583 hyp.
Z₀ = 359.4 ∠3° 3' 16" ohms/wire
Neutral or Zero Potential
G
G
sinh Θ/Θ = 0.832 L 1° 12'
by charts
Equivalent π
tanh(Θ/2)/(Θ/2) = 1.10 ∠0° 37'
by charts
26.6 + j 308.4 = 309.6 L 85° 4' ohms
(0.017 + j 1.585)10⁻³m
(0.017 + j 1.585)10⁻³m
1.585 × 10⁻³ L 89° 23' mho
1.585 × 10⁻³ L 89° 23' mho
G'
G

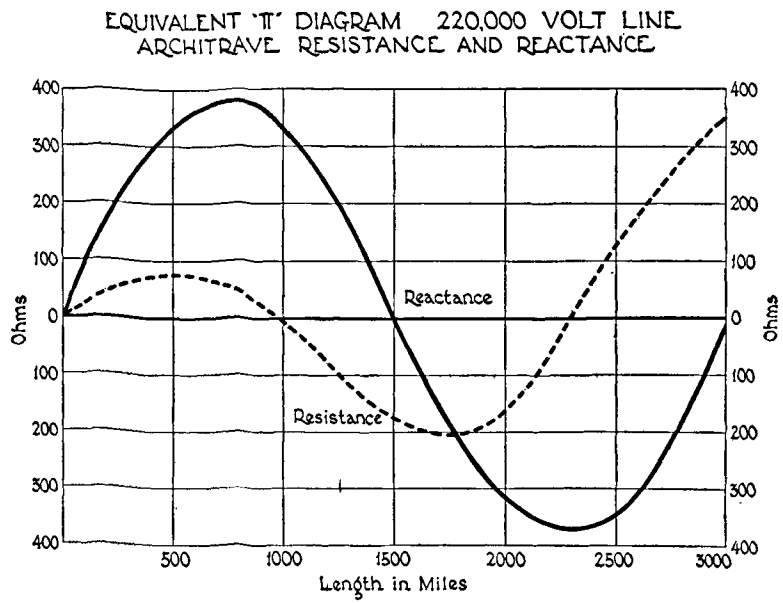
1. Kennelly Chart—Difference between Nominal and Actual.

The numerical values of (P) and (Z_0) for the particular 220,000-volt line, which has been taken as a basis for discussion in this paper, have already been given hereinabove.

The reversal of sign in the resistance component of the architrave impedance was unexpected to the writer and may prove a surprise to many other students. It might lead one to

suppose that transfer of electrical power can be made for a particular length of high tension line (976 miles actually for the particular line considered herein) without loss of real power in the line. This conclusion would be incorrect since the pillar impedance has a positive resistance component equal to 2,000 ohms for this particular line. The reversal of

FIG. 2.



2. Equivalent Diagram—Architrave Impedance, 500-Mile Line—220,000 Volts.

sign above referred to is of such general interest that the following reference is made here to make its derivation clear to those who may be interested. If formula (1) is amplified, the following relation is obtained:

$$\begin{aligned} Z \sinh Pl &= (R - jX)(\sinh al \cos bl + j \cosh al \sin bl), \\ &= (R \sinh al \cos bl + X \cosh al \sin bl), \\ &\quad - j(X \sinh al \cos bl - R \cosh al \sin bl). \end{aligned} \tag{3}$$

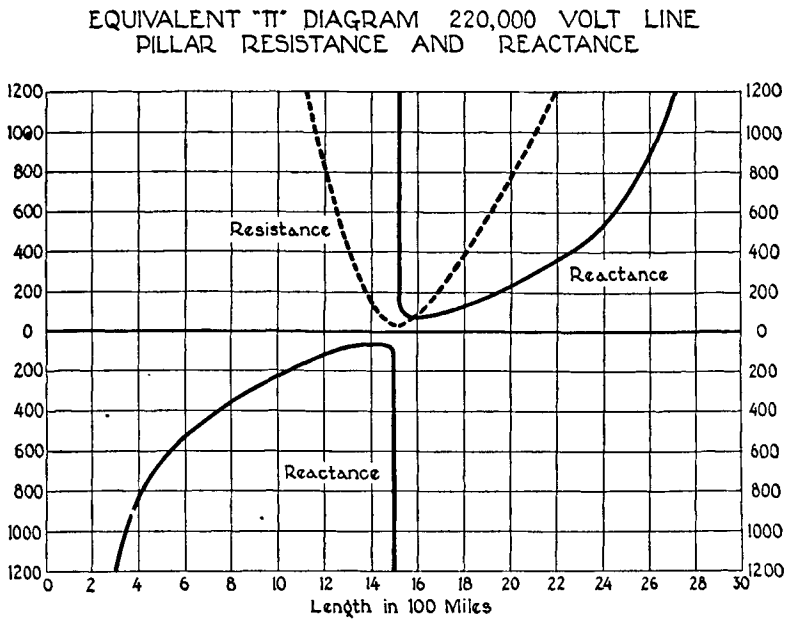
The resistance component is naturally equal to zero where

$$R \sinh al \cos bl = - X \cosh al \sin bl,$$

$$\tan bl = \frac{- R \sinh al}{X \cosh al}. \tag{4}$$

The reversal of sign for the particular 220,000-volt line considered here is for a length of 976 miles, in excess of one-quarter wave-length, the wave-length as shown by the computed propagation constants being equal to 3,013 miles.

FIG. 3.



3. Equivalent Diagram—Pillar Resistance and Reactance—500-Mile Line—220,000 Volts.

The change in the reactance component of the architrave impedance is also of great interest. We are so used to thinking of line reactance in connection with high tension lines, particularly from the standpoint of such line reactance being the determining factor in the amount of power that can be physically transported over a transmission line, that it is somewhat of a surprise to find that it can be zero in amount or, perhaps what is still more interesting, it can actually become equivalent to a series capacity. It should not, of course, be concluded that the equivalent series line impedance is actually equal to zero for some particular length of line. This is not true as the two components of the architrave impedance do not reach zero value for the same length of line.

The results, as shown on Figure 3, are equally as interesting. It is assumed that the pillar impedance will be represented by two branch impedances, a resistance branch and a reactive branch. It is interesting to note that for a line having a length slightly in excess of one-half wave-length, the value of the pillar resistance becomes extremely small, of the order of only 25 ohms. The reactance branch, on the other hand, is represented by a capacity, as well known, for short lengths of line. However, for a line close to one-half wave length this capacity effect almost suddenly disappears and is replaced by an inductance effect.

It is customary to refer to the importance of the "charging current" or "capacity current" in connection with high tension lines. It is certainly of interest to know that a line close to one-half wave-length would have practically no charging current. In addition, it would practically have no series line reactance. On the other hand, it would have an equivalent pillar resistance quite small in value—about 23 to 25 ohms. As well known, such a line with no series line reactance can not be operated on a constant potential basis. For the benefit of those who are interested in the quantitative aspects of this problem, the following formulæ, which have been used for the numerical computations, are reproduced below.

$$\text{Tanh}\frac{Pl}{2} = \frac{\sinh al + j \sin bl}{\cosh al + \cos bl}, \tag{5}$$

where (a) and (b) indicate the components of the propagation constant (P).

The values of the resistance and the reactance of the pillar impedance, considered as a two-branch impedance of a resistance in parallel with a reactance, may be obtained from the following:

$$\text{Resistance} = (R^2 + X^2)\frac{\cosh al + \cos bl}{R \sinh al - X \sin bl}, \tag{6}$$

$$\text{Reactance} = -j(R^2 + X^2)\frac{(\cosh al + \cos bl)}{X \sinh al + R \sin bl}. \tag{7}$$

The reactance as given by relation (7) reaches infinity (with consequent disappearance of the "charging current)"

where

$$X \sinh al + R \sin bl = 0,$$

$$\sin bl = -\frac{X}{R} \sinh al. \tag{8}$$

LIMITATIONS DUE TO STABILITY OF ROTATING APPARATUS.

The figures (2 and 3) descriptive of the series and shunt characteristics of the equivalent π circuits of high tension lines of various lengths must be read, of course, in the light of practical possibilities of the art. On account of the fact that the generator at the power house and the motor at the generator include rotating parts, the transmission of electrical power can be carried with stability up to a certain length of line. At the present stage in the development of the art, this length does not exceed one-quarter of a wave-length. In the case of a typical 220-kilovolt line, the wave-length is approximately equal to 3,013 miles. (For check, refer to computed value of the imaginary term in the propagation constant as given above.) The limit of stability, as usually accepted, would not therefore exceed 750 miles, roughly. Noteworthy progress has lately been made in this direction. Wagner and Sels (*A. I. E. E. Transactions* 1928, page 101) show that the maximum angle of stability is given by the following relationship:

$$\tan \phi_0 = \frac{W_b + W_a}{W_b - W_a} \tan \phi_1, \tag{9}$$

where

ϕ_0 = maximum angle of stability,

ϕ_1 = angle of vector $\frac{\text{Sinh } Pl}{Z_0}$,

W_a = stored mechanical energy at generating end,

W_b = stored mechanical energy at receiving end.

A method of extending the maximum angle of stability is described below as the "neutralization method," which is believed to be new. Since the angle of stability, as given by the above relation, depends upon the value of the angle ϕ_1 of the line, a decrease in the hyperbolic angle per unit length of

line will naturally result in the same relative stability for a longer length of line. It is possible, for instance, by providing for 50 per cent. neutralization of the line constants, to double the wave-length from 3,013 to 6,026 miles and, correspondingly, increase the limit of stability from 750 miles to somewhere near 1,500 miles.

The limitation just described as to the length of transmission line which offers possibilities of stable operation is distinctive of the power art. In the communication art no such limitation holds. A cable circuit, such as ordinarily used, between Philadelphia and Wilmington has a length equal to two wave-lengths. A line between Philadelphia and Chicago has the equivalent of 45 wave-lengths, when speech frequencies of the order of 1,000 cycles are being transmitted over them.

POWER TRANSFER LIMITATIONS—PRESENT ART.

Important contributions have recently been made to this subject. Present power transmission practice calls for a synchronous condenser at the receiving end of the transmission line, not only for purposes of voltage control, but also for the purpose of increasing the power transfer over a line operating at a given constant potential. It has been shown that the power limit (P_r) is given by

$$P_r = E_r \frac{(E_s - E_r \cos \theta)}{Z}, \tag{10}$$

where

E_s = generator potential,

E_r = receiving end potential,

Z = series impedance of line.

(See Steinmetz, "Alternating Current Phenomena," page 102.)

The above formula admittedly holds for short lines only. For long lines, the following formula, as given by F. Terman (*A. I. E. E. Transactions*, 1926, page 1091) is reproduced hereunder.

$$\text{Maximum Power} = \frac{E_r\{E_s \cos(\phi-\beta) - aE_r \cos(\alpha-\beta)\}}{b}, \tag{11}$$

where

α = angle of vector a or (cosh Pl),

β = angle of impedance b or (Z_0 Sinh Pl),

Z_0 = characteristic line impedance,

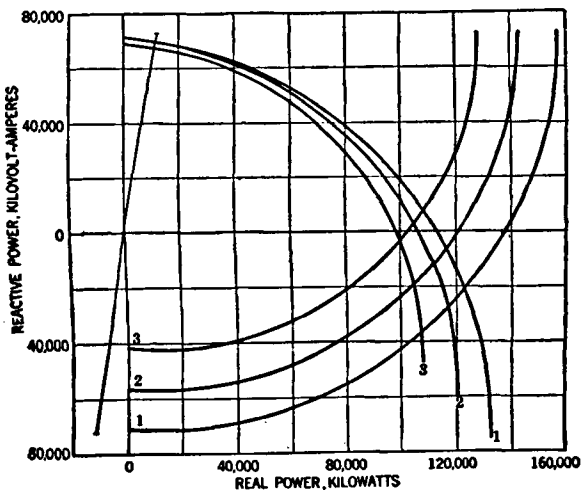
P = propagation constant of line per unit length,

l = length of transmission line,

ϕ = angle by which the sending voltage leads the receiving voltage.

It is interesting to note here that the denominator in the above formula is the architrave impedance of the equivalent π circuit of the transmission line. Values for both the resistance and the reactance components of the architrave impedance have been computed for various lengths of 220,000-volt high

FIG. 4.

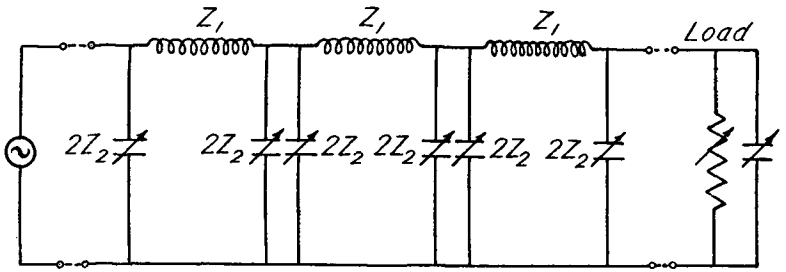


4. Philip's Chart—Circle Diagram.

tension line and shown on Figure 2. The figure referred to will therefore prove of interest from the standpoint of applications of formula (11) of maximum power to particular cases. Power limits may also be obtained by the famous Circle Diagram (see Figure 4), due to R. A. Philip (see *A. I. E. E.*

Transactions, 1911, and the symposium papers read at the Midwinter Convention of the A. I. E. E., 1924). Since there is a power limit to a line of given length using a synchronous condenser at the receiving end, a great deal of real interest has been aroused by the suggestion of Mr. Baum that, in addition to the terminal synchronous condenser, the use of intermediate synchronous condensers would result in increasing the power limit of a line and incidentally in increasing its stability.

FIG. 5.



5. Power Transmission System—Intermediate Synchronous Condensers.

The procedure now used for obtaining the power limit of a line with intermediate condensers is a step-by-step method. The power limit of the first section is determined, then that of the second section, etc. I find that I can get a clearer mental picture of the performance of a power line with a number of intermediate condensers and the need for such condensers in the following manner. Assume that, instead of the synchronous condenser at the receiving end being considered as part of the load, it be considered as part of a composite line formed of sections of artificial line and actual line, shown on Figure 5. Let the line be terminated by an impedance equal to the iterative or characteristic impedance of the network. Then, in that case, the drop in potential along the line will be relatively small and will be given by e^{-Pl}. For a line with no synchronous condensers, this would be equal to about 5 per cent. on a 500-mile line and would be somewhat larger, although still small, for the case where the usual amount of synchronous capacity is added. Assume, as the load changes, that some kind spirit changes simultaneously the values of the capacities of the condensers bridged across

the line so that the load is still equal to the iterative impedance of the network. In practice, the kind spirit just referred to would be the voltage regulator associated with the synchronous condenser. This may require, for very light loads, that the synchronous condensers be operated as reactances; and, finally, they may be sufficiently large reactively to wipe out the capacity of the line. Then, in such a case, there will be very small, if any, drop of potential at the receiving end, with extremely wide changes in load. The line is then "regulated."

The interesting question comes up—what is the heaviest load such a line will carry? The circle diagram method can of course be used on a step-by-step basis, as already referred to. There is another method which shows some interesting results. It can be shown that, if capacity is added, the iterative impedance of the network follows a curve as shown in Fig. 6 below. It is infinity, of course, for $C = 0$, reaches a minimum, then increases and reaches infinity again. A further increase in capacity stops all power transfer. This is the cutoff point of the line, as we call it in the communication art. The value of the capacity which results in the lowest value for the iterative impedance is inversely proportional to the series impedance included between successive condensers. By installing condensers at a greater number of points, we gradually approach the condition of a line with uniformly distributed constants. The total amount of synchronous capacity can take a value as high as we please, in fact can approach infinity when added at a sufficiently large number of points—leaving aside all questions of cost. The second interesting and important point in this connection, however, is this: As the capacity installed increases, the attenuation constant of the line also increases rapidly and, in fact, approaches infinity ultimately. This will be clear by reference to the fundamental formula

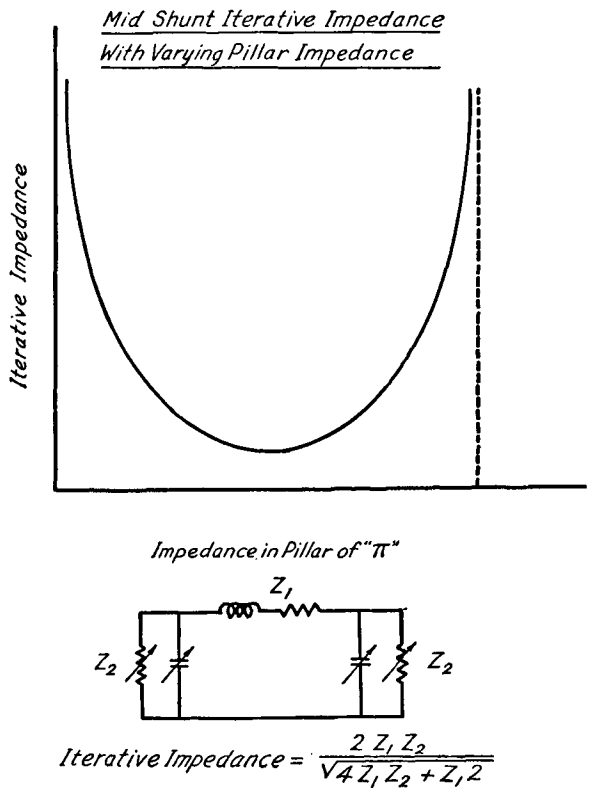
$$a = \frac{R}{2}\sqrt{\frac{C}{L}}, \tag{12}$$

when C is given gradually increasing values.

It is clear, therefore, that increased power transfer, through the use of synchronous condensers located at intermediate

points along the line, when designed for maximum power transfer, results in decreased efficiency of transmission.

FIG. 6.


Mid Shunt Iterative Impedance
With Varying Pillar Impedance
Iterative Impedance
Impedance in Pillar of "π"
Z_1
Z_2
Z_2
Iterative Impedance = $\frac{2 Z_1 Z_2}{\sqrt{4 Z_1 Z_2 + Z_1^2}}$
Note: Assume that the phase angles of Z_1 and Z_2 differ by 180°

6. Midshunt Iterative Impedance—Varying Pillar Impedance.

EFFICIENCY OF LINE TRANSMISSION AS RELATED TO POWER TRANSFER.

One question which, perhaps, has not received as much attention as the question of power transfer limitation is that of efficiency of transmission. The use of synchronous condensers, both at intermediate points along the line and at the receiving end, when used for the practical purpose of increasing the power transfer over a given high tension line, is

electrically equivalent to adding capacity at intermediate points along the line and at the receiving end. This will be clear by referring to the circle diagram (Figure 4) which, for maximum transfer of real power, shows leading kilovolt amperes required as reactive power. The same situation holds at intermediate synchronous condenser stations. The question properly arises: what is the effect upon efficiency of transmission of the arrangement proposed to secure increased power transfer? The study of this question shows that a loss in efficiency of transmission is the result, as perhaps might have been expected. For instance, the 500-mile line has the following efficiency:

	Transmission Efficiency %
1. With synchronous condensers arranged for voltage control purposes only	95
2. With synchronous condenser at receiving end, arranged for maximum power transfer	89
3. With synchronous condensers at intermediate point and also at receiving end. (Midway location.)	77

DISCUSSION OF GENERAL METHODS FOR SECURING HIGH EFFICIENCY IN TRANSMISSION.

The above situation leads immediately to the following general question. Under what conditions can high efficiency of transmission be obtained? It should, of course, be clearly understood that under certain conditions it may be desirable, from an economical point of view, to operate a line at a point lower than its highest possible efficiency. Notwithstanding this, the inquiry as to possible maximum limits of transmission efficiency loses none of its interest. This is particularly true in case it is found economically practicable to transport electrical power to greater distances than now contemplated. It is well known that the attenuation constant (a) of a line is given by

$$a = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}, \tag{13}$$

where R, C, L and G are, respectively, the resistance, capacity, inductance and the dielectric conductance constants of the line. The above relation is known to hold where the reactive

components of the line constants are large as compared with the resistance components. In the case of the typical 220-kilovolt line, the series reactive component is ten (10) times larger than the resistance component, and power engineers hold that G is negligible. The above relation therefore holds and may be expressed still more simply by the relation

$$a = \frac{R}{2}\sqrt{\frac{C}{L}}.$$

To obtain high transmission efficiency it is necessary to make (a) as small as possible.

In the communication art, the trend of progress has consisted in increasing (L). A second method would consist in decreasing (C). A third method would consist in reducing equally (C) and (L) or reducing (C) more than (L). Referring to the second method, we can not carry this out at the present time in the communication art, as it would require a device which has the exact opposite characteristics to that of a capacity for the entire range of frequencies involved in the band of intelligible speech. It is possible, however, to apply it in the power art, since single frequency transmission only is involved. Strange as it may seem at first sight, the present tendency in the power art is in the opposite direction; The tendency is toward increasing (C) as already explained, with the resulting disadvantage of loss in transmission efficiency. But, as already stated, the increase in power transfer is a justification for this method.

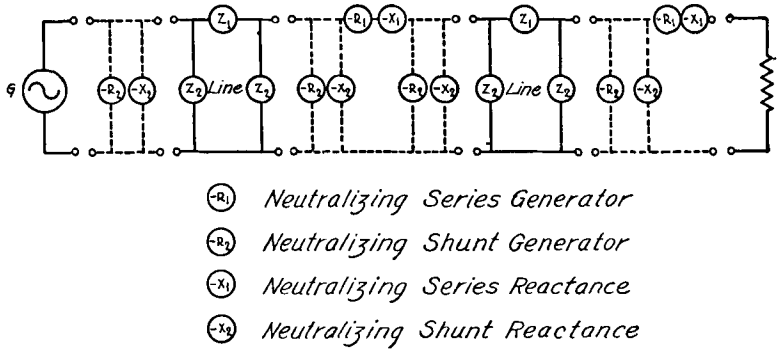
METHOD OF NEUTRALIZING NETWORKS.

The second and third methods for obtaining high efficiency of transmission with increase in power transfer also lead to the arrangement shown on Figure 7, which is believed to be new. As shown on this figure, networks consisting of series condensers and shunt reactances with series and shunt, generators are bridged at intermediate points along the line, thus "neutralizing" the series line and shunt capacity characteristics of the line. The neutralization may be more or less complete, depending upon the economic factors involved which will not be discussed here. There are several physical

and technical questions that are of such general interest that it seems worth while to outline them briefly.

The first difficulty which must be met is that of series resonance. This difficulty can be overcome by installing the series condensers at sufficiently frequent intervals. I find that the most desirable arrangement consists in providing

FIG. 7.



7. Transmission System with Neutralizing Networks.

half-spacing at both the power house and substation ends and full spacing at intermediate points where neutralizing networks are installed. With this arrangement in effect, the necessary number of neutralizing networks is given by the formula shown below if it is desired to reduce the increase due to series resonance to less than c times the line potential.

$$n = \frac{P}{2E^2} \cdot pL \cdot \frac{1}{2c}.^2$$

As might have been expected, the spacing must be more frequent, the greater the power transfer desired. Using the physical data as to line constants, a practical spacing seems to be a neutralizing network for every 150 miles of high tension line. The second question that must be answered is: how can the use of "neutralizing networks" be worked out to secure high transmission efficiency combined with high transfer of power? It can be very readily shown that the

[2] P is expressed in watts and pL = total linear reactance of the line.

method of "neutralizing networks" has no power limit if the underlying fundamental idea is carried through to its extreme possibilities which, of course, include neutralization of the resistance as well as the reactive components of the line. Neutralization of the resistance components of a high tension line in the power art would be equivalent to the use of amplifiers in the communication art. So far as the limited study made by the writer indicates, there seems to be no real field for this type of neutralization up to 3,000 miles of line having the characteristics of a typical 220-kilovolt line. The use of "neutralizing networks" with the exception as just noted eliminates power limitations physically if the neutralization is 100 per cent. But it is unnecessary and in many cases undesirable to carry neutralization to its extreme limits.

Figure 7 shows, in addition to the series capacity and shunt reactances, both series and shunt generators. In practice it seems unnecessary to have recourse to the relatively complicated arrangement shown on that chart, the arrangement shown on the succeeding figure, 8, representing the arrangement which will likely find application in practice. It is interesting to show, however, that the neutralizing series power generators can be entirely eliminated and resistance neutralization obtained by means of shunt generators only. If R, C, L and G represent, as heretofore, the characteristics of the "equivalent" line after neutralization, then, in order that power may be transmitted without any "net" losses between power house and substation, it is necessary and sufficient that the attenuation constant of the "equivalent" line, composed of sections of actual line and of sections of neutralizing network, shall be equal to zero. This condition requires that the following relation be satisfied:

$$\frac{R}{2}\sqrt{\frac{C}{L}}+\frac{G}{2}\sqrt{\frac{L}{C}}=0,$$

or

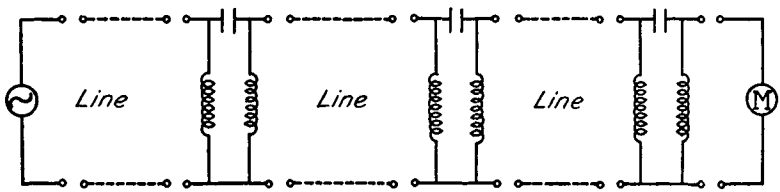
$$G=-R\left(\frac{C}{L}\right).$$

The kilowatt capacity of the shunt generator required will be obtained approximately by $E^2Gl \times 10^{-3}$, where l is the spacing

desired between neutralizing generators. Accurate computations can be carried out by determining the constants of the equivalent π of a length (l) of the line and computing a value for a negative shunt resistance such that the total attenuation of the equivalent π diagram, including line and corresponding neutralizing network, is equal to zero.

Referring now to the simpler arrangement shown on Figure 8, it may be shown how high power transfer may be obtained. On this chart, only reactance neutralization elements are shown. Since, as already indicated, the transfer of power is inversely proportional to the architrave impedance, $Z_0 \text{ Sinh } Pl$

FIG. 8.



8. Transmission System with Reactance Neutralizing Networks.

of the equivalent π diagram, the use of neutralization networks, of the type shown on Figure 8, will permit greater transfer of power on account of the possibility it affords of reducing both of the terms, Z_0 and Sinh Pl, of the architrave impedance Z_0 Sinh Pl. It may be mentioned here, as a matter of real interest, that where Z_0 (for the composite line illustrated on Figure 8) is maintained at the same value as the characteristic impedance of the line without neutralizing networks, then the transmission efficiency of the composite line will be practically the same as that of the actual line not equipped with neutralizing networks. The power limit, however, of the composite line will be greater on account of the reduction in the value of Sinh Pl.

In order to establish as broad and as simple a comparison as possible between the two methods of increasing the power limit of a line—between that which uses an increase in the equivalent capacity of the line and that which uses neutralization of the series line reactance and of the shunt capacity reactance of the line, the following simplified case is submitted.

Assume an actual line of length (l) and determine the power limit of the line, restricting the use of synchronous condensers at the substation end to voltage control purposes only. Suppose we wish to double the power transfer. We can attain this objective by increasing fourfold the natural capacity of the line. The transmission efficiency of this same line will be equal to the square of the transmission efficiency of the actual line. As an illustration, if the efficiency is 85 per cent. for the original simple line, the efficiency of the line with increased capacity will be $0.85 \times 0.85 = 0.7225$. This efficiency will obtain provided, as assumed, the load at the substation is equal to the characteristic line impedance.

If, now, use is made of the neutralization method and the series line reactance is reduced to a fourth of its original value, the characteristic line impedance of the composite line so obtained will be approximately one-half that of the original line. The power limit of the line, when transmitting a "normal" load, will be the same as that of the line with its normal capacity increased fourfold. The maximum power limit of the line equipped with neutralizing networks, however, will be much greater than that of the line equipped with intermediate synchronous condensers for the reason that the architrave impedance of the equivalent π diagram will be considerably smaller. This point is almost obvious since, in the expression $Z_0 \operatorname{Sinh} Pl$ for the architrave impedance, the term Z_0 has exactly the same numerical value for the two solutions under discussion but the term $\operatorname{Sinh} Pl$ is considerably smaller for the system using neutralizing networks. The value of the propagation constant (P), as already indicated, is equal to $(a + jbl)$, and the wave-length factor (b) is equal to $\sqrt{LC}$, approximately. In the solution which adds capacity to the line the wave-length of the equivalent line is decreased one-half, while in the solution utilizing neutralizing networks the wave-length is twice as large as that of the original line. It therefore follows that the power limit of the line equipped with neutralizing networks is considerably greater than that of the line equipped with bridged condensers, when these two systems are adjusted to have equal characteristic impedances and equal attenuation factors. Furthermore, the transmission efficiency of the method using neutralizing networks

will be higher than that of the method using synchronous condensers for all loads in excess of the "normal" load. This is due to the fact that the series impedance interposed between the power house and the substation is smaller for the solution using neutralizing networks. The numerical value of this impedance is given by

$$Z_0 \operatorname{Sinh} Pl = \sqrt{R^2 + X^2}\,\sqrt{\operatorname{Sinh}^2 al + \sin^2 bl}$$

and we have already shown that (b) is considerably smaller for the solution using neutralizing networks as compared with the solution which "adds" capacity to the line.

A complete analysis of the relationship between power transfer and transmission efficiency for the various solutions discussed above is quite complicated and perhaps unnecessary to consider at this time. The above discussion has been limited to the simplest aspects of the case. It is of real interest, however, to indicate one of the outstanding merits of the method of neutralizing networks. Consider a transmission line of length (l). Such a line can be represented by a π diagram. We can, by adding impedances of the correct sign, completely neutralize the two pillar impedances. Such a procedure may require reactances at both ends of the line and, in addition, resistance compensation or generators. If this has once been done, we have the equivalent of what is usually designated in the transmission art as a "short line." The transmission characteristics of such a short line are well known and completely covered in standard text-books. It is known, for instance, that the power limit of a short line is definitely given by

$$P_r = E_r \frac{E_s - E_r \cos\theta}{Z}.$$

In the case of transmission lines operated at constant potential $E_r = E_s$ and the expression for the power limit in this case is

$$P_r = E_s^2 \frac{1 - \cos\theta}{Z}.$$

It has also been shown in standard text-books that if $Z = R + jX$, then the power limit of the line reaches a maximum when

$$X = \sqrt{3}R.$$

(Steinmetz, Alternating Current Phenomena, pages 102 and 103). When and if the above relationship can be satisfied, then

$$P_r = E_s^2 \frac{1 - \frac{R}{Z}}{Z} = \frac{E_s^2}{4R}$$

and it is possible to transmit over an alternating current system the same amount of power as over a direct current system operated at the same potential, except for the comparatively small losses required for pillar resistance neutralization. In the case of short lines, the relation between X and R is far from satisfying the optimum condition $X = \sqrt{3}R$, as the ratio $\frac{X}{R} = 10$, approximately, for typical 220,000-volt lines. It is possible, by reference to Figure 2, showing the characteristic resistance and reactance values for lines of various lengths to see readily that for 220,000-volt lines it is not practicable to realize the optimum condition for maximum power transfer. However, if the system of neutralizing networks is applied and series capacity reactance is added to the line, it is possible to realize the condition for maximum power transfer within wide ranges of line length.

COMPARATIVE POWER TRANSFER AND EFFICIENCY FIGURES FOR HIGH VOLTAGE LINES USING INCREASED LINE CAPACITY OR REDUCED LINE CONSTANTS.

The above brief description gives an idea of the comparative characteristics of high voltage lines which have capacity added in shunt, approximating the condition of synchronous condensers added at intermediate points along the high voltage line and of lines equipped with neutralizing networks. It is known, of course, that the power limit of a line equipped with static capacity, in some form, bridged across the line is not exactly the same as the similar limit for lines provided with synchronous condensers. Nor is the transmission efficiency the same in both cases since synchronous condensers consume real energy while, with static condensers, the corresponding consumption of energy is negligibly small. The performance analysis of lines equipped with synchronous condensers at a great number of intermediate points is quite complicated and, for that reason, the

simpler case of bridged static condensers has been considered here.

Quantitative data as to power transfer and transmission efficiency have been worked out for a 500-mile, 220,000-volt line for the following two cases:

1. A 220,000-volt line with bridged static condensers at a sufficiently large number of points, increasing the normal capacity of the line four times.
2. A 220,000-volt line with series static condensers, reducing the series reactance of the line, 0.80 ohms per mile, to 0.20 ohms per mile, or to a quarter of its original value.

The computed characteristics of these two lines would be as follows:

	1	2
(*a*) Characteristic Impedance in Ohms.	191.85 − j9.35	195.6 − j36.88
(*b*) Pillar Resistance in Ohms.....	645.2	1918
(*c*) Pillar Reactance in Ohms.....	− j112.7	− j776
(*d*) Propagation Constant.....	0.0002 + j0.004170	0.0002 + j0.00106
(*e*) Architrave Impedance in Ohms....	− 1.44 + j168.4	36.57 + j67.69

Solutions 1 and 2 were worked out so as to obtain the same characteristic impedance and the same attenuation factor so far as practicable. The power transfers for these two solutions, for normal loads equal to the characteristic impedance of the line, are also practically the same for these two solutions:

Line with bridged capacity = 205,000 K.W.
Line with neutralized networks = 195,000 K.W.

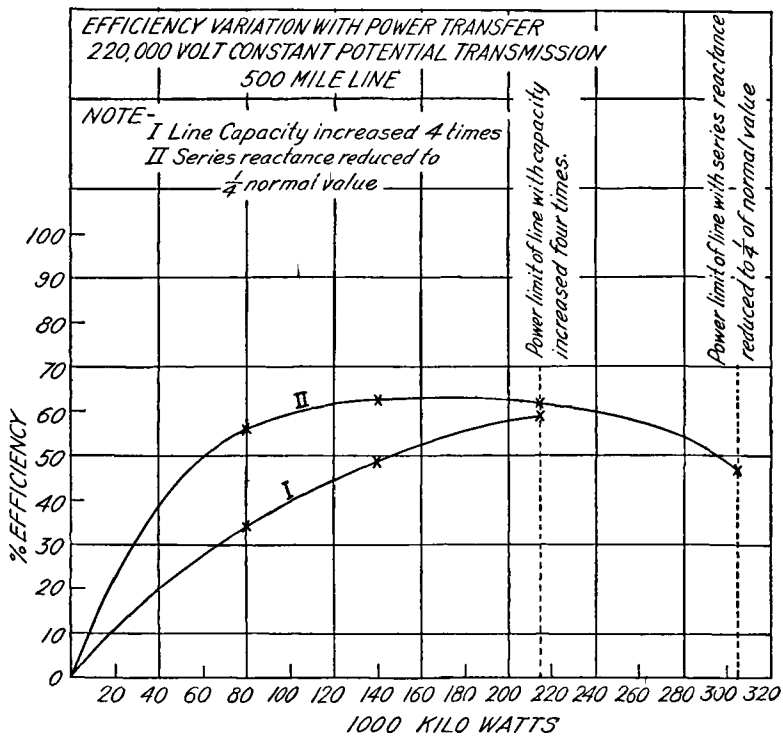
Under the assumptions just made, there would be a drop in potential of 9.7 per cent. and a line transmission efficiency of 81 per cent. for both solutions.

In the case of the normal line the corresponding power limit is 113,000 kilowatts.

The important differences between these two solutions obtain in conjunction with the maximum power limits and the transmission efficiency when operating on a constant potential basis. The maximum power limit of the high capacity line is 215,000 kilowatts. The maximum power limit of the line equipped with neutralizing networks (Solution 2) is 305,000 kilowatts. The efficiency of transmission for the high

capacity line transmitting 215,000 kilowatts is 59.6 per cent. The efficiency of transmission of the line equipped with neutralizing networks, transmitting the same power, is 62 per cent. It should be stated here that all figures given above are for the transmission line only, disregarding the limitations as to the maximum amount of power that can be transmitted

FIG. 9.



9. Efficiency Variation with Power Transfer.

due to the impedance of the generating apparatus, and the effect of the step-up and of the step-down transformers. These could easily have been included, and the methods for including them are well known but were omitted for the sake of simplicity.

Transmission line efficiency figures were computed for a number of different loads at the substation end. The results of these computations are shown on Figure 9, which shows at a

glance the advantage of the use of the system using neutralizing networks over that using bridged capacity across the line.

LIMITATIONS IN POWER LIMITS—LINES WITH BRIDGED CAPACITY AT INTERMEDIATE POINTS ALONG TRANSMISSION LINE.

It might be thought at first sight that if bridged capacity is added at sufficiently frequent intervals, the power limit of such a line might increase indefinitely. A closer view of this situation shows that the increase in attenuation resulting from the capacity added to the line, definitely limits the power transfer over a high tension line operated at a given potential with a "normal" termination equal to the characteristic impedance of the line.

The study of the quantitative value of this limit, for the case of distributed capacity, is extremely interesting. It shows that the value of this limit is given by

$$\text{Power Limit} = \frac{E^2}{2lR} \cdot \frac{1}{e}\left(1 + \frac{pL}{\sqrt{R^2 + p^2L^2}}\right),$$

where, as before, the significance of the symbols is

l = length of line,

R = resistance per mile,

pL = reactance per mile,

e = basis of Naperian logarithms.

It is surprising to find that the maximum power limit is quite high; it is equal to 454,000 kilowatts, approximately. This compares with a figure of 113,000 kilowatts for the normal 220,000-volt line operated under similar conditions. This high limit is obtained for an optimum value of distributed line capacity given by the following formula:

$$\sqrt{pC} = \frac{1}{l\sqrt{2(\sqrt{p^2L^2 + R^2} - pL)}}.$$

The optimum value for the capacity (C) thus decreases with length of the line (l). Another important conclusion that can be immediately drawn is that, when a high tension line is long enough to have a total attenuation of 0.50 in

numerical value, the addition of "bridged" capacity will decrease its power limit instead of increasing it.

The above conclusions, as definitely stated, refer only to transmission systems with bridged capacity. It seems also evident that, notwithstanding how closely intermediate synchronous condensers may be placed upon a given transmission line, there will be a maximum power limit to such a line. The second interesting and important conclusion is that the advantage that may be expected through the use of synchronous condensers will be less and less effective for very long lines. It would be interesting to have this second point established by direct analysis rather than by inference.

POSSIBILITIES OF TRANSCONTINENTAL TRANSMISSION OF POWER.

In the above detailed outline of the possibilities of the transportation of electric power, a 500-mile line was taken as a basis of discussion. It should be understood, of course, as already stated, that the procedure consisting in the use of alternate sections of transmission line and neutralizing network can be carried out for any distance. Transportation of electric power may thus be physically carried out across the continent of the United States. The neutralizing networks, as shown on the drawings, may be partial or complete. The length of each section of transmission line may also differ, depending upon economical conditions obtaining in each specific case. Assuming, for illustration, the distance of 3,000 miles and that the pillar neutralization is complete and is provided for at every 500 miles, while the architrave neutralization is partial and is provided for at every 100-mile interval (in order to limit the potential rise along the line), it is readily seen that the corresponding equivalent π network of the 3,000-mile line will be equal to that of a 500-mile line if the architrave neutralization is 5/6 complete. Therefore, the amount of power that can be transmitted over the 3,000-mile line will be equal to that which can be transmitted over a 500-mile line. The cost of transportation will, of course, be greater, since the investment in transmission line conductors and in neutralizing equipment will be correspondingly greater.

The above statement assumes, of course, that neutral-

ization has been provided, not only for the reactance component, but also for the resistance component of the architrave impedance. Resistance neutralization requires the use of either series or shunt generators spaced at intervals of, roughly, 500 miles. The resistance neutralization can be avoided by raising the potential of transmission from 220,000 volts to 330,000 volts and providing a compound line, substantially, as described above, with provision for

(*a*) Complete pillar susceptance neutralization.
(*b*) Partial architrave reactance neutralization.

To show more definitely the practical application of the system disclosed herein, for transcontinental transmission, the following data are submitted. The increase in the transmission potential will require raising the outside diameter of the transmission line conductors in the ratio of 3 to 2 (corresponding to ratio of potentials, 330,000 to 220,000 volts) in order to avoid corona. The line conductors can, however, be in hollow copper, avoiding in this manner the use of extra metal in the line. The spacing of the conductors will have to increase practically in the same proportion, thereby increasing the line reactance constant per mile. Assuming that the resistance per mile of the more widely spaced hollow copper conductors is the same as that of the typical 220,000-volt line described hereinabove, and carrying out the neutralization of the series line reactance to the point where maximum power limit is realized for the system as a whole, the characteristics of this "compound" line would be

Resistance per mile	.0782 ohm
Reactance ($\sqrt{3}$ times the resistance)	.1360 ohm
Susceptance per mile	0.0000 mho

The architrave impedance of a 2,000-mile line, constructed in accordance with the above specifications, would be

$$\text{Resistance} = 234.6 \text{ ohms},$$

$$\text{Reactance} = 408.0 \text{ ohms}.$$

The power limit (P) of this line would be

$$P_{\max.} = \frac{(330{,}000)^2 \times 10^{-3}}{4 \times 234.6} = 116{,}000 \text{ kilowatts, approximately.}$$

The above limit is practically the same as that of a 500-mile line, operated at 220,000 volts, constant potential, not equipped with neutralizing networks. It is thus seen that a system of "compound" conductors could be built, connecting the great power centers in the United States, which would allow power transfer of the order of 100,000 kilowatts, or more, from end to end, up to a distance of 3,000 miles, when operated at 330,000 volts and equipped with neutralizing networks substantially as described.

APPLICATION OF NEUTRALIZING NETWORKS TO COMMUNICATION CIRCUITS

The application of neutralizing networks to communication circuits also offers a wide and interesting field. It was noted that in power transmission application reactance neutralization was important. In the communication art, the reverse holds true. Resistance neutralization in the form of π or τ networks will undoubtedly be found useful. It is also possible that phase correction may be obtained. Such an application would prove of real importance on very long cables.

Such neutralization would represent the equivalent of amplification, as at present so successfully carried out in communication practice.

The application of neutralization to communication circuits is so complex in character that it will not be considered further in the present paper.

THE COST OF LIGHTING INDUSTRIAL BUILDINGS.

BY

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WITH the advent of the industrial age, man was forced indoors for the protection of his work from the ravages of the elements. This change of work-place was made regardless of the inadequacy of the natural lighting within his crude building, and regardless of the expense and limited controllability of the artificial light-sources at his command. As time progressed, buildings were constructed with larger and larger windows. Finally, in order to obtain a higher and more uniform intensity of illumination upon the work-plane, buildings were constructed with high continuous windows in the walls and elaborate windows in the roof.

Within the last few decades, artificial light-sources have been made dependable, much less expensive than formerly, and highly controllable.

As a consequence, most industrial and commercial buildings are equipped for both natural and artificial lighting. In uncongested districts one-story buildings constructed with monitor or sawtooth windows in the roof employ artificial lighting for a relatively small part of the usual work-hours of the day; whereas multi-story buildings in congested districts employ artificial lighting many so-called daylight hours.

A habit has been formed of using daylight in all types of buildings regardless of the fact that in certain places artificial light could better fill the requirements. Many years ago Dr. M. Luckiesh began emphasizing the undue persistency of this "daylight habit" in lighting museums and other places. Later, in order to bring the economic aspect of the matter forcefully to the attention of architects, engineers and the public, Dr. Luckiesh and the writer [1] investigated the relative

[1] "The Cost of Daylight," M. Luckiesh and L. L. Holladay, *Trans. I. E. S.*, 18, 1923, p. 119.

cost of natural and artificial lighting in residences, hotels and art museums, and found their costs quite comparable.

With the increasing economic stress of industrial competition and the advancement of the lighting art, it appears desirable to determine the most economical type of building from a lighting viewpoint, and to determine the most economical conditions under which it may be operated. Therefore, the purposes of this investigation are to devise ways and means of computing the complete cost of lighting industrial buildings, to determine from a number of suitable types of buildings for a given industry, which one can be most economically lighted, and to ascertain the most economical conditions of maintenance and operation.

INVESTMENT COST OF NATURAL LIGHTING EQUIPMENT.

Natural light is free outdoors, but when delivered indoors it has a fairly definite cost. Upon consideration, it is apparent that windows, awnings, shades, curtains, light-courts, a more expensive type of building to admit more light and distribute it better, certain losses of heat incident to windows, and the heating equipment to supply these losses are avoidable expenses directly chargeable against natural lighting.

Windows have a psychological value in permitting one to see out; however, in many industrial buildings this advantage is sacrificed by glazing the windows on the level of the workmen's eyes with "obscuring" glass. Windows may be a disadvantage in summer in admitting and trapping heat; however, this only reduces the pleasantness of the building somewhat without increasing its cost unless measures are taken to remove the heat.

In order to have a definite basis for computing the cost of natural lighting, a standard procedure which seems logical and fair has been adopted and followed throughout this investigation. The cost of the standard building of given type and size with all of its natural lighting equipment, including windows, light-courts, heating system, etc., is computed. Then the cost of an equally satisfactory building of the same size, of analogous type of construction, with no windows and no light-courts, but provided with heating and ventilating systems of the same types and of equal perform-

ances to the ones in the standard building is computed. The difference in the costs of the two completely equipped buildings is considered the investment cost of the natural lighting system.

Throughout this investigation no charges have been made against natural lighting for shades, awnings, painting, and other accessories for diffusing or eliminating direct sunlight, since their lack of standardization or of general acceptance indicates their unsatisfactoriness.

The problem of determining the cost of natural lighting may be complicated by the fact that certain things serve more than one purpose. For example, land surrounding a building and used to admit light to it, may be used for platforms, railroad sidings, yard roads, parking of automobiles, etc. In all such cases the cost of these things must be apportioned to the various purposes in accordance with the value of the service rendered.

ANNUAL COST OF NATURAL LIGHTING.

The annual cost of natural lighting is computed in a manner analogous to the one just described. Thus the difference between the annual cost of the standard building with windows, and the annual cost of the substituted building without windows is the annual cost of natural light.

In computing annual costs, uniform rates are used for the various types and sizes of buildings. For interest, depreciation, repairs, taxes and insurance, a charge of 13 per cent. of the initial cost is made for the one-story buildings of brick and steel construction; and a charge of $10\frac{1}{2}$ per cent. on multi-story buildings of reinforced-concrete construction. A charge of 13 per cent. is made for interest, depreciation, repairs, taxes, etc., on the steam-heating system. It is assumed that during the seven months of operation of the heating system, the attendance of a fireman is required for 300 hours, plus two hours for each ton of coal burned, for which he received $.65 an hour of service. It is also assumed that for seven months of the year the inside of the building is maintained at 65° F. during work-hours when there are two changes of air per hour, and at 50° F. during other times when there is no change of air. The capacity of the

heating system is such that these temperatures can be maintained with a temperature of 0° F. outdoors. Coal, delivered in the bin, is assumed to cost $5.00 a ton. It is assumed that two per cent. of the windows are broken per year and that they are replaced at $.50 per square foot, and that the washing of the windows on both sides costs 1½ cents per square foot.

COST OF ARTIFICIAL LIGHTING.

Throughout this investigation, all buildings are assumed to have similar types of electric lighting equipment. The electric wiring is assumed to be in iron conduit and to be done in accordance with the National Electric Code. The lighting-units are assumed to be porcelain-enameled (RLM dome) steel reflectors, fitted with high-efficiency white bowl tungsten lamps mounted 11 feet six inches above the floor and spaced uniformly on 10-foot centers. The initial cost of the electric lighting equipment includes the costs of the wiring system and lighting-units complete in every particular.

The annual cost of electric lighting is assumed to consist of an overhead charge of 12 per cent. on the wiring, 21 per cent. on the lighting-units, six cleanings of the lighting-units per year at from five to ten cents per cleaning (depending upon their size), lamp renewals, and electric energy at the approximate average price of two cents per kilowatt-hour, unless otherwise stated.

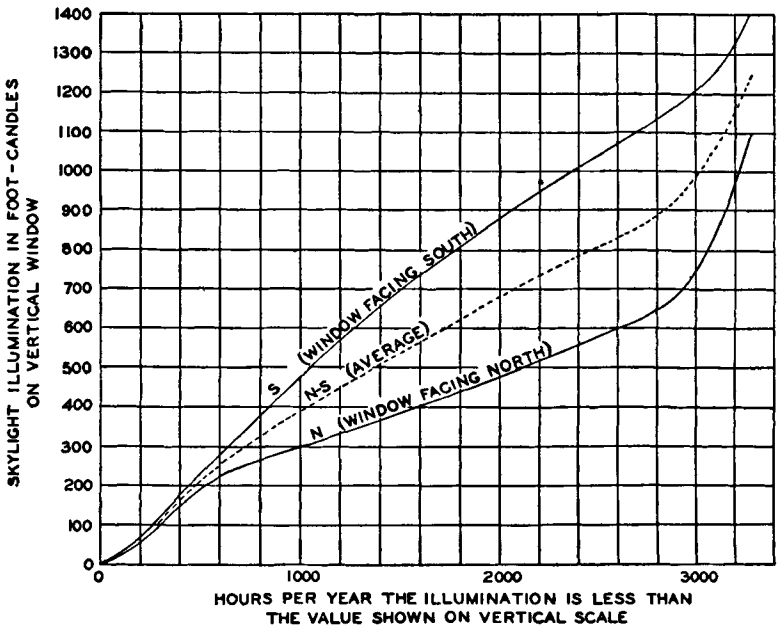
DETERMINATION OF NATURAL ILLUMINATION.

In order to find the intensity of natural illumination at any given point upon the work-plane within the building, it is first necessary to find the brightness of, or the illumination upon, the window through which the light comes. Professor H. H. Kimball [2] has given the intensity of skylight illuminations upon plane surfaces having various orientations for any time of the year and any time of day for clear and cloudy days. Mr. M. W. Davis of the U. S. Weather Bureau in Cleveland has furnished tables showing the number of clear and cloudy days in each month of the year for Atlanta,

[2] "Daylight Illumination on Horizontal, Vertical and Sloping Surfaces," H. H. Kimball, *Trans. I. E. S.*, 18, 1923, p. 434.

Birmingham, Boston, Chicago, Cleveland, Cincinnati, New York City, Philadelphia, Pittsburgh, San Francisco, Seattle and St. Louis; and Mr. C. F. Marvin, Chief of the Weather Bureau in Washington, has furnished some data obtained in Chicago which is of use in estimating the intensity of skylight illumination on dark and rainy days.

FIG. 1.



Showing the computed number of hours per year between 7:30 A.M. and 4:30 P.M. in an average industrial center during which the intensity of skylight illumination upon an unobstructed vertical window is less than the number of foot-candles given. *N* is for windows having a northern exposure; and *S*, for windows having a southern exposure; and *N-S*, the average for northern and southern exposures.

By the use of these data the number of hours per year (between the hours of 7:30 A.M. and 4:30 P.M.) during which the intensity of illumination upon an unobstructed vertical window is less than a certain value has been computed for an average of the industrial centers mentioned above. Curves of these results for windows having northern and southern exposures are plotted in Fig. 1.

The intensity of illumination upon a vertical window may be reduced by the presence of obstacles outdoors. When the

obstacle is a parallel building of relatively great length at a distance of W feet and its skyline is H feet above the center of the window, it may be shown that when the sky is of uniform brightness the intensity of illumination due to it upon the window is reduced to

$$1 - H/\sqrt{H^2 + W^2} \quad \text{or} \quad 1 - \cos(\tan^{-1} W/H)$$

of its unobstructed value.

Since light from the sky makes angles of incidence with the normal to the surface of a plane unobstructed vertical window that vary from zero to 90 degrees, it may be shown by use of Fresnel's equation that about 32 per cent. of the light incident upon the outer surface of such a window is lost by reflection. When the glass is of some diffusing type and the light is more or less obstructed by adjacent buildings, it is somewhat problematical as to just what portion of the incident light is lost by reflection. However, for the sake of uniformity, it has been assumed throughout this investigation that under all circumstances the loss by reflection is 32 per cent. Therefore, taking into consideration the amount of light absorbed by the accumulation of dirt upon the window [3] and the loss of light by reflection, the overall transmission of a vertical window is assumed to be 55 per cent. when it is washed four times a year, 51 per cent. when washed twice a year, and 38 per cent. when washed once a year.

Finally, suitable equations for computing the intensities of illumination from vertical and inclined windows were used for determining the intensities of natural illumination at various points upon the work-plane.[4, 5]

By the use of these data, the intensities of natural illumination upon a work-plane three feet above the floor were computed for points underneath the various lighting-units at various distances from the windows for various intensities of illumination upon the outer surface of the window.

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[3] "Making Your Windows Deliver Daylight," W. C. Randall and A. J. Martin, *Trans. I. E. S.*, 22, 1927, p. 239.

[4] "Prediction of Daylight from Vertical Windows," H. H. Higbie, *Trans. I. E. S.*, 20, 1925, p. 433.

[5] "Prediction of Daylight from Sloping Windows," H. H. Higbie and A. Levin, *Trans. I. E. S.*, 21, 1926, p. 273.

HOURS LIGHTING-UNITS OPERATE A YEAR.

The size of artificial lighting-unit employed in any given installation is such that it gives the "prescribed minimum" intensity of illumination upon the work-plane. The lighting-units are placed in rows parallel with the windows and each row is lighted only when the intensity of natural illumination under that row has fallen to the prescribed minimum.

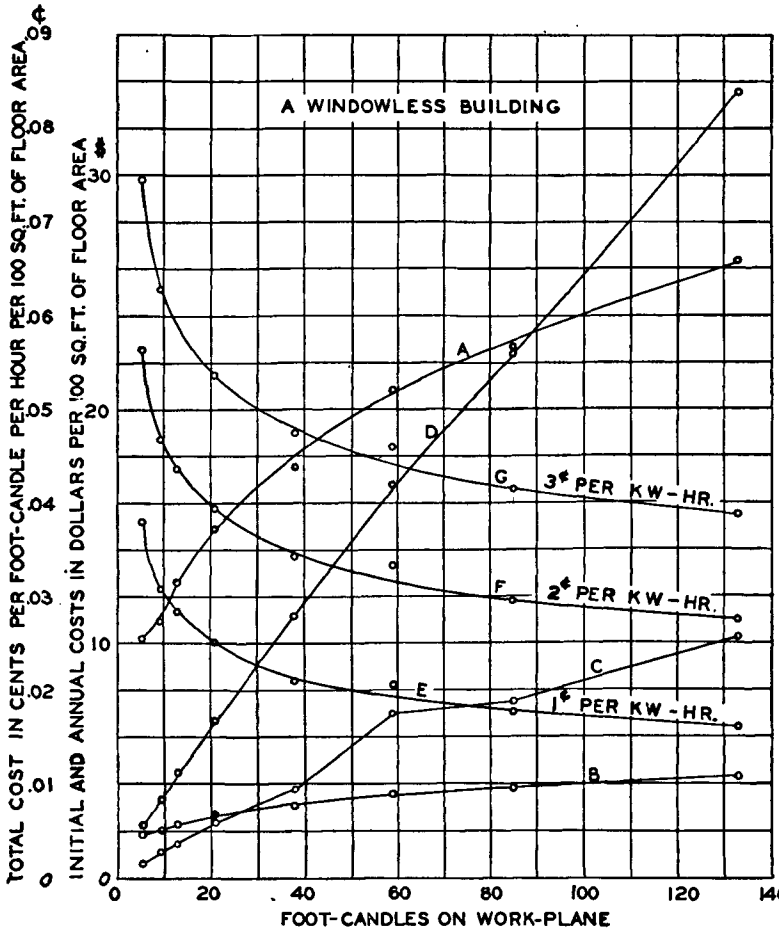
Having determined the intensity of natural illumination under a given row of lighting-units for a given intensity of skylight illumination upon the window, it is possible to compute, from the prescribed minimum intensity of illumination upon the work-plane, the intensity of skylight illumination upon the window required to give the minimum intensity of illumination under the given row of lighting-units. Knowing the required intensity of skylight illumination upon the window, the hours of operation per year of each lighting-unit in a given row may be obtained by scaling the appropriate curve in Fig. 1 and making suitable deductions for holidays, Sundays, half-day Saturdays, and noon-hours. Throughout this investigation, the number of work-hours in a year has been assumed to be 2,240. In this way the hours of operation per year of every lighting-unit in the building may be obtained, and from them their average for any prescribed minimum intensity of illumination upon the work-plane.

CHARACTER OF DATA USED.

It has been the aim to use data fairly representative of the average, but it is recognized that prices and conditions vary greatly. For example, the cost of an industrial building may vary from $1.50 to $3.50 per square foot of floor area; the cost of real estate may vary from a negligible amount to $3.00 or more per square foot; the cost of heating system may vary from $1.50 to $2.00 per square foot of radiation; the average and maximum differences of temperature between indoors and outdoors may vary greatly with the section of country; due to obstructions, the intensity of skylight illumination upon a window may vary from zero to 100 per cent. of its unobstructed value; the average sky-brightness varies with weather conditions and the sections of country; the average transmission of a window may vary with the

type of glass used, the exposure, and frequency of washing from 5 per cent. up. Also, in certain instances costs may have to be pro-rated and thus be subject to differences of

FIG. 2.



Costs of lighting 100 sq. ft. of floor area of a building illuminated solely by means of electric lighting-units spaced uniformly on 10-foot centers. *A* shows total investment in wiring and lighting-units, complete per lighting-unit or per 100 sq. ft. of floor area; *B*, the annual overhead cost of lighting equipment; *C*, the cost of lamp renewals; *D*, the cost of electric energy at one cent per kw.-hr.; *E*, *F*, *G*, the total cost per foot-candle per hour per 100 sq. ft. of floor area for energy at respectively one, two and three cents per kw.-hr.

opinion. Therefore, in order that the reader may be able to make such recomputations, or allowances, for other conditions, costs have been itemized insofar as space permitted.

All unit costs employed throughout this investigation have been obtained from responsible building contractors, heating engineers, realtors, and others, many of whom enjoy a national reputation.

COST OF LIGHTING VARIOUS TYPES OF INDUSTRIAL BUILDINGS.

Six types of industrial buildings have been studied. Four are single-story, and are designated as Buildings *A*, *B*, *C* and *D*; and two are multi-story, and are designated as Buildings *E* and *F*.

Building A.—A single-story structure of brick and steel, 60 ft. by 180 ft., with 10 ft. 3 in. continuous side windows, and flat roof, as shown in Fig. 3.

The costs of natural light and the costs of artificial light needed to supplement the natural light in maintaining a minimum of 13 foot-candles upon a horizontal work-plane three feet above the floor are presented in the table below. The space between buildings is 20 feet, the windows are washed semi-annually and the work-hours are from 7:30 A.M. to 11:30 A.M. and from 12:30 P.M. to 4:30 P.M.

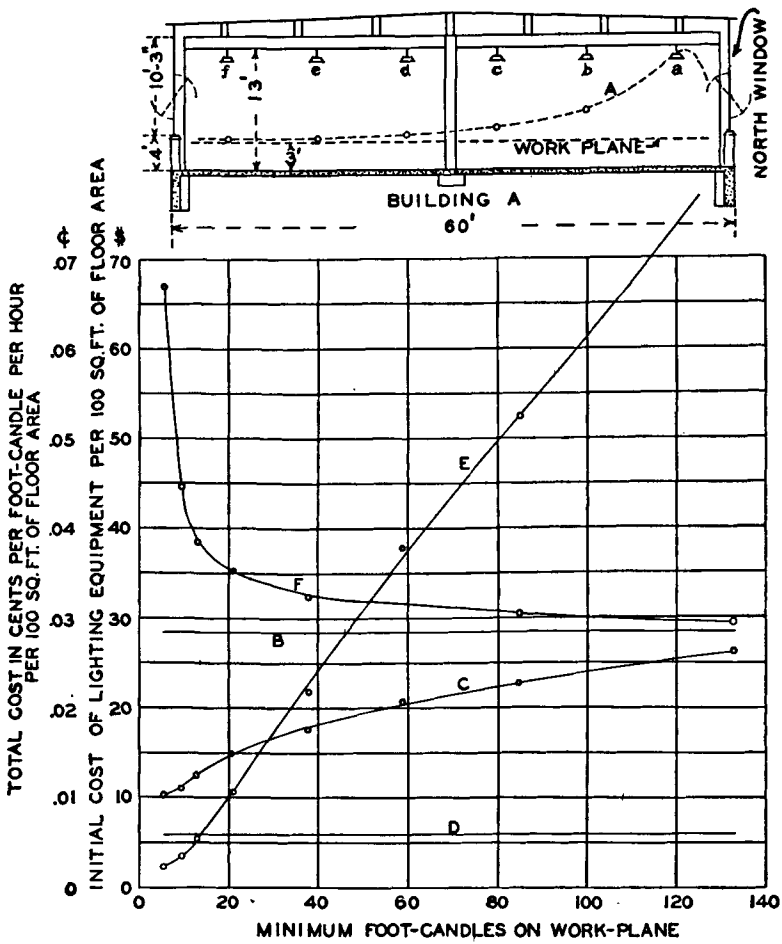
	Natural Lighting.	Artificial Lighting.
Initial cost of installation	$3,088	$1,371
Annual cost of lighting	630	578

This building is representative of one of the standard types erected by several large firms. It has a width of 60 feet and a length of 180 feet, being made up of nine standard 20-foot units. The roof is supported by 30-foot I-beams 13 feet above the floor. The walls are of brick. The windows, 10 ft. 3 in. high, contain 4,700 sq. ft. of hammered glass or the equivalent; and they are constructed with continuous sash and hand-operated ventilating panels. The floors consist of a 5-inch concrete base with monolithic finish.

Through the coöperation of various building contractors and engineers, the cost of this building complete with heating, ventilation, plumbing, and electric lighting was estimated to be $21,600 or $200 per 100 sq. ft. of floor area. The estimated cost of an equally satisfactory building without windows but containing hand-operated ventilating panels, etc., was estimated to be the same as the standard structure, Building *A*.

A ventilating-court, two feet wide, was assumed to surround the windowless building.

FIG. 3.



Cross-section of Building *A* and costs of natural and artificial lighting per 100 sq. ft. of floor area. The building is surrounded by a light-court 20 feet wide, costing $.40 a sq. ft. The windows are washed twice a year at a cost of 1½ cents per sq. ft. per washing, and electric energy costs two cents per kw.-hr. *A* shows the foot-candles under each lighting-unit per 100 foot-candles of illumination upon the unobstructed windows to the right; *B*, initial cost of natural lighting equipment; *C*, initial cost of electric lighting equipment; *D*, annual cost of natural lighting; *E*, annual cost of electric lighting; and *F*, the cost per minimum foot-candle per hour per 100 sq. ft. of floor area.

In Building *A* it was estimated from data supplied and used by heating engineers that 880 sq. ft. of steam radiation

was necessary to supply the heat losses incident to the windows. If the cost of the steam-heating system complete per square foot of radiation is $1.70, it follows that the initial cost of the heating equipment chargeable to natural lighting is $1,496 or $13.85 per 100 sq. ft. of floor area.

It is assumed that areas 20 ft. wide on the two sides and rear of the building are necessary for light-courts and that after making suitable deductions for the other uses made of these areas, one-half of their remaining cost is chargeable equally against the adjoining buildings. A windowless building would require a space about 2 feet wide as an air-shaft for ventilation. With a pro-rated cost of real estate of $.40 per sq. ft., which is probably a fair average price when all things are considered, the initial cost of real estate chargeable against the natural system of lighting would be $1,592 or $14.71 per 100 sq. ft. of floor area.

Thus the investment charges against the natural system of lighting per 100 sq. ft. of floor area are $0.00 for the windows and structure, $13.85 for the heating system, and $14.71 for light-courts or a total of $28.56.

With the assumptions made, the items of annual cost of natural lighting of Building *A* are:

Interest, depreciation, repairs, taxes, etc., on windows and structure, $0.00 × 13%	$ 0.00
Window renewals, 4,700 sq. ft. × $0.50 per sq. ft. × 2%	47.00
Washing windows, 4,700 sq. ft. × 2 × $0.015 per sq. ft.	141.00
Interest, depreciation, repairs, taxes, etc., on heating system—$1,496 × 13%	194.48
Fuel to supply window losses, 21.7 tons at $5.00 per ton	108.50
Boiler attendance, 43 hours × $.65 per hour	27.95
Interest and taxes on light-court, $1,592 × 7%	111.44
Total	$630.37

Therefore, the annual cost of natural lighting for Building *A* is $630 for the items considered, or is $5.83 per 100 sq. ft. of floor area. This itemized statement makes it possible for anyone to alter various items according to his own judgment and experience or for a special case. For example, the charge for light-courts is subject to wide variations due to the price of real estate and possibly to other uses being made of the courts.

It being assumed that the building is surrounded on its sides and rear by other similar buildings, distant 20 feet,

the intensity of skylight illumination upon its windows is only about 64 per cent. as much as if there were no buildings nearby. Also, since about 32 per cent. of the skylight upon the windows is lost by reflection and does not enter the building, and about 25 per cent. of the remainder is lost by absorption when the windows are washed only twice a year, it follows that only about 51 per cent. of the light falling upon the windows is transmitted to the interior.[6, 7] Thus the apparent diffuse brightness of the window as viewed from the interior is $b = .51 \times 100/3.14$ candles per square foot per 100 foot-candles of illumination upon the outside of the window due to light from the sky. By using this value of window brightness b in the equation derived by Higbie,[4] the intensity of natural illumination at locations under each lighting-unit of the artificial lighting system may be computed. This value is increased by ten per cent. in the case of natural lighting to allow for reflected light from walls, ceilings, etc. In this way the intensity of natural illumination at stations under each artificial lighting-unit per 100 foot-candles of illumination upon the exterior of a window due to light from the sky was computed and is as shown plotted in curve *A*, Fig. 3. With this intensity of natural illumination under each lighting-unit per 100 foot-candles of sky illumination upon the exterior of each window, and with the curves of the number of hours per year below which the intensity of illumination upon unobstructed northern and southern windows have the values shown respectively on curves *N* and *S* in Fig. 1, the number of hours the lighting-units in each row operate per year has been computed. Thus for a minimum intensity of illumination of 13 foot-candles upon the work-plane, the number of work-hours per year each lighting-unit in each row (Fig. 3) operates is:

Row *a*	245	hours
" *b*	660	"
" *c*	1,310	"
" *d*	960	"
" *e*	500	"
" *f*	230	"
Average	651	"

[6] "Transmission of Light through Window Glass," E. H. Hobbie and W. F. Little, *Trans. I. E. S.*, 22, 1927, p. 258.

[7] "Light and Work," M. Luckiesh, D. Van Nostrand Company, 1924.

TABLE I.

Data upon the Cost of Lighting Various Types of Inaustrial Buildings Operating under Approximately Normal Conditions.

The windows are washed twice a year; the minimum intensity of illumination upon the work-plane is 13 foot-candles; and other conditions are as stated.

Building	*A*	*B*	*C*	*D*	*E*	*F*	*O*
Type	Flat Roof.	6 ft. Monitor.	34 ft.-V-type *Monitor.*	Sawtooth.	Multi-story.	Multi-story.	Flat Roof Windowless.
Ground dimensions in feet	60 × 180	60 × 180	100 × 200	120 × 180	60 × 180	60 × 180	60 × 180
No. of stories	1	1	1	1	6	5	1
Type of construction	Brick and steel	Brick and steel	Brick and steel	Brick and steel	Rein. con.	Rein. con.	Brick and steel
Space between buildings in feet	20	20	20	20	80	80	2
Square ft. of floor area	10,800	10,800	20,000	21,600	64,800	54,000	10,800
Square ft. of window area	4,700	6,560	10,935	10,860	20,800	27,750	0
Ratio of window area to floor area	.435	.607	.546	.503	.321	.514	0
Proportion of sky illuminating side windows	.64	.64	.64	.55	.31 to .89	.30 to .90	—
Proportion of sky illuminating monitor windows	—	.95	.95	1.00	—	—	—
Cost of building complete	$21,600	$22,680	$41,000	$49,680	$194,400	$183,600	$21,600
Investment cost of natural lighting equipment:							
Windows and structure	$ 0	$1,080	$4,000	$9,720	$ 0	$21,600	$0
Heating system to supply window losses	1,496	2,660	6,410	5,740	6,630	12,400	0
Real estate at $40 per sq. ft. chargeable to light-courts	1,592	1,592	1,880	1,810	7,840	7,840	0

TABLE I (*Continued*).

Investment costs for natural lighting per 100 sq. ft. of floor area:

Building	*A*	*B*	*C*	*D*	*E*	*F*	*O*
Type	Flat Roof.	6 ft. Monitor.	34 ft.-V-type Monitor.	Sawtooth.	Multi-story.	Multi-story.	Flat Roof Windowless.
Windows and structure	$ 0.00	$10.00	$20.00	$45.00	$ 0.00	$40.00	$0.00
Heating system	13.85	24.60	32.10	26.50	10.30	22.95	0.00
Real estate at $.40 per sq. ft.	14.71	14.71	9.40	8.40	12.09	14.50	0.00
TOTAL	$28.56	$49.31	$61.50	$79.90	$22.39	$77.45	$0.00
Annual cost of natural lighting per 100 sq. ft. of floor area:							
Int., dep., rep., taxes, etc., on windows and structure	$0.44	$ 1.91	$ 3.15	$ 6.35	$ .32	$ 4.71	$0.00
Heat losses through windows	3.06	5.42	6.71	5.69	2.28	4.85	0.00
Real estate—7%	1.03	1.03	.66	.58	.85	1.02	0.00
Washing windows twice	1.30	1.82	1.64	1.5.	.96	1.54	0.00
TOTAL	$5.83	$10.18	$12.16	$14.13	$4.41	$12.12	$0.00
Investment cost of electric lighting system per 100 sq. ft. of floor area for 13 f.c.:							
Electric wiring,—complete	$ 8.65	$ 8.65	$ 8.65	$ 8.65	$ 8.65	$ 8.65	$ 8.65
Lighting-units, installed	4.05	4.05	4.05	4.05	4.05	4.05	4.05
TOTAL	$12.70	$12.70	$12.70	$12.70	$12.70	$12.70	$12.70

TABLE I (*Continued*).

Annual cost of electric lighting per 100 sq. ft. of floor area for a minimum of 13 f.c.:

Building	*A*	*B*	*C*	*D*	*E*	*F*	*O*
Type	Flat Roof.	6 ft. Monitor.	34 ft.-V-type Monitor.	Sawtooth.	Multi-story.	Multi-story.	Flat Roof Windowless.
Int., dep., rep., etc., on electric equip.—12%	$1.04	$1.04	$1.04	$1.04	$1.04	$1.04	$1.04
Int., dep., rep., etc., on lighting-units—21%	.85	.85	.85	.85	.85	.85	.85
Cleaning lighting-units six times a year	.42	.42	.42	.42	.42	.42	.42
TOTAL OVERHEAD COST	$2.31	$2.31	$2.31	$2.31	$2.31	$2.31	$2.31
Annual no. of hours of operation per lighting-unit	651	379	256	253	1187	704	2240
Lamp renewals	$0.44	$ .25	$ .17	$ .17	$ .80	$ .47	$ 1.50
Electric energy at 2¢ per kw.-hr.	2.60	1.52	1.02	1.01	4.75	2.81	8.96
TOTAL COST OF ELECTRIC LIGHTING	$5.35	$4.08	$3.50	$3.49	$7.86	$5.59	$12.77

Total annual cost of lighting per 100 sq. ft. of floor area for a minimum of 13 f.c.:

Natural lighting	$ 5.83	$10.18	$12.16	$14.13	$ 4.41	$12.12	$ 0.00
Electric lighting	5.35	4.08	3.50	3.49	7.86	5.59	12.77
TOTAL FOR NATURAL AND ELECTRIC LIGHTING	$11.18	$14.26	$15.66	$17.62	$12.27	$17.71	$12.77
Cost per min. foot-candle per hour per 100 sq. ft. of floor area	.0384¢	.0489¢	.0537¢	.0605¢	.0421¢	.0608¢	.0438¢

Thus when natural lighting in Building *A* is supplemented by artificial lighting during the assumed work-hours, the lighting-units operate on an average of 651 hours per year out of a total of 2,240 in maintaining a minimum of 13 foot-candles upon the work-plane.

The artificial lighting equipment is designed to give a uniform intensity of illumination of 13 foot-candles upon a horizontal work-plane three feet above the floor and consists of 108 porcelain enameled (RLM dome) steel reflectors, each fitted with a 200-watt white bowl high-efficiency tungsten lamp. The cost of the wiring complete at $8.65 per outlet is $934; and the cost of lighting-units installed at $4.05 each is $437. Thus the total investment cost for artificial lighting is $1,371, or a total cost of $12.70 per 100 sq. ft. of floor area.

With the assumptions made, the items of annual cost of artificial lighting used to supplement natural lighting in maintaining the prescribed minimum of 13 foot-candles upon the work-plane are:

Interest, depreciation, repairs, taxes, etc., on wiring, $934 × 12%.....	$112.00
Interest, depreciation, repairs, taxes, etc., on lighting-units, $437 × 21%	91.80
Cleaning lighting-units six times, 108 × 6 × $.07.....	45.36
Lamp renewals, 108 × 651 × $.67/1000.....	47.10
Electric energy at two cents per kw.-hr., 108 × 651 × 200/1000 × $.02.	281.24
Total.....	$577.50

Therefore, the total annual cost of artificial lighting is $577.50 or an annual cost of $5.35 per 100 sq. ft. of floor area.

A summary of all these costs is presented at the beginning of this section and in the second column of Table I.

Building B.—A single-story structure of brick and steel, 60 ft. by 180 ft., with 4,700 sq. ft. of 10 ft. 3 in. continuous side windows and 1,860 sq. ft. of 5 ft. 2 in. windows in a monitor 6 ft. wide.

The costs of natural light and the costs of artificial light needed to supplement the natural light in maintaining a minimum of 13 foot-candles upon a horizontal work-plane three feet above the floor are presented in the table below. The space between buildings is 20 feet, the windows are washed semi-annually and the work-hours are from 7:30 A.M. to 11:30 A.M. and from 12:30 P.M. to 4:30 P.M.:

	Natural Lighting.	Artificial Lighting.
Initial cost of installation	$5,332	$1,371
Annual cost of lighting	1,100	440

This building is similar in design and construction to Building *A* excepting that it has a monitor 6 feet wide down the center of its roof. The cost of this monitor is about $1,080 and is entirely chargeable to natural lighting. The heating system requires about 1,565 sq. ft. of radiation to supply heat losses due to the windows. Assuming the installation cost of the heating system per sq. ft. of radiation to be $1.70, the initial cost of the heating system chargeable to natural lighting is thus 1,565 × $1.70, or $2,660. With an intervening space between buildings of 20-foot width, the cost of real estate at $.40 per sq. ft. chargeable against natural lighting is $1,592. Therefore, the initial cost of the natural lighting equipment is $1,080 for windows and structure, $2,660 for heating system, and $1,592 for light-courts, or a total of $5,332, as stated above.

With light-courts of 20-foot width, the intensity of skylight illumination upon the windows is only about 64 per cent. of its unobstructed value upon the side windows, and 95 per cent. of it upon the monitor windows. Therefore, with the windows washed twice a year, and with a minimum intensity of illumination upon the work-plane of 13 foot-candles, the average time of operation of a lighting-unit is 379 hours per year.

Following the method of computation previously outlined, the costs of lighting are as given at the head of this section and as presented in the third column of Table I.

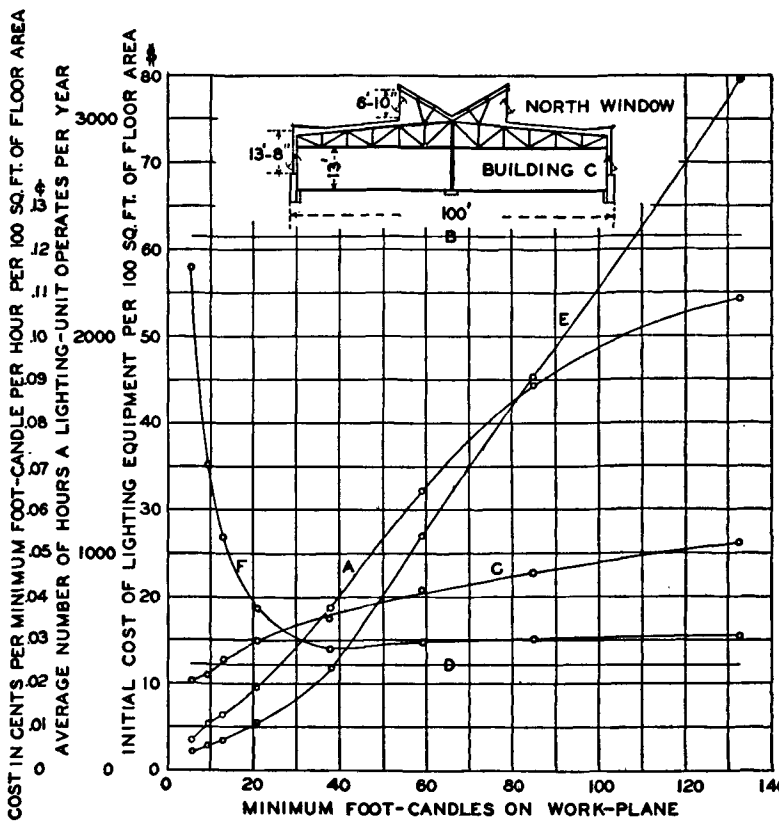
Building C.—A single-story structure of brick and steel, 100 ft. by 200 ft., with 8,200 sq. ft. of 13 ft. 8 in. side windows and 2,735 sq. ft. of 6 ft. 10 in. vertical windows in a V-type monitor, as shown in Fig. 4.

With 20 feet between adjacent buildings, the windows washed twice a year, and a minimum of 13 foot-candles upon the work-plane, the costs of lighting are:

	Natural Lighting.	Artificial Lighting.
Initial cost of installation	$12,290	$2,540
Annual cost of lighting	2,432	700

In this building the cost of windows and structure chargeable to natural lighting is about $4,000. The radiation required to supply losses due to windows is 3,770 sq. ft.

FIG. 4.



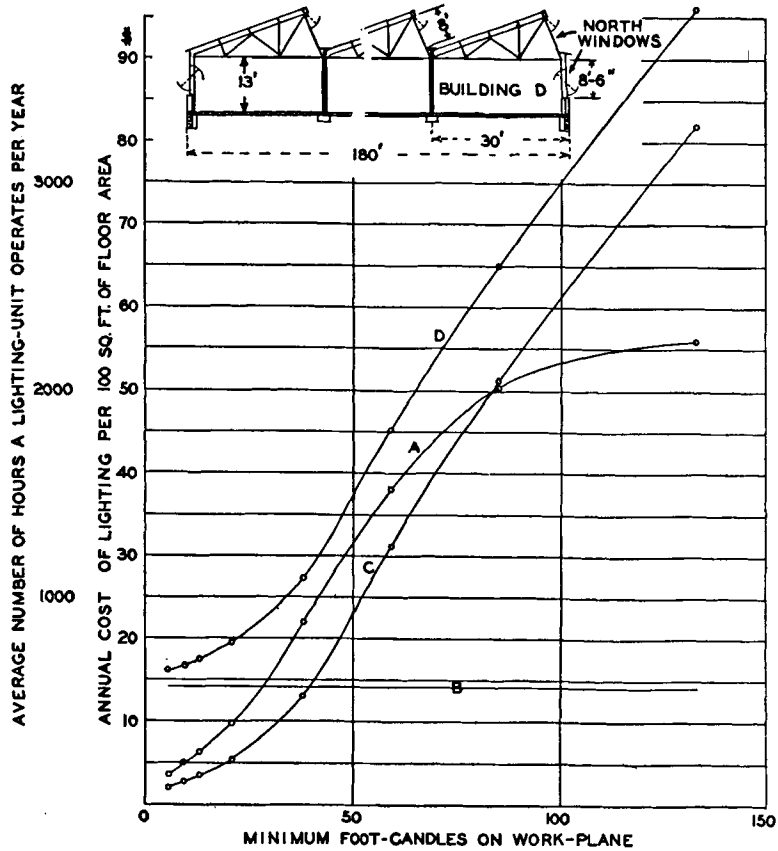
Costs of lighting Building *C* when it is surrounded by a light-court 20 feet wide costing $.40 a sq. ft. and when windows are washed twice a year. *A* shows the average number of hours per year lighting-units operate; *B*, the initial cost of equipment for natural lighting per 100 sq. ft. of floor area; *C*, the initial cost of the electric lighting equipment; *D*, the annual cost of natural lighting; *E*, the annual cost of electric lighting; *F*, the total cost of lighting per minimum foot-candle per hour per 100 sq. ft. of floor area.

and the initial cost of heating system chargeable to natural lighting is 3,770 × $1.70 or $6,410. The cost of real estate in light-courts at $.40 per sq. ft. is $1,880.

With the light-courts of 20-foot width the intensity of skylight illumination is about 64 per cent. of its unobstructed

value upon the side windows and 95 per cent. of it upon the monitor windows. With the undersurface of the V-monitor having a reflection-factor of 63 per cent., and with the windows washed twice a year, the average time of operation

FIG. 5.



Lighting-cost data for sawtooth Building *D*, when surrounded by a light-court 20 ft. wide, costing $.40 per sq. ft. The windows are washed twice a year, and electric energy costs two cents per kw.-hr. *A* shows the average number of hours of operation per year per lighting-unit; *B*, the annual cost of natural lighting per 100 sq. ft. of floor space; *C*, the annual cost of electric lighting; and *D*, the annual cost of natural and electric lighting.

of a lighting-unit for a minimum of 13 foot-candles upon the work-plane is 256 hours per year. With these data the costs of natural and electric lighting are as given at the head of this section and as tabulated in the fourth column of Table I.

It may be noted here that with this building surrounded by other buildings of about its own height and distant from it 20 feet or more, and with its windows of a diffusing type of glass, the intensity of illumination upon its work-plane is fairly high and uniform; but if it were surrounded by high buildings and if its windows were of a clear plane type of glass, the reflecting undersurface of the V-monitor would be comparatively useless.

Building D.—A single-story brick and steel structure, 120 ft. by 180 ft. with 5,100 sq. ft. of 8 ft. 6 in. windows in its side walls and with 5,760 sq. ft. of 8-ft. windows having an inclination of 30 degrees to the vertical in a roof of sawtooth construction, as shown in Fig. 5.

With 20 feet between adjacent buildings, with windows washed twice a year, and with a minimum of 13 foot-candles upon the work-plane, the costs of lighting are:

	Natural Lighting.	Artificial Lighting.
Initial cost of installation	$17,270	$2,743
Annual cost of lighting	3,042	754

In this building the cost of windows and structure chargeable to natural lighting is about $9,720. The amount of radiation required to supply the heat lost on account of the windows is 3,375 sq. ft.; therefore, the cost of heating equipment chargeable to natural lighting is $5,740. The cost of land used in light-courts is $1,810. Thus the total cost of the natural lighting equipment is $17,270 or $79.90 per 100 sq. ft. of floor area.

Due to the adjacent buildings of about the same height as itself being 20 feet distant, the intensity of skylight illumination is about 55 per cent. of its unobstructed value on the side windows and 100 per cent. on the sawtooth windows. The diffuse reflection-factor of the undersurface of a sawtooth is assumed to be 63 per cent. With the windows washed twice a year, the total transmission of the side windows was assumed to be 51 per cent., and the transmission of the inclined sawtooth windows, 49 per cent. When it is assumed that the sawtooth windows face the north, the computed average time of operation of a lighting-unit is 253 hours per year for a minimum intensity of illumination upon the work-plane of 13 foot-candles.

Using these data, the costs of lighting the building are as given at the head of this section and in the fifth column of Table I.

Building E.—A six-story reinforced concrete structure, 60 ft. by 180 ft., with pitch of ceiling in each story 13 feet high and the windows uniformly 8 ft. 6 in. high, containing 20,800 square feet of glass.

With 80-foot spacings between buildings, with real estate costing $.40 a square foot, with windows washed twice a year, and for a minimum of 13 foot-candles upon the work-plane, the costs of lighting are:

	Natural Lighting.	Artificial Lighting.
Initial cost of installation	$14,470	$8,230
Annual cost of lighting	2,858	5,093

This structure costs about the same whether made with or without windows. The amount of steam radiator surface to supply heat losses due to windows is about 3,900 sq. ft., which, with accompanying heating equipment, costs $6,630. The cost of the light-courts at $.40 per sq. ft. is $7,840.

With similar buildings spaced 80 feet apart, the intensity of skylight illumination upon the windows of each story in per cent. of its unobstructed value is from the first to the sixth story respectively as follows: 31, 38, 48, 59, 73 and 89. When the windows are washed twice a year and the minimum intensity of illumination upon the work-plane of each story is 13 foot-candles, the average time of operation of the lighting-units on each story is 1,615 hours per year on the first story, 1,470 on the second, 1,300 on the third, 1,105 on the fourth, 915 on the fifth, and 715 on the sixth, or an average for the building of 1,187 hours per year.

With these data, the costs of lighting are as given at the head of this section and in the sixth column of Table I.

Building F.—A five-story reinforced concrete structure, 60 ft. by 180 ft., with the height of windows on each floor such that the amount of natural lighting in each story is substantially the same when the light-courts are 80 feet wide. The height of windows in each story from first to fifth is 20, 16, 13, $10\frac{1}{2}$, and $8\frac{1}{2}$ feet respectively; and the total window surface, allowing for 3-foot mullions, is 27,750 sq. ft.

With 80 feet between adjacent buildings, the windows washed twice a year, and a minimum of 13 foot-candles upon the work-plane of each story, the costs of lighting are:

	Natural Lighting.	Artificial Lighting.
Initial cost of installation	$41,840	$6,858
Annual cost of lighting	6,545	3,019

The cost of the windows and structure chargeable to natural lighting is $21,600. The amount of steam radiation required to supply the heat loss due to the windows is about 7,290 sq. ft., giving a cost of heating equipment chargeable to natural lighting of $12,400. The cost of the 80-foot light-court at $.40 per sq. ft. chargeable against natural lighting is $7,840. Thus the total cost of natural lighting equipment is $41,840 or a total cost of $77.45 per 100 sq. ft. of floor area.

With the 80-foot light-court between buildings the intensity of skylight illumination upon the windows of each story in per cent. of its unobstructed value is for each story from the first to the fifth respectively as follows: 30, 42, 57, 73 and 90. When the windows are washed twice a year and the minimum intensity of illumination upon the work-plane of each story is 13 foot-candles, the average time of operation of a lighting-unit on each story is within about plus or minus five per cent. of 704 hours per year.

With these data the costs of lighting are as given at the head of this section and in the seventh column of Table I.

It does not appear that this type of building is justifiable from the standpoint of economy in lighting.

COST OF LIGHTING SOLELY BY ARTIFICIAL MEANS.

Since it would be interesting to know not only the investment costs but the total annual cost of lighting a building solely by artificial means, these costs have been worked out for various intensities of illumination ranging from 5.5 to 133 foot-candles upon a work-plane 3 feet above the floor. It is assumed that the general dimensions of the building, the wall and ceiling finishes, height of ceiling, the type and cost of electric wiring, type and cost of electric lighting-units, mounting height, and spacings are the same as in all the standard types of buildings investigated for the same

TABLE II.

Showing Details of Cost of Lighting a Windowless One-Story Industrial Building to Various Levels of Illumination.

The building has a ceiling height of thirteen feet, a fairly light ceiling and dark walls. The lighting-units are porcelain enameled (RLM dome) steel reflectors fitted with white bowl high-efficiency tungsten lamps, mounted 11½ feet from the floor and spaced uniformly on 10-foot centers. The lighting-units are assumed to be cleaned six times a year and the lamps are in an average condition. The number of work-hours per year is assumed to be 2240.

Foot-candles / Watts per Lamp	5.5 100	9.4 150	13 200	21 300	38 500	59 750	85 1,000	133 1,500
Installation costs per lighting-unit or per 100 sq. ft. of floor area:								
Electric wiring—complete	$ 6.85	$ 7.48	$ 8.65	$ 9.93	$12.50	$14.41	$16.24	$19.72
Lighting-unit—installed	3.35	3.52	4.05	4.97	5.00	6.39	6.46	6.58
TOTAL PER OUTLET	$10.20	$11.00	$12.70	$14.90	$17.50	$20.80	$22.70	$26.30
Annual cost per lighting-unit or per 100 sq. ft. of floor area:								
Int., dep., etc., on electric wiring—12%	$ 0.82	$ .90	$ 1.04	$ 1.19	$ 1.50	$ 1.73	$ 1.95	$ 2.37
Int., dep., etc., on lighting-unit—21%	.70	.74	.85	1.04	1.05	1.34	1.36	1.38
Cleaning six times	.30	.36	.42	.48	.48	.54	.54	.60
Cost of lamp renewals	.63	1.14	1.50	2.40	3.81	6.99	7.53	10.26
Cost of electric energy at 2¢ per kw.-hr.	4.48	6.72	8.96	13.44	22.40	33.60	44.80	67.20
TOTAL ANNUAL OPERATING COSTS PER LIGHTING UNIT	$ 6.93	$ 9.86	$12.77	$18.55	$29.24	$44.20	$56.18	$81.81
Cost per f.c. per hour per 100 sq. ft. of floor area	.0563¢	.0468¢	.0438¢	.0394¢	.0343¢	.0335¢	.0295¢	.0275¢

minimum intensity of illumination upon the work-plane. The total time of operation of each lighting-unit is, of course, the entire work-year. When work-hours are from 7:30 A.M. to 11:30 A.M. and 12:30 P.M. to 4:30 P.M., and when holidays, Sundays, and half-day Saturdays are deducted, there remain approximately 2,240 work-hours per year, the value used throughout this investigation.

The items of cost for each level of illumination are presented in Table II and shown graphically in Fig. 2. It may be observed that when a building is lighted solely by artificial means, the cost per foot-candle per hour per 100 sq. ft. of floor area decreases rapidly at first and later more gradually as the intensity of illumination upon the work-plane is increased.

LIGHTING COSTS VERSUS INTENSITY OF ILLUMINATION.

In order to ascertain how the annual costs of natural and artificial lighting vary with the minimum intensity of illumination upon the work-plane, these costs have been computed for each building for various intensities of illumination. The results for the four single-story buildings for intensities of illumination ranging from 5.5 to 133 foot-candles are presented in Table III. The assumed conditions are that these buildings are surrounded by 20-foot light-courts costing $.40 per sq. ft., that the windows are washed twice a year at a cost of $1\frac{1}{2}$ cents per sq. ft. per washing, and that electric energy costs two cents per kw.-hr. A plot of the results for Buildings *A*, *C*, and *D* are shown respectively in Figs. 3, 4 and 5.

The annual costs of natural and artificial lighting were computed for Building *E* for the minimum intensities of illumination of 13 and 21 foot-candles with the width of light-court surrounding the buildings varied from 2 to 150 ft., and the cost per sq. ft. of light-court at $1.50 and $.40. The windows which are uniformly $8\frac{1}{2}$ feet high on all six stories are assumed to be washed twice a year. These results are presented in tabular form in Table IV, and graphically, in Fig. 6.

The annual costs of natural and artificial lighting were computed for Building *F* for the minimum intensities of

TABLE III.

Annual Cost of Lighting per 100 Square Feet of Floor Area of Various Types of Single-Story Industrial Buildings.

The buildings are spaced 20 feet apart; windows are washed twice a year; and one-half of the cost of the intervening space at the sides and rear of the buildings is charged to natural lighting at $.40 per square foot. The annual costs for lighting are given in dollars per 100 sq. ft. of floor area; and the costs per minimum foot-candle per hour per 100 sq. ft. are given in cents.

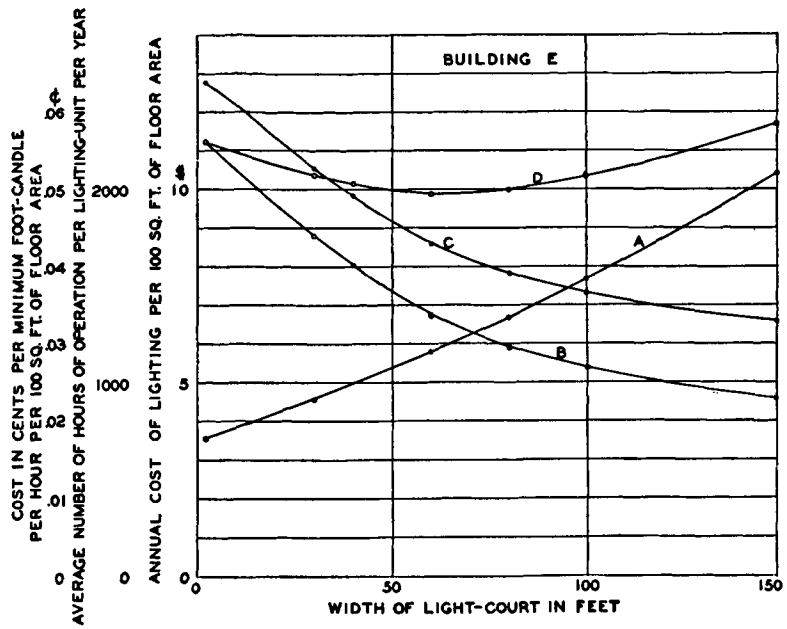
Min. Foot-candles Watts per Lamp	5.5 100	9.4 150	13 200	21 300	38 500	59 750	85 1,000	133 1,500
Building A								
Ave. no. of hours lamps operate per yr.	267	448	651	1,140	1,605	1,902	2,080	2,240
Annual cost of natural lighting	$ 5.83	$ 5.83	$ 5.83	$ 5.83	$ 5.83	$ 5.83	$ 5.83	$ 5.83
Annual cost of electric lighting	2.43	3.57	5.35	10.77	21.81	38.08	52.44	81.81
Annual cost of natural and electric lighting	8.26	9.40	11.18	16.60	27.64	43.91	58.27	87.64
Cents per min. f.c. per hour per 100 sq. ft. of floor area	.0670¢	.0447¢	.0384¢	.0353¢	.0325¢	.0332¢	.0306¢	.0294¢
Building B								
Ave. no. of hours lamps operate per year	195	298	379	663	1,352	1,860	2,053	2,225
Annual cost of natural lighting	$10.18	$10.18	$10.18	$10.18	$10.18	$10.18	$10.18	$10.18
Annual cost of electric lighting	2.26	3.04	4.08	7.40	18.85	37.32	51.81	81.24
Annual cost of natural and electric lighting	12.44	13.22	14.26	17.58	29.03	47.50	61.99	91.42
Cents per min. f.c. per hr. per 100 sq. ft. floor area	.1010¢	.0628¢	.0489¢	.0374¢	.0341¢	.0359¢	.0326¢	.0307¢

TABLE III (*Continued*).

Min. Foot-candles Watts per Lamp	5.5 100	9.4 150	13 200	21 300	38 500	59 750	85 1,000	133 1,500
Building C								
Average no. of hrs. lamps operate per yr	141	205	256	385	748	1,290	1,776	2,173
Annual cost of natural lighting	$12.16	$12.16	$12.16	$12.16	$12.16	$12.16	$12.16	$12.16
Annual cost of electric lighting	2.14	2.72	3.50	5.43	11.78	26.99	45.32	79.50
Annual cost of natural and electric lighting	14.30	14.88	15.66	17.59	23.94	39.15	57.48	91.66
Cents per min. f.c. per hr. per 100 sq. ft. floor area	.1160¢	.0706¢	.0537¢	.0374¢	.0281¢	.0296¢	.0302¢	.0307¢
Building D								
Average no. of hrs. lamps operate per year	142	203	253	390	878	1,523	2,025	2,240
Annual cost of natural lighting	$14.13	$14.13	$14.13	$14.13	$14.13	$14.13	$14.13	$14.13
Annual cost of electric lighting	2.14	2.71	3.49	5.47	13.20	31.21	51.15	81.81
Annual cost of natural and electric lighting	16.27	16.84	17.62	19.60	27.33	45.34	65.28	95.94
Cents per min. f.c. per hr. per 100 sq. ft. floor area	.1320¢	.0800¢	.0605¢	.0417¢	.0321¢	.0343¢	.0343¢	.0322¢

illumination of 9.4, 13, and 21 foot-candles with the light-courts 80 feet wide and with real estate at $1.50 and $.40 per sq. ft. The results obtained are presented in Table V.

FIG. 6.



Cost data on six-story Building *E* when surrounded by light-courts of various widths, costing $1.50 per sq. ft. The windows are washed twice a year and electric energy costs two cents per kw.-hr. *A* shows the annual cost of natural lighting per 100 sq. ft. of floor area; *B*, the average number of hours of operation a year of the 200-watt lighting-units in maintaining a minimum intensity of illumination of 13 foot-candles on the work-plane of each story; *C*, the annual cost of electric lighting: and *D*, the cost per minimum foot-candle per hour per 100 sq. ft. of floor area.

When the minimum intensity of illumination desired upon the work-plane is less than about 10 foot-candles or more than 75, artificial illumination is the more economical one. Between these limits, usually at about 38 foot-candles, the combined cost of natural and artificial illumination per foot-candle per hour attains a minimum. The determination of which one should be employed to supply the major portion of light would naturally depend upon the relative costs of building, real estate, and electric energy.

TABLE IV.

Cost of Lighting Building E, a Six-story Building Having a Ground Area 60 Feet by 180 Feet, when Surrounded by Other Buildings of Similar Dimensions, Spaced at Various Distances Apart.

The spaces intervening between the building and adjacent ones at its sides and back are assumed to be light-courts and their cost to be divided equally between adjacent buildings. The windows are assumed to be washed twice a year.

Building E—when minimum illumination is 13 foot-candles and the cost of real estate is $1.50 per sq. ft.:

Width of Court between Buildings in Feet.....	2	30	40	60	80	100	150
Annual cost of natural lighting per 100 sq. ft. of floor area when real estate is $1.50 per sq. ft.:							
Windows and structure.....	$ 0.32	$ 0.32	$ 0.32	$ 0.32	$ 0.32	$ 0.32	$ 0.32
Heating.....	2.28	2.28	2.28	2.28	2.28	2.28	2.28
Real estate at $1.50 per sq. ft.....	0.00	1.02	1.42	2.26	3.17	4.14	6.86
Washing windows twice.....	.96	.96	.96	.96	.96	.96	.96
TOTAL.....	$ 3.56	$ 4.58	$ 4.98	$ 5.82	$ 6.73	$ 7.70	$10.42
Annual cost of electric lighting per 100 sq. ft. of floor area for min. illum. of 13 f.c.:							
Av. no. hours lamps operate a year.....	2,240	1,762	1,615	1,347	1,187	1,077	918
Lamp renewals—(200-watt).....	$ 1.50	$ 1.18	$ 1.08	$ 0.90	$ 0.80	$ 0.72	$ 0.61
Elec. energy at 2¢ per kw.-hr.....	8.96	7.05	6.46	5.39	4.75	4.31	3.67
Int., dep., etc., on elec. lighting system.....	2.31	2.31	2.31	2.31	2.31	2.31	2.31
TOTAL.....	$12.77	$10.54	$ 9.85	$ 8.60	$ 7.86	$ 7.34	$ 6.59
Total ann. cost of nat. and elec. lighting....	$16.33	$15.12	$14.83	$14.42	$14.59	$15.04	$17.01
Total cost per min. f.c. per hr. per 100 sq. ft. floor area.....	.0561¢	.0519¢	.0509¢	.0495¢	.0501¢	.0516¢	.0584¢

TABLE IV (*Continued*).

Width of Court between Buildings in Feet.....	2	30	40	60	80	100	150
Building E—when min. illum. is 21 f.c. and real estate is $1.50 per sq. ft.:							
Av. no. hrs. lamps operate a yr.....	2,240	1,974	1,822	1,689	1,564	1,478	1,350
Ann. cost of nat. lighting.....	$ 3.56	$ 4.58	$ 4.98	$ 5.82	$ 6.73	$ 7.70	$10.42
Ann. cost of elec. lighting.....	18.55	16.66	15.59	14.65	13.76	13.16	12.26
Total ann. cost of nat. and electric lighting.	22.11	21.24	20.57	20.47	20.49	20.86	22.68
Cost per min. f.c. per hr. per 100 sq. ft. floor	.0470¢	.0452¢	.0437¢	.0435¢	.0435¢	.0444¢	.0482¢
Building E—when real estate is $.40 per sq. ft.:							
Cost per min. f.c. per hr. per 100 sq. ft. of floor area when min. illum. is 13 f.c....	.0560¢	.0493¢	.0473¢	.0438¢	.0421¢	.0411¢	.0111¢
Cost per min. f.c. per hr. per 100 sq. ft. of floor area when min. illum. is 21 f.c....	.0470¢	.0436¢	.0415¢	.0400¢	.0386¢	.0378¢	.0375¢

TABLE V.

Cost of Lighting Building F,—a Five-story Building with Window Heights of 20, 16, 13, 10½ and 8½ Feet or with Window Heights Such as to Necessitate about Equal Amounts of Electric Lighting to Supplement the Natural Lighting in Order to Maintain the Same Minimum Intensity of Illumination upon the Work-plane of Each Story.

The building is assumed to be spaced 80 ft. from adjacent ones and to be charged with one-half of the cost of the light-courts at its sides and rear.

	Costs with Real Estate at $1.50 per Sq. Ft.			Costs with Real Estate at $.40 per Sq. Ft.		
Min. foot-candles	9.4	13	21	9.4	13	21
Watts per lamp	150	200	300	150	200	300
Av. no. of hours lamps operate a year	455	704	1,219	455	704	1,219
Annual cost of natural lighting per 100 sq. ft. of floor area:						
Windows and structure	$ 4.71	$ 4.71	$ 4.71	$ 4.71	$ 4.71	$ 4.71
Real estate	3.81	3.81	3.81	1.02	1.02	1.02
Heating	4.85	4.85	4.85	4.85	4.85	4.85
Washing windows	1.54	1.54	1.54	1.54	1.54	1.54
Total cost of natural lighting	$14.91	$14.91	$14.91	$12.12	$12.12	$12.12
Annual cost of electric lighting per 100 sq. ft. of floor area:						
Overhead cost of electric lighting equipment	$ 2.00	$ 2.31	$ 2.71	$ 2.00	$ 2.31	$ 2.71
Lamp renewals	.23	.47	1.30	.23	.47	1.30
Electric energy at 2¢ per kw.-hr.	1.37	2.81	7.32	1.37	2.81	7.32
Total cost of electric lighting	$ 3.60	$ 5.59	$11.33	$ 3.60	$ 5.59	$11.33
Total annual cost of nat. and elec. lighting	$18.51	$20.50	$26.24	$15.72	$17.71	$23.45
Cost per min. f.c. per hr. per 100 sq. ft. floor	.0879¢	.0704¢	.0556¢	.0747¢	.0608¢	.0498¢

TABLE VI.

Cost of Lighting Various Types of Industrial Buildings when the Rather Unusual and Limiting Condition Obtains in which the Entrance of Light is Wholly Unobstructed or the Buildings are Spaced Very Far Apart and No Real Estate is Chargeable Against the Natural Lighting System.

The windows are assumed to be washed twice a year.

	Cost of Lighting per 100 Sq. Ft. of Floor Area for a Minimum of 13 Foot-candles.					
Building	*A*	*B*	*C*	*D*	*E*	*O*
Average Number of Hrs. of Electric Lighting with 200-watt Lamps	390	283	209	253	600	2,240
Annual cost of natural lighting:						
Windows and structure	$ 0.44	$ 1.91	$ 3.15	$ 6.35	$ 0.32	$ 0.00
Heating	3.06	5.42	6.71	5.69	2.28	0.00
Real estate	0.00	0.00	0.00	0.00	0.00	0.00
Washing windows twice	1.30	1.82	1.64	1.51	.96	0.00
TOTAL	$ 4.80	$ 9.15	$11.50	$13.55	$ 3.56	$ 0.00
Annual cost of electric lighting for a min. of 13 f.c.:						
Lamp renewals—(200 watt)	$ 0.26	$ 0.19	$ 0.14	$ 0.17	$ 0.40	$ 1.50
Electric energy at 2¢ per kw.-hr.	1.56	1.13	.84	1.01	2.40	8.96
Int., dep., etc., and six lamp cleanings	2.31	2.31	2.31	2.31	2.31	2.31
TOTAL	$ 4.13	$ 3.63	$ 3.29	$ 3.49	$ 5.11	$12.77
Total ann. cost of natural and electric lighting	$ 8.93	$12.78	$14.79	$17.04	$ 8.67	$12.77
Cost per min. f.c. per hr. per 100 sq. ft. floor area	.0306¢	.0438¢	.0507¢	.0585¢	.0297¢	.0438¢

TABLE VI (*Continued*).

	Cost of Lighting per 100 Sq. Ft. of Floor Area for a Minimum of 21 Foot-candles.					
Building	*A*	*B*	*C*	*D*	*E*	*O*
Av. no. of hrs. of elec. lighting per yr. with 300-watt lighting-units	660	447	302	390	1,065	2,240
Ann. cost of natural lighting	$ 4.80	$ 9.15	$11.50	$13.55	$ 3.56	$ 0.00
" " " electric "	7.38	5.87	4.84	5.47	10.24	18.55
Total ann. cost of natural and electric lighting.	$12.18	$15.02	$16.34	$19.02	$13.80	$18.55
Cost per minimum foot-candle per hr. per 100 sq. ft. of floor area	.0259¢	.0319¢	.0347¢	.0404¢	.0293¢	.0394¢

WINDOW OBSTRUCTIONS AND REAL ESTATE AS COST FACTORS.

If it were possible to have light-courts of infinite width without cost we would have ideal conditions for cheap natural lighting. With a view of determining the combined influence of these two factors, computations have been made of the costs of natural and electric lighting for each of the types of buildings investigated with no window obstructions and with no charge for light-courts. The results are presented in Table VI.

COST OF LIGHTING VERSUS FREQUENCY OF WINDOW WASHING.

In order to investigate the economic advantage of frequent window washing, the annual costs of natural and artificial lighting were computed for Buildings *A*, *B*, and *C*, for one window washing a year instead of two and the results presented in Table VII.

TABLE VII.

Decrease in the Annual Cost of the Natural Lighting per 100 Square Feet of Floor Area Due to the Practice of Washing the Windows Once a Year Instead of Twice, Compared with the Increase in Annual Cost of Electric Lighting when the Cost of Washing a Square Foot of Window on Both Sides is 1.5 Cents and the Cost of Electric Energy is 2 Cents per Kw.-hr.

	Comparative Annual Changes of Costs of Natural and Electric Lighting Due to One Window Washing a Year Instead of Two.							
Min. foot-candles.....	5.5	9.4	13	21	38	59	85	133
Building A								
Decrease in cost of natural lighting.....	$.65	$.65	$ .65	$ .65	$ .65	$ .65	$ .65	$ .65
Increase in cost of electric lighting.....	.21	.68	1.37	1.77	2.51	2.83	2.57	.00
Building B								
Decrease in cost of natural lighting.....	$.91	$.91	$ .91	$ .91	$ .91	$ .91	$ .91	$ .91
Increase in cost of electric lighting.....	.13	.22	.68	2.31	4.48	3.01	3.31	.57
Building C								
Decrease in cost of natural lighting.....	$.82	$.82	$ .82	$ .82	$ .82	$ .82	$ .82	$ .82
Increase in cost of electric lighting.....	.08	.16	.32	1.06	4.35	7.60	7.12	2.31

As may be observed, the advantage is in favor of two window washings a year instead of one, for Building *A* for a minimum intensity of illumination upon the work-plane of more than about 10 foot-candles; for Building *B*, for more than 15 foot-candles; and for Building *C*, for more than about 20 foot-candles.

INFLUENCE OF ENERGY COST ON LIGHTING COSTS.

There is an energy rate for every level of illumination below which it is cheaper to make no provision or at least make no extra expenditure to procure natural lighting but rely solely upon artificial lighting. As shown in Table VIII,

TABLE VIII.

Cost of Electric Energy Below Which It is Cheaper to Have a Building of Equal Size without Windows or Light-courts and Illuminated Solely with Artificial Light, Rather than a Standard Building Illuminated with a Combination of Natural and Electric Lighting.

Windows of each type of building are assumed to be washed twice a year.

	Cost of Electric Energy in Cents per Kw.-hr. Below Which It is Cheaper to Have a Simpler Building with Electric Lighting Alone.						
Min. foot-candles	5.5	9.4	13	21	38	59	85
With 20-ft. light-courts at $.40 per sq. ft.:							
Building *A*	$2.68	$1.83	$1.50	$1.41	$1.50	$1.89	$3.30
" *B*	4.70	3.15	2.40	1.80	1.95	3.16	5.10
" *C*	5.51	3.64	2.73	1.83	1.29	1.29	2.28
" *D*	6.44	4.28	3.22	2.19	1.72	2.21	6.23
With 20-ft. light-courts at $0.00 per sq. ft.:							
" *A*	2.15	1.45	1.18	1.10	1.17	1.48	2.66
" *B*	4.19	2.80	2.12	1.58	1.72	2.80	4.56
" *C*	5.19	3.43	2.56	1.71	1.20	1.20	2.14
" *D*	6.17	4.10	3.07	2.08	1.63	2.10	6.10
With 80-ft. light-courts at $.40 per sq. ft.:							
Building *F*		4.19	3.60	3.60			
With 80-ft. light-courts at $1.50 per sq. ft.:							
Building *F*		5.23	4.52	4.51			

TABLE VIII (*Continued*)

Various widths of light-courts at $.40 per sq. ft.

Width of Light-court in Feet.....	30	40	60	80	100	150
Bldg. *E* for 9.4 min. ft.-candles				1.85		
" " " 13 " " "	3.68	2.81	2.00	1.76	1.66	1.70
" " " 21 " " "	4.43	2.78	2.16	1.81	1.67	1.66

Various widths of light-courts at $1.50 per sq. ft.:

Bldg. *E* for 9.4 min. ft.-candles				3.00		
" " " 13 " " "	4.36	3.65	2.92	2.86	2.98	3.60
" " " 21 " " "	5.36	3.61	3.16	2.95	3.01	3.62

Infinite width of light-court without cost:

Buildings.....	*A*	*B*	*C*	*D*	*E*
For 13 min. ft.-candles.....	.96	2.00	2.50	3.07	.75
" 21 " " "	.66	1.35	1.62	2.08	.65

this rate has been computed for each type of building under a great variety of conditions as to minimum intensity of illumination, width of light-court, cost of real estate, etc.

Judged merely from a standpoint of economy in lighting when the conditions of operation are approximately an average, it appears uneconomical to have a single-story building equipped with more than the simplest side windows unless energy costs more than 1.4 cents per kw.-hr., or to have it equipped with monitor windows unless energy costs more than 1.8 cents per kw.-hr., or to have sawtooth windows unless energy costs more than 2.2 cents per kw.-hr. From a similar standpoint, it appears uneconomical to have the light-court surrounding a six-story building of a greater width than its height when real estate costs $.10 per sq. ft., unless electric energy costs more than 1.5 cents per kw.-hr.; or when real estate costs $.40 per sq. ft., unless electric energy costs more than 1.7 cents per kw.-hr. or when real estate costs $1.50 per sq. ft., unless electric energy costs more than about 2.9 cents per kw.-hr.

INFLUENCE OF THE NUMBER OF WORK-HOURS PER DAY ON LIGHTING COSTS.

In order to study the influence of the number of work-hours per day upon lighting costs, the annual costs of lighting

Building *D* to a minimum intensity of 21 foot-candles have been computed for one, two and three shifts of labor per day. The building is assumed to be spaced a distance of 20 feet from other buildings of similar height but no charges are made against natural lighting for light-courts. With but one shift of labor per day, work begins at 7:30 A.M. and ends at 4:30 P.M.; with two shifts a day, work begins at 6:00 A.M. and ends at midnight; and with three shifts a day, each shift is on duty eight hours a day. With the windows washed semi-annually, a lighting-unit operates on an average of about 390, 2,390 and 3,970 hours per year respectively with one, two and three shifts a day. Electric energy is assumed to cost two cents per kw.-hr.

Under the conditions stated and for the items considered, the annual costs of natural light per 100 sq. ft. of floor area and the costs of the artificial light needed to supplement the natural light in maintaining a minimum of 21 foot-candles upon a horizontal work-plane three feet above the floor are:

	One Shift a Day.	Two Shifts a Day.	Three Shifts a Day.
Annual cost of natural lighting.....	$13.55	14.11	14.80
Annual cost of artificial lighting.....	5.47	19.62	30.78
Total.....	$19.02	33.73	45.58

SUMMARY.

With the growth of the science of light and vision, the engineer has been putting forth some effort to use daylight to a greater advantage. In our endeavor to use natural and artificial light in a more economical manner, it becomes necessary to know the initial and operating costs of each. The determination of the initial cost of natural lighting is made by computing the difference in cost between a building equipped with natural and artificial lighting and a building of similar construction equipped merely with artificial lighting.

Though it may be possible to dispense with natural lighting, it would hardly be possible to dispense with artificial lighting if work is to be pursued for eight hours a day through-

out the year; for natural light is of no greater constancy than daylight itself.

The major items entering into the initial cost of natural lighting equipment per 100 sq. ft. of floor area under normal conditions vary from a minimum in one type of standard building to a maximum in another type as follows:

	Minimum.		Maximum.
Windows and structure	$ 0.00	to	$45.00
Heating equipment	10.30	"	32.10
Light-courts	8.40	"	45.30
Total for all three items for standard buildings	28.56	"	79.90

The investment cost of heating equipment to supply heat losses incident to windows varies from $.32 to $.45 per sq. ft. of window surface for buildings having side windows only, and from $.41 to $.59 for buildings having windows in the roof also.

The annual cost of maintaining and operating the heating system to supply heat losses incident to windows varies from a minimum of 7 cents per square foot of window surface in Building *A* to a maximum of 12.3 cents in Building *C*.

The initial total cost of electric lighting equipment per outlet or per 100 sq. ft. of floor area with lighting-units spaced on 10-foot centers, varies from $10.20 for 5.5 foot-candles on the work-plane to $17.50 for 38 foot-candles.

Under average conditions the ratio of the initial costs of natural to electric lighting equipment varies from 2.25 for a single-story flat roof building to 6.3 for a sawtooth building.

Under the same conditions the ratio of the annual cost of natural lighting to the annual cost of electric lighting varies from 1.09 for a single-story, flat roof building to 4.03 for a sawtooth building.

For a minimum intensity of illumination of 13 foot-candles upon the work-plane the average time a lighting-unit operates a year varies from 651 hours for a single-story, flat roof building to 253 hours for a sawtooth building. For a minimum of 13 foot-candles, the average time a lighting-unit operates per year in a six-story building surrounded by buildings of its own height varies from 918 hours for 150-foot light-courts to 1,762 hours for 30-foot light-courts.

The combined cost of natural and artificial lighting per

hour per minimum foot-candle upon the work-plane attains a minimum at about 38 foot-candles for most types of standard buildings.

Since the cost of heat loss due to windows is generally offset by the cost of electric energy saved by the incoming light, it would seem usually justifiable to equip industrial buildings with simple side windows. However, it would seem unjustifiable from a viewpoint of economy in lighting to incur a great expense for light-courts, or windows in the roof of the building unless the minimum intensity of illumination upon the work-plane is to exceed about 15 foot-candles.

It appears that a practice of washing windows less frequently than twice a year is uneconomical excepting when the minimum intensity of illumination upon the work-plane is maintained at less than 10 foot-candles.

When determined merely by the cost of lighting, it appears from these results that for any type of standard building investigated and operating under average conditions that no appreciable expenditure for natural lighting equipment is justifiable when electric energy can be purchased as low as 1.4 cents per kw.-hr.

PLASTIC CELLULOSE IN SCIENTIFIC RESEARCH.*

BY

K. HICKMAN AND D. E. HYNDMAN.

Experimenters in the time of Newton or Priestly were proud to boast that, from a jam jar, some string, and a piece of sealing wax, they could construct almost anything.

Nowadays, although our laboratories are supplied with a great variety of intricate devices and pieces of glassware, nearly every chemical and physical problem demands the construction of some special apparatus. The methods at our disposal vary with our constructive skill, but rarely embrace more than:

Carpentering,
Glass blowing,
Soldering and sheet metal working,
Plumbing,
Elementary electric wiring.

Of these, with the advent of Pyrex, it may be said that glass working is by far the most useful, and the least dispensable. To be able to see what is happening inside the apparatus is the great advantage, outweighing a hundred minor drawbacks. If metal working could produce transparent articles, the value of soldering and plumbing would be enormously enhanced, and many experimental problems would be simplified. This article does not describe such a miracle, but it does compromise by showing how transparent cellulose compounds may be built into useful laboratory apparatus.

Transparent cellulosic sheeting is generally made from cellulose nitrate or cellulose acetate. The more usual nitrate material is commonly known as celluloid. Since, however, this is a trade name for a special product, we shall speak of the material as pyroxylin;[1] and refer to cellulose acetate sheeting as acetate base.

* Communication No. 370 from the Kodak Research Laboratories.

[1] Plasticized nitrocellulose sheeting.

A word as to the properties of these two substances: Pyroxylin is a mixture of cellulose nitrates plasticized with camphor. It is not only inflammable, but it also decomposes in the absence of air. Decomposition begins between 140–170° C. and inflammation occurs at a higher temperature. Below the boiling point of water it softens sufficiently for moulding.

Acetate base, on the other hand, since it contains no camphor, is not so plastic or easily moulded. It can be bent at 100° C. and will retain its new shape. It does not decompose spontaneously in the absence of air at temperatures below its ignition point, and it does not burn with special vigor. Hence, its use for safety cinematograph film and home motion pictures. Both materials are softened by certain organic solvents and two pieces may be stuck together by placing a drop of solvent between and squeezing into contact. The solvent softens and partially dissolves the surfaces and is then absorbed by the mass of the material, as water is absorbed by gelatin. Later it evaporates. The process, however, can produce a perfectly autogenous weld of a strength equal to that of the original material.

Pyroxylin and acetate base may therefore be bent and welded into the most complicated shapes. Since they may be stuck to glass, elaborate apparatus may be built up of dimensions impossible with glass tubes alone. Examples will be described later.

Besides the inability to resist heat, neither material will withstand strong liquids. The vague term "strong liquids" may be interpreted as:

Pyroxylin	*Acetate Base*
Concentrated nitric acid	Moderately concentrated acids
Concentrated hydrochloric acid	2 per cent. alkali
Concentrated sulfuric acid	Acetone
10 per cent. or stronger caustic alkali	Alcohol
Acetone	Ether
Alcohol	Amyl acetate
Amyl acetate	

Both will stand water, solutions of metallic salts, dilute acids, carbonates, benzene, chloroform, and xylene. Pyroxylin will resist 20 per cent. sulfuric acid indefinitely.

CONSTRUCTION WITH PYROXYLIN.

The operations involved are four in number and are extremely simple. They are:

(1) Cutting
(2) Shaping
(3) Welding:
    (*a*) to like material,
    (*b*) to glass and other substances.
(4) Laminating.

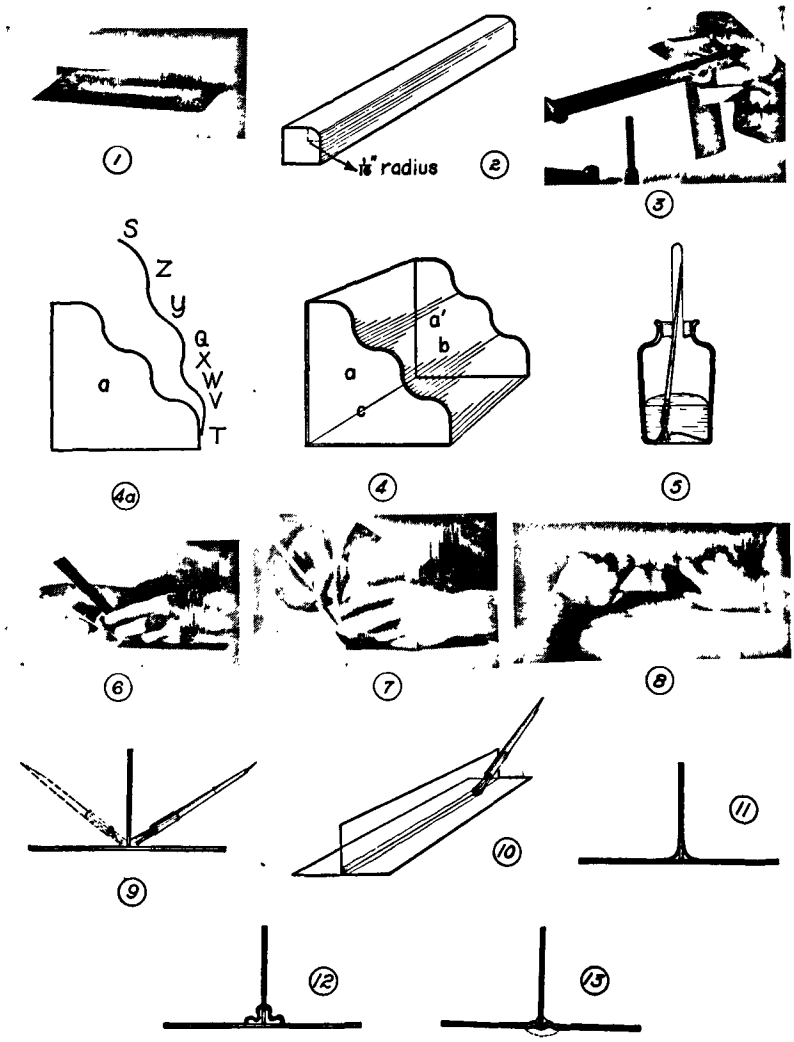
Directions given below for pyroxylin serve, in practically every instance, for acetate base.

(1) *Cutting.* Pyroxylin which is to be cemented perpendicularly to the surface of a second piece (Fig. 1) should never be cut. The scissors should be reserved for trimming the edges of finished articles. No matter how carefully the cut is made, the edge will deviate slightly from a straight line or pure curve. The most convenient way to part pyroxylin is with a sharp point drawn in pencil fashion along a straight edge. A "Moore" glass push pin works well and may be discarded when the point is blunt. Old razor blades are not suitable because they tend to depart from the line. When a good clean scratch has been made, the material is bent backwards until, with a slight crack, it parts along the entire length. Circles are cut with the dividers, placing a piece of scrap pyroxylin under the center leg to avoid making a large pivot hole. Two or three revolutions with the dividers leaning in the direction of travel will make a deep enough scratch. If the internal circle is to be used, two or three scratches are ruled tangentially to the circumference and the outer portion broken away. When it is the outer ring that is wanted, the inner circle must be detached by careful pressure with the thumbs. Gentle working backwards and forwards is better than using excessive force.

Other shapes are easy to produce. An ellipse is made by guiding the cutting point with a loop of string held between two fixed pivots. Wavy contours are produced with french curves. The line is first built up with a grease pencil from various positions of the curve and is then retraced, still using the curve, with the sharp point.

(2) *Shaping.* The many cellulose nitrate articles available commercially are evidence that with moulds and dies any shape can be produced. Moulds are not common in the

FIGS. 1–14.



laboratory so that the shaping operation is limited to simple bending. Variations of the cube and cylinder are possible, but not the sphere.

There are three requisites for bending: a square metal bar, a selection of metal rods, and a vessel of hot water. The metal bar is by far the most useful.

The inflammable nature of pyroxylin would suggest using hollow bars and tubes which could be warmed with steam from an electric kettle. We have found it safe, however, to use a Bunsen burner, making sure that the pyroxylin and cement are pushed out of the way.

The bar has available four edges and four sides. At least one edge should be rounded, forming part of a circle of about 1/16-inch radius (Fig. 2). All four sides may be at right angles, or the bar may be a right-angled triangle. In this case the other two angles should be rounded as well. The bar is going to supply heat to the pyroxylin, and no matter how sharp a bend it is desired to make, there must be sufficient bearing surface to convey the heat.

The bar is warmed by waving the Bunsen flame along its length until the moistened finger just sizzles when touching the upper surface. The flame is now extinguished and the pyroxylin brought into contact. A line should previously have been ruled with the grease pencil where the bend is to come. On no account must a scratch be made. The pyroxylin is grasped by the ends with the two hands, and when it has begun to yield, the regions near the bend should be pressed with wood strips, or smothered tightly with a cloth. When the desired angle has been reached, the sample is removed, taking care to maintain the angle until cold (Fig. 3). The procedure is similar when making a less sharp bend around a rod.

Sometimes it is necessary to produce an irregular shape to fit some special contour. In Fig. 4 the pieces *a*–*a'* are similar and have two edges at right angles, while the third follows the special contour; *a* and *a'* form the side pieces to the angular strip *c*. Let us suppose it is necessary to bend *b* so that it may be cemented in place to complete the box. *b* is cut in a strip ¼ inch wider than called for and at least an inch longer. Placing the piece *a* beside a bowl of water hotter than the hands can bear, *b* is held with pliers and immersed. In a minute it is withdrawn and given a quick pressure with the hands at *V* and allowed to spring away

again. It will have taken a slightly permanent shape. If it does not match the curve of *a* sufficiently, the warming and pressing are repeated until a slightly *exaggerated* bend is secured. The curves at *W*, *X*, *Y*, and *Z* now receive attention, exaggerating each one, in order, a little less than its predecessor. In this way the final sample is found to have curves at the right places but owing to recovery in warm water each curve is a little *less* pronounced than needed. This is an advantage, for when fixture is made at the point *T* (Fig. 4*a*), pressure at *S* brings the whole strip into contact with *a* and allows the cementing to be accomplished in one operation.

(3) *Welding.* (*a*) *Autogenous: Pyroxylin to Pyroxylin.* The requirements are *thin cement*, and *thick cement* in suitable containers, and some *hog's hair brushes*, in holders from which the varnish has been scraped. The containers are easily made from 6-ounce wide necked bottles supplied with large corks which do not fit too tightly. Through a central hole in each cork a stout walled test tube ¾ inch in diameter and 6 inches long is pushed from below. This is much better than having the brush itself fixed in the cork (Fig. 5).

Many formulas for film cement have been published, but since small quantities only are needed, we have found it convenient to use the ready-made Eastman cement without inquiring into its composition. This is the thin cement referred to above.

The thick variety is made by dissolving scrap pyroxylin, cut in strips an inch and a half long by 1/16-inch wide, in the thin cement. A spare bottle is filled one-third full of the chips and cement poured in until the bottle is more than half filled. The mixture is stirred vigorously with a stout glass rod every ten minutes throughout the day and left to settle over night. In the morning the clear thick syrup is decanted into the container from a slight residue of gelatinized strips. A brush is selected, placed in the syrup, and the cork thrust home.

It should be understood at the outset that pyroxylin sticks to itself by virtue of its own powers of adherence. The cement is used merely to prepare the surfaces. After these have momentarily been flooded, the best joins are

obtained when the surfaces are pressed together tightly enough to extrude every trace of unabsorbed liquid. Obviously, the thick cement will not be pressed out as efficiently as the thin and will therefore not make as good a join. The thick fluid must never be used except for external reinforcement of the join.

Joins are of two kinds, perpendicular and parallel, or end to face and face to face. The latter is lamination on a small scale.

The parallel join is very easy. One sample is stood at an angle on the other and a few drops of cement from a pipette or a brush are laid in position. The upper sample is quickly laid down and an even pressure applied all over with the flat of the hand or a cloth pad. A piece of blotting paper underneath will absorb excess cement pressed from the edges. The operations are shown in Figs. 6 and 7, while the finished join is being tested in Fig. 8.

The perpendicular join is more difficult but much more useful. At its simplest, the straight edge of a piece of pyroxylin is pressed tightly against a pyroxylin surface, while a little thin cement is run along the angle at each side (Fig. 9). The pressure is maintained for 30 seconds after all trace of liquid has disappeared and the sample left undisturbed for an hour. It is then held in an inclined position (Fig. 10), while a drop of thin cement is placed at the top of the angle on each side and allowed to run down, which it should do quickly. Immediately a big drop of thick cement is placed in the same position and allowed to fall slowly along each crevice. At every inch or two of fall it should be replenished, not at the top, but at the head of the traveling drop. Finally the sample is put away to dry for one day on a level surface, after which a section of the join should present the appearance in Fig. 11. The object of the pre-wetting with thin cement is to prevent the inclusion of air bubbles.

If the thick cement had been applied to the corner with a brush, it would have formed ridges where it was least needed, yielding a finished product like Fig. 12.

The reinforcing cement must be used with caution. It sets after partial drying into the shape of the join, but later when more solvent has evaporated, it contracts and pulls the

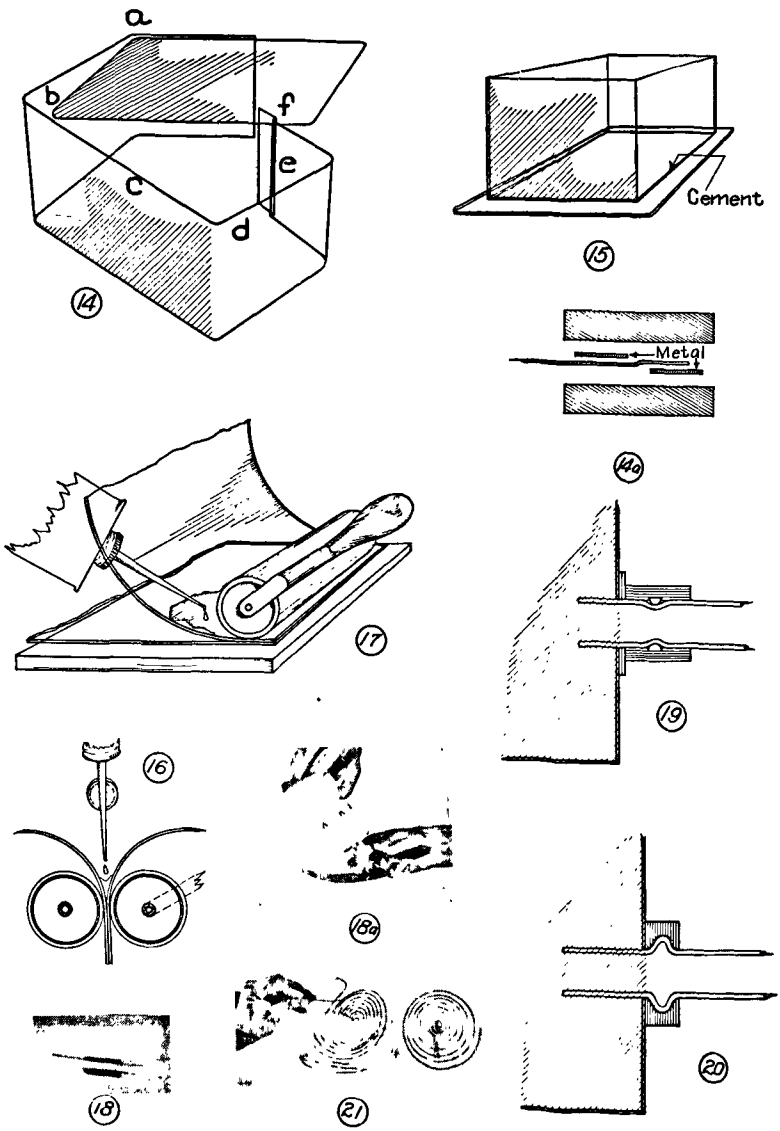
pieces to a more acute angle than was intended. The reinforcing must always be done on two sides of the join and sometimes, if the appearance of Fig. 13 is to be avoided, on three. The properly finished join is shown in Fig. 11.

Most perpendicular joins are less simple. They are the kind involved when an end, a side, or a partition is cemented in a shaped container. Fitting a circular disk into the end of a cylinder is perhaps the simplest case. The disk, cut to fit tightly, is pushed into place and thin cement run all round. Since there is no tendency for the join to come apart, the thick cement may follow immediately. A large tear is placed in one crevice and the cylinder rotated until the syrup has fallen all around. The other side is then treated.

Fastening the end into a rectangular box is a little more difficult. The sides of the box may be completed before or after insertion of the end. In the first case the end piece is cut accurately to size but the corners are rounded off more than would appear necessary. It is thrust into position and cement applied at the center of the long sides, holding them against the end piece for at least a couple of minutes. The centers of the short sides next receive treatment and cement is finally brushed all around. One hour must elapse before the application of thick cement.

When the rectangular box has to be built around the end piece, it should be fitted by trial and error before any cementing is done. The join is most conveniently in the middle of a long side. Cementing of the top plate is begun with one-half of the split long side (*a* in Fig. 14), being careful that the plate is pushed snugly up to the corner. After a few minutes, sides *b*, *c*, and *d* may be brought into position and secured. Finally side *e*, previously given a rebate *f* by pressing between two thin metal plates in a vise (Fig. 14*a*), is cemented in place and a parallel joint made all the way down. If necessary, the operation on this end plate may be repeated step by step on another plate at the other end, thus forming a closed box. The simplest way of all of closing the rectangle is to stand it on a plate larger than actually required, run cement all around the crack, and follow this, within and without, by thick cement (Fig. 15).

FIGS. 14–21.



(4) *Lamination.* Although joining pyroxylin to glass and other materials is next in order for consideration, it will be convenient first to describe lamination.

If two thin pyroxylin sheets are placed face to face, with their ends between the rollers of a horizontal mangle (Fig. 16), cement may be poured in the junction and they may then be rolled into one homogeneous piece. In place of a mangle one may use a drawing board and a gelatin composition roller of the kind favored by printers for inking up halftone blocks (Fig. 17). With a ten-inch roller, pieces 9 by 20 inches may be built up with ease. One sheet 11 by 25 inches is laid on the board. A thin line of cement drops is placed at one end and a second sheet secured above it along this edge. Holding the upper sheet at 45°, a liberal pool of cement is poured into the angle, after which the roller is pushed smoothly but quickly forward. Enough cement must be used to stick the entire length. It is convenient to squirt the liquid from the original can.

Laminating is a messy operation. The cement exudes from sides and end and is very likely to get on the board or under the clean bottom face of the pyroxylin. This may be prevented by doing the operation on blotting paper. It is important that the freshly laminated material should have the uncemented side portions cut away at once. It should be allowed to hang vertically for some hours, because any bend suffered at this stage will impose a permanent curvature.

Pyroxylin is readily obtained in thicknesses of 5, 10, and 20 thousandths (1/200, 1/100, 1/50) of an inch. The 20 thousandths is the most useful material, and by laminating two or three thicknesses of this, a sheet sturdy enough for most requirements can be made.

The need is soon encountered for pyroxylin tubing. Unfortunately we have been unable to obtain this in lots of less than 50 pounds, and have been forced to make it by lamination, or else press glass tube in service. Short lengths can be made very simply after the manner shown in Figs. 18 and 18*a*. Cement must not get between the pyroxylin and the glass, otherwise the mandrel cannot be withdrawn.

(3*b*) *Welding Pyroxylin to Other Materials.* Pyroxylin does not make good shafts or bearings, or good narrow tubes. Whenever moving parts are involved, or fluids have to be conveyed to containers, glass or metal spindles or tubes have to be fixed to the pyroxylin.

Pyroxylin cemented to smooth glass or metal adheres fairly well until one corner becomes lifted. Then any slight strain strips it away. This happens particularly easily under water. Consequently the pyroxylin and glass (or metal) have to be shaped so that one surrounds the other completely and without the possibility of rotation.

Cementing a Glass Tube into a Pyroxylin Container. Two methods have been used successfully. In the first the glass is indented in the blow pipe flame and when cold, a chip of pyroxylin is thrust into the cavity. A thick tube is then laminated over this, wetting the glass with much cement. When the walls are a quarter of an inch thick, the whole assembly is cemented to the side of the vessel. Fig. 19 shows the procedure in detail.

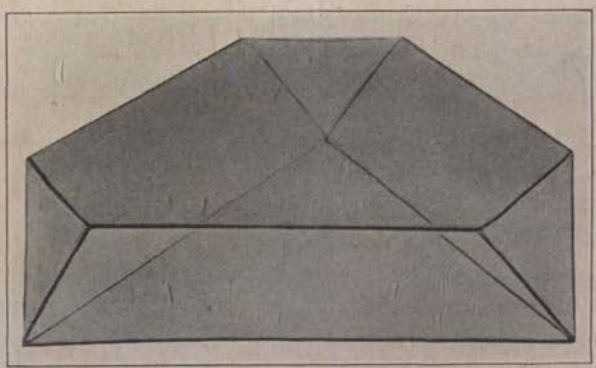
The other way uses an unsymmetrical expansion in the glass to secure grip. A flat uneven bulb is blown, and a number of washers are cut to fit its contours, the outer ones just fitting the undistorted tube. These are cemented in position one by one, giving the effect shown in Fig. 20. Such built up collars give extremely strong joins. Shafts may be pushed through pyroxylin Pelton wheels, or film developing drums, and secured at each contact by some modification of the procedure. Good contact can often be secured between a glass tube and a single plate of the rotor by merely denting the glass at the junction and painting it liberally with thick cement. This dries into the hollow and produces firm adhesion.

Steel and copper offer no especial difficulties except that the cement must on no account contain acetic or other acid.

A device which has proved useful is shown in Fig. 21. A spiral of narrow pyroxylin tube was wanted. Accordingly a coil of fine copper piping, having an outside diameter equal to the internal diameter of the finished pyroxylin, was wound into a well-spaced spiral, and supported by a wire handle. It was then dipped about 30 times into thick pyroxylin cement, allowing at least three hours for drying between successive dips. The dipping required rigid adherence to a certain sequence of operations. First the spiral was dipped with a slanting motion into thin cement and quickly withdrawn. It was then instantly lowered *slowly* in a slanting

manner into the thick cement. When under the surface it was rotated through 180° and removed at an angle, being careful that each convolution left the surface without forming a drop. Once out in the air it was rotated in all

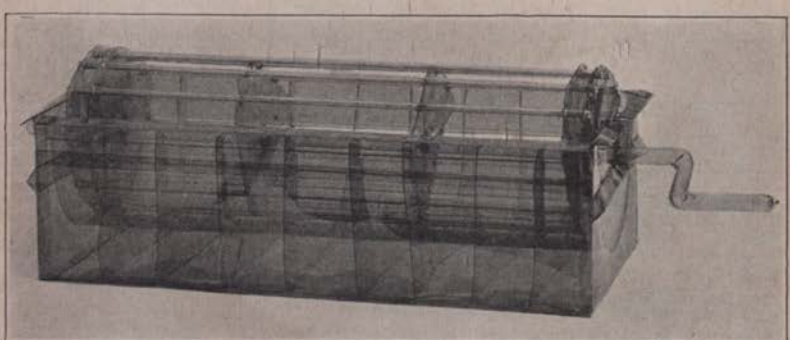
FIG. 22.



directions for a minute or two until the new layer had set. It was then hung up to dry.

After the thirtieth coat, a day's drying was given in the 60° C. oven. The copper spiral was then dissolved out with

FIG. 23.

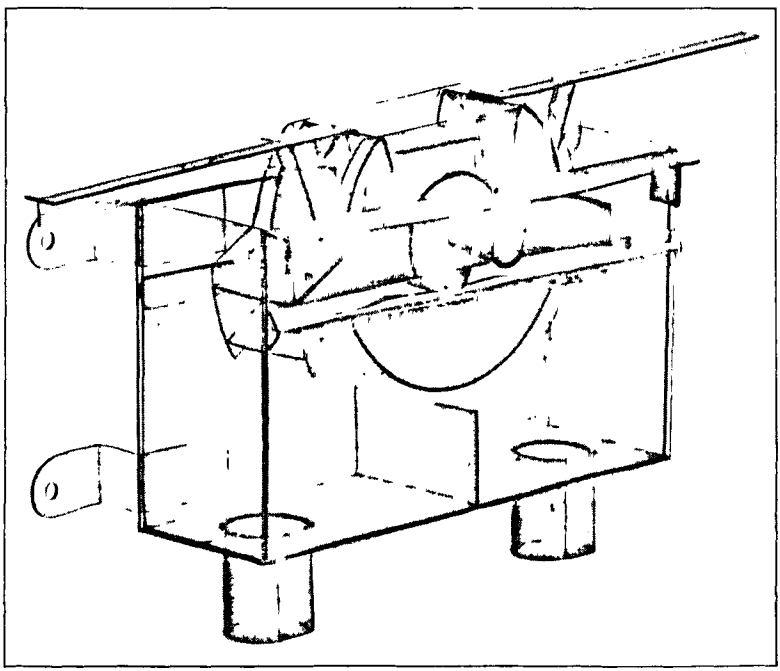


1 : 1 nitric acid. This was a tedious operation owing to the gases driving the acid out of the spiral. It has to be done quickly or degradation of the pyroxylin sets in. Finally when the copper was entirely removed, a mixture of water

with 25 per cent. of industrial alcohol was drawn through, a few c.c. per minute, for a day. The spiral, when dried, served its purpose excellently. One is shown, minus the copper core, at the right of Fig. 21.

Strength and Rigidity. Thin pyroxylin sheet is too flexible to give mechanical rigidity unless it is bent and secured in many directions. A careful design will include sufficient

FIG. 24.



cross pieces. When these are not admissible, right-angle struts and bracings may be put across the straight sides. They will be observed in many of the later figures.

Tinting. Pyroxylin may be colored by dissolving dyes in a mixture of alcohol and acetone. The particular mixture of solvents must be found to suit the individual worker. Generally speaking, the more acetone, the more strongly the color will take.

Sphere of Usefulness. In the laboratory we have found pyroxylin construction invaluable in making:

Models of crystals and atoms where it is desired to see top, bottom, and contents all at the same time (Fig. 22).

Film developing drums in those photographic researches where it is inadvisable to let the developer touch anything but glass and film substance (Fig. 23).

Containers in hydraulic systems where it is necessary to observe the path taken by water.

Certain industrial instruments where the solutions attack all materials but pyroxylin (Fig. 24).

There is one use, however, now being developed in our spare time, which would appear to fill an outstanding need of years.

When an architect designs any building more ambitious than a four-room villa, he is forced to build a model of paper or pasteboard to satisfy his patron's curiosity. Pasteboard does not allow the interiors to be seen, so that when the appearance of these is in question, a special section has to be built. Furthermore, pasteboard is not so plastic or so easily assembled as pyroxylin.

We have experimented in building a cottage in transparent cellulose, and find it possible, besides giving a superior external finish, to show the layout of the interior down to such details as upper floors, stairs, fireplaces, radiators, bathroom fittings, and even furniture.

Pyroxylin is easily tinted and thus the roof may be red, and the walls painted with ivy without impairing the essential transparency. This conduces to a most realistic finished appearance.

No example is shown here because it is intended to deal with the matter more fully in the future.

THE EXPERIMENT IN STATICS.

BY

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Buenos Aires,
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THE experience, familiar in almost any field of science, that problems might be easier, quicker and even more accurately solved by the visible experiment than by abstract reasoning only, has largely been overlooked in at least one line of its application of importance in daily life, viz. the calculation of resistance of structures. That calculation so far generally is being based on abstract mathematics with the result that even today a fair approximation to actual conditions in the field is practically unknown, because it would lead to formulas complicated beyond usefulness for application in practical life. It will be of interest to adepts in any field of natural science to learn how statical problems may be solved by one visible, simple experiment, which the mathematical Statics could solve only by numberless equations and formulas.

In my article of July 1926, of the JOURNAL OF THE FRANKLIN INSTITUTE, "Mechanical Calculations of Elastic Systems," I have shown how flexible models of structures will automatically form the influence lines of stresses at any section of the structure, when subjected to such visible displacements at that section, which will occur after those stresses have overcome resistance, that is to say that the stress may be read right from the model, which a load, acting at any other section, will produce in the displaced or disrupted section. I described in that article how in the "Continostat," invented and developed by me for the purpose, models may conveniently be built up of steel splines, which reproduce structures of the daily practice of the engineer, e.g. continuous beams, portal frames and frame buildings of any number of spans and stories, trusses without diagonals, any other structure with members joining at right angle, even

one span arches, etc., altogether not less than 95 per cent. of the cases occurring in the daily practice of the engineer. In the present article, the most interesting and spectacular applications to irregular problems of statics, that is to say, to oblique and compound structures may be illustrated.

1. COMMON STRUCTURES.

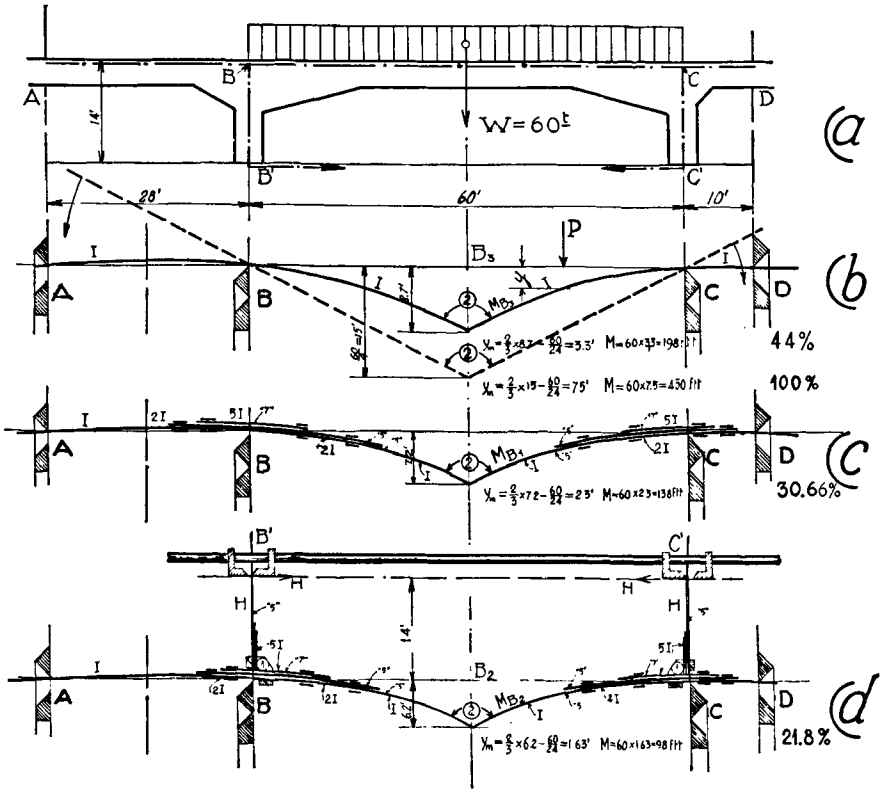
To properly understand the mechanostatical solution of irregular structures by means of visible deformations, it may be well to show first the great scope and importance in technical practice of the theories set forth in the former article, explaining their application to an example from the field. In Fig. 1 is shown the cross section of a reinforced concrete building supported at the ends on party walls and, between them, on two rigidly joined columns, forming spans of 28′, 60′ and 10′ and a total height of floor of 14′. The main span BC is loaded uniformly with a total load of 60 tons. The height in center of BC and consequently the bending moment must be reduced as much as possible to leave greatest clear space possible.

Commonly now the designer would consider the beam BC as freely supported and as a "rule of thumb" deduct any amount between 10 per cent. and 30 per cent. from the bending moment thus obtained and call that Statics. The bending moment for the simple beam would be $M_E = \frac{1}{8}Wl = \frac{1}{8} \times 60 \times 60 = 450$ ft. t. The influence line for the bending moment at the center of the freely supported beam BC is shown as the dotted line in Fig. 1b, it is a triangle, the height of which in center is $h = \frac{1}{8}l$, the area $F = \frac{1}{8}l^2$ and the average height or ordinate $y_m = F/l = \frac{1}{8}l$.

Suppose now we join two steel splines with the No. 2 clamp of the Continostat so that they will form the broken dotted line in Fig. 1b; we hold the broken beam thus formed at B and C and bend the extending ends until they pass through A and D. It will easily be seen that by doing that the No. 2 clamp will shift automatically towards BC until the spline will form the heavy line curve shown in Fig. 1b, which is the influence line of bending moments M_E at center E of the span BC, assuming a constant section of the beam from A to D. We may take into consideration now the

increase of section near the supports by simply adding short steel splines near B and C, whereupon the influence line will flatten more (Fig. 1*c*); joining further by means of the No. 1

FIG. 1.



right angle clamps at B and C two short splines at right angle, which represent the interior columns, holding the column feet upside down, at original distance by means of the sliding rule, the bending moment will be further reduced as shown in Fig. 1*d*.

A concentrated load P acting at any section of the structure will produce then at E the bending moment $M_E = Py$, where y is the ordinate (Fig. 1*b*) of the influence line at P. A load W however uniformly distributed over the span BC of $l = 60'$ length will produce the bending moment

$M_E = Wy_m$, wherein $y_m = F/l$ is the average ordinate of the influence area F, enclosed by the influence line and the axis BC. I have been able to prove that in order to know F it is only necessary to know the ordinate h in center of the loaded length and that

$$y_m = \frac{2}{3}h - \frac{1}{24}l.$$

That simple equation does away with all the complicated mathematical methods of integration used so far; it is mathematically correct for constant section, and nearly so, when beam is reinforced near supports. We obtain then, and may read off the cross section paper, in less time than it takes to describe it, the bending moment at center of beam BC, the length of which is $l = 60'$ and therefore $1/24l = 2.5'$, and which is loaded by $W = 60t$, as follows:

Beam BC.	Ordinate h.	In Center $2/3h$.	Average Ordinate $y_m = \frac{2}{3}h - \frac{1}{24}l$, feet.	Bending Moment $M_E = Wy_m$.	
	Feet.			Foot-tons.	%.
Free supports at B & C...	15	10	$10 - 2.5 = 7.5$	450	100
Continuous, constant section.....	8.7	5.8	$5.8 - 2.5 = 3.3$	198	44.0
Continuous, sections increased near supports..	7.2	4.8	$4.8 - 2.5 = 2.3$	138	30.7
do. do. fixed columns....	6.2	4.13	$4.13 - 2.5 = 1.63$	98	21.8

It will not require a specialized Engineer to appreciate that the reduction obtained nearly to one-fifth the original value of the stress at the most exposed section of the structure may be of greatest economical and scientific importance. It should further be noted that to obtain those results the way shown in Fig. 1 will not take more minutes than the mathematical stressanalysis requires hours with all the errors possible in abstract mathematical procedures. With variation of story height, of relative spans, etc. results may change entirely and possible rules of thumb based on results as obtained above may easily lead to disaster. It is then the enormous progress and advantage of visible mechanostatical methods in the "Continostat" that they actually permit figuring each case by taking into account its actual conditions.

2. DEVICES FOR IRREGULAR STRUCTURES.

Irregular structures in Mechanostatics may be called those structures the members of which joint at oblique angles; that distinction is convenient on account of their entirely different mechanical requirements, from those necessary for common structures as described in the former article and in the chapter No. 1.

FIG. 2.

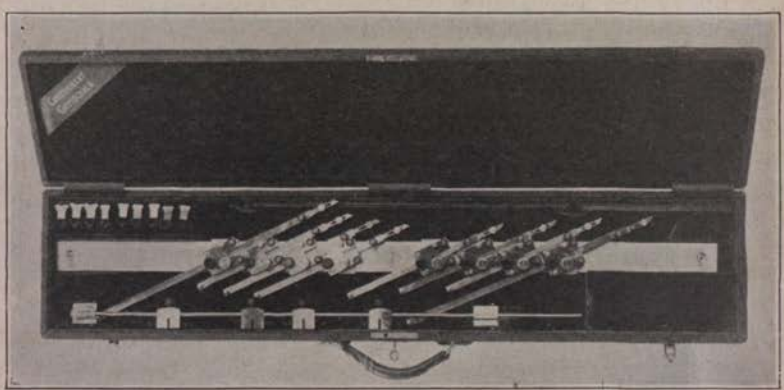
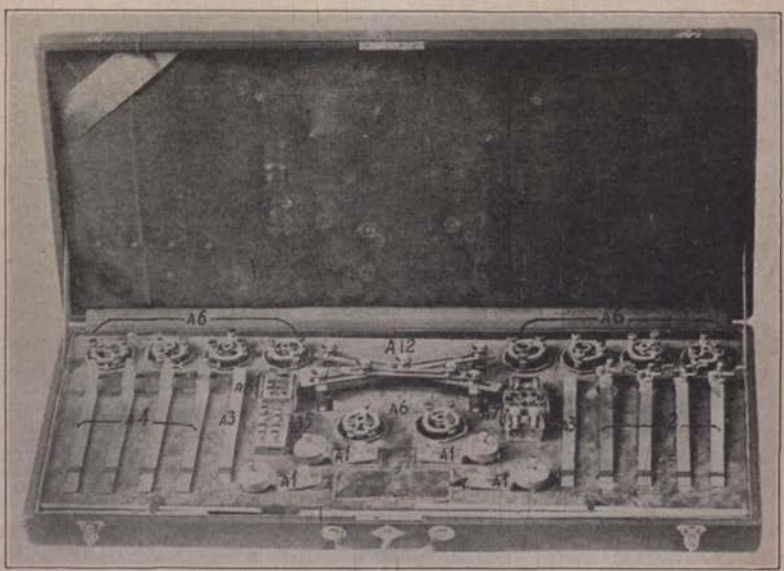


Fig. 2 shows the "Constinostat" used for common structures, whilst Fig. 3 is the "Continostat-A" or "Mechanostat," containing a complete set of devices, which in combination with the "Continostat" make it possible to apply the visible methods developed by me even outside of building work to the complicated structures which occur i.e. in shipbuilding, airplanes, parts of engines etc. As will be more clearly shown in the photographs hereafter the "Continostat-A" consists mainly of attaching devices A-1 and A-5 to the several parts of the "Continostat," of holders and rods A-2, A-3 and A-4, of insert clamps A-7 for tension rods and of ringclamps A-6, the holders of which are automatically held in exact radial direction, but otherwise may freely gyrate or be fixed in any position.

The material of which models are to be composed will preferably be steel splines the same as before; however at irregular structures it frequently has become convenient to use plastic metal splines, which could be bent into shape

by hand, without losing elasticity outside of the sharply bent places. Specially rolled plastic metal splines are therefore being added to the "Continostat-A" to be used for what may be called plastic models. It may be said that generally plastic models give satisfactory results and approximations

FIG. 3.



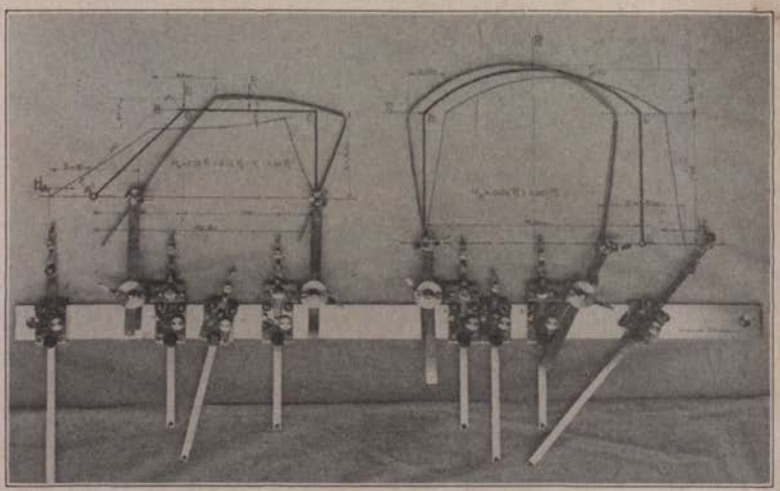
within 5 per cent. of analytical results, although they do not permit as large deformations as do the models composed of steel splines and cannot be deformed as great a number of times as the latter without harm to their elasticity.

3. PLASTIC MODELS.

As said before, plastic metal may be bent into shape right on top of the drawing; we also may use combined plastic metal for curved parts and steel splines for straight members, taking into consideration the modulus of elasticity E of the plastic metal is approximately half of that of steel splines. It therefore must have a thickness of $\sqrt[4]{2}$ or very nearly 5/4 of that of steel splines in order to have the same specific resistance to bending. It should be noted that when

model is not bent absolutely true to plan, errors neutralize one another when subjecting models to two equal but opposite displacements each time.

FIG. 4.



In Fig. 4 to the left has been shown an oblique portal frame, to the right a portal with curved beam; the "Constinostat" has been photographed in the position of displacing horizontally the foot of the left and the right column respectively by any distance $s/2$. The slide of the "Constinostat" then, holding the columnfeet in the hinged A-2 heads will be moved by $s/2$ the opposite way in order to obtain the influence line of horizontal thrusts. Taking the arched portal frame to the right of Fig. 4 the resulting thrust H_A at foot of left column and H_D at foot of right column may be read off the plan as follows:

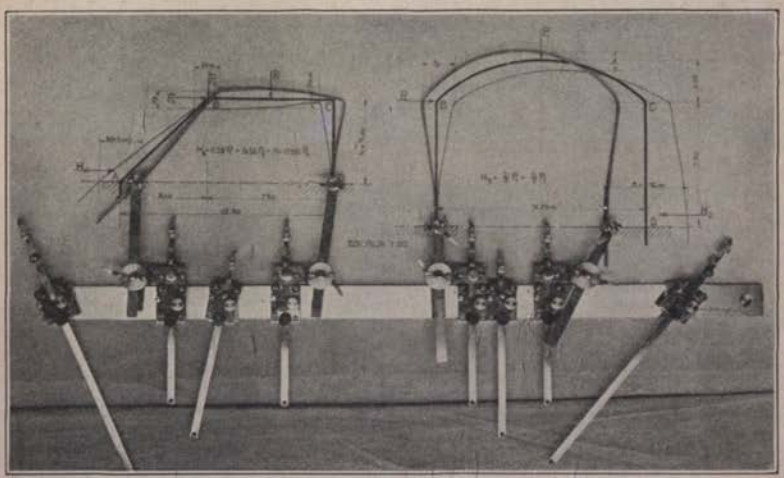
$$H_A = \frac{1.34}{10} P_1 - \frac{10 - 3.95}{10} P_2, \qquad = 0.134 P_1 - 0.605 P_2,$$

$$H_D = \frac{1.34}{10} P_1 + \frac{3.95}{10} P_2, \qquad = 0.134 P_1 + 0.395 P_2,$$

1.34 and 3.95 cm. being the distance the model has moved at P_1 and P_2 respectively when shifting D each way $\frac{s}{2} = 5$ cm.

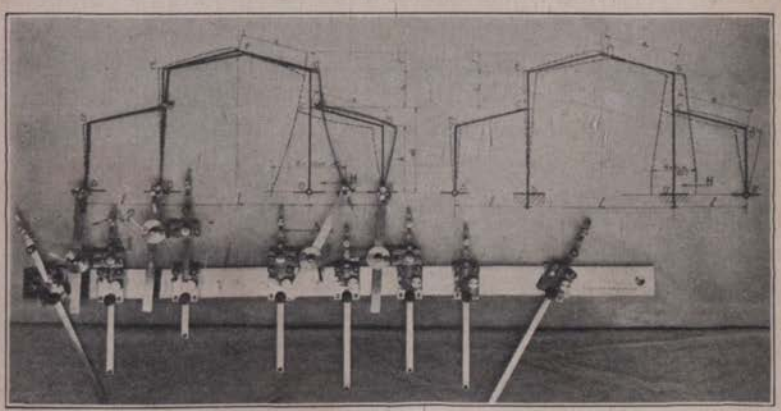
By merely pulling the small lever at the swivel head of the A-2 rods which hold the feet of the columns, those heads may be fixed and when sliding the A-2 rods along the main

FIG. 5.



bar of the "Continostat" the model will automatically mark the influence lines of the horizontal thrust for fixed column

FIG. 6.



feet (Fig. 5). Note the most instructive picture of the difference between influence lines for hinged supports (Fig. 4) and fixed supports (Fig. 5).

Several parts of plastic models can be easily joined with small clamps A-10 in order to form only one model. In Fig. 6 has been shown a three span portal frame, the shape frequently used in factory building, the model having been composed of the complete central part with the two aisles attached by small clamps. At the left of Fig. 6 the supports of the columns have been supposed hinged and the photograph shows the "Continostat" holding the model in the swivel heads of the A-2 rods with the foot of the third column shifted to the right by any not too large distance $s/2$, forming that way the influence line of the horizontal thrust at the foot of the third column. By pulling a small lever at the swivel head of the A-2 rod those heads cannot gyrate and will produce automatically the same influence lines for fixed supports as indicated in dotted lines to the right of Fig. 6.

Plastic models are especially convenient for quick estimating work; with reasonable care the results with the same will stay within 5% of those obtained by exact analysis.

4. STEEL SPLINE MODELS.

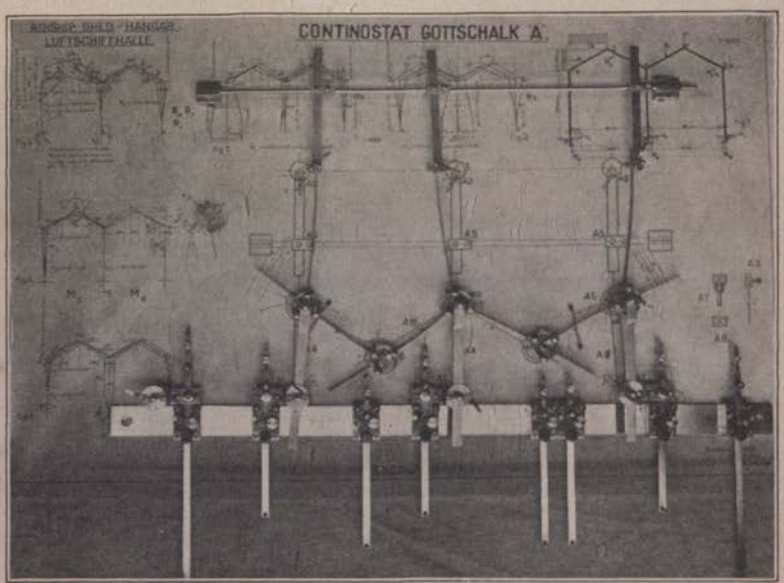
Steel splines do not lend themselves to curving or breaking, they admit however very large deformations, returning with great force to their original shape however long time they may have been bent; consequently they should be used for very accurate work and especially where models may be composed of straight splines, or as indicated in the following chapter.

As an example Fig. 7 shows the statical calculation of an airship shed. The model has been composed of short, straight and strong splines joined with A-6 ring clamps. The model is being held upside down, at the column feet by means of the A-2 rods in the sliding rod of the "Continostat" and at the heads of the columns by the A-4 gooseneck rods, which on their part slide and gyrate in the A-1 devices attached to the rods of "Continostat" itself. The interior A-4 rod has been fixed, the two exterior ones may swing freely. The photograph Fig. 7 shows how the thrust at the interior column is being obtained by shifting the foot of the same any distance $s/2$ towards the column to the left, whereupon the frictionless sliding rule swings automatically to the right until the elastic equilibrium of the model has been regained, that is to say,

until the model has bent into the exact shape of the influence line for the horizontal thrust at the foot of the interior column.

In the same photograph Fig. 7 have been marked to the left in small scale the influence lines of bending moments at corners obtained by gyrating one leg of the ring clamps with respect to the others by $2 \times$ "½" = "1." No part of the model should touch the paper until deformation has taken

FIG. 7.



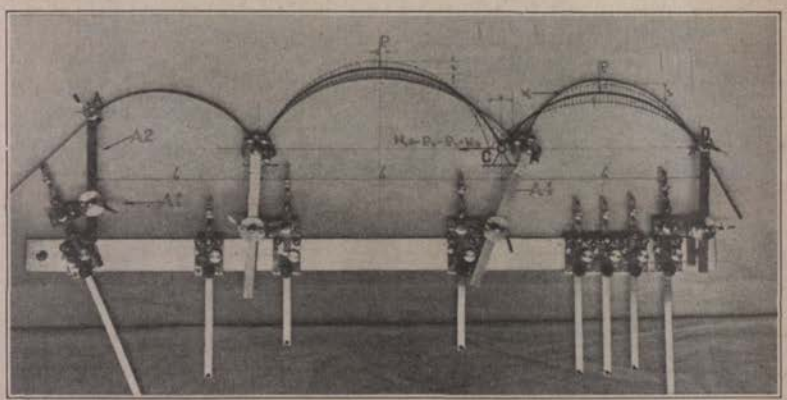
place; plastic models might be lifted a bit from the table and stay without rubbing on paper, whilst when using ring clamps the small frictionless rolls may be slipped underneath the model at strategical places, to keept the model from touching the table.

5. ARCHES.

Frequently it is possible to form arches, holding under stress straight steel splines by approaching two points of the same, until rise and shape of arch desired have been obtained. The continuous three span arch supported at different levels Fig. 8 has been formed that way. Angles between arches at interior supports have been fixed by means

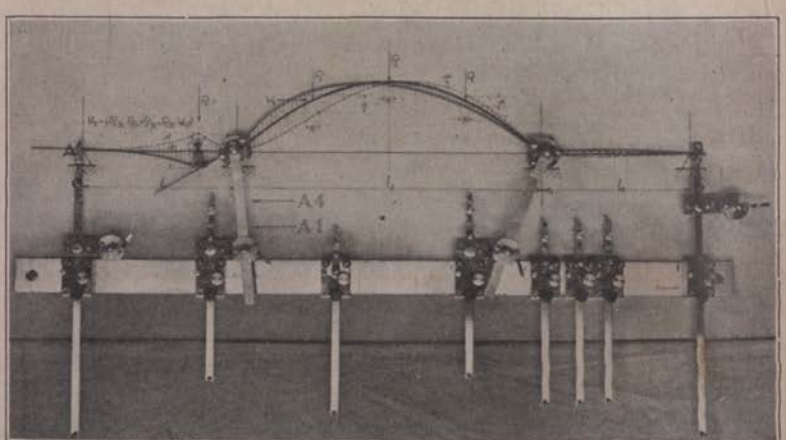
of the ring clamps, which however may gyrate freely around their axis, held in A-4 rods. The photograph shows the rod

FIG. 8.



holding third support *C* shifted to the right in order to obtain immediately the horizontal thrust at that support.

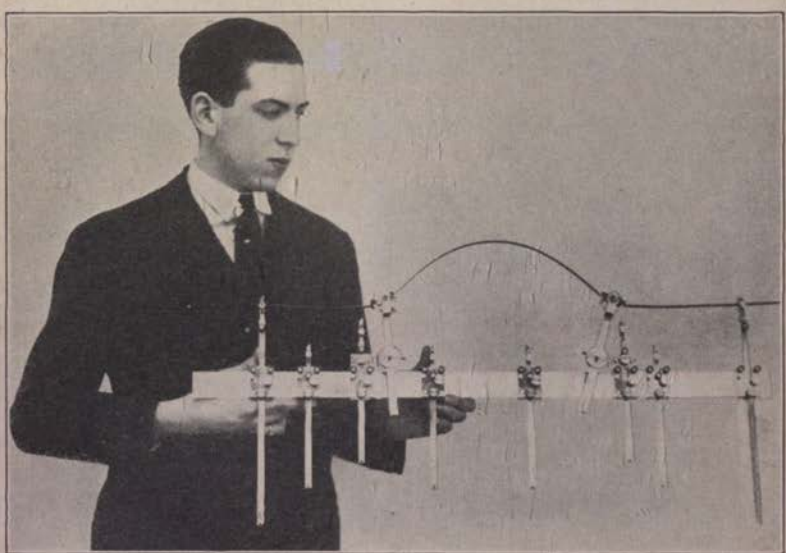
FIG. 9.



Similarly in Fig. 9 an archbridge with two straight end-spans has been reproduced with straight steel splines. By inserting the No. 2 clamp of the "Continostat" at any section, e.g. in the left span as shown, the influence line of

the bending moment at that section is obtained. Fig. 10 again shows the model of the same structure, however with the second support from left shifted a small distance to the right, obtaining that way the influence line of horizontal thrust at the second support; note how the model is being

FIG. 10.



held freely in the "Continostat," so that deformations may take place accurately without danger of rubbing on drafting table or any other exterior hindrance.

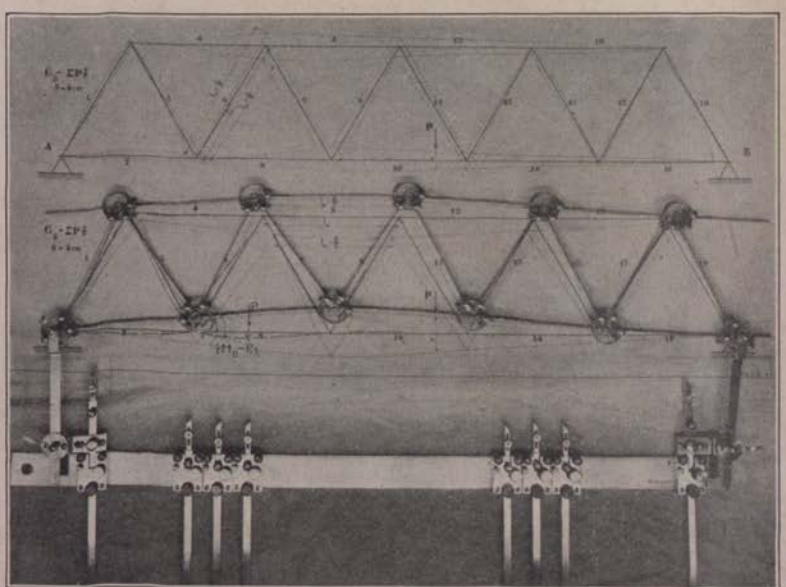
6. LATTICED TRUSSES.

The "Continostat-A" and especially the A-6 ring clamps open up new fields which do not lend themselves to analytical investigation; as an example, secondary stresses may be calculated due to deviation of shopwork from theoretical assumptions e.g. when using riveted, or welded, joints in latticed trusswork, where for statical calculations hinges without friction are supposed.

Fig. 11 shows a plain truss composed of steel splines joined by A-6 clamps, the holders of which are fixed to the rings. By spacing farther apart the second and third clamp

on the upper chord, the model is forced into the shape of the real influence line of compression in second panel of the upper chord, for rigid joints. Now we may go along loosening two of the legs of each ring clamp whereupon the diagonals of the truss straighten themselves forcefully and eventually the model forms the influence line for hinged joints. The difference from the influence line obtained before for rigid joints show automatically the secondary stresses due to the same.

FIG. 11.



Secondary bending moments at rigid joints are automatically shown by gyrating the one leg of ring clamp where moment is wanted with respect to the others. To the left of the lower chord in Fig. 12 the influence line of secondary bending moments in chord has been obtained that way as shown in dotted line.

The simpleness and efficiency of this method may be more clearly illustrated in Fig. 12, showing the cross section through a three chord truss. By inserting the No. 2 clamp of the "Constinostat" in the center of the lower crossbeam we get the heavy influence line of bending moments there shown in

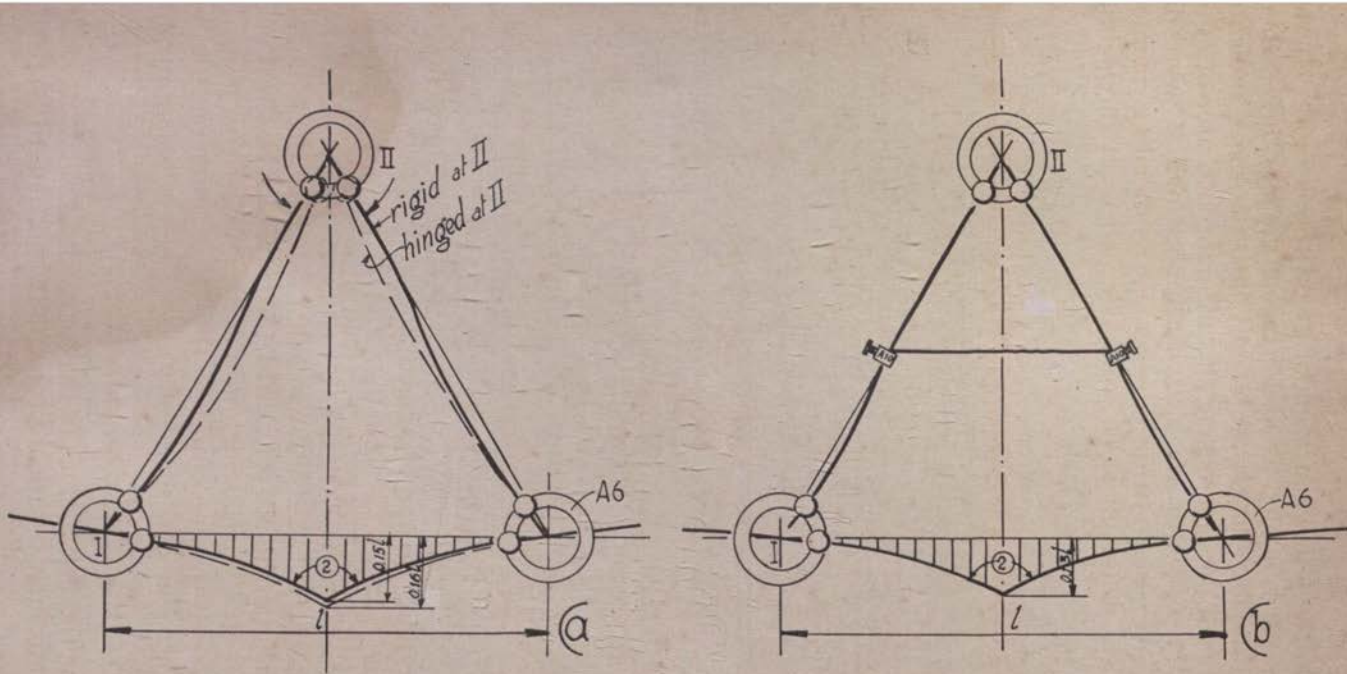
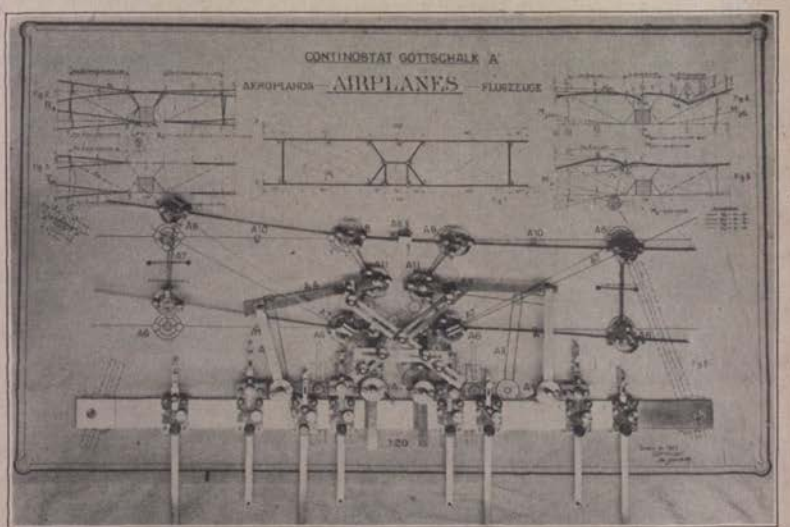


FIG. 12.

Fig. 12 to the left when joint at II on top is rigid. If that joint however is hinged we loosen one of the legs there, whereupon the model will snap forcefully into the shape indicated in fine dotted line, increasing the ordinate in center of crossbeam from $0.15l$ to $0.16l$. If therefore a load W were distributed over the entire crossbeam the bending moment produced in center of same would be, when all three joints

FIG. 13.



are fixed, $M = (2/3 \times 0.15l - 1/24l)W = 0.058lW$ and when the joint on top is hinged

$$M = (2/3 \times 0.16l - 1/24l)W = 0.065lW;$$

that means an increase of 11 per cent. Bracing the triangular frame with a strut at half height the influence line will be as shown in Fig. 12*b*, reducing ordinate in center to $0.13l$; in this case however the loosening of the A-6 clamp on top viz. introducing a hinged joint on top will not alter appreciably the result.

7. COMPOUND STRUCTURES.

The methods explained in this article have made it possible to calculate many statical problems which before had to be

solved by empirical formulas, e.g. in shipbuilding, in parts of engines, etc. Fig. 13 shows an application for calculating stresses in airplanes. The four hinges forming the supports of the statical system are so close together that small difference there may multiply and produce relatively large errors on the outside. To prevent that, the mutual position of those supports is being especially fixed by means of the stabilizer A-12. The photograph shows how the stress in one of the diagonal wires is being obtained by lengthening or shortening of the respective spline, which is inserted on top of two A-6 clamps by means of the tension rod inserts A-7.

November 9, 1928.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

UNIDIRECTIONAL RADIO BEACON FOR AIRCRAFT.

DURING the development by the bureau of a directional beacon for guiding aircraft, it appeared that the usefulness of the beacon could be increased by making the radiated field unidirectional. This was accomplished by the combined use of a vertical antenna with the two crossed-coil antennas utilizing the directive and non-directive fields simultaneously, with the proper phase and amplitude relations between them to secure unidirectional transmission.

The system is explained in a paper "Unidirectional Radio Beacon for Aircraft" by E. Z. Stowell in the Bureau of Standards Journal of Research for December, 1928, Research Paper No. 35. Reprint copies of this paper may be obtained for ten cents from the Superintendent of Documents, Government Printing Office, Washington, D. C.

A LABORATORY INTERNAL MIXER FOR RUBBER.

A SMALL internal mixer for rubber has been installed recently. Various extraction and drying operations have been performed with the use of the mixer, in addition to the compounding of rubber for which it was designed.

The mixer consists essentially of shaped rotors moving in opposite directions at different speeds in a closed chamber. Temperature control is effected by a jacket for steam and water at the bottom of the chamber. The working capacity of the mixer is 500 to 2,500 grams of rubber. The machine is driven by a motor with variable speed control.

A variety of stocks has been successfully compounded with the mixer. Those containing large proportions of graphite have been mixed by the machine with greater facility, ease, and accuracy than on the usual laboratory roll mill. Sulphur, accelerators, and even ultra-accelerators are

* Communicated by the Director.

usually incorporated in batches in the mixer. Temperature control is not difficult, as in case of factory size internal mixers on account of the relatively larger surface per unit volume of rubber.

No critical study has, as yet, been made of the performance of the laboratory internal mixer or of the properties of rubber compounded in it. However, such an investigation is contemplated.

The mixer has been used as an extraction apparatus by mascerating rubber or other gums with successive portions of solvent in the closed mixing chamber. Since a large and continually renewed surface of the gum is presented to the action of the solvent, the extraction process is relatively rapid and efficient. Furthermore it is possible to work with very soft and tacky gums that would be difficult to handle with usual laboratory extraction equipment. The machine has also been used to make special rubber cements having a wide range of consistencies.

The internal mixer has also been found convenient and effective for the drying of rubber. During this operation an inert gas, such as carbon dioxide, is usually circulated through the mixing chamber to carry out vapors and to avoid oxidation by air. The machine is designed to permit its operation with the chamber evacuated, but this feature has not yet been tested.

COLLOIDAL CONTENT OF CHINA CLAYS.

INCLUDED in the program of the china clay investigation is a study of grain size distribution. While this has been carried out by the sedimentation process and application of Stokes' law down to a particle size diameter of 1.7 microns, the data indicated that from 35 to 60 per cent. of the total clay had a grain size of less than 1.7 microns. Since this large quantity of finely divided material undoubtedly has a pronounced effect upon the properties of the clay, it seemed desirable to make a further classification as regards size.

After reviewing the literature and studying the methods of various investigators, it was decided that centrifuging was the best way to effect the separation. Hand and motor driven

centrifuges were tried, but the speed was insufficient and a Sharples laboratory super-centrifuge was obtained. It was found that for a suspension of 1.6 specific gravity and a range of flow of 20 liters per hour, the centrifuge would throw out of suspension all particles greater than one micron in diameter when rotating at 12,000 r.p.m.

The particle size was checked with a microscope using a 12.5 micrometer eyepiece and an oil immersion objective. The second separation consisted in taking the suspension from the first and centrifuging it at a speed of 18,000 r.p.m. and a rate of flow of about 8 liters per hour. The material remaining in suspension after the second separation was removed either by centrifuging at 40,000 r.p.m. and a very slow rate of flow, or running through a set of Pasteur-Chamberland filters. The ultra filters have a slight advantage over the centrifuge in that they remove every particle of suspended matter while the centrifuge leaves a trace in suspension.

The Bureau of Chemistry and Soils has done considerable work on the estimation of colloidal material by obtaining the adsorption of water or other liquids, by the clay and by a sample of the total colloid. The adsorption in grams per gram of the total clay divided by the corresponding value for the total colloid gives the percentage of colloid. This method will be used to estimate the total colloid present in the clays. The figure obtained should check roughly the amount indicated in the earlier work as being under 1.7 microns. Then to go farther, it is desired to compare the quantities of material obtained from the second and final extractions of any one clay with like quantities from any other clay. If the same volume and the same concentration of suspension are used for the various clays, then under similar conditions of centrifuging, the actual amounts of material thrown out at any one speed should be relative and comparable. The separated material can be carefully removed, dried, and weighed.

Thus far data have been collected on a few clays only, but it is expected that considerable difference between one clay and another will be shown in the quantities of these fractions.

ABSORPTION OF CLAY BRICK.

IN connection with an investigation, sponsored by the American Face Brick Association, of the factors controlling the moisture transmission of brick masonry, a study has been made of the absorption properties of nine different types of clay brick. All bricks of one type were made at the same plant and differed from those of other types in the clay used and in certain details of manufacture. Among the nine different types investigated were included bricks made by the stiff mud and dry press processes. The samples of each type used in the absorption tests were selected to give as wide a range in the properties as could be obtained from several hundred marketable products. Those selected were free from large cracks or laminations. The absorption tests included measurements of the water absorption of brick by total immersion at room temperature, total immersion in boiling water for 7 hours, and partial immersion at room temperature for several periods. The absorption by partial immersion was determined by noting the increase in weight of an initially dry brick when one surface was kept in contact with water and the other surfaces were surrounded by an atmosphere saturated with water vapor.

The results of the tests indicated that the relations between the amounts of water absorbed by total immersion at normal room temperature and by boiling were given closely by the empirical equation

$$Y = KX^3 \qquad \text{or} \qquad Y = K\left(\frac{Y}{Y+Z}\right)^3,$$

where Y = volume of water absorbed per unit of bulk volume of brick after total immersion at room temperature. (Taken at a period when the gain in the weight of the brick was less than 1 gm. per 24 hours.)

$Y + Z$ = volume of water absorbed per unit of bulk volume of brick after 7 hours total immersion in boiling water.

K = a constant for one make of brick.

$X = \frac{Y}{Y+Z}$ = ratio of absorption by cold immersion to absorption by boiling.

The absorption constant K was different for different types of brick but was essentially constant for any one make of brick. Since the values of K were independent of the degree of burning and were constant for brick made from the same clay by the same manufacturing process, there seems to exist the possibility that clays may be classified in accordance with these values, though it has not been determined definitely to what extent (if any) they are affected by the manufacturing process.

With any one type of brick the values of the ratio X decreased with a decrease in the apparent porosity of the brick, but there was not a close relation between these two values with brick of different type. Since the quantity Z is that part of the total open pore space which can be entered during the cold immersion test, it may be termed the residual porosity. By combining the equation $Y = KX^3$ and $X = \frac{Y}{Y+Z}$ and solving for Z, it can be deducted mathematically that Z is a maximum when $X = \frac{2}{3}$. This relation was borne out by the results of the tests the values of the residual porosity Z being greatest with any one type of brick when the value of the ratio X was about 0.67.

When a definite surface area of a brick is in contact with water at normal room temperature, the rate of gain in weight of the brick because of water absorbed by capillarity has been in all cases such that $\frac{d}{dT}\left(\frac{W}{T^{1/3}}\right) =$ a constant, B. W is the gain in weight of the brick at any definite time interval T. In all cases wherein the rate of absorption was slow, thus making possible an error due to evaporation, the tests were conducted in an atmosphere saturated with moisture.

Performing the indicated operation $\frac{d}{dT}\left(\frac{W}{T^{1/3}}\right)$, there results the equation, $\frac{dW}{dT} - \frac{W}{3T} = BT^{1/3}$, which when integrated, gives the equation, $C = \frac{W}{T^{1/3}} - BT$, where C is an integration constant. This latter equation has been applied to each of the nine types of brick studied. The quantity $\frac{W}{T^{1/3}} - BT$,

has been found to be constant within the limits of experimental error for any one single individual brick among all of the nine types of brick studied.

Among brick of any one type, the magnitude of C is directly proportional to Y, the apparent maximum volume of water absorbed at room temperature per unit bulk volume of brick. This relation is expressed by equation (1), $C = K'Y^3$ or $Y = \left(\frac{C}{K'}\right)^{1/3}$. But since (2) $Y = KX^3$, the value for Y in equation (1) may be substituted for Y in (2) and there results equation (3) $\left(\frac{C}{K'}\right)^{1/3} = KX^3$. Equation (3) contains every absorption property characteristic of any single brick selected from any of the nine manufacturers' marketable brick.

It is expected that noticeable deviations from these relationships will obtain in the case of any brick that is either distinctly under-burned or over-burned.

DIATOMACEOUS SILICA AS ADMIXTURE IN CONCRETE.

A STUDY of the representative diatomaceous silicas commercially produced in the United States for use as admixtures in concrete is now being made at the bureau; eleven representative samples having been obtained from nine producers. The purpose of the study is the development of Federal specifications for this material.

The petrographic analysis, fineness through the No. 200 sieve, and weight per cubic foot, have been determined. The material runs from a minimum of approximately 70 per cent. diatomaceous silica mixed with quartz and clay to practically pure diatomaceous silica. All samples are finely divided from 86.2 to 99.5 per cent. passing a No. 200 sieve.

As usual, the determination of the weight per cubic foot introduced difficulties, the weight of any one material varying according to the method whereby it is placed in the measure.

A $\frac{1}{3}$d cubic foot aluminum measure, 8 inches in diameter, was made expressly for the measurement of the weight per cubic foot. The dried diatomaceous silica was always screened through a sieve into the measure. It was found that

a $\frac{1}{8}$ or $\frac{1}{4}$ inch opening gave the same results. The $\frac{1}{16}$ inch opening was too small, the material caking and blocking the sieve opening.

Hand sieving required such considerable time that motor driven apparatus was built. A 1/100 H.P. motor drives through worm and wheel a vertical shaft, at the end of which another horizontal shaft forms a *T*. The horizontal shaft rotates over the screen surface and about $\frac{1}{16}$ inch above it. If the distance above the screen is too great the material will cake and if too close it will touch the screen at some spot, cause movement of the screen and thus shake the material through, with a resultant change in unit weight. The weight per cubic foot will vary with the speed of the rod, and this speed will vary with the amount of material on the sieve. For this reason the diatomaceous silica is slowly added to the screen from a scoop, so that never more than $\frac{1}{2}$ inch of the material is on the screen. The mean velocity of the rod is about 120 r.p.m.

The weights per cubic foot of the materials varied from 7.5 to 34.7 pounds. Only one specimen, however, weighed more than 16.2 pounds per cubic foot, six of the specimens varying from 14.3 to 16.2 pounds per cubic foot.

PANEL TESTS OF LIME PLASTER.

IN March, 1922, a series of panels of lime plaster was erected, using several types of limes and intentionally including in the finish coat various impurities. The panels were allowed to age under ordinary laboratory conditions and at the end of a year an inspection was made to determine the effect of the various impurities. The results of the first inspection were published by Emley and Berger in the Journal of the American Ceramic Society, Vol. 6, No. 9; September, 1923, under the title "Panel Tests of Lime Plaster."

A recent inspection, made nearly seven years after the panels were erected, shows that while the majority of the panels are apparently in the same condition as in 1923, certain ones have developed defects not noticeable at that time.

In the construction of the panels four sets of duplicates were made containing, respectively, hydrated lime, lime plus coarse impurity, lime plus medium impurity, and lime plus fine impurity.

For convenience in reference, the 1923 report on the panels which have undergone change since then is given below, together with comments on their condition in December, 1928.

Hydrate No. 5 plus iron carbonate.

(1923) The hydrate itself shows a few pops and cracks. The additions of iron carbonate seem to have no effect.

(1928) Decided map cracking has developed on the panels containing lime without impurity. Numerous cracks and pops have occurred on the panels containing coarse and medium impurity; some have occurred on the panels containing fine impurity.

Hydrate No. 6 plus quicklime.

(1923) The fine quicklime shows no noticeable effect; the medium shows a few additional pops, and the popping of the coarse particles is quite apparent.

(1928) Numerous pops have developed on the panels containing fine quicklime.

Hydrate No. 10 plus lime burned during hydration.

(1923) The coarse particles caused a few pops; with the medium size the pops are almost negligible; and the fine material caused no popping.

(1928) Considerable cracking appears in the panels containing medium and fine burned lime.

Hydrate No. 11 plus tannic acid (fine).

(1923) The surface is badly checked and discolored.

(1928) Several large pops are noticeable.

The results of this later inspection indicate that while in the majority of cases defects which develop in lime plaster are noticeable at the end of a year, certain impurities may cause defects which are only apparent after several years' aging.

FIRE RESISTANCE OF HOLLOW LOAD-BEARING WALL TILE.

THE results of comprehensive tests to determine the fire resistance value of hollow clay wall tile conducted in cooperation with the Hollow Building Tile Association are

reported in the January number of the Bureau of Standards Journal of Research. (Research Paper No. 37, 334 pages. Reprints obtainable from Superintendent of Documents at 75 cents per copy.)

While hollow clay tile has been used in this country for about 50 years in floor arches, partitions and as protection for structural members, only during the past 20 years has it been extensively applied for load-bearing purposes in walls. The safe and economical use of this material as of other masonry materials, requires knowledge of its strength and fire resistance under conditions imposed by severe fires. Fire resistance of walls is rated mainly on ability to keep the temperatures of the side away from the fire to a point where they will not ignite combustible materials in contact with the wall. Ability to carry load throughout the fire and to withstand the impact from hose streams are also necessary conditions. The amount of damage incurred and methods of making repairs are also of interest.

The work reported in the paper consists in the main of three series of fire tests of wall construction built of tile of representative designs and clay materials. The first tests were made on wall sections 1 foot wide and 6 feet high that were subjected to fire on one side while under working load, the main object of the tests being to determine how the fire exposure affects the individual tile units. This was found to vary with the type of clay used, hardness of burning and design of the unit, from minor cracking of a few of the units in the specimen, to failure under load a little after 2 hours.

The second group of tests was with wall specimens 4 feet wide and 4 feet high tested exposed to fire on one side, under load or restrained within the containing frames, and was made to study the effect of changes in design of the unit, and in the constitutents and preparation of the clay.

Addition of ground burnt clay (grog) to the raw clay in amounts up to 2 per cent. was found to be without effect, but higher percentages increased the fire damage, apparently because of initial shrinkage cracks radiating from the grog particles. Additions of sawdust to the raw clay in amounts up to 15 per cent. by volume decreased the tendency to cracking when exposed to fire without seriously decreasing the

normal strength of the tile. Fine grinding and greater amount of pugging of the raw clay increased the strength of the tile and consequently its load-carrying ability when exposed to fire.

Of modifications in the design of the unit, one with double outside shells improved the fire resistance for all but very dense tile in confining the cracking mainly to an outer thin shell. Increased shell thickness was also found to decrease the fire effects, as also fillets of up to $\frac{1}{4}$ inch radius at the junctions of shells and webs.

The final series consisted of 167 fire endurance tests and four fire and water tests of typical wall constructions, 71 of which were made with walls between 10 and 11 feet high and 8 to 16 feet wide. The thickness ranged from 8 to 16 inches, and plaster, stucco, or brick facing was applied on a number of the walls. The walls were subjected to constant applied loads of 70 to 120 lbs./in.^{2} during the fire test, were restrained within the containing frames, or were tested unrestrained, the latter condition being representative of walls in low buildings or the upper story walls of higher buildings. The results are summarized as fire resistance periods, which are determined by the time the walls sustained load under fire exposure and prevented the average temperature rise on the unexposed side from exceeding 139° C. (250° F.), or the maximum rise at any point where temperature measurements are taken from exceeding 180.6° C. (325° F.). Freedom from cracks or openings large enough to transmit flame or ignite combustible materials is also required both in fire endurance tests and fire and water tests.

The final grouping of test results is in three classes, determined by the design of the tile. Fire resistance periods are given for 8, 12, and 16 inch walls in each class, the values varying with the thickness and design from 1 to 11 hours for unplastered walls and from 4 to 15 hours for walls plastered on both sides. Periods are also given for walls assumed to have combustible members projecting into them 4 inches from the unexposed side, this being determined by the time required for temperatures high enough to ignite wood to obtain at the given depth in the wall. Fire resistance periods for this condition ranged from one hour with 8-inch unplastered walls to 10 hours with 16-inch plastered walls built of tile of the designs giving the highest results.

Gypsum and portland cement plasters applied on one or both sides of the wall constructions not only increased their fire resistance periods but that on the side exposed to fire also appreciably decreased the fire effects on individual units and the construction generally. Where applied on split furring tile, plaster was found to prevent all fire damage to the load-bearing tile in fire tests lasting as long as six hours.

Tests for Hydrocyanic Acid. G. C. SHERRARD of the U. S. Public Health Service (*Pub. Health Rep.*, 1928, *43*, 1016–1022) has studied methods for the detection of gaseous hydrocyanic acid (prussic acid) in the air.

He considers the benzidine-cupric acetate test to be too rapid and too sensitive for practical purposes in fumigation. In this test, a test paper containing benzidine acetate and cupric acetate yields a blue color with hydrocyanic acid; a pale blue results in 10 seconds if 2.8 grams of hydrocyanic acid be present in 1,000 cubic feet of air. Chlorine and the oxides of nitrogen also respond to this test.

For the methyl orange-mercuric chloride test, two solutions are prepared; one contains 1.25 grams of mercuric chloride in 250 cc. of distilled water; the other contains 0.60 gram of methyl orange in 250 cc. of distilled water. The reagent is obtained by mixture of 10 volumes of the mercuric chloride solution, 5 volumes of the methyl orange solution, and 1 volume of glycerol. Test papers are prepared from this reagent, and retain their reactivity for a period of 14 to 30 days if kept in a dark, tightly stoppered container. They are most effective when their moisture content lies between 7 and 8 per cent. They react with hydrocyanic acid to produce a pink or red color. The color depends upon the concentration of the hydrocyanic acid, and the time of exposure. Thus a faint pink color is produced by 1.67 grams of hydrocyanic acid per 1,000 cubic feet of air acting for 2 minutes, by 3.35 grams per 1,000 cubic feet acting for 1.5 minutes, or by 6.7 grams per 1,000 cubic feet acting for 1 minute. Chloropicrin, in a concentration of 0.02 per cent. by weight has no action in this test in a period of 6 minutes. The test paper may be introduced into fumigated holds of ships by means of a clip and string.

Albino rats may be used to detect the presence of hydrocyanic acid. Thus, 3.35 grams of hydrocyanic acid per 1,000 cubic feet of air is the minimum lethal dose in a gas-tight compartment in an exposure of 12 hours.

The sense of smell reveals the presence of as little as 0.25 cc. of liquid hydrocyanic acid diffused in 1,000 cubic feet of air.

J. S. H.

NOTES FROM THE U. S. BUREAU OF MINES.*

BENTONITE, A MINERAL OF MANY POSSIBILITIES.

ONE of the oddest and least known mineral substances is bentonite. Certain bentonites have such strong affinity for water that they are capable of absorbing more than ten times their volumes of water. Owing to its peculiar physical properties bentonite has been suggested as a component material in the manufacture of a great variety of commodities as diversified as paper, rubber, putty, phonograph records, pencil leads and soaps. On the other hand, because of its tendency to swell greatly when wet, underground deposits of bentonite have caused great difficulties in the drilling of oil wells, it frequently becoming necessary to take special steps to combat the nuisance.

The properties, mining, preparation and utilization of bentonite have formed the subject of an investigation recently conducted under a coöperative agreement between the United States Bureau of Mines and the Mackay School of Mines, University of Nevada. A large number of samples of bentonites from many sections of the United States were studied in the course of this investigation.

Bentonite is a rock that contains 75 per cent. or more of the crystalline claylike minerals montmorillonite or beidellite.

Investigators have recognized the occurrence of a peculiar claylike substance which, when wet with water, resembled soft soap and was called "mineral soap" or "soap clay" for a long time. Early reports show that such material had long ago been used at the Hudson Bay posts in Canada for washing blankets.

The first shipments for commercial purposes were made in 1888 by William Taylor of Rock Creek, Wyo., after whom the material was called "taylorite." In 1898, however, it was learned that the name taylorite had already been used as a mineral name for potassium ammonium sulphate, so that this

* Published by permission of the Director, United States Bureau of Mines.

substance was then designated as bentonite from its occurrence in the Fort Benton formation of the Rock Creek district.

At the time considerable interest in bentonite was aroused, but it quickly subsided and the price of $25 per ton was soon reduced to $5 per ton. Within the last few years the interest in bentonite has revived, resulting in investigations that have developed industrial processes and products in which bentonite apparently has an important part. The prices today at western points for the finely pulverized material approaches the first quoted price.

Of the many uses suggested only a few have been tested, and investigators have not worked out the properties of bentonite that determine its usefulness or made tests to determine what type of material is best suited to the different uses.

Bentonite deposits occur in beds from a few inches to many feet thick, mainly in the Tertiary but to some extent in the Paleozoic and Mesozoic rocks in many parts of the United States and Canada, and deposits have been reported from Mexico, China, and France. Most bentonite deposits are the result of devitrification and partial decomposition of glossy volcanic ash.

Bentonite outcrops are unique and striking. As little vegetation will grow on them, they are barren, and because of the peculiar physical properties of bentonite these weathered outcrops often present a crinkled coral-like appearance. If the internal portion of bentonite is dry, it may finally absorb water and swell greatly, producing cracks in the surface layer, which has meanwhile become more or less dry. After a rain many bentonite outcrops are covered with a thick mass of slippery jelly, but in dry weather the surface may be dry and fluffy or may have a peculiar granular appearance.

The bentonite of Wyoming is usually considered a standard type in studying other so-called bentonites. One of the important producers of the material has deposits at Medicine Bow and a grinding plant at Cheyenne. Another important producer in Wyoming has a mine at Clay Spur, near Newcastle. A large new deposit of bentonite has recently been reported in Johnson County.

The largest known California bentonite deposits occur in

the arid desert region along the Amargosa River in Inyo County. A material closely resembling Wyoming bentonite occurs at Belle Fourche, South Dakota. On claims owned by one company at this place, there is estimated to be millions of tons of high-grade bentonite. Other deposits are located in Nevada, New Mexico, Idaho, Tennessee, Kentucky, Alabama, and other states.

Nearly all deposits of bentonite contain gritty or sandy inclusions, which are considered impurities and must be removed before marketing. Pulverized bentonite is fine grained and commonly light colored, ranging from cream to olive green; but it may be pink, dark brown, or even black, the color usually but not always becoming darker on wetting. When fired the color is white, buff, or brown. When cut the fresh material usually has a waxy luster which may become dull or powdery on drying. Some varieties may be cut into thin shavings.

Prospectors or others who become interested in the production of bentonite should make a careful investigation of the commercial possibilities of the deposit in question before investing heavily. Such a study should include the possibility of marketing the product, the specifications demanded, the price obtainable, the cost of mining, treating, and shipping to market, the size of the deposit, and the nature of the crude bentonite. One should also realize that most consumers desire a homogeneous product that will remain uniform over a period of years and that deposits containing bentonite which will do this are not common; that most crude bentonites contain appreciable quantities of objectional impurities such as sand, gypsum, carbonaceous matter, or soluble salts that must be removed at considerable expense by washing; and that the purified bentonites from different deposits and even from different levels or parts of the same deposit may have very different properties, such as variation in color, colloid content, and ease of hydration. In general, consumers want a clean, homogeneous, finely divided substance.

Purified, ground alkali bentonites should make admirable fillers, binders or plastics in the manufacture of numerous industrial materials due to its general characteristics, such as extreme state of fine division, property of remaining in

suspension, plasticity, and high absorbent power. Thus bentonite should be useful in the manufacture of paper, linoleum, curtain cloth, cordage, rubber, ceramics, Portland cement, lubricating greases, putty, phonograph records, crayons, plasters, paste, glue, shoe and stove polishes and numerous other materials.

The characteristics of bentonite as an adsorbent, emulsifier or peptizer, should make it available in the manufacture of soaps and detergents, horticultural sprays, animal dips, insecticides, fungicides, paints, inks, water-proofing plasters and dynamite; in the de-inking of printed papers, the refining of oils and fats and in the removal of water from petroleum. As a chemical reagent bentonite should be of value as a water softener, as an aid to soil fertility and in the treatment of molasses. It is also suggested for use as a medical dressing in certain drugs and in beauty clays.

COAL MINE DRAINAGE MAY SOMETIMES AID IN PURIFYING STREAMS.

IT is the common opinion that with rare exceptions all active coal mines produce drainage which contains acid, and that the discharge of that drainage into natural streams contributes to pollution from the standpoint of the use of the water for domestic and industrial purposes; as well as a habitat for aquatic life. While this is true in varying degrees for the majority of coal mines, it has been disclosed in a study of coal mine drainage that is being made by the Pittsburgh Experiment Station of the Bureau of Mines that the water from a considerable number of mines in the Thick Freeport Bed in the vicinity of Pittsburgh is rather strongly alkaline, and instead of contributing to the pollution of the streams into which the mine water drains, it is actually an agent which aids in neutralizing acid that may already be present, or which may enter later from other sources. The absence of acid in the drainage from these mines is apparently due to the fact that the water which enters at the depth of the workings contains more than enough alkaline compounds in solution to neutralize all the acid with which it comes in contact in the mine. In some instances this has been substantiated by

finding both acid water in one section of the mine and alkaline in the other, whereas the combined effluent stream of water was alkaline.

A STUDY OF THE FORMS IN WHICH LEAD IS NOW BEING LOST IN SMELTER SLAGS.

As the studies made by the United States Bureau of Mines at its North Central Experiment Station at Minneapolis have resulted in furnishing a large amount of data which have been of assistance to iron blast furnace operators in understanding the chemical phenomena of the iron blast furnace and the circumstances which govern the successful operations of such a furnace, the Bureau decided, in connection with its work on increasing the efficiency of metallurgical practices, to apply similar studies to the operations of the lead blast furnace. This study was assigned to the Intermountain Experiment Station at Salt Lake City, Utah, and is now being conducted in coöperation with one of the local smelting companies.

In connection with this work the following studies have been made:

(*a*) Location and chemical composition of the accretions in lead blast furnaces.

(*b*) The rate of charge movement in the furnace.

(*c*) The liberation of sulphur from blast furnace charges in the form of SO_3 and SO_2.

(*d*) The zone of maximum reduction of lead.

(*e*) The relation of temperature, CO concentration and slag composition to reduction of lead.

Along with the other studies that are being made, particular attention is being given to accretions. Chemical analyses are made of them. From the difference in composition and texture of the various zones of the samples, an attempt is being made to study the history of the formation of the accretions and from the data obtained, to devise a method to prevent their occurrence.

Interesting data are also being obtained in connection with the studies which are being made on the composition of lead blast furnace slag. The results of the investigation to date show in general that the lead blast furnace is much less

uniform in operating behavior than is the iron blast furnace. The temperature and composition of the gases obtaining at the various tuyeres vary both from each other and at various stages in the cycle of operation. These fluctuations influence profoundly the composition of the slag, the loss of unreduced lead, and the rate of smelting at the various tuyeres.

BARIUM OXIDE AS A BLAST-FURNACE FLUX.

A comparatively expensive flux could be used in blast-furnace practice provided small amounts of it would produce a substantial increase in the desulphurizing action of the slag without making the slag more viscous. Such a flux would be particularly beneficial to operators using coke containing more than 1 per cent. sulphur, because as the sulphur in the coke exceeds this amount changes in operation necessary to keep the sulphur in the metal within specifications become increasingly expensive.

Results from laboratory tests made at the North Central Experiment Station of the United States Bureau of Mines, Minneapolis, Minn., indicate that slags containing barium oxide do not desulphurize more readily than normal slag containing calcium and magnesium oxides. A comparison of the desulphurizing action of slags was made by holding contiguous layers of the molten slags and high-sulphur metal in contact at temperatures from 1,400 to 1,600° C. Small streams of metal were also passed through layers of the various slags to assimilate the passage of globules of metal through the slag bath in the crucible of the blast furnace. Due to the high molecular weight of barium oxide it is only equivalent to about one-third its weight of calcium oxide as a base for fluxing silica. A replacement of about one-fourth the calcium oxide with barium oxide increased the rate of sulphur elimination. However, the cost of barium oxide necessary to replace this fraction of the calcium oxide in blast-furnace slag would be prohibitive.

THE FRANKLIN INSTITUTE.

ANNUAL MEETING, WEDNESDAY, JANUARY 16, 1929.

The annual meeting of The Franklin Institute was called to order at eight-thirty p.m. by the Acting President, Mr. Henry Howson.

The Secretary reported that the minutes of the December meeting of the Institute had been published in full in the January number of the Journal of the Institute. He therefore moved that the minutes be approved as printed. The motion was seconded and adopted. The minutes were then declared approved.

The Secretary stated that the detailed reports of the various committees of the Institute and of the Board of Managers would be published in full in the Year Book, for those interested in the details of the administration of the Institute. To read these at this time would require far too much time.

The Secretary then read, on behalf of the Acting President, his report as President, which also will be published in full in the Year Book. He supplemented the statement of the Acting President by the announcement that the Mayor of Philadelphia intended to celebrate Franklin's birthday by introducing into the City Council the Ordinance authorizing the use of the plot of ground bounded by Twentieth, Winter, Twenty-first and Race Streets, for the new Franklin Institute and Benjamin Franklin Memorial.

The report of the Tellers of the election, Drs. Henry Leffmann, Wilbur M. Stine, and Mr. W. N. Jennings, was then read by the Secretary. It showed that the following gentlemen had been elected to the offices indicated:

For President (to serve one year), Nathan Hayward.

Vice-President (to serve three years), C. C. Tutwiler.

Treasurer (to serve one year) Benjamin Franklin.

Managers (to serve three years), James Barnes, G. H. Benzon, Jr., G. H. Clamer, Theobald F. Clark, A. W. Goodspeed, Charles Penrose, G. D. Rosengarten, W. C. Wetherill; (to serve two years), Malcolm Lloyd.

The Chairman of the meeting then declared the candidates named to have been elected to their several offices.

The statement was made by the Secretary that the Tellers had devoted four hours to the reception and tabulation of the ballots and requested that a vote be passed, thanking the Tellers for their intelligent and helpful efforts. Such a motion was presented and was unanimously adopted.

There being no other business, the Acting President introduced as the speaker of the evening Professor Charles P. Olivier, Director of the Flower Astronomical Observatory of the University of Pennsylvania, who spoke convincingly and delightfully on "The Influences of Meteoric Astronomy on Evolution." His paper was followed by discussion.

The meeting was adjourned at nine-thirty-four, with a rising vote of thanks to Professor Olivier for his delightful talk.

HOWARD MCCLENAHAN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, January 2, 1929.)

HALL OF THE COMMITTEE,
PHILADELPHIA, January 2, 1929.

DOCTOR GEORGE S. CRAMPTON *in the Chair.*

The following reports were presented for first reading:

No. 2840: Klydonograph.
No. 2885: Work of Sir James C. Irvine.
No. 2903 and No. 2904: Franklin Medal.

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, January 9, 1929.)

RESIDENT.

DR. STUART MUDD, Medical Investigation, Henry Phipps Institute, Philadelphia, Pa. For mailing: Villa Nova, Pa.

NON-RESIDENT.

MR. GEORGE A. HENDRICKSON, Engineer, 2000 Second Avenue, Detroit, Michigan.

DR. ELMER O. KRAEMER, Research Chemist, Director-Research in Colloids, E. I. duPont de Nemours Experimental Station, Wilmington, Delaware. For mailing: 2322 West Fourteenth Street, Wilmington, Delaware.

STUDENT.

MR. CHARLES A. COHEN, Student, College of Pharmacy and Science, Philadelphia Pa. For mailing: 516 Greenwich Street, Philadelphia, Pa.

CHANGES OF ADDRESS.

JOHN J. CATON, Esq., 16140 Fairfield Avenue, Detroit, Michigan.

DR. E. A. ECKHARDT, Gulf Oil Corporation, Research Department, 903 Law and Finance Building, Pittsburgh, Pa.

MR. H. LINWOOD GODFREY, Fidelity-Philadelphia Trust Building, 123 South Broad Street, Twenty-third Floor, Philadelphia, Pa.

MR. E. S. HIGGINS, Motor Mileage Corporation, 1113 Packard Building, Philadelphia, Pa.

CHARLES HENRY HOWSON, Esq., Fidelity-Philadelphia Trust Building, 123 South Broad Street, Twenty-third Floor, Philadelphia, Pa.

HENRY HOWSON, Esq., Fidelity-Philadelphia Trust Building, 123 South Broad Street, Twenty-third Floor, Philadelphia, Pa.

RICHARD HOWSON, Esq., Fidelity-Philadelphia Trust Building, 123 South Broad Street, Twenty-third Floor, Philadelphia, Pa.
MR. J. C. JOHNSON, 332 North Lansdowne Avenue, Lansdowne, Pa.
DR. JOHN JOHNSTON, Research Laboratory, United States Steel Corporation, Kearny, N. J.
MR. F. B. LYSLE, 4921 Pulaski Avenue, Germantown, Phila., Pa.
DR. J. C. MCLENNAN, 88 Prince Arthur Avenue, Toronto, Canada.
MR. L. W. MACLEAN, Atlantic City Railroad, Camden, N. J.
MR. H. H. PLATT, Villa Bachlyk, Cap d'Ail, A. M., France.
MR. DEXTER N. SHAW, Fidelity-Philadelphia Trust Building, 123 South Broad Street, Twenty-third Floor, Philadelphia, Pa.
MR. GEORGE H. TABOR, 4114 Bigelow Boulevard, Pittsburgh, 13, Pa.

NECROLOGY.

Dr. William J. Williams, Philadelphia.

LIBRARY NOTES.

RECENT ADDITIONS

ALEXANDER, JEROME, editor. Colloid Chemistry: Theoretical and Applied. By selected international contributors. Volume 2: Biology and Medicine. 1928.
American Annual of Photography, 1929. Volume 43. 1928.
BUTLER, RUTH LAPHAM. Doctor Franklin, Postmaster General. 1928.
CAREY, LEWIS J. Franklin's Economic Views. 1928.
EISELEN, MALCOLM R. Franklin's Political Theories. 1928.
HABER, FRITZ. Fünf Vorträge aus den Jahren 1920–1923. 1924.
MACLAURIN, LOIS MARGARET. Franklin's Vocabulary. 1928.
MACLEOD, J. J. R. The Fuel of Life. 1928.
VAIL, JAMES G. Soluble Silicates in Industry. 1928.
WIEN, W., AND F. HARMS, editors. Handbuch der Experimentalphysik. Band 7, 2 Teil. 1928.

BOOK REVIEWS.

COLLOID CHEMISTRY, THEORETICAL AND APPLIED. By selected international contributors, collected and edited by Jerome Alexander. Volume II, Biology and Medicine. 1029 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1928. Price $15.50.

Notice of the general nature of this work was given in the review of the first volume. The present volume is devoted largely to the biological relations of colloids, a field which is of great extent and necessarily capable of treatment only by an array of specialists. These have been selected from many nations and thus

has been obtained not only an immense amount of detailed information but a multiplicity of view-points. The nearly three score essays have been subjected to the editorial scrutiny of Mr. Alexander, so well-known in this department of physical chemistry.

The text proper is introduced by a preface which takes a more literary tone than usual in the prefaces of scientific works, about a score of lines being quoted from the conversation between Mephistopheles and Wagner, rather unwisely given in the original. Goethe's German is not the simplest and there are excellent standard translations of the drama, which would be very welcome to most of the users of the book. After all perhaps the tempter's insinuations have no more force than the brief sentence of Hobbes that "Words are wise men's counters, they do but reckon by them, but they are the money of fools."

In discussing the bearing of facts upon inference, the editor remarks that "ordinary observation proves that the earth is flat." A strict critic would be inclined to say that ordinary observation *suggests* that the earth is flat, for even comparatively slight careful observation indicates the contrary. Pliny, the Elder, in his essay "That there are or may be Antipodes" instances the curved surface of the sea as an evidence of the spherical form of the earth.

It is impossible to enumerate the many essays in the volume, but one especially attracts attention, because it is contributed by a whilom Philadelphian, in his student days a pupil of the reviewer, who noted and stated at that time, that the boy was destined to occupy a prominent place in the American science. This is Dr. Edgar T. Wherry, who contributes a brief but very important essay on the advantage of stating at times reactions in terms of concentration instead of pH. Dr. Wherry has given much attention to this subject and has pointed out a number of instances in which trained workers have applied the Sörenson method wrongly, from failure to appreciate the fact that the data are reciprocal logarithms and not simple arithmetic relations. Dr. Wherry suggests a system, largely his own, by which relative hydrion-concentrations may be expressed in a simpler way than by the generally used pH method. In connection with Dr. Wherry's paper the editor adds a somewhat extended note, which explains in detail points about the Sörenson method. In this note the expression "pH value" is used, but it seems that "pH number" would be preferable.

A paper of somewhat unusual type is that of Bredig on "Inorganic Ferments." It is interesting to note that one of the earliest instances of catalytic action was reported in 1832 by Döbereiner, namely, the property of manganese dioxide to promote the decomposition of potassium chlorate. It was not until 1836 that Berzelius coined the word "catalysis" to designate such phenomena, now familiar in many phases both in pure and applied chemistry and employed in many industrial procedures.

The now only too familiar lack of contributors from Philadelphia in compendiums of research-work is exhibited in this book. Dr. Wherry is indeed originally from that city but he has for many years been an emigré. One is reminded of the satirical remark (probably of some New Yorker) that the three greatest Pennsylvanians of Colonial times were Robert Morris of England, James Wilson of Scotland, and Benjamin Franklin of Massachusetts.

A work of most extensive and comprehensive type upon a subject of the greatest importance in both theoretical and practical physical chemistry.

HENRY LEFFMANN.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 294. The Measurement of Maximum Cylinder Pressures, by Chester W. Hicks. 11 pages, illustrations, quarto. Washington, Government Printing Office, 1928.

The work presented in this report was undertaken to determine a suitable method for measuring the maximum pressures occurring in aircraft engine cylinders. The study and development of instruments for the measurement of maximum cylinder pressures has been conducted in connection with carburetor and oil engine investigations on a single cylinder aircraft-type engine. Five maximum cylinder-pressure devices have been designed, constructed, and tested, in addition to the testing of three commercial indicators.

Values of maximum cylinder pressures are given as obtained with various indicators for the same pressures and for various kinds and values of maximum cylinder pressures, produced chiefly by variation of the injection advance angle in a high-speed oil engine. It is the high pressure of short duration that is most difficult to measure, because the time of its duration is so short that little work can be done to operate an indicator.

The investigations conducted thus far indicate that the greatest accuracy in determining maximum cylinder pressures can be obtained with an electric, balanced-pressure, diaphragm or disk-type indicator so constructed as to have a diaphragm or disk of relatively large area and minimum seat width and mass.

Report No. 300. The Twenty-Foot Propeller Research Tunnel of the National Advisory Committee for Aeronautics, by Fred E. Weick and Donald H. Wood. 16 pages, illustrations, quarto. Washington, Government Printing Office, 1928, price ten cents.

This report describes in detail the new propeller research tunnel of the Committee at Langley Field, Va. This tunnel has an open jet air stream twenty feet in diameter in which velocities up to 110 M.P.H. are obtained. Although the tunnel was built primarily to make possible accurate full-scale tests on aircraft propellers, it may also be used for making aerodynamic tests on full-size fuselages, landing gears, tail surfaces, and other aircraft parts, and on model wings of large size.

DIE TECHNIK ELEKTRISCHER MESSGERÄTE. Von Dr. Ing. Georg Keinath. Dritte, vollständig umgearbeitete Auflage. Erster Band: Messgeräte und Zubehör. Zweiter Band: Messverfahren. 2 volumes, illustrations, 8vo. München & Berlin, R. Oldenbourg, 1928, price in cloth, 59.50 marks, paper 55.50 marks.

The present third edition of this work has been thoroughly revised from the text of the former editions. The general request made in former reviews for the more extended treatment of the subjects has been carried out by the author as far as possible. By reason of this enlargement it has been necessary to publish the work in two parts, the texts of which have been made to include in one volume the measuring apparatus and accessories, and in the other, procedures. A special addition to the first volume is a section on the materials for the construction of electrical measuring apparatus, comprising many of the special forms. Though

the author has gone rather fully into the construction of the apparatus it is not his intention to present a manual in that field, but only to acquaint the reader with the details of the various forms that he may be more familiar with their nature and use thus giving more information than in most of the works at hand. This perhaps involves some critical examination of the apparatus and procedures, the inclusion of which adapts the book to use in high schools and professional institutions.

In the second volume the procedures of measurement are described fully. In addition to the usual procedures for the measurement of electrical units there is included information concerning non-electrical data such as velocity, acceleration, pressure and magnetic measurement. In both volumes the general conditions for accurate determinations are carefully set forth and the source of error and uncertainty are pointed out so that the highest degree of accuracy may be secured.

In this as in former editions the author has endeavored to present a work suitable to the practicing engineer without many complicated formulas or unrelated matters such as are often found in works of this type. The book, therefore, is suited to those who wish to carry out their own measurement and have not been able to follow closely the technical literature of the subject. The work is excellently printed and liberally illustrated. It is gratifying to note the extensive quotation of British and American literature.

HENRY LEFFMANN.

INDUSTRIAL CARBON. By C. L. Mantell, Ph.D., Pratt Institute, Brooklyn, Member, American Institute of Mining and Metallurgical Engineers, American Institute of Chemical Engineers. 410 pages, illustrations, 8vo. New York, D. Van Nostrand Company, Inc., 1928. Price, $4.50.

This book is loaded with interesting and valuable information. It covers only forms of elemental carbon, or substances in which elemental carbon is the active constituent. The immense field of the carbon compounds is, therefore, untouched. Its field of application is extensive and is increasing at a rapid rate. The use of asphyxiating gases in the late war gave a great impetus to the study of the adsorptive powers of the known forms of char and the information thus gained has found valuable application in industrial operations. Carbons of very high adsorptive powers are now on the market and are employed in many ways. The methods of manufacture, properties, uses, and nature of the several forms are set forth in detail. Many illustrations add to the value of the book.

A brief chapter is given on the Diamond with illustrations of scenes at the great mines in South Africa which have so profoundly modified the economic and aesthetic relations of the gem. The curious fact is noted that the more important precious stones are so indifferent to wetting, that a mixture of them with the common crystals such as quartz, zircon, magnetite, can be carried by a current of water over a greasy cloth while the precious stones will be caught in the grease. This property is utilized with much success in separating diamonds from the inferior stones.

Graphite next claims attention, both the natural and artificial forms being described and their properties and uses set forth.

As might be expected the larger portion of the book is devoted to carbon in the amorphous form as obtained from animal and vegetable materials. The methods

of activating these chars and the principles on which this activation is based are given full attention. A brief chapter on charcoal as a fuel is included, but the fuel uses of carbon are wholly outside the scope of the book. The different forms of amorphous carbon have found an astonishing number and variety of uses, such as inks, paints, and in producing a hard exterior on iron. The latter procedure is very old, but has found much employment of late. The most remarkable chapter in the history of carbon black is its extensive use during the last ten years in the rubber trade. After long research, chemists of the rubber companies found that carbon black will make tires tougher and more serviceable. The use began tentatively with some American manufacturers but is now world-wide. Carbon black is also used in many other rubber compositions in which resistance to wear is especially important such as hose, flooring material and surgical rubber articles. Lampblack also receives full attention in the book.

So many and so varied are the uses of elemental carbon as set forth in this work that it reads like one of the familiar popular introductions to science, but it is strictly scientific, though the editor indulges in a little rhetoric in the preface. In some cases careless phraseology has been noted. On p. 138 it is stated that "in the form of wood ashes," carbon has been used as an eliminant, and on p. 192 it is stated that charcoal is obtained by "burning wood in a closed vessel." These slips are of minor moment and do not detract from the high practical value and excellent character of the book.

HENRY LEFFMANN.

THE FUNDAMENTALS OF QUANTITATIVE ANALYSIS. Third edition. By Walter C. Blasdale, Ph.D., Professor of Chemistry in the University of California at Los Angeles. 403 pages, illustrations, 8vo. New York, D. Van Nostrand Company, Inc., 1928. Price, $3.75.

The first and second editions of this work appeared under the title "Principles of Quantitative Analysis." The text has largely been rewritten and this revision is regarded by the author as justifying the change of title. It does not seem that anything of value to the user of the book has been obtained by this change, for the primary object of the present edition is the same as that of the early ones, namely to give in some detail the features of a series of exercises in quantitative analysis, presenting the usual material arranged for the year's course of students who have had at least one year's work in general chemistry. There is a very large number of such books now available in this country. The time seems to be near at hand for the application of the principles of standardization such as are so much in vogue in many of our industries, indeed we might also hope for a development of the "merger spirit" which is affecting many of our financial and public utility corporations.

Chemistry is an exact science and while in analysis there may be several good methods for certain purposes there is generally a best one and would seem that this should be used to the exclusion of the others unless there was some very serious reason to the contrary. In this edition a section of the earlier editions dealing with the use of physio-chemical processes has been eliminated being considered of sufficient importance to be treated in a separate volume, and because the author has found the treatment of them within the year-period not satisfactory. Questions are given in connection with the various topics but it seems to the reviewer

that these might be best left to the suggestion of the teacher. There is a very large amount of valuable information contained in the book.

HENRY LEFFMANN.

HANDBUCH DER EXPERIMENTALPHYSIK. Band 13, 2. Teil. Physik der Glühelektroden, by Prof. Dr. W. Schottky and Dr. Ing. H. Rothe, 55 illustrations; Herstellung der Glühelektroden, by Dr. Rer. Techn. H. Simon, 36 illustrations; Technische Elektronenröhren und ihre Verwendung, by Dr. Ing. H. Rothe, 88 illustrations. 492 pages. Edited by W. Wien and F. Harms with the collaboration of H. Lenz. Leipzig, Akademische Verlagsgesellschaft M. B. H. 1928.

This volume which is one of the series comprising the "Handbuch der Experimentalphysik" (Handbook of Experimental Physics) is typical of the extensive and thorough treatment accorded the subjects of its contemporary volumes. The subject matter has been divided into three parts. Part one, The Physics of Incandescent Electrodes is presented in an able and comprehensive manner by W. Schottky and H. Rothe. The authors have sought to treat the problem of the emission of electrically charged particles from hot bodies in the light of thermodynamical considerations rather than statistical. The term. "μ-thermodynamics" is employed to designate the application of the regular thermodynamic equations to the characteristics of individual electrified particles, Particular attention is paid to the correlation between theoretically calculated values and those obtained experimentally. In this connection the investigation of thermionic emissions by Langmuir and associates received particular attention.

Part two, on the "Preparation of Incandescent Electrodes," written by H. Simon, has a preliminary study upon the characteristic emissions of polished metals, metals coated with other metals possessing a high emissivity and metals having a coating of metallic oxide. Methods for producing a high vacuum, measuring the same, determining temperature and preparing a glowing cathode tube are treated all with considerable detail.

Part three, by H. Rothe, on Technical Electronic Tubes and their Applications, embraces a discussion upon the present day widespread use of vacuum tubes as amplifiers, oscillators and detectors. Characteristic curves and equations are given indicating the performance of various types of tubes and their seemingly infinite variety of hookups.

The book is an excellent treatise and a general work of reference upon the subjects in question and the authors are to be commended for their compilation and critical treatment of the subject matter presented therein.

T. K. CLEVELAND.

VIBRATION PROBLEMS IN ENGINEERING. By S. Timoshenko, Professor of Engineering Mechanics, University of Michigan. vi–351 pages, illustrations, 8vo, cloth. New York, D. Van Nostrand Company, Inc., 1928. Price, $4.50.

Among the numerous specialties of modern engineering practice, the study of the vibration of machines and of structures which carry moving loads, and problems of kindred nature may justly claim to have achieved a position of

recognized importance. In the past, vibrations of machines, and their more or less injurious and sometimes decidedly unpleasant effects upon the immediate surroundings were more often endured than cured. Now, they are more often cured or prevented by the application of century-old principles which have only needed the quickening touch of modern vision. A member of this Institute, Mr. N. W. Akimoff, one of the initiators of the modern practice of vibration engineering, during the trying months of our entry into the Great War, by the application of these principles, it is related, won the distinction of putting into service a much-needed naval vessel which had proved quite useless for its intended purpose on account of destructive vibration at high speeds.

Professor Timoshenko develops fully the theory of vibrations, its formulation and its application to the solution of technical problems. This he illustrates by various examples, in many cases, taken from actual experience with vibration of machines and machines in service. The subject-matter is based upon a series of lectures on vibration which he delivered before the mechanical engineers of the Westinghouse Electric and Manufacturing Company during the year 1925. The treatment is rational and deductive in first dealing with general equation of motion and then passing to specific cases. The list of topics is long and representative, and their treatment thorough and practical.

LUCIEN E. PICOLET.

ENZYME ACTIONS AND PROPERTIES. By Ernst Waldschmidt-Leitz, Institut für Biochemie, Deutsche Technische Hochschule, Prag., formerly at the University of Munich. Translated and extended by Robert P. Walton, Department of Organic Chemistry, Columbia University. 255 pages, illustrations, 8vo. New York, John Wiley & Sons, Inc., 1929. Price, $4.

The nature and mode of action of enzyms are among the most interesting and difficult problems of organic chemistry. The subject is especially associated with bio-chemistry, since a large group of enzyms are produced only in living tissues. The present work is limited to this group, but the enzymist must not overlook the part played by inorganic bodies in actions essentially similar in result to those produced by those of organized origin. Broadly speaking, all substances that affect the speed or products of a reaction without being themselves changed may be classified as catalysts, and there is no necessity for culling out the organic phenomena and designating them by a special name. It is the province of science to connect objects and phenomena that are analogous as well as to distinguish them when the analogy is only apparent.

The present work is a translation from German with considerable additions. The original is not available to the reviewer, and no judgment can be made as to the general accuracy of the translation nor as to the extent of the additions, but as the translator has been closely associated with the author and has had the latter's direct assistance during the preparation of the book, it is evident that the original is correctly represented. In the course of reading, a single inadvertence has been noted, the adjectives "formed" and "unformed" are used to designate the old distinctions between "organized" and "unorganized" ferments, and the former terms are given as the accepted English terms, but they were not so used.

The work covers a wide field within the specific limits assigned to it, that is, the enzyms producing changes in organic substances. Modern research, partly

encouraged by an earnest search for facts and partly by economic impulse has brought to our knowledge a very large number of substances that can be classified as catalysts, a notable portion of which are operative only in the inorganic field. As far as possible the reactions which are brought about by the action of enzyms on organic substances are set forth fully in this book, and the details of molecular changes also when known. It does not appear clear to the reviewer why the reaction of urease on urea should be described as resulting in tetrahydroxymethane and ammonia, when the reaction can be more simply and satisfactorily expressed as a conversion of the urea into ammonium carbonate which then ionizes under standard conditions.

Some interesting and important information is given concerning Pasteur's work in contrast to that of Buchner. There has been some sharp discussion lately on this subject, it being claimed that Buchner has been badly treated by those who are presenting Pasteur's work to the world. It seems established that Buchner first introduced the work "zymase" and gave it the proper signification.

It is an advantage to American chemists to have this book in their own language. The world owes much to Germans, and chemistry is especially under obligations, but while those of us who are familiar with their language, appreciate its merits and enjoy its vast literature, there is no doubt that it is a difficult tongue to all who are not "native and to the manner born."

Science has enjoyed about a decade of peaceful internationalism in which all workers are coöperating for the discovery of truth and its applications to the benefit of mankind, but were Cassandra alive to-day she could say:

"Eine Fackel seh' ich gluehen,
Aber nicht in Hymen's Hand;
Nach dem Wolken seh' ich's ziehen,
Aber nicht wie Opferbrand."

"Coming events cast their shadows before" and the dark shadow of war lies over the laboratories.

HENRY LEFFMANN.

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Colloid Chemistry Theoretical and Applied, by selected International Contributors. Collected and edited by Jerome Alexander. Vol. II Biology and medicine. 1029 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1928, price $15.50.

Memoirs and Addresses of Two Decades, by Dr. J. A. L. Waddell, Consulting Engineer. Edited by Frank W. Skinner, Consulting Engineer. First edition. 1174 pages, illustrations, 8vo. Easton, Penna., Mack Printing Company, 1928.

Die Technik Elektrischer Messgeräte, von Dr. Ing. Georg Keinath. Erster band, Messgeräte und Zubehör; zweiter band, Messverfahren. 2 vols, illustrations, 8vo. München, R. Oldenbourg 1928, price, bound, 59.50 marks.

Le Temps Absolu et l'Espace a Quatre Dimensions (La gravitation—La masse—La lumière), par Emile Sevin. 127 pages, illustrations, 8vo. Paris, Albert Blanchard, 1928, price 22 marks.

National Advisory Committee for Aeronautics. Fourteenth annual report, 1928. Administrative report without technical reports. 80 pages, quarto. Washington, Government Printing Office, 1928, price twenty cents.

Deforested America. Major George P. Ahern. Foreword by Gifford Pinchot. 79 pages, 12mo. Washington, D. C., 1928, price twenty cents.

Introduction a la Physique des Rayons X et Gamma, par MM. Maurice de Broglie et Louis de Broglie. 201 pages, illustrations, 8vo. Paris, Gauthier—Villars et Cie, 1928, price 50 francs.

The Spectroscopy of the Extreme Ultra-Violet, by Theodore Lyman, Ph.D. Second edition. 160 pages, illustrations, 8vo. London and New York, Longmans, Green & Company, 1928, price $3.75.

CURRENT TOPICS.

The Green Flash at Sunset. S. J. BARNETT. (*Nature*, August 4, 1928). For decades observers have published in *Nature* their descriptions of this supposedly rare sky phenomenon. Now Professor Barnett dispels the illusion of its rarity, at least in California, saying: "Here in Southern California in the last two years I have seen the flash many times as the sun has set over the Pacific Ocean, or over the Santa Monica Mountains a few miles west of our residence. At least once I have seen the color of the flash change distinctly from green to blue or blue-green before disappearance."

G. F. S.

The Critical Point of Monel Metal. L. F. BATES and R. C. BROWN. (*Nature*, Aug. 18, 1928.) Monel, metal made from a natural ore found in Ontario, is an alloy of 67 per cent. nickel, 28 per cent. copper and 5 per cent. of other metals. It shows great resistance to corrosion. While it is ferromagnetic its critical point is unusually low. "It may conveniently be used for a laboratory experiment to illustrate the loss of ferromagnetism with rise of temperature. The magnetic critical temperature varies from specimen to specimen, and is stated to lie between 100° and 150°. This appears to apply to specimens in the form of the stout bars supplied commercially, but specimens in the form of anchor rings of diameter 10 cm. and 1 cm. thickness supplied to us possess critical temperatures of 70° C."

G. F. S.

The Stark Effect at Very High Field. YOSHIO ISHIDA. (*Nature*, Aug. 25, 1928.) Ishida has attained an electrical field of 650,000 volts per cm. and believes that the limit has not yet been reached. A quartz tube of the Lo Surdo type was used. The excitation was furnished by a transformer with a synchronous motor rectifier having its vanes set for the most favorable phase.

With this apparatus certain lines of the helium and lithium spectra were affected that have been hitherto obdurate to the electrical field.

G. F. S.

The Waves of an Electron. GEORGE P. THOMSON. (*Nature*, Aug. 25, 1928.) In the science of physics it is not common for

both a father and a son to be investigators. In the United States we have Henry Crew and his son while in Great Britain the two Rayleighs come to mind. At the present time there are also Sir J. J. Thomson and his son G. P. Thomson, of the University of Aberdeen, who delivered at the Royal Institution, June 8, 1928, the lecture of which this article is a report. Some account of the same matter was published in the JOURNAL OF THE FRANKLIN INSTITUTE, Sept., 1928, page 417.

At the beginning of the present century the wave theory of light was regarded as impregnably established. It had driven the corpuscular theory not only from the field but even into contempt. All the evidence of interference and diffraction was in favor of the wave theory. Then new phenomena clamored for recognition. In the field of photoelectricity it was found "that light could communicate an amount of energy to an electron which only depended on the kind of light and not at all on the intensity. If light consists of continuous waves this seems almost impossible; but it is just what we should expect if, as Newton firmly held, light is due to a stream of corpuscles. In such a case, if one of these hit an electron, it would give to it an energy which would be quite independent of the number of other corpuscles present in the stream, that is, of the intensity of the light. Louis de Broglie has hit on a brilliant synthesis of the two views. Put briefly, it comes to saying that light is indeed corpuscular, but that the corpuscles are accompanied and guided by waves." One of de Broglie's results is that the wave-length of the waves escorting any particle equals Planck's constant $h(6.57 \times 10^{-27})$ divided by the momentum of the particle. By this relation the wave-length of an electron with the energy of 150 volts is 10^{-8} cm., about that of a soft X-ray. Another deduction from de Broglie's theory is that the square of the velocity of the waves equals the square of the velocity of light increased by a quantity proportional to the square of the lengths of the accompanying waves. Thus the velocity of the waves varies with their length, i.e., there is dispersion even in a vacuum.

The predictions of de Broglie's theory were confirmed by the experimental results of Davisson and Germer in the United States ("Are Electrons Waves?" this JOURNAL, May, '28). They projected a stream of electrons upon the face of a nickel crystal and observed that they are reflected much as a beam of light is reflected. Thompson, in turn, undertook the study of the possible diffraction of cathode rays. "De Broglie's theory is quite a general one, and should apply to any sort of corpuscular motion." He, working with Reid, let cathode rays impinge on very thin films of metals and celluloid. The rays were received on a photographic p ate after

they had been diffracted. The measurement of the images there formed made possible a comparison of theory and result. From the size of the rings on the plate it was calculated that the wave-length was really inversely proportional to the momentum, as by theory it should be. Diffraction of cathode rays by mica also is reported by Nishikawa and Kikuchi, Tokyo, in *Nature*, June 30, '28. They obtained a pattern consisting of "three sets of parallel bands intersecting each other at an angle of 60°, forming a net of triangular mesh, of which the net points are arranged into an array of spots." A change in the applied voltage caused a change in the distribution of the spots. Since cathode rays are deflected by a magnetic field, the application of such a field to the path of the rays shifts the photographic impression.

"What are these waves? Are they another name for the electron itself? How many waves in series does an electron have? Here we have a little direct evidence. The sharpness of the rings depends, among other things, on this length. A short train of waves results in blurred rings. I calculate that there must be at least 48 ordered waves in the series."

"As regards the relation between electron waves and light waves: they are certainly not the same, even when their wave-lengths are equal. They go at different speeds: one is refracted by electric and magnetic fields, while the other is not; their penetrating powers in ordinary matter are widely different. If they are the actual motion of an ether, it must differ in some way in the two cases.

"The easiest way of looking at the whole thing seems to be to regard the waves as an expression of the laws of motion. The uniform motion of Newton's first law is replaced by a simple plane wave, and so on. On this view, the electron remains the reality, and this seems right, for after all it is the electron as a particle which is actually detected in any practicable experiment, and the waves come to bear the same sort of relation to it that Newton's or Einstein's law of gravitation bears to the planets which obey it."

G. F. S.

The Constitution of Zinc. F. W. ASTON. (*Nature*, Sept. 8, 1928.) Dempster concluded that zinc is a complex element having three strong isotopes 64, 66 and 68 and one weak one, 70. There was also a suggestion of 67. Aston now gets results by using zinc methyl agreeing in general with Dempster's conclusions. "The mass-spectra indicate that zinc consists of seven isotopes 64(*a*), 65(*e*), 66(*b*), 67(*d*), 68(*c*), 69(*g*), 70(*f*). The letters in brackets indicate the order of intensity. Three of these mass-numbers, 65, 69, 70, are isobaric with those of copper, gallium, and germanium respectively."

G. F. S.

Some Recent Work on the Light of the Night Sky. LORD RAYLEIGH. (*Nature*, Sept. 1 and 8, '28.) The light of the night sky with the sun 18° below the horizon and the moon not yet up has little resemblance to the light of the sky by day. Light from the night sky shows very little polarization and is so faint as to be below the threshold of color vision so that ascriptions of color are illusory. By the use of colored glasses it has been found that the day sky is much richer in blue than the night sky.

When a spectroscope resolves the light of the night sky the eye sees the green auroral line of wave-length 5577 Å. against a background of a seemingly continuous spectrum. Some observers can always see this line but Lord Rayleigh can see it only sometimes while others never are able to perceive it. Photographic examination of the light is reduced in value by the very long exposures requisite with such faint illumination. The green line can be photographed by an exposure of one night but several nights are needed to get records of certain emission and absorption lines. "An exposure of many nights on an Ilford panchromatic plate failed to show anything in the red region of the spectrum. It is noteworthy that the negative bands of nitrogen, which are the most important features in the photographs of the auroral spectrum, are not present in the night sky."

The difficulties presented by the application of spectroscopy and photography to the problem of observation have caused Lord Rayleigh to turn to photometry. To analyze the light of the night sky three color filters were used. One transmitted almost no light but that of the green line, a second transmitted the light falling in the spectrum between this line and the red limit while the third let the light pass that lies to the short wave-length side of the same line. Self luminous crystals of potassium-uranyl sulphate furnished the standard light for comparison. The radioactivity of the uranium is the cause of the fluorescence of the salt which in a sealed vessel is constant for years.

With a Lummer cube the light of the night sky was matched with the standard, neutral glasses being used to reduce the intensity of either component light until a match was obtained. An unfortunate typographical error confuses the understanding of the tabulated results of observations. Observations were made at about fifteen stations in Europe, North and South America, Asia, Africa and Australia as well as on two voyages. "Fairly normal values can be stated for the intensity of each component at any part of the world." The normal value for the auroral light lies in intensity between those for blue and red but much nearer to that of the red. "There is a strong correlation between the red and

the auroral intensities on any given occasions, and a rather less strong but still marked correlation between these and the blue. It is, however, definitely established that this correlation is not complete."

When the intensity of the auroral light is studied from month to month it is seen that in general a marked fall occurs from October to December. There is partial recovery with a secondary maximum in March. The main annual maximum probably would be found not far from July but residual daylight interferes with observations at that time of year. The intensity in August is about 1.6 that in April. 1923 was a year of sunspot minimum while 1928 should have a maximum. During this five year interval there has been a general increase in the brightness of the night sky.

The author compares the polar aurora limited to zones with the non-polar aurora observable all over the earth. Both have the green line 5577 Å. due to oxygen. The polar aurora alone shows the negative nitrogen bands, the forms of arcs and draperies and rapid changes of intensity.

A better way of studying night light with a photoelectric cell is under development. With it some hints of diurnal periodicity in the intensity have appeared. G. F. S.

The Newly Announced Raman Effect. (See this JOURNAL for August, 1928, page 276.) *Nature*, Sept. 8, '28, contains the following comment by R. W. Wood: "Professor Raman's brilliant and surprising discovery that transparent substances illuminated by very intense monochromatic light scatter radiations of modified wave-length, and that frequency difference between emitted radiation and one exciting medium is identical with frequency of infra-red absorption bands, opens up wholly new field in study of molecular structure. I have verified his discovery in every particular, using improved apparatus which makes it possible to photograph strongest lines in few minutes." The conciseness of expression is due to transmission by cable. Like Raman Professor Wood found in the scattered radiation not only wave-lengths longer than those of the exciting radiation but also shorter wave-lengths, "anti-Stokes' terms" he calls them. In the photograph of the spectrum of the light scattered by benzene illuminated by the 4358 Å. group of lines of the mercury arc, published by Raman and Krishnan, *Nature*, July 7, '28, two such modified lines of shorter wave-length are seen. "Measurements show that their frequencies exceed that of the latter (the exciting 4358 line) by the infra-red frequencies of the molecule." They interpret the occurrence of these lines of shorter wave-length as furnishing experi-

mental confirmation of the process of the negative absorption of energy predicated by Einstein in his derivation of Planck's radiation formula and also employed in the Kramers-Heisenberg theory of dispersion. "The presence of these lines proves simultaneously the existence in the liquid of molecules at levels of energy correspondingly higher than the normal, and the fact that the incident radiation induces a return to a lower state of energy; in other words that there is a negative absorption of the radiation." This interpretation is sustained by Ruark, *Nature*, Sept. 1, '28, but disagreement is expressed by Saha, Kothari and Toshniwal, *Nature*, Sept. 15, '28, who say: "We wish to point out that Raman and Krishnan's experiment is not, strictly speaking, a confirmation of Einstein's negative or stimulated emission, as mentioned by these authors, but of the analogous case of negatively modified scattering discussed by Smekal." In 1923 Smekal, basing his conclusion on the considerations advanced by Einstein and Ehrerfest, pointed out that three different wave-lengths of light should be scattered from an assembly of particles on which falls radiation of a single wave-length. These three kinds are (*a*) light of the same frequency as the incident light, (*b*) light whose frequency equals the incident frequency increased by a certain frequency, and (*c*) light whose frequency equals the incident frequency diminished by the same frequency as is added in (*b*). This added or subtracted frequency is calculated by dividing h, Planck's constant, into the difference between the energies of the scattering particles in the excited and the normal states.

Raman and Krishnan (*Nature*, Aug. 25, 1928) state: "The appearance in the spectrum of monochromatic light diffused by fluids, of new lines of modified frequency, gives us a powerful, accurate and convenient method of exploring molecular spectra, especially in the near and extreme infra-red regions. We have only to photograph the spectrum of the scattered light, and the frequency-differences between the incident light and the new radiations excited by it give us the molecular frequencies." Wood in the article quoted above adds "Raman's discovery thus makes possible investigations in remote infra-red regions hitherto little explored owing to experimental difficulties. It appears to me that this very beautiful discovery, which resulted from Raman's long and patient study of phenomena of light scattering, is one of the most convincing proofs of the quantum theory of light which we have at the present time."

Ruark (loc. cit.) calls for a quantitative proof that the Raman effect is not merely fluorescence. This question could be settled by finding whether any interval of time elapses between the recep-

tion of the incident light and the emission of the scattered radiation. The interval in the case of rhodamine is 2×10^{-8} secs.

Ramdas (*Nature*, July 14, '28) reports that he has obtained the Raman effect in the vapor of ether. With the plates and apparatus used in getting this result he photographed the spectrum of the zodiacal light with an exposure of less than an hour. Saha, Kothari and Toshniwal (loc. cit.) in considering the emission in the scattered light of wave-lengths shorter than that of the incident light raise the question of the proper application of the word 'scattering' to the phenomenon. In scattering by fog particles the process causes no change in the particles. In absorption of radiation the energy absorbed produces a change in the configuration of the atom by separating an electron from the nucleus. "This phenomenon (the emission of waves of higher frequencies) is just intermediate between the two, as the incident light reacts on the particle, and robs it of its internal energy and is re-emitted as a new radiation of increased frequency."

International Conference on Bituminous Coal, 1928.—Twelve major topics for discussion at the Second International Conference on Bituminous Coal to be held under the auspices of the Carnegie Institute of Technology, Pittsburgh, Pa., November 19–24, 1928, are tentatively announced.

Although the Second International Conference will be similar in purpose to the first Congress held in 1926, preliminary plans for this year's meeting show that its scope will be considerably enlarged and the program will be more important and more international in character. The discussion of Fixed Nitrogen is one of the topics which will receive close attention. The liquefaction of coal, which was one of the principal subjects of discussion at the first meeting, will again occupy a prominent place in the deliberations. Low temperature distillation will be treated by representatives of at least a half dozen countries. High temperature distillation, power from coal, coal tars and oils, complete gasification of coal, origin of coal, coal washing, pulverized coal, catalysts, and the general aspects of the bituminous coal industry are other topics that will be considered.

The Economics of the Coal Industry will be discussed by Lord Melchett. Dr. Friedrich Bergius, Professor Dr. Franz Fischer, Germany; Andre Kling, and E. Audibert, France; Engineer Guardabassi, Italy; will deal with the subject of the liquefaction of coal.

Low Temperature Distillation will be treated by George E. K. Blythe, Dr. C. H. Lander, Harald Nielsen, Dr. E. W. Smith, England; Dr. A. Herz, Joseph Plassmann, Professor F. P. Kersch-

baum, Germany; Henri Lafond, Professor Paul Lebeau, Antonie Vonk, A. Leaute, France; Professor Dr. Granigg, Austria; Professor Yoskikiyo Oshima, Japan; Professor Samuel W. Parr, Professor Alfred H. White, and F. C. Greene, United States.

High Temperature Distillation: Jean Bing, France; Professor Ernest Terres, Germany; Edgar C. Evans, England.

Power from Coal: Dr. J. E. Noeggerath, Germany; C. Simon, France; A. T. Stuart, Canada; W. B. Chapman and Professor A. G. Christie, United States.

Coal Tars and Oils: Henri Winckler, France; Dr. L. Edeleanu and Professor Dr. Fritz Frank, Germany; and Gustaf Egloff, United States.

Gasification of Coal: Dr. Karl Bunte and Dr. Alfred Pott, Germany; Paul Weiss, France.

Origin of Coal: Professor George L. Stadnikoff, Russia; Dr. Chozo Iwasaki, Japan; Dr. Reinhardt Thiessen and Professor E. C. Jeffrey, United States.

Fixed Nitrogen: A representative of L'Air Liquide Societe, France; Rudolf Battig, Germany; Professor Harry A. Curtis, Louis C. Jones, and Charles J. Brand, United States.

Coal Washing: A. France, Belgium; Professor Dr. Glinz, Germany; Dr. R. Lessing, England; F. R. Wadleigh, Dr. F. W. Sperr, James B. Morrow, and Byron Bird, United States.

Pulverized Coal: Dr. I. P. Goosens, Rudolph Pawlikowski, and Dr. P. Rosin, Germany; C. J. Jefferson, United States.

Catalysts: Professor A. Mailhe, France; Professor Hugh S. Taylor, United States.

Rubber from Coal: Professor Fritz Hoffmann, Germany.

The list of speakers will be considerably enlarged during the coming months, and the final program details will be announced in October.

There were nearly 1,800 delegates at the Conference in 1926 and it is expected that the meeting of 1928 will be more largely attended.

Moisture Content of Oleomargarine. ROBERT H. KERR of the Bureau of Animal Industry, U. S. Department of Agriculture (*Jour. Asso. Official Agric. Chemists*, 1928, *11*, 382–386) reports the results of the analysis of 159 samples of oleomargarine, representing 100 different brands, produced by 37 establishments located in 18 cities of the United States. In most of the samples, the moisture content was above 8 per cent. and below 14 per cent. Three types of oleomargarine were examined. The first type included 98 samples and represented 46 brands manufactured in part from the body fats of meat animals, and intended for general use as

butter substitutes. The second type included 45 samples and represented 39 brands manufactured from the same raw materials as those in the preceding class, but intended for use in cooking and baking. The third type included 16 samples and represented 15 brands of so-called vegetable oleomargarine or nut oleomargarine which is made by churning vegetable fats with skimmed milk and contains no fat derived from the carcasses of meat animals. The oleomargarines of the second type usually contained less moisture than those of the other types.

J. S. H.

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Journal
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Vol. 207 MARCH, 1929 No. 3

PROBE AND RADIATION MEASUREMENTS IN THE COPPER ARC.*

BY

W. B. NOTTINGHAM, B.S., M.A.,

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BARTOL RESEARCH Foundation
Communication No. 34.

Abstract. The Langmuir probe has been used to explore the copper arc. The cathode fall of the normal arc has been found to be 20.5 volts, which equals the second ionization potential of copper Cu (II), while the cathode fall for the hissing arc has been found to be 14.0 volts. The potential gradient along the arc stream is constant and therefore the space charge is zero and the concentrations of positive ions and electrons are equal. The average energy of the electrons throughout the arc is between 2.1 and 2.7 volts. The space potential, when measured with reference to the cathode, depends on the current flowing in the arc in the same way as the over all potential, i.e., $V = A + \frac{B}{I^n}$ where A, B, and n are constants. The positive ion concentration in the arc stream increases as the arc current increases according to the equation $n_+ = b\epsilon^{aI}$ where a and b are constants. The light per unit volume has been measured for twelve copper arc spectrum lines and found to

* Presented to the Graduate Committee of Purdue University as the thesis for the graduate degree of Electrical Engineer.

(Note.—The Franklin Institute is not responsible for the statements and opinions advanced by contributors to the JOURNAL.)

follow the same sort of equation as above, i.e., $I = ce^{paI}$, where c is a constant, a the constant appearing above, and p takes on values 2, 3, 4, etc. Eliminating the current between these two equations, it is shown that the light intensities in lines λ3248, λ3274, and λ5105 of the copper arc are proportional to the *square* of the electron concentration while the intensities of λ2618 and others are proportional to the *cube* of the concentration, etc. Thus it is demonstrated that the relative intensities of the lines in the arc spectrum are not fixed but depend in a definite manner on the arc current flowing since the excitation is produced as a result of the cumulative action of the electrons in which 2, 3, 4 or more electrons take part.

INTRODUCTION.

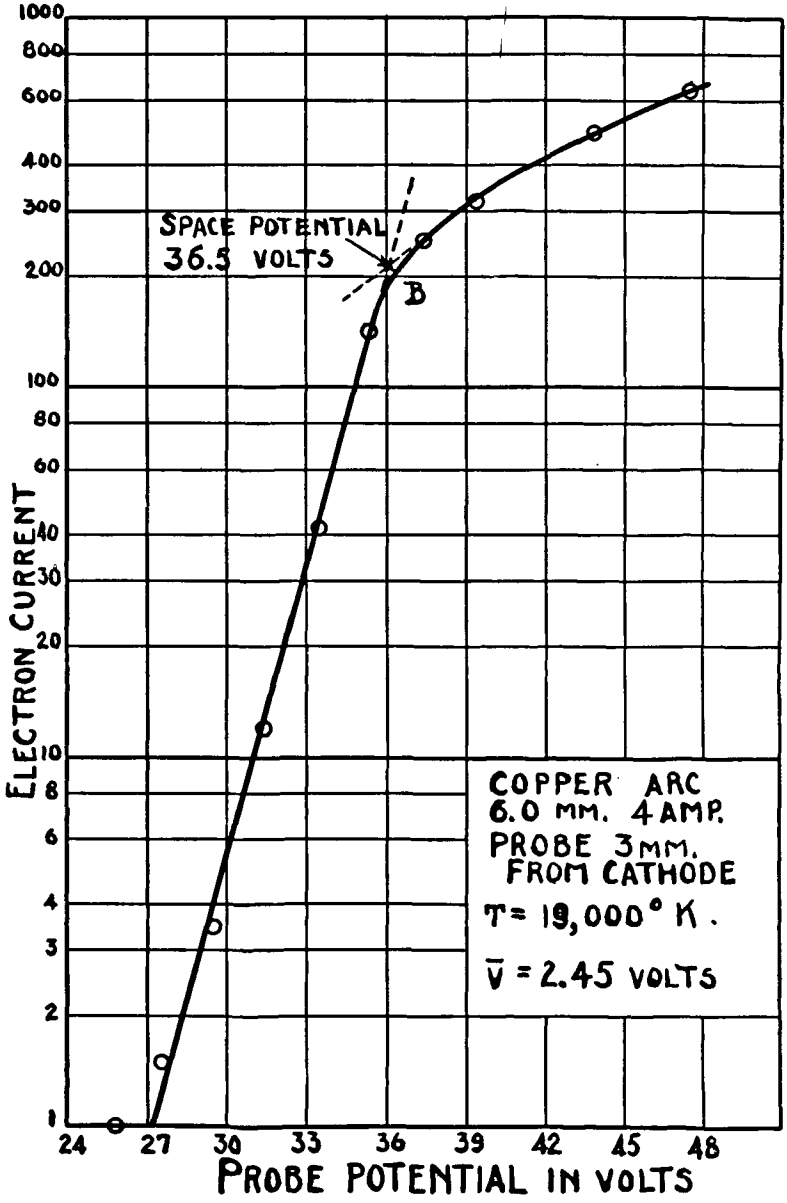
In a recent issue of this JOURNAL, a report was made on the use of the Langmuir probe for the study of the normal electric arc.[1] Those methods have been used to study the normal arc between copper electrodes in order to find (1) the average energy of the electrons in this arc; (2) the distribution of potential along the arc stream; and (3) the positive ion concentration in the central portion of the arc with different currents flowing. A method was worked out by which the variation in the intensity of certain of the lines of the copper spectrum could be measured when the arc current was changed. These results combined with those of the probe measurements have revealed many interesting facts concerning the detailed mechanism of the copper arc which may possibly be common to other arcs between metals at atmospheric pressure.

PROBE MEASUREMENTS.

The experimental procedure may be recalled to mind as follows: a copper wire probe 0.01 inch in diameter is moved through the arc stream at practically a constant velocity of about 60 cm. per second, while the potential of this probe is maintained at a definite value with reference to the negative electrode by means of a low resistance battery. The current which flows through the probe circuit during the passage of the probe through the arc stream is registered on a ballistic galvanometer. When the probe is very negative with respect

[1] JOURNAL OF THE FRANKLIN INSTITUTE, 206, 43 (1928).

FIG. 1.



Semi-logarithmic plot of electron current.

to the space in the arc through which it travels only positive ions strike the probe but when the probe is only slightly negative with respect to the space, electrons with high kinetic energy are able to reach the probe against this adverse negative potential. Thus by changing the probe potential in small steps the energy distribution of the electrons can be determined. In case the electrons have a Maxwellian distribution the corresponding temperature and average energy can be determined from the slope of the straight line part of the curve obtained by plotting the logarithm of the electron current (i_-) against the probe potential.[2] A typical example of such a curve is shown in Fig. 1. From the Boltzman equation

$$i_- = a\epsilon^{eV/kT} \tag{1}$$

we see that

$$\log_\epsilon i_- = \frac{e}{kT} V + \log_\epsilon a. \tag{2}$$

Thus with the slope of the straight line in Fig. 1 equal to $\frac{e}{kT}$, the temperature can be calculated ($e = 4.774 \times 10^{-10}$ e.s.u. the electronic charge; and $k = 1.372 \times 10^{-16}$ ergs per degree, Boltzman's constant). The potential of the space is given by the point B just above the termination of the straight line. The average energy of the electrons in volts $\bar{v}$ corresponding to an observed temperature (T) is given by the equation

$$\frac{e\bar{v}}{300} = \frac{3}{2}kT, \tag{3}$$

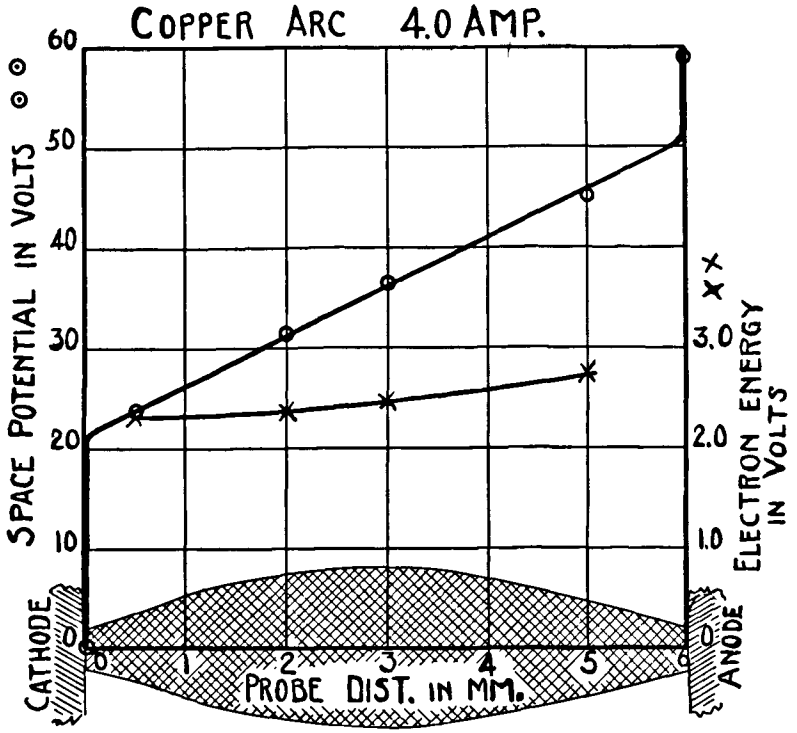
$$\bar{v} = \frac{T}{7,732}. \tag{4}$$

PROBE MEASUREMENTS AT POINTS ALONG THE ARC STREAM.

The potential of the space and the average energy of the electrons have been determined at a number of places in a copper arc 6 mm. in length and carrying a current of 4.0 amperes. The results are summarized by means of curves in Fig. 2.

[2] For the complete theory of the probe see Langmuir and Mott-Smith, *G. E. Rev.*, **27**, 449, 538, 616, 762, 810 (1924).

FIG. 2.



Distribution of potential and electronic energy along the copper arc.

Cathode fall: The most important fact brought out by these results is that the potential drop in the immediate neighborhood of the cathode is very close to 20.5 volts. This observation is corrobrated spectroscopically by the fact that the entire spark spectrum of copper can be observed at the surface of the cathode in the normal copper arc if the image of the arc is carefully focused on the slit of the spectroscope.[3] Fig. 3 serves to prove this point.

Over the range of wave-length λ2330 to λ2850 every one of the 35 lines listed by Shenstone[4] can be seen easily on the negative. In arcs previously studied, the cathode fall has been found to be practically equal to the ionization potential of

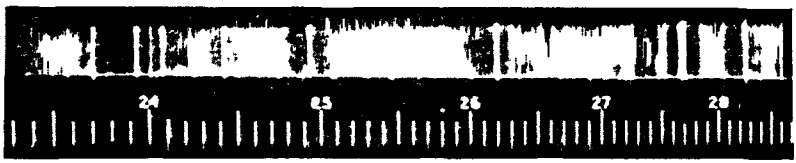
[3] Observed also by Hagenbach, *Phy. Zs.*, **12**, 1015 (1911).

[4] Shenstone, *Phys. Rev.*, **29**, 380 (1927).

the electrode material (the cyanogen molecule for the carbon arc) but in this case we have a cathode fall exactly equal to the second ionization potential of the copper atom Cu (II) of 20.5 volts,[4] that is the potential required to remove an electron from a singly charged copper ion. Thus the presence of a large number of doubly charged atoms at the surface of the cathode may be responsible for the reduction of the work function to such a low value as to make the observed current

FIG. 3.

Anode



Cathode

Spectrum plate showing copper spark lines at the surface of the cathode.

densities of about 3,000 amperes per square centimeter possible even at a cathode temperature of 2500° K. or less. Two forms of the copper arc in air can be observed when the arc current is within the range 2.0 to 8.0 amperes and the cathode sufficiently large. The one taken on by the arc the greater part of the time has been considered as the "normal" arc and is the one which has received the greater attention. The second form, some times called the "hissing arc," is characterized by greater luminosity in the core and also in the red flame sheath which surrounds the arc while the over all voltage between the electrodes is 7.0 to 8.0 volts less than that required to operate the "normal" arc. If the cathode is only two or three millimeters in diameter the hissing arc is usually the only form found. The fact that the greater part of this change in voltage takes place in the cathode fall has been demonstrated by probe measurements made by Hagenbach [2] in which a high resistance voltmeter was used to measure the potential of a probe immersed in the discharge. Langmuir has shown that the voltmeter does not show the potential of the space but shows the potential at which equal numbers of positive and negative ions reach the probe. The cathode fall

in the hissing arc was measured using the Langmuir method and found to be about 14.0 volts which is 6.5 volts less than that in the normal arc. If the cathode fall in the hissing arc is to correspond to the ionization of the active vapor it must be associated with that of copper oxide or cuprous oxide (Cu_2O) for it is much too high to correspond to the first ionization potential of copper which is 7.7 volts. Since copper has one valence electron in this compound, two copper atoms and one oxygen atom should give a molecule with eight electrons in the outer shell and an ionization potential not far from krypton at 13.3 volts. As further evidence that the cathode fall of the hissing arc is to be associated with the oxide we can take Hagenbach's observation that only the normal form of the arc can be produced in an atmosphere of nitrogen using clean copper electrodes.[5]

The arc stream: The potential gradient along the arc stream from the boundary of the cathode fall to that of the anode fall is practically constant. Since Poisson's equation states that

$$\frac{d^2V}{dx^2} = -4\pi\rho, \tag{5}$$

we see that the space charge (ρ) is zero and therefore the concentration of electrons must exactly equal the concentration of positive ions. The total current flowing along the arc stream is the sum of the electron current (I_-) and the positive ion current (I_+) and these stand in the ratio

$$\frac{I_-}{I_+} = \frac{v_-}{v_+}, \tag{6}$$

where v_- and v_+ are the corresponding ion velocities in a field of 47 volts per cm.

Anode fall: The anode fall is between 7 and 8 volts. The factors which serve to determine its value are not immediately obvious.

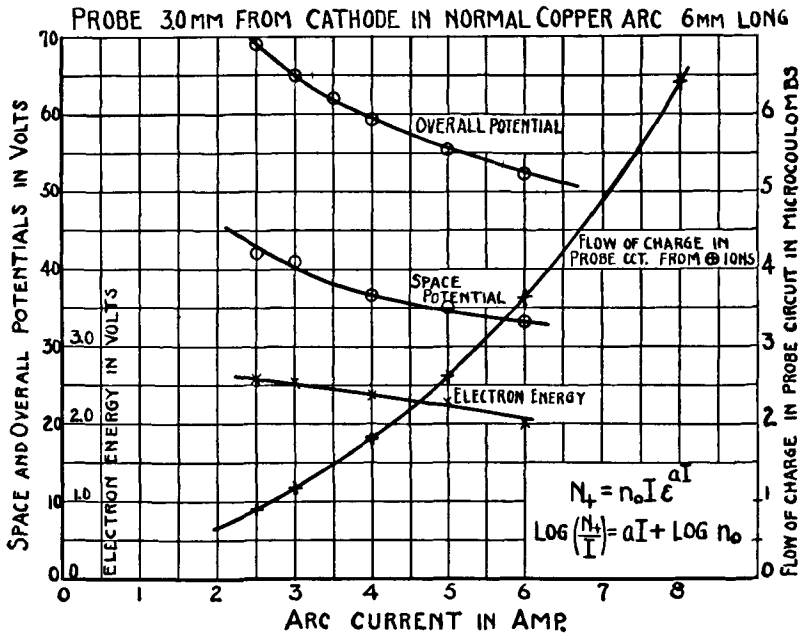
Electron energy: The average energy of the electrons increase slightly as they proceed from the cathode to the anode. These energies are 2.3 ± 0.1 volts near the cathode and 2.7 ± 0.1 volts near the anode.

[5] Hagenbach & Veillon, *Phy. Zs.*, 11, 833 (1910).

COMPARISON OF PROBE MEASUREMENTS IN ARCS CARRYING DIFFERENT CURRENTS.

Copper arcs 6.0 mm. in length and carrying currents ranging from 2.5 to 6.0 amperes were studied with a copper probe moved through the arc stream at a distance of 3.0 mm. from the cathode. The average electron energy, the space potential, and a measure of the positive ion current to the probe were determined in each arc. These results are shown by the different curves of Fig. 4.

FIG. 4.


PROBE 3.0 MM FROM CATHODE IN NORMAL COPPER ARC 6 MM LONG
SPACE AND OVERALL POTENTIALS IN VOLTS
ELECTRON ENERGY IN VOLTS
FLOW OF CHARGE IN PROBE CIRCUIT IN MICROCOULOMBS
ARC CURRENT IN AMP.
OVERALL POTENTIAL
SPACE POTENTIAL
ELECTRON ENERGY
FLOW OF CHARGE IN PROBE CCT. FROM ⊕ IONS
$N_+ = n_0 I \varepsilon^{aI}$
$\text{LOG}\left(\frac{N_+}{I}\right) = aI + \text{LOG}\, n_0$

Probe measurements in the copper arc with the probe half way between the electrodes.

Space potential The space potential at the point half way between the electrodes follows the same form of function in its dependence on the arc current as the over all potential and is given by the equation

$$V = A + \frac{B}{I^n}. \qquad (7)$$

The corresponding values for the constants are $A = 20$,

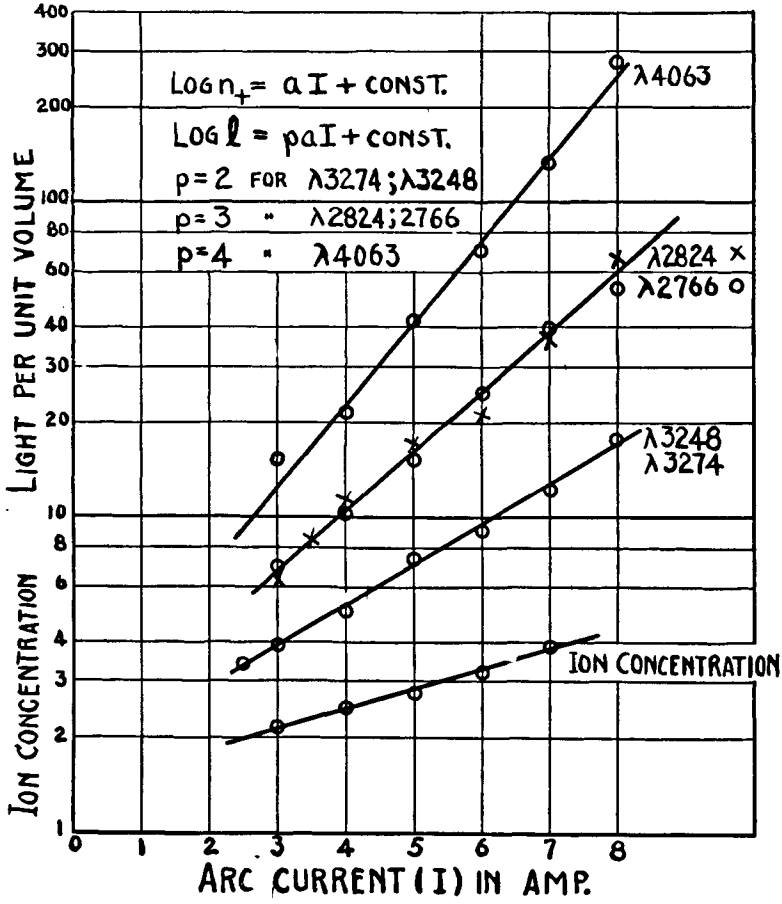
$B = 41$, and $n = 0.64$ for the space potential while $A = 36.3$, $B = 59$, and $n = 0.67$ for the over all potential. Since in general the value of n depends upon the absolute temperature, the lower value of n of 0.64 may result from the fact that the average temperature of the arc is lower near the cathode than it is near the anode.

Average electron energy: The average energy of the electrons has been found to decrease slightly as the current flowing through the arc is increased. From Fig. 4 we see that the average electron energy decreases from about 2.5 volts to 2.0 volts for the range of arc current of 2.5 amperes to 6.0 amperes. This electron energy is probably determined by the electron transition in the copper atom from the metastable "D" levels to the $2^2P_{1,2}$ levels involving an inelastic impact which takes either 2.14 or 2.4 volts energy. (See Fig. 6.) The $2^2P_{1,2}$ state in the copper atom is the lowest radiating level. The important thing to note at this point is that the energy required to ionize a copper atom is three to four times the average energy of the electrons. Since we know that ionization takes place in the arc stream, it must be the result of impacts of the few electrons with energies three or four times the average energy, or else it must result from the cumulative action of three or four successive impacts of average velocity electrons. Evidence to be presented later in this paper indicates that the cumulative action is responsible for the production of most of the positive ions.

Positive ion current to probe: The charge which flows through the probe curcuit in a single swing of the probe due to positive ions coming to the probe increases very noticeably as the arc current is increased. Owing to the fact that the area of cross-section of the arc is directly proportional to the arc current flowing, the positive ion charge flowing through the probe circuit would increase linearly with the current in case the positive ion concentration in the arc stream remained unchanged. The fact that this curve is convex toward the current axis (see Fig. 4), shows that the concentration of positive ions increases as the arc current is increased. The ordinates plotted show the positive ion charge flowing through the probe circuit when at the potential of the space. This is obtained by extrapolating the "voltage-current" curve ob-

served over a range of voltage which is so negative with respect to the space that the electron current is too small to measure. The positive ion probe current, which is a measure of the number of positive ions which strike it and give up

FIG. 5.



Curves showing dependence of light and ion concentrations on arc current in copper arc.

their charges, can be represented by the simple empirical equation

$$N_+ = n_0 I \epsilon^{aI}, \tag{8}$$

in which n_0 and a are constants. The charge which flows

through the probe circuit when the probe passes through the arc stream of radius r at a constant velocity v can be shown to be given by the equation

$$N_+ = \frac{\pi r^2 i_+}{v}, \tag{9}$$

where i_+ is the current to the probe per unit length. Since the motion due to thermal agitation is large compared to that due to the drift in the direction of the field, the current i_+ is proportional to the positive ion concentration n_+. From equations (8) and (9) and the fact that the area of the arc is proportional to the current flowing, we can express the relation between the positive ion concentration and the arc current by equation (10).

$$n_+ = b\epsilon^{aI}. \tag{10}$$

Here a and b are constants and n_+ is proportional to $\frac{N_+}{I}$. Graphically this relationship is shown by the straight line of Fig. 5 designated "ion concentration." Since

$$\log_\epsilon n_+ = aI + \log_\epsilon b, \tag{11}$$

the value of a is given by the slope of this straight line.

INTENSITY OF SPECTRAL LINES AS A FUNCTION OF ARC CURRENT.

Method of measurement: In order to determine how the intensity of certain spectral lines in the copper arc depends on the current flowing in the arc, it is necessary to have a device whose response is proportional to the light intensity falling on it. The light from the arc was passed through a quartz lens and two quartz right angle prisms adjusted to form an image of the arc on the slit of a D 41 Hilger monochromatic illuminator. The axis of the image was turned into the horizontal position by prisms so that the light which passed through the monochromatic illuminator all came from the small cylinder in the arc syream bounded by two planes passed perpendicular to the axis of the arc and separated by a distance equal to the width of the slit of the monochromator. The arc was so adjusted that this cylinder occupied the samc region which was swept through by the probe when it was set

to swing through half way between the electrodes. The light intensity for the lines between λ4023 and λ5220 was measured by a potassium-hydride photoelectric cell made in our laboratory [6] coupled with a vacuum tube amplifier with a sensitivity of 24,000 mm. per volt and an input resistance of about 2,000 megohms. To measure the light intensity in the ultraviolet from λ2441 to λ3274, the light was made to fall on a glass container with a quartz window and filled with a saturated solution of esculin in water. A part of the fluorescent light produced by the ultraviolet passed into the photoelectric cell and gave a current which could easily be measured. The fact that the fluorescence is proportional to the intensity of the ultraviolet light falling on the container was proved by measuring the change in photoelectric current when the intensity of the ultraviolet light was reduced by interposing various wire screens at the lens. The absorbing power of the screens measured with blue light was found to be the same as that for the ultraviolet assuming the intensity of fluorescence proportional to the incident ultraviolet radiation.

Experimental results: The first lines to be examined were the two resonance lines λ3274 and λ3248 which correspond to the $1^2S - 2^2P_{1,2}$ transitions. Although these two lines could not be definitely resolved, no harm should result since they correspond to transitions so similar in all respects. The results of these measurements showed that the light intensity L followed the same simple empirical relationship found for the positive ion current and given by equation (8). This equation in more suitable terms is given in equation (12)

$$L = l_0 I \epsilon^{2aI}. \tag{12}$$

Here the arc current is I, and l_0 and a are constants. Since a negligible part of the radiation of this wave-length originating throughout the volume of the arc is absorbed in the arc, we can take the light per unit volume l as being proportional to $\frac{L}{I}$ and write equations (13) and (14).

$$l = c\epsilon^{2aI}, \tag{13}$$

[6] "Instruction on the Making of Potassium-hydride Photoelectric Cells," W. B. Nottingham, JOURNAL OF THE FRANKLIN INSTITUTE, 206, 637 (1928).

$$\log_e l = 2aI + \log_e c. \tag{14}$$

From Fig. 5, we see that the slope of the straight line designated "λ3274; λ3248" is twice that for the positive ion concentration and therefore the constant a is the same in equations (10) and (13). From these two equations, together with the fact that the positive ion concentration is equal to the electron concentration, we see at once that

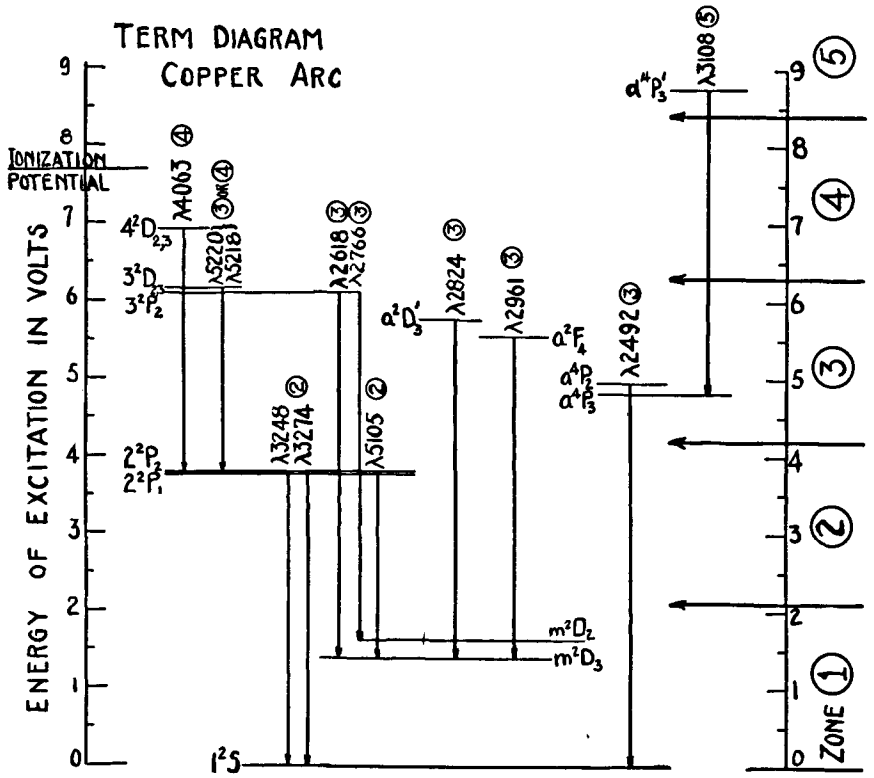
$$l = \frac{b^2}{c} n_-^2. \tag{15}$$

This equation expresses the fact that the light per unit volume emitted as radiation λ3274 *and* λ3248 *is proportional to the square of the electron concentration.* The next line studied was λ5105 which also originates in the 2^2P level of the copper atom as is shown by the level diagram in Fig. 6. Here too the light per unit volume was approximately proportional to the square of the electron concentration although the proximity of the line λ5153 slightly distorted the curve. Altogether twelve lines were measured in this way and from the slopes of the semi-logarithmic plots the dependence on the electron concentration was found for each. In every case it was found that the slope of the straight line of the semi-log plot was very nearly an integral multiple of that found for the positive ion concentration. The lines λ2961, λ2824, λ2766, and others designated in Fig. 6 by a three, each had slopes of almost exactly $3a$ while λ4063 and a few others had slopes of $4a$. Lines of wave-length shorter than λ2441 could not be studied because here the absorption of the light in the hot gas around the arc can not be neglected. Intensity measurements at λ2392 and λ2293 showed very little change when the arc current was changed as a result of this increased absorption.

Interpretation of results: These results can be easily explained if we assume that cumulative impacts of electrons on atoms are largely responsible for the excitation of the atoms. Let us imagine that there are certain discrete excitation energy levels in the atoms making up the gas of our arc discharge and designate these by 0, 1, 2, 3, etc. Let the difference in energy between consecutive states equal the average energy of the electrons. Assume that state "0" is the normal un-

excited state and that state "1" can be produced by a single impact of an electron with an energy equal to the average

FIG. 6.



Term diagram showing certain of the levels and lines of the copper arc spectrum.

energy of all the electrons. Within a unit volume the rate at which atoms enter or leave state "1" will be given

$$\frac{dN_1}{dt} = a_{01}N_0n - b_1N_1 - c_{12}N_1n. \qquad (16)$$

Here N_1 is the number of atoms per unit volume in state "1," n is the concentration of electrons, a_{01} is the constant of proportionality which when multiplied by N_0, the concentration of unexcited atoms, and n gives the rate at which atoms are excited to the state "1." The term $-b_1N_1$ represents the rate

at which atoms leave state "1" by a spontaneous giving up of energy in radiation. The term $-c_{12}N_1n$ gives the rate at which atoms leave state "1" to enter state "2" as a result of impacts with electrons. This rate certainly must be proportional to the concentrations N_1 and n with c_{12} as the constant of proportionality. In the steady state we must have $\frac{dN_1}{dt} = 0$ and therefore we can write

$$a_{01}N_0n - b_1N_1 - c_{12}N_1n = 0 \tag{17}$$

and in the same way we can write

$$c_{12}N_1n - b_2N_2 - c_{23}N_2n = 0, \tag{18}$$

$$c_{23}N_2n - b_3N_3 - c_{34}N_3n = 0. \tag{19}$$

These equations can be solved for N_1, N_2, etc., and we find

$$N_1 = \frac{a_{01}N_0}{b_1 + c_{12}n}n, \tag{20}$$

$$N_2 = \frac{a_{01}N_0c_{12}}{(b_1 + c_{12}n)(b_2 + c_{23}n)}n^2, \tag{21}$$

$$N_3 = \frac{a_{01}N_0c_{12}c_{23}}{(b_1 + c_{12}n)(b_2 + c_{23}n)(b_3 + c_{34}n)}n^3. \tag{22}$$

From these equations we see that the number of atoms in state "1" is proportional to the concentration of electrons; the number in state "2" is proportional to n^2 etc., if the number of atoms which leave a given state by radiating energy is large compared with those which leave it to enter a higher state due to electron impacts. We should therefore expect the intensity of light per unit volume which is given off the arc stream to be proportional to the square of the electron concentration if the atoms are each excited as result of two impacts of electrons. This theory is supported by the fact that the level diagram of Fig. 6 can be divided into "zones." Lines resulting from transitions originating in the second zone have intensities proportional to the square of the electron concentration, while those from the third zone are

proportional to the cube of the concentration and so on. It is important to note that the width of these zones, measured in volts, is about 2.1 volts and the average energy of the electrons as given by the probe measurements is also slightly greater than 2.1 volts. (See Fig. 4.)

GENERAL CONCLUSIONS.

These experiments have brought out a few simple facts concerning the mechanism of the electric arc which may be worth reviewing. The cathode fall in potential is usually equal to the ionization potential of the active gas but may, as in the case of the normal copper arc, be equal to the potential required to remove the second electron from an atom already singly ionized. The presence of a large concentration of singly or doubly charged atoms in the immediate neighborhood of the cathode surface is no doubt partly responsible for the high current density at the cathode. The electrons are found to have approximately a Maxwellian distribution of velocity with an average energy determined by critical potentials of the atom in case the gas or vapor playing the controlling part is monatomic. In case the gas is diatomic, as in the carbon arc in which we have the cyanogen molecule, the average energy of the electrons is not determined sharply and is not constant along the arc stream. This accounts for the "striations" found in the carbon arc.[1] The electrons produce the excitation responsible for the emission of light as a result of the cumulative action of successive impacts of electrons on atoms. It follows from this that the relative intensities of the lines observed in an electric arc are not fixed but depend on the current flowing because the concentration of electrons is a function of the current.

It is with pleasure that I acknowledge my indebtedness to Dr. W. F. G. Swann for many valuable suggestions.

MODULATION OF LIGHT WAVES.

BY

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BARTOL RESEARCH FOUNDATION

Communication No. 33.

WHEN light of a given frequency ν is passed through a dielectric, transparent to that frequency, which forms part of a parallel plate condenser, the light wave can be changed in character in two ways when electromagnetic waves of frequency P are impressed on the condenser.

First, the light, if it is initially plane polarized, will become elliptically polarized due to the medium becoming double refracting. If the frequency P of the electromagnetic waves is such that the medium has an absorption band at this frequency, then the double refraction will be considerable, or in other words, the Kerr constant will be abnormally large.

Second, the light train will be modulated, i.e., besides the original frequency ν, there will appear light trains of frequencies $\nu + P$, $\nu + 2P$, $\cdots$ and $\nu - P$, $\nu - 2P$, $\cdots$ etc. If the medium does not become double refracting, then the amplitudes of the modulated wave trains would be the same for corresponding frequencies, i.e., the amplitude of the light train of frequency $\nu - nP$ would be the same as that of frequency $\nu + nP$, so that the energy would be equally distributed towards the long and short wave-length side of the original line. When, however, account is taken of the double refraction of the medium, the energy will be no longer equally distributed among the modulated wave trains towards the long and the short wave-length side of the line, but there will be a greater amount of energy divided among the modulated waves on the long wave-length side of the line than towards the other side.

In these experiments, the light of an iron arc which passed through a Kerr cell containing water was photographed with a quartz spectrograph; the lines in the region from 2480 to 2500 Å. were examined with a microphotometer. It was found

that of the fifteen or more lines in this region only one showed any difference when the high frequency oscillations of wave-length, 8 meters approx., passed through the cell. This particular line at 2492.7 Å. was displaced towards the red by an amount .06 Å. On account of the complexity of the iron arc and the unknown structure of the lines, the mercury arc was substituted for it and the two lines at 2536.5 and 2534.7 Å. examined with a densitometer. The line at 2536.6 showed a shift towards the red with respect to the other line when the high frequency field was passing through the condenser. This effect was independent of the temperature of the water within the Kerr cell from 20° to 60° C., and independent also of the amount of the oscillatory current which passed through the cell but dependent on the wave-length of the high frequency waves. If the wave-length of the oscillations was changed from the value used in the above experiments, 8 meters approx., to 14 meters, the effect disappeared entirely. On varying the capacity of the system by small amounts, it was found that, as the frequency of the oscillations varied from one absorption band of the water to another, the effect reappeared and disappeared depending on whether the frequency of the electromagnetic waves was the same as or different from, that of the absorption frequency.

THEORY OF LIGHT MODULATIONS.

Before we discuss in detail the experimental results, we shall review the theory of light modulations which is based on Drude's theory of optically active media.

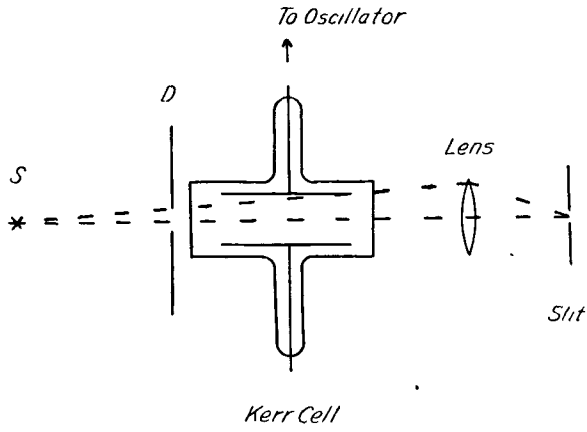
In the classical dispersion theory we suppose that when a light ray with electric vector E traverses a transparent medium it sets into motion the virtual oscillators which have a frequency characteristic of the medium, or expressed in different terms, it induces in the molecules of the medium a variable electric moment which emits secondary rays. If the frequency of the light train is ν and the frequency of the virtual oscillators is p, then besides wave trains of frequency ν, the primary radiation's frequency, there will appear wave trains of frequency $\nu - p$, $\nu - 2p \cdots \nu + p$, $\nu + 2p \cdots$, and the intensity of the wave trains of corresponding frequencies, i.e., the intensity of the rays of frequency $\nu - np$ and $\nu + np$ will

be in the ratio of $\left(\frac{\nu - np}{\nu + np}\right)^4$, which is essentially unity if the induced frequency $p << \nu$ (Carbannes 5. R. 186, p. 1714) so that the light intensity is equally divided among the long and short wave-length components of the line.

In the experiment we are describing we have a somewhat similar situation, except that here the induced electric moment of frequency p arises from the action of an electromagnetic field distinct from the primary light train; in this case, the external electromagnetic field acting on the medium was generated by the high frequency radio oscillator.

In the case considered by Carbannes, the secondary rays of frequency $\nu - p$, $\cdots$ are due to the interaction of the primary light wave of frequency ν and the secondary rays of frequency p emitted by the virtual oscillators. The phenomenon is here coherent, whereas in the other case where the radio waves cause the induced electric moment of the medium, the interaction of the induced moment and the primary light wave is non-coherent.

FIG. 1.

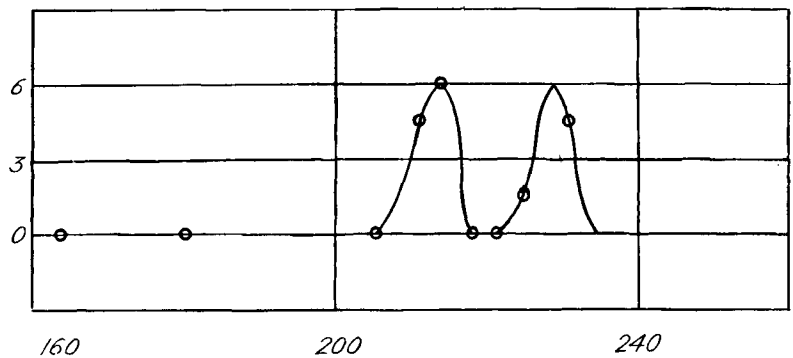


The diagram of the apparatus used to modulate the light is shown in Fig. 1. The light from the source S, in the first experiments with an iron arc, was passed through the Kerr cell, which contained water as the dielectric, and was then focused on the slit of the spectrograph by a quartz lens. The Kerr cell which had a capacity of 5 e.s.u., was the plate con-

denser of a high frequency oscillator which could oscillate at a wave-length as short as four meters. The diaphragm *D*, which was placed before the Kerr cell, allowed only the light which passed between the plates of that cell to enter the slit of the spectrograph.

On examining the spectrogram taken of the iron arc, it was found that several of the lines of the spectrum were displaced towards the red from their normal position when the light from the iron arc was photographed with the high frequency oscillations impressed on the cell. In order to measure this

FIG. 2.



shift, a spectrogram of the arc lines was taken with the light passing through the cell, but without the high frequency generator operating and the distance of the lines from one another was measured with a microphotometer, then a second spectrogram was taken with the same arrangement, but with the generator oscillating and the distance of the lines from one another again measured. In the two regions examined in detail from 2480 to 2500 Å. and from 2380 to 2400 Å. only one line in each region was displaced while the others retained their separations unchanged within the accuracy of the instrument, which was .01 Å. These two lines at 2498.9Å. and 2382 Å. respectively were displaced towards the red by .06 Å.

On account of the sensitiveness of the lines of the iron spectrum to external conditions and lack of information concerning the character of the spectrum, in the following experiments the mercury arc was used in place of the iron arc.

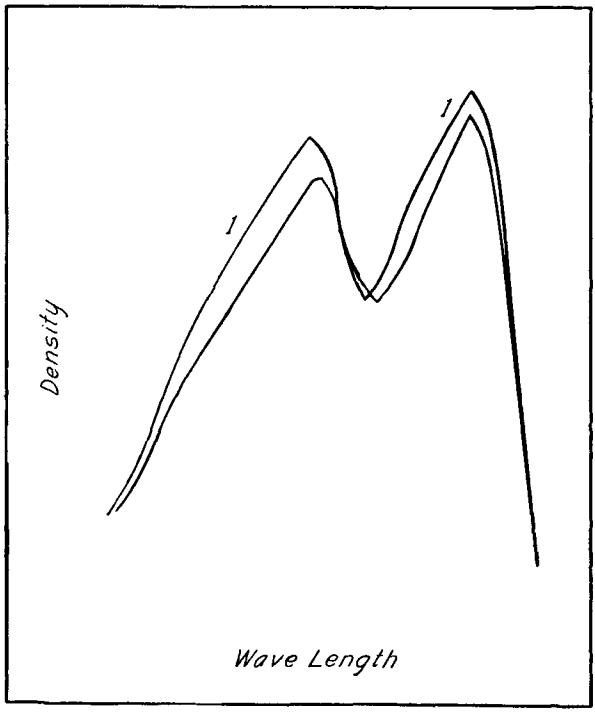
On examining the separation of the two mercury lines 2534.7 and 2536.5 Å., the latter showed a shift towards the red by the same amount as the iron lines, i.e., .06 Å. when the oscillations were passing through the cell. Fig. 2 shows the distribution of density for these two lines on the photographic plate, plotted against wave-length. It is seen from curve 1 which represents the distribution of intensity of the lines when the oscillator was operating that the line 2536.5 is displaced towards the red with respect to the distribution of density of the same line when the generator was not operating, i.e., with respect to its normal position.

These spectrograms were normally taken with an oscillatory current in the plate circuit which contained the Kerr cell of 3.0 milliamperes but the same results were obtained when the plate current was changed from this value to 9.0 milliamperes, the maximum current obtainable with the tube. On heating the cell by applying 1800 volts, 60 cycles, to the plates of the Kerr cell and allowing the current to flow for several hours, the results were found to be independent of this applied a.c. voltage and also of the variation of the temperature of the cell from 20° C. to 60° C., the higher limit being as far as it was practical to heat the Kerr cell, since the quartz windows were sealed on by cement. We would not have expected any effect due to the change in temperature of the water, first, because the change in the refractive index of water with temperature is small in this region, and secondly, because the source was so far away from the diaphragm *D*, see Fig. 1, that the light passing through the cell was practically parallel. These experiments show that any effect due to the change in the focusing of the light on the slit of the spectrograph is negligible. There existed the possibility, however, that the high frequency radiation from the generator might interact with the atoms emitting the light in the source. In order to test this possibility, the Kerr cell was removed from the path of the light beam. The spectrograms taken of the mercury arc in this way were the same whether the oscillator which was still connected to the Kerr cell was operating or not.

Finally the frequency of the system was changed by placing a variable air capacity in parallel or series with the Kerr

cell depending on the frequency change desired. Fig. 3 shows the change in separation of the lines from their normal separation when the oscillator is not acting as a function of the frequency of the oscillation.

FIG. 3.



The abscissæ represents the wave number $\times 10^{-6}$ of the radio frequency oscillations, the ordinates the frequency change of the line 2536.5 in unity of 1/100 Å. for these frequencies.

The curve in this figure, which shows the change in separation of the lines 2534.7 and 2536.5 plotted against frequency, shows a series of maxima and minima which correspond to the positions of the maxima and minima for the double refraction of light from the same frequency scale. (Kerr Effect in water for radio frequencies, J. FR. IN., Aug. 1928.) In that experiment the retardation of the elliptically polarized light produced by passing plane polarized light through a Kerr cell which was part of a high frequency oscillating circuit was measured for various frequencies of the oscillator. The

retardation was found to go through a series of maxima and minima as the frequency of the oscillations was changed. The retardation of the elliptically polarized light thus exhibited the same type of behavior as the displacement of the spectral lines. The two phenomena are also closely associated with the absorption bands for high frequency oscillations which correspond to the positions of the maxima of the retardation upon displacement of the spectral lines with regard to their frequency.

In conclusion, I wish to express my indebtedness to Professor James Barnes of Bryn Mawr College who kindly placed at my disposal his microphotometer and to the Department of Physics of Princeton University for allowing me the use of their high dispersion quartz spectrograph.

Crystallographic Study by X-Rays of the Structure of the Layer of Two Metals Deposited Simultaneously by Electrolysis. A. ROUX AND J. COURNOT. (*Comptes Rendus*, June 12, 1928.) Several pairs of metals were deposited together and the diffraction spectra of the resulting mixed layer were examined by X-rays to see whether each metal kept its personality or whether modifications occurred. The conclusion is "Examination of the results obtained show distinctly that the spectra of the deposits made simultaneously are not merely due to the superposition of the spectra of the constituent metals and consequently that during electrolysis the metals unite to form either solutions or combinations."

When copper and zinc were deposited together the strongest diffracted ray was obtained when the double angle of incidence was 21.46°. The strongest ray for copper was for the double angle of 22.29° while for zinc there were two strong rays for 18.03° and 21.20° respectively. The combined deposit seems to show no evidence of the two maxima displayed by zinc.

G. F. S.

Deviations from the Vertical in Corsica. P. HELBRONNER. (*Comptes Rendus*, June 25, 1928.) The work here reported was done in connection with the measurement of an arc of meridian. For the study of the effect of mountain masses in deviating a plumb line from its normal direction Corsica offers the advantage of being 100 km. from the mainland and of having within its borders quite high mountains. It is rather well isolated from distant gravitational attractions. At six stations on the island comparison was made between astronomical and geodesic latitude. The largest difference amounted to 61″, while the greatest divergence of the geodesic from the astronomical longitude was 64″. The deflecting forces acted in general toward the interior of the island in the direction of the nearest elevated masses, even when these seemed relatively unimportant in size and height. Any effect depending on geological formation was masked by the influence of altitude.

G. F. S.

THE HEAT CAPACITIES OF ORGANIC COMPOUNDS AT LOW TEMPERATURES. II. A LOW TEMPERATURE THERMOSTAT. THOUSANDTH DEGREE COPPER-CONSTANTAN THERMOCOUPLE REFERENCE TABLES FROM 85° TO 310° K.

BY

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AND

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Communication No. 32.

IN connection with the work on the heat capacities of organic compounds described in paper I of this series [1] it was found necessary to obtain an improved temperature—e.m.f. scale for copper-constantan thermocouples in order that the specific heat measurements might have an accuracy of *one per mille;* that is to say, an absolute accuracy of 0.01° C. and a relative accuracy of at least 0.002° C. was required in the temperature measurements. Furthermore, it was desirable that this temperature scale should be based as directly as possible on the so-called International Temperature Scale of the University of Leiden, Reichsanstalt, and the U. S. Bureau of Standards. In order to accomplish this, it was necessary to devise a thermostat capable of maintaining the temperature constant to within 0.001° at any point from the boiling point of liquid air to room temperature. Moreover there seemed to be considerable general interest in devising a comparatively simple yet precise low-temperature thermostat. These considerations led to the research described herewith.

A comprehensive review of the work on low temperature thermostats up to 1925 is given by Walters and Loomis.[2] Maass and Barnes [3] have constructed an automatic thermostat

[1] D. H. Andrews, J. FRANKLIN INST., 206, 285.

[2] Walters and Loomis, *J. A. C. S.*, 47, 2302 (1925).

[3] Maass and Barnes, *J. A. C. S.*, 49, 360.

accurate to 0.1°, and Simon [4] describes one accurate to several tenths of a degree. In addition to these an automatic thermostat with an accuracy of 0.01° was described in the first paper of this series.[5] However, this latter thermostat is so constructed that it is adaptable only to such work as can be carried on inside a steel vacuum bomb and the type of resistance thermometer in vogue at the present time at the Bureau of Standards cannot be completely enclosed in such a bomb. This fact led us to develop the following comparatively simple yet precise low temperature thermostat.

THE LOW TEMPERATURE THERMOSTAT.

The thermostat bath is of pentane contained in a cylindrical copper vessel (*C*, Fig. 1) with walls 2 mm. thick. A heating unit of 40 ohms resistance is wound around the outside of this vessel and the whole placed inside a small Pyrex Dewar flask (*B*) made for the purpose. All this in turn is placed in a bath of liquid air contained in a larger (quart) Dewar (*A*). The usual objection to this type of thermostat in the past has been the danger of the inner Dewar breaking and thus forming a highly explosive mixture of liquid air and pentane. This defect has been remedied by the use of the copper container.

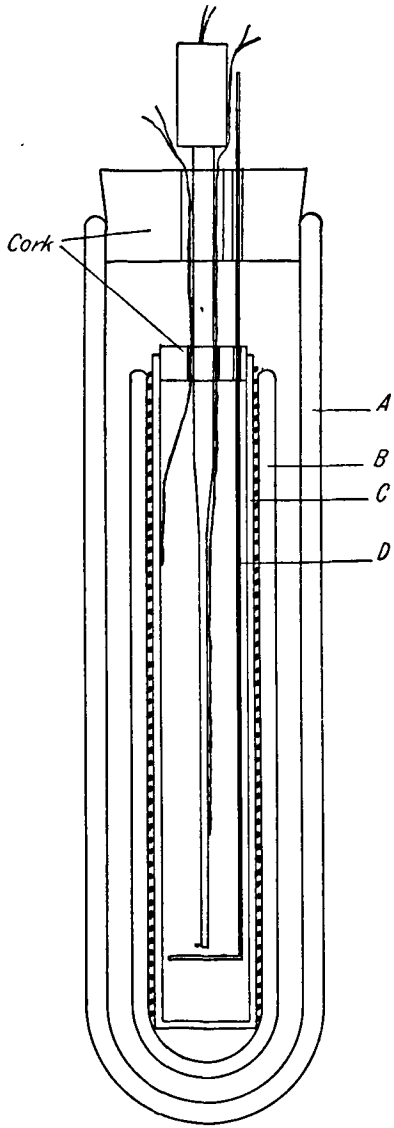
Thermal equilibrium is maintained to a fair degree by the heavy copper walls of the thermostat since it is of such dimensions that no point within it is more than 1.5 cm. from the walls. Sometimes a hand stirrer (*D*) is used but more often a heavy copper tube (not shown) of a diameter of about half that of the thermostat employed to further minimize temperature gradients. Obviously only the latter can be used below about 140° C. at which point pentane begins to solidify.

Temperature regulation is accomplished in the following way. A copper-constantan thermocouple is soldered to the inside of the copper container about one third of the way down from the top. The e.m.f. from this is opposed with that from a Type *K*, Leeds and Northrup potentiometer which can be set to correspond to the desired temperature. The thermo-

[4] Simon and col., *Ber.*, **60**, 568 (1927).

[5] *Loc. cit.*

FIG. 1.



couple and potentiometer are connected in series with a high sensitivity Leeds and Northrup galvanometer. Obviously when the temperature of the thermostat differs from that

corresponding to the e.m.f. set on the potentiometer there will be a galvanometer deflection. The deflection of the galvanometer is made to regulate the heating in the following manner. An image of the filament of a 100 watt lamp is focused by means of a lens on the galvanometer mirror which reflects this onto a photoelectric cell about $\frac{1}{2}$ meter away. The cell is of the potassium hydride type and was made by the authors according to the directions of Mr. W. B. Nottingham of the Bartol Research Foundation. It is enclosed in a black box which has a square hole with sharp edges so placed that the hole is in front of, but smaller than the window in the cell. The image of the filament falling on the cell box produces a sharp line of light about 1 mm. wide and the zero point of the galvanometer is so adjusted that the edge of this line of light falls exactly on the edge of the square hole over the photoelectric cell window. When the temperature drops below that required the line of light moves about 0.2 mm. onto the cell thus causing the latter to operate a relay which shunts a resistance thus increasing the heat input into the thermostat. Conversely, when the temperature becomes too high the image moves off the cell, the relay is released and the heat input cut down. In this manner the temperature of the walls of the thermostat is regulated to about a thousandth of a degree. Even this small fluctuation could not be detected in the center of the thermostat with the B. of S. platinum resistance thermometer by means of which a temperature change of 0.001° was easily observable.

We are also indebted to Mr. W. B. Nottingham for the photoelectric cell circuit (Fig. 2) which was found to be very satisfactory. All the batteries were carefully insulated against grounds by paraffined blocks, and the cell, radio tubes, and grid leaks were enclosed in a lead covered moisture proof box. This, together with sheets of metal under the batteries, was grounded. When this was not done the operation of the cell seemed to cause the galvanometer to behave erratically. The adjustment of the photoelectric cell battery is quite critical as are also those of the other batteries to a lesser degree.

The thermostat heating circuit (Fig. 2) was so designed that the voltage drop across the heating coil and shuntable

resistance as well as the current through them could be adjusted. This was accomplished by placing directly across the 110 V., D.C. line a 65-ohm variable resistance and connecting one lead of the heater to the movable contact while the other was connected to one end of the resistance. It was necessary to have the resistance which the relay shunted

FIG. 2.

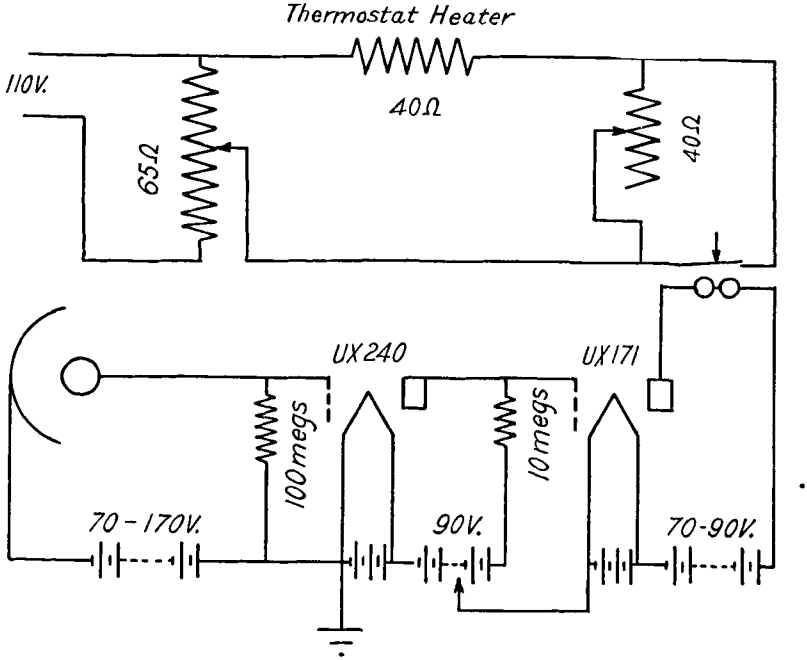


Photo electric Cell Circuit

variable in order that the change in heat with the operation of the cell could be adjusted to give the best temperature regulation.

In order to cool down the thermostat, the cork in the top of the outer Dewar is raised a short distance and liquid air blown in until its level is above that of the copper container, which is of such dimensions that it projects slightly above that of the Dewar which holds it. In this way the thermostat is quickly cooled and as soon as the slight excess of liquid air evaporates it is ready for use.

The advantages of this type of low temperature thermostat may be summed up as follows:

1. The temperature can be maintained constant to 0.001° for an indefinite length of time if proper stirring is provided.

2. Small quantities of liquid air are used. A day's work including cooling down, requires about two liters.

3. It requires only fifteen minutes to an hour to change from one temperature to another, and reëstablish equilibrium.

4. It is safe, since the inflammable pentane is encased in a copper container.

5. No vacuum pumps, etc., are required.

6. It is quickly cooled down to liquid air temperatures.

THE COPPER-CONSTANTAN THERMOCOUPLE TABLES.

Until something over a year ago it was generally thought sufficient to compare a thermocouple with the reference thermometer at three temperatures below zero, and to interpolate the intermediate temperatures by means of an equation or by a graph, in order to secure temperature readings of about 0.1 to 0.01° accuracy. Eastman and Rodebush [6] proposed an equation which represented the data obtained experimentally by them at the freezing point of mercury, the sublimation point of CO_2, and the boiling point of oxygen. Also, Adams in the International Critical Tables presents a temperature e.m.f. reference table based on observations made at the same three points in which the intermediate temperatures were obtained by analytical and graphical interpolation and given to 0.01°. However, any kind of interpolation over a span of 100° could hardly be expected to be accurate over the whole range to one part in ten thousand (0.01°), which precision we thought necessary for our work.

Recently Giauque, Buffington and Schulze [7] have compared a copper-constantan thermocouple at intervals of 10° from 25° to 283° K. with a hydrogen thermometer of their own construction and have presented a more or less empirical reference table from which they plot a deviation curve of one of their thermocouples. This deviation curve seems to be certain to only about 0.05° C., and weaves back and forth

[6] Eastman and Rodebush, *J. A. C. S.*, 40, 489.

[7] Giauque, Buffington and Schulze, *J. A. C. S.*, 49, 2343.

across the axis. It seemed to us desirable to have a reference table that is truly representative of the e.m.f. of an actual thermocouple so that a deviation curve such as would be necessary in using any other thermocouple would be regular, and could be obtained by making comparisons at say only 40° intervals. Also in order to reduce the labor of calculation the table should be extensive enough so that changes in the dE/dT and related quantities would be so small between tabulated temperatures that they could be neglected. Of course the table must have an absolute accuracy of 0.01° with respect to the International Temperature Scale.

EXPERIMENTAL.

As a comparison thermometer a Leeds and Northrup platinum resistance thermometer which had been calibrated by the Bureau of Standards, was used. This was checked before and after runs against another of the same type which had been calibrated by the Bureau of Standards less than a month before use. This last mentioned thermometer was reserved exclusively for checking purposes. In this way we made certain that no strains had developed during the experiments in the comparison thermometer, which would cause it to become faulty.[8]

Four thermocouples were calibrated, two being of constantan wire from one spool, and two from another, having a slightly different composition.[9] The wire was tested by drawing it through liquid air in the usual way and observing

[8] The "Comparison Thermometer" upon which the Standard Reference Tables are based is Leeds and Northrup Co.'s No. 128016 and B. and S. No. 2736, calibrated June 15, 1927. The "Checking Thermometer" with which the "Comparison Thermometer" was checked before and after each run is Leeds and Northrup Co.'s No. 137184, and B. and S. No. 312, calibrated November 5, 1928. These thermometers checked each other consistently, No. 128016 reading 0.002° higher than No. 137184 at −180° C.

It should be noted that the equation by which temperature is calculated for these thermometers expresses the result in ° C.; thus while our thermocouple readings are really based on the Centigrade scale, we have converted them to ° K., as the latter is the more useful scale for thermodynamic purposes. We have used the value 273.1° K. for the ice point and while the accuracy of this value seems to be in dispute, a change will merely mean adding a small constant correction to the tables.

[9] The wire is the so-called "ideal" wire made by the Driver-Harris Co.

e.m.f.'s due to nonuniformity. The thermocouples were bakelited to the case of the thermometer for practically its whole length in such a manner that their junctions lay about opposite the center of the thermometer coil and their leads had no turns or kinks in them. The thermometer was placed in the center of the thermostat bath as is shown in Fig. 1.

The resistance of the thermometer was easily observed to one ten-thousandth of an ohm by comparing it with a ten ohm coil calibrated by the Bureau of Standards. This was accomplished by connecting the two in series with rheostat and battery and comparing the potential drops across each of them by means of a White double potentiometer. The galvanometer was about 15 feet from the scale enabling the e.m.f. to be read to 0.1 μ V, and the thermometer to 0.001° with ease. The e.m.f. of the thermocouple was read on the same potentiometer.

Table I gives the experimental data from which the Standard Reference Tables have been calculated. Each point represents the average of from four to ten readings. Since the thermostat was constant to 0.001°, and the resistance thermometer could be read to the same fraction, the chief error was in reading the e.m.f. of the thermocouple. This was equivalent to from 0.002 to 0.006° depending on the part of the scale.

TABLE I.

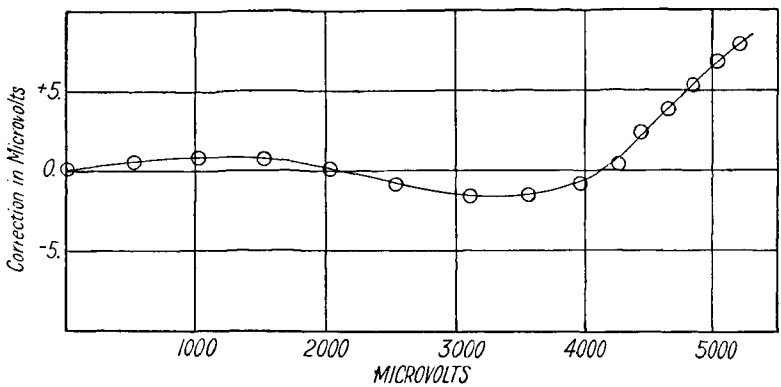
Temp. ° K.	E.M.F. in Microvolts.	Temp. ° K.	E.M.F. in Microvolts.
90.074	5216.6*	199.704	2541.7
90.186	5214.9	215.739	2033.2
99.890	5038.1	230.863	1529.2
109.786	4845.3	245.294	1026.5
119.225	4649.9	259.050	528.1
129.062	4434.0	273.1	0.2**
136.681	4259.6	285.757	− 489.9
149.151	3956.9	298.482	− 998.0
164.882	3548.2	311.170	− 1518.8
180.355	3117.9		

The reading marked (*) was taken in a bath of liquid oxygen. The one marked (**) was taken in a bath of finely crushed ice and distilled water such as is used for the ice junction. The fact that these readings fall on the curve with those taken in the thermostat is proof that there was no temperature gradient between the thermocouples and the resistance thermometer.

DERIVATION OF THE STANDARD REFERENCE TABLES.

The equation $E = E_0 - 0.447\ T^{1.699}$ was found to fit the above data approximately. E is the observed e.m.f., E_0 a constant, and T the temperature in degrees Kelvin. A deviation curve (Fig. 3) shows that excepting three points the maximum error is less than 0.1 μ V, which corresponds to 0.002 to 0.006°.

FIG. 3.



From this curve a correction was read and added algebraically to the above equation which was then solved for T. In this manner the temperature was calculated for every ten microvolts from 85° to 273.1° K. and every twenty microvolts from 273.1° to 310° K.

Temperature correction curves of the thermocouple made from wire of the second spool mentioned above were plotted from our table and from Adams's table. The corrections calculated from our table all lie on a perfectly smooth curve while some points calculated from Adams's lie as much as 0.04° off. The deviations of the data in Paper I treated in the same manner are about 0.01°. The e.m.f. of the thermocouple of Giauque is so radically different from that of ours that the corrections were too large to be plotted satisfactorily. Our original intention was to use the data in Paper I in conjunction with Adams's tables to establish the temperature scale. The above results show the advantage we have gained by means of the present investigation.

STANDARD REFERENCE TABLES FOR THE COPPER-CONSTANTAN THERMOCOUPLE.

Part I—Above 0° C.

E.M.F.	Temp. ° K.	Diff.	E.M.F.	Temp. ° K.	Diff.
0	273.1		800	293.567	.499
20	273.624	.524	820	294.066	.499
40	274.148	.524	840	294.564	.498
60	274.672	.524	860	295.062	.498
80	275.195	.523	880	295.559	.497
100	275.717	.522	900	296.055	.496
120	276.239	.522	920	296.551	.496
140	276.760	.521	940	297.046	.495
160	277.280	.520	960	297.541	.495
180	277.800	.520	980	298.035	.494
200	278.319	.519	1000	298.529	.494
220	278.837	.518	1020	299.022	.493
240	279.354	.517	1040	299.515	.493
260	279.870	.516	1060	300.007	.492
280	280.385	.515	1080	300.499	.492
300	280.900	.515	1100	300.991	.492
320	281.414	.514	1120	301.482	.491
340	281.928	.514	1140	301.973	.491
360	282.441	.513	1160	302.463	.490
380	282.953	.512	1180	302.953	.490
400	283.465	.512	1200	303.442	.489
420	283.976	.511	1220	303.931	.489
440	284.487	.511	1240	304.419	.488
460	284.997	.510	1260	304.907	.488
480	285.496	.509	1280	305.394	.487
500	286.015	.509	1300	305.881	.487
520	286.523	.508	1320	306.367	.486
540	287.031	.508	1340	306.853	.486
560	287.538	.507	1360	307.338	.485
580	288.044	.506	1380	307.823	.485
600	288.549	.505	1400	308.307	.484
620	289.053	.504	1420	308.791	.484
640	289.557	.504	1440	309.274	.483
660	290.060	.503	1460	309.757	.483
680	290.563	.503	1480	310.239	.482
700	291.065	.502			
720	291.567	.502			
740	292.068	.501			
760	292.568	.500			
780	293.068	.500			

Part II—Below 0° C.

E.M.F.	Temp. ° K.	Diff.	E.M.F.	Temp. ° K.	Diff.
0	273.1		450	261.163	.269
10	272.838	.262	460	260.893	.270
20	272.576	.262	470	260.623	.270
30	272.314	.262	480	260.353	.270
40	272.052	.262	490	260.083	.270
50	271.790	.262	500	259.813	.270
60	271.528	.262	510	259.542	.271
70	271.265	.263	520	259.271	.271
80	271.002	.263	530	259.000	.271
90	270.739	.263	540	258.729	.271
100	270.476	.263	550	258.458	.271
110	270.213	.263	560	258.187	.271
120	269.950	.263	570	257.915	.272
130	269.687	.263	580	257.643	.272
140	269.424	.263	590	257.372	.272
150	269.161	.263	600	257.099	.272
160	268.897	.264	610	256.827	.272
170	268.633	.264	620	256.554	.273
180	268.369	.264	630	256.281	.273
190	268.105	.264	640	256.008	.273
200	267.841	.264	650	255.735	.273
210	267.576	.265	660	255.462	.273
220	267.311	.265	670	255.188	.274
230	267.046	.265	680	254.914	.274
240	266.780	.266	690	254.640	.274
250	266.514	.266	700	254.366	.274
260	266.248	.266	710	254.091	.275
270	265.982	.266	720	253.816	.275
280	265.716	.266	730	253.541	.275
290	265.450	.266	740	253.266	.275
300	265.183	.267	750	252.991	.275
310	264.926	.267	760	252.715	.276
320	264.649	.267	770	252.439	.276
330	264.382	.267	780	252.163	.276
340	264.115	.267	790	251.887	.276
350	263.848	.267	800	251.610	.277
360	263.580	.268	810	251.333	.277
370	263.312	.268	820	251.056	.277
380	263.044	.268	830	250.779	.277
390	262.776	.268	840	250.502	.277
400	262.508	.268	850	250.225	.277
410	262.239	.269	860	249.947	.277
420	261.970	.269	870	249.669	.278
430	261.701	.269	880	249.391	.278
440	261.432	.269	890	249.113	.278

Part II—Below 0° C.—*Continued.*

E.M.F.	Temp. ° K.	Diff.	E.M.F.	Temp. ° K.	Diff.
900	248.835	.278	1350	236.074	.289
910	248.556	.279	1360	235.785	.289
920	248.277	.279	1370	235.496	.289
930	247.998	.279	1380	235.207	.289
940	247.719	.279	1390	234.917	.290
950	247.440	.279	1400	234.627	.290
960	247.160	.280	1410	234.337	.290
970	246.880	.280	1420	234.047	.290
980	246.600	.280	1430	233.756	.291
990	246.320	.280	1440	233.465	.291
1000	246.040	.280	1450	233.174	.291
1010	245.759	.281	1460	232.883	.291
1020	245.478	.281	1470	232.592	.291
1030	245.197	.281	1480	232.300	.292
1040	244.915	.282	1490	232.008	.292
1050	244.633	.282	1500	231.716	.292
1060	244.351	.282	1510	231.424	.292
1070	244.069	.282	1520	231.131	.293
1080	243.787	.282	1530	230.838	.293
1090	243.504	.283	1540	230.545	.293
1100	243.221	.283	1550	230.251	.294
1110	242.938	.283	1560	229.957	.294
1120	242.655	.283	1570	229.663	.294
1130	242.372	.283	1580	229.338	.295
1140	242.088	.284	1590	229.073	.295
1150	241.804	.284	1600	228.778	.295
1160	241.520	.284	1610	228.483	.295
1170	241.235	.285	1620	228.187	.296
1180	240.950	.285	1630	227.891	.296
1190	240.665	.285	1640	227.595	.296
1200	240.380	.285	1650	227.299	.296
1210	240.095	.285	1660	227.002	.297
1220	239.909	.286	1670	226.705	.297
1230	239.523	.286	1680	226.408	.297
1240	239.237	.286	1690	226.110	.298
1250	238.951	.286	1700	225.812	.298
1260	238.675	.286	1710	225.514	.298
1270	238.378	.287	1720	225.216	.298
1280	238.091	.287	1730	224.918	.298
1290	237.804	.287	1740	224.619	.299
1300	237.516	.288	1750	224.320	.299
1310	237.228	.288	1760	224.021	.299
1320	236.940	.288	1770	223.722	.299
1330	236.652	.288	1780	223.422	.300
1340	236.363	.289	1790	233.122	.300

Part II—Below 0° C.—Continued.

E.M.F.	Temp. ° K.	Diff.	E.M.F.	Temp. ° K.	Diff.
1800	222.822	.300	2250	209.010	.314
1810	222.522	.300	2260	208.696	.314
1820	222.221	.301	2270	208.382	.314
1830	221.920	.301	2280	208.067	.315
1840	221.619	.301	2290	207.752	.315
1850	221.317	.302	2300	207.437	.315
1860	221.015	.302	2310	207.121	.316
1870	220.713	.302	2320	206.805	.316
1880	220.410	.303	2330	206.488	.317
1890	220.107	.303	2340	206.171	.317
1900	219.804	.303	2350	205.854	.317
1910	219.501	.303	2360	205.537	.317
1920	219.197	.304	2370	205.219	.318
1930	218.893	.304	2380	204.901	.318
1940	218.589	.304	2390	204.582	.319
1950	218.285	.304	2400	204.263	.319
1960	217.980	.305	2410	203.944	.319
1970	217.675	.305	2420	203.624	.320
1980	217.370	.305	2430	203.304	.320
1990	217.064	.306	2440	202.983	.321
2000	216.758	.306	2450	202.662	.321
2010	216.452	.306	2460	202.344	.322
2020	216.145	.307	2470	202.022	.322
2030	215.838	.307	2480	201.700	.322
2040	215.531	.307	2490	201.377	.323
2050	215.224	.307	2500	201.054	.323
2060	214.917	.307	2510	200.731	.323
2070	214.609	.308	2520	200.408	.323
2080	214.301	.308	2530	200.084	.324
2090	213.992	.309	2540	199.760	.324
2100	213.683	.309	2550	199.436	.324
2110	213.374	.309	2560	199.111	.325
2120	213.064	.310	2570	198.786	.325
2130	212.754	.310	2580	198.461	.325
2140	212.444	.310	2590	198.135	.326
2150	212.133	.311	2600	197.809	.326
2160	211.822	.311	2610	197.483	.326
2170	211.511	.311	2620	197.156	.327
2180	211.199	.312	2630	196.829	.327
2190	210.887	.312	2640	196.501	.328
2200	210.575	.312	2650	196.173	.328
2210	210.263	.312	2660	195.845	.328
2220	209.950	.313	2670	195.516	.329
2230	209.637	.313	2680	195.186	.330
2240	209.324	.313	2690	194.856	.330

Part II—Below 0° C.—Continued.

E.M.F.	Temp. ° K.	Diff.	E.M.F.	Temp. ° K.	Diff.
2700	194.526	.330	3150	179.236	.350
2710	194.195	.331	3160	178.885	.350
2720	193.864	.331	3170	178.534	.351
2730	193.532	.332	3180	178.182	.352
2740	193.200	.332	3190	177.830	.352
2750	192.868	.332	3200	177.477	.353
2760	192.536	.332	3210	177.124	.353
2770	192.203	.333	3220	176.770	.354
2780	191.870	.333	3230	176.416	.354
2790	191.536	.334	3240	176.062	.354
2800	191.202	.334	3250	175.707	.355
2810	190.868	.334	3260	175.352	.355
2820	190.533	.335	3270	174.996	.356
2830	190.198	.335	3280	174.640	.356
2840	189.862	.336	3290	174.283	.357
2850	189.526	.336	3300	173.936	.357
2860	189.190	.336	3310	173.568	.358
2870	188.853	.337	3320	173.210	.358
2880	188.516	.337	3330	172.851	.359
2890	188.178	.338	3340	172.491	.360
2900	187.840	.338	3350	172.131	.360
2910	187.501	.339	3360	171.770	.361
2920	187.161	.340	3370	171.409	.361
2930	186.821	.340	3380	171.047	.362
2940	186.480	.341	3390	170.685	.362
2950	186.139	.341	3400	170.322	.363
2960	185.798	.341	3410	169.958	.364
2970	185.456	.342	3420	169.594	.364
2980	185.114	.342	3430	169.230	.364
2990	184.772	.342	3440	168.865	.365
3000	184.430	.342	3450	168.499	.366
3010	184.087	.343	3460	168.133	.366
3020	183.744	.343	3470	167.766	.367
3030	183.400	.344	3480	167.399	.367
3040	183.055	.345	3490	167.031	.368
3050	182.710	.345	3500	166.662	.369
3060	182.364	.346	3510	166.293	.369
3070	182.018	.346	3520	165.924	.369
3080	181.672	.346	3530	165.554	.370
3090	181.325	.347	3540	165.183	.371
3100	180.978	.347	3550	164.812	.371
3110	180.631	.347	3560	164.440	.372
3120	180.283	.348	3570	164.067	.373
3130	179.834	.349	3580	163.693	.374
3140	179.585	.349	3590	163.319	.374

Part II—Below 0° C.—Continued.

E.M.F.	Temp. ° K.	Diff.	E.M.F.	Temp. ° K	Diff.
3600	162.944	.375	4050	145.393	.407
3610	162.569	.375	4060	144.985	.408
3620	162.193	.375	4070	144.576	.409
3630	161.818	.376	4080	144.167	.409
3640	161.442	.376	4090	143.757	.410
3650	161.065	.377	4100	143.346	.411
3660	160.687	.378	4110	142.934	.412
3670	160.308	.379	4120	142.521	.413
3680	159.928	.380	4130	142.107	.414
3690	159.548	.380	4140	141.692	.415
3700	159.167	.381	4150	141.276	.416
3710	158.786	.381	4160	140.859	.417
3720	158.404	.382	4170	140.441	.418
3730	158.021	.383	4180	140.022	.419
3740	157.677	.384	4190	139.602	.420
3750	157.255	.384	4200	139.181	.421
3760	156.868	.385	4210	138.759	.422
3770	156.483	.385	4220	138.337	.422
3780	156.097	.386	4230	137.914	.423
3790	155.710	.387	4240	137.490	.424
3800	155.323	.387	4250	137.066	.424
3810	154.935	.388	4260	136.641	.425
3820	154.546	.389	4270	136.215	.426
3830	154.157	.389	4280	135.788	.427
3840	153.767	.390	4290	135.360	.428
3850	153.376	.391	4300	134.931	.429
3860	152.984	.392	4310	134.500	.431
3870	152.592	.392	4320	134.068	.432
3880	152.199	.393	4330	133.635	.433
3890	151.806	.393	4340	133.201	.434
3900	151.402	.394	4350	132.766	.435
3910	151.008	.394	4360	132.330	.436
3920	150.613	.395	4370	131.893	.437
3930	150.017	.396	4380	131.555	.438
3940	149.821	.396	4390	131.016	.439
3950	149.424	.397	4400	130.576	.440
3960	149.026	.398	4410	130.135	.441
3970	148.627	.399	4420	129.693	.442
3980	148.226	.401	4430	129.250	.443
3990	147.824	.402	4440	128.806	.444
4000	147.421	.403	4450	128.361	.445
4010	147.017	.404	4460	127.915	.446
4020	146.612	.405	4470	127.468	.447
4030	146.206	.406	4480	127.020	.448
4040	145.800	.406	4490	126.571	.449

Part II—Below 0° C.—Continued.

E.M.F.	Temp. ° K.	Diff.	E.M.F.	Temp. ° K.	Diff.
4500	126.121	.450	4900	107.044	.505
4510	125.670	.451	4910	106.537	.507
4520	125.217	.453	4920	106.028	.509
4530	124.763	.454	4930	105.517	.511
4540	124.308	.455	4940	105.004	.513
4550	123.852	.456	4950	104.490	.514
4560	123.394	.458	4960	103.974	.516
4570	122.935	.459	4970	103.456	.518
4580	122.475	.460	4980	102.937	.519
4590	122.014	.461	4990	102.416	.521
4600	121.551	.463	5000	101.894	.522
4610	121.087	.464	5010	101.370	.524
4620	120.621	.466	5020	100.835	.525
4630	120.154	.467	5030	100.308	.527
4640	119.686	.468	5040	99.780	.528
4650	119.217	.469	5050	99.250	.530
4660	118.747	.470	5060	98.719	.531
4670	118.276	.471	5070	98.186	.533
4680	117.804	.472	5080	97.651	.535
4690	117.331	.473	5090	97.113	.538
4700	116.856	.475	5100	96.573	.540
4710	116.380	.476	5110	96.030	.543
4720	115.902	.478	5120	95.484	.546
4730	115.422	.480	5130	94.935	.549
4740	114.940	.482	5140	94.383	.552
4750	114.457	.483	5150	93.829	.554
4760	113.973	.484	5160	93.273	.556
4770	113.488	.485	5170	92.715	.558
4780	113.001	.487	5180	92.155	.560
4790	112.513	.488	5190	91.593	.562
4800	112.023	.490	5200	91.029	.564
4810	111.532	.491	5210	90.463	.566
4820	111.039	.493	5220	89.895	.568
4830	110.545	.494	5230	89.325	.570
4840	110.049	.496	5240	88.753	.572
4850	109.552	.497	5250	88.178	.575
4860	109.054	.498	5260	87.600	.578
4870	108.554	.500	5270	87.020	.580
4880	108.052	.502	5280	86.438	.582
4890	107.549	.503	5290	85.853	.585

EFFECT OF TEMPERATURE GRADIENTS.

Giauque, Buffington and Schulze [10] suggest that the irreversible heat flow which is necessarily present in thermo-

[10] *Loc. cit.*

couples may be contributing appreciably to the e.m.f. and if so the amount due to this factor may depend on the character of the gradient. To investigate this a piece of constantan wire was tested by drawing it through liquid air in the usual manner and a portion selected which did not cause a galvanometer deflection corresponding to more than one μV. Without removing the wire from the bath two copper strips $1\frac{1}{2}''$ wide and $2\frac{1}{2}'$ long were clamped on the wire and dipped into the liquid air. This distributed the temperature difference of about 300° over a distance of two feet on one end of the wire while on the other end there was the same temperature difference in a few inches. If temperature gradient affected the e.m.f. noticeably the absolute e.m.f. in the latter case would be different from that observed with equal temperature gradients on either end. No change was observed and furthermore pulling the wire carefully through the copper strips in the latter case did not cause a galvanometer deflection corresponding to more than was observed without them namely, $1\ \mu\ V$. The experiment was repeated with a different piece of wire and the same results were obtained. Hence it is concluded that the temperature gradient per unit length between the "hot" and "cold" junctions of thermocouples does not affect the e.m.f. noticeably *provided* the wire is homogeneous.

The authors wish to express their appreciation to Mr. R. F Deese for the very helpful assistance which he gave them in making the measurements. The authors are also deeply indebted to Mr. W. B. Nottingham for his advice on the photoelectric equipment which played such a vital part in bringing about the success of the project.

Intense Ionization of the Air in Localities often Struck by Lightning. C. DAUZÈRE AND J. BOUGET. (*Comptes Rendus*, June 18, 1928.) In a previous paper the authors advanced the idea that the predilection of lightning for certain places depends upon geological constitution and that there may be a greater ionization of the air in such places than in others less visited by lightning. They now proceed to test their hypothesis by experiment. Using an Elster and Geitel apparatus they recorded the time required for the charge to leak away so that the potential of the cylinder sank from one value to another. From this was calculated the conductivity of the air due to ions opposite in sign to the charge of the cylinder. In strictness ions of both signs should be taken into account but simple apparatus was essential since the observations were conducted in the Pyrenees under difficult conditions. The experiments were uniformly carried out at a distance of 15 cm. above the surface of the earth. Quiet weather was preferred, but measurements were made also just before storms. It was sought to make the observations at places of the same altitude and not far apart and also with short intervals of time intervening. In one case there was immediately before a heavy rain a conductivity of 1.24×10^{-4} electrostatic units at a place where lightning fell whereas it was only about 70% as much at a point 200 m. southeast of the first or at a point at the same distance northeast of the first. The investigators examined many places and found conditions analogous to those just cited. They draw the conclusion that "there are places where the ionization of the air near the ground is constantly more intense than at neighboring places at the same altitude and under the same physical conditions. The spots frequently struck by lightning coincide with the places of maximum ionization. Their position depends on the geological composition of the ground." When a storm comes the ascending current of air carries up ions from the place near the earth where they have been produced in the greatest numbers. Thus there is a somewhat vertical column of ions that acts like a lightning rod in attracting the discharge and providing it with a path of small resistance.

Places of intense ionization will assist the leakage from transmission lines, influence the progress of thunder storms and have some connection with atmospherics in radio transmission. The electrometer can render further aid in furnishing information relating to the composition of the geological strata beneath. It seems possible that the electrometer for the measurement of ionization at the surface of the earth may associate itself with the other physical instruments that furnish information of the inaccessible parts of the earth. G. F. S.

THE MEASUREMENT OF SOUND ABSORPTION COEFFICIENTS.*

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INTEREST in quantitative data on the absorption of sound by materials arises almost wholly in connection with the problems of architectural acoustics. In the thirty years that have elapsed since the pioneer work in this field by Professor Wallace C. Sabine, it has come to be almost universally recognized that reverberation is by far the most important factor affecting hearing conditions within rooms. The control of reverberation lies in providing the proper areas of sound absorbent surface of known absorbing efficiency. The determination of the relative efficiencies of various materials used for this purpose becomes thus a matter of considerable practical importance.

At the present time there is rather a wide divergence in the values of the "absorption coefficients" obtained by measurements on the same materials by different observers, using in some instances the same, and in other instances different, methods. Apart from the intrinsic difficulties of sound measurements in general, and the consequent lack of precision in such measurements, it appears that there are also hitherto unrecognized factors that affect the measured values of sound absorption coefficients of materials, the neglect of which doubtless accounts for much of this difference of results. A careful study of all the factors that affect these measured values is therefore of importance both for scientific and practical reasons.

From time to time during the last nine years, investigations of various aspects of this problem have been made at the Riverbank Laboratories. Within the last year, more intensive work has been done, and some of the results are here presented with the hope that the reason for the differences just mentioned may appear, and that a fuller knowledge of

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the various factors involved may lead to a somewhat more standardized practice in carrying out laboratory measurements of sound absorption coefficients.

THEORY OF THE REVERBERATION METHOD.[1]

For the sake of definiteness in discussion, let us assume within a room of volume V, a source of sound whose acoustic output is E ergs per second. The rate of change of the average sound intensity due to absorption at the bounding surfaces of the room is given by the equation

$$\frac{dI}{dt} = -AI, \tag{1}$$

I being expressed in ergs per cubic meter. Then the rate of change of energy density in the room with the source speaking is

$$\frac{E}{V} - AI = \frac{dI}{dt}. \tag{2}$$

In the steady state, in which production at the source is just balanced by dissipation at the boundaries, the intensity I_0 is given by the expression

$$I_0 = \frac{E}{AV}. \tag{3}$$

Suppose the source is sounded for a time sufficiently great that the steady state is practically reached, and then suddenly stopped, and that the time, t, is measured from this instant. Integrating (1) and supplying the constant $I = I_0 = E/VA$, when $t = 0$, we have

$$At = \log_e \frac{I_0}{I} = \log_e \left(\frac{E}{VAI} \right).$$

If t_1 is the time required for the energy density to decrease to i, the threshold of audibility, then

$$At_1 = \log_e \left(\frac{E}{VAi} \right). \tag{4}$$

[1] Throughout the discussion, the nomenclature is that given by Wallace Sabine. The analysis follows that given by Franklin, Jaeger, Eckhardt and Buckingham.

We may express equation (4) in terms of what has been called the "total absorbing power" of the room and defined by the expression

$$a = \Sigma a_1 s_1 + a_2 s_2 + a_3 s_3 + \text{etc.}, \tag{5}$$

in which s_1 is the area in square meters of the surface whose *absorption coefficient* is a_1, the summation being extended to include all the surfaces in the room at which sound is reflected. If S is the total surface, a/S is the mean absorption coefficient. The average number of reflections per second is v/p, v being the velocity and p the mean free path of any "soundelement" between reflections. By methods of the kinetic theory, p has been shown to be $4V/S$, so that

$$A = \frac{v}{p} \cdot \frac{a}{S} = \frac{av}{4V}. \tag{6}$$

Equation (4) thus becomes

$$at_1 = \frac{4V}{v} \log_e \left(\frac{4E}{avi} \right) = 9.2 \frac{V}{v} \left(\log_{10} \frac{4E}{avi} \right). \tag{7}$$

Certain assumptions are involved in the derivation of the foregoing. Some of these have been set forth in an excellent critical paper on the subject by Dr. Edgar Buckingham.[2] Since the discrepancies in experimental results may be in part due to the doubtful validity of some of these assumptions, it may be well to state them explicitly:

1. It is assumed that E, the acoustic energy generated per second at the source is constant and is not influenced by the reaction of the standing wave system in the room.

2. In the steady state, the sound energy in the room is uniformly distributed in space, and is diffuse as regards direction of propagation. This assumption ignores the inequalities of energy distribution due to interference.

3. The level of the steady state intensity is that at which the rate of emission of acoustic energy from the source is equal to the rate of absorption at the bounding surfaces.

4. The time rate of change of intensity at any instant due

[2] "Theory and Interpretation of Experiments on the Transmission of Sound through Partition Walls," Sci. Paper, Bur. of Strand. 506.

to absorption is strictly proportional to the intensity at that instant.

5. At any given surface, a certain fraction of the energy is absorbed at each reflection of sound from that surface. This fraction, the absorption coefficient, is constant for a pure tone of given pitch, and is not a function of intensity. It may or may not be a function of the angle of incidence. It is only assumed that as a function of the angle of incidence it has a mean statistical value for sound incident at the random angle of incidence assumed in the theory.

6. The contribution to the total absorbing power of the room by any surface is the product of the area and the absorption coefficient of that surface. This assumes that the actual energy absorbed per unit area is uniform over the entire surface. It follows from these assumptions that duration of audible sound after the source has ceased should be independent of the position of the source in the room, and that the energy absorbed by a given surface is independent of the position of the surface.

7. The dissipation of sound energy in the atmosphere, due to the viscosity of the air itself, is negligibly small in comparison with the dissipation at the boundaries of the room.

Turning now to the implications of equation (7): (1) It is apparent that the actual time t_1 as measured by any observer for sound from a given source will vary linearly with the logarithm of the acoustic power of the source; (2) that all other conditions being constant, the measured reverberation time is a linear function of the logarithm of the sensitivity (reciprocal of the threshold intensity) of the ear of the observer. (3) Knowing a, for any given room, equation (7) affords a means of evaluating E/i for any given constant source of sound and any observer, from the observed reverberation time. From this, one may plot a as a function of t, for the given room, source, and observer. From such a curve the total absorbing power of the room when altered by the introduction of absorbing material is known by observation of the reverberation time in the altered room.

APPLICATION OF THE METHOD.

The Sound Chamber.

Figure 1 gives a view of the Sound Chamber of the Riverbank Laboratories. As sources of sound, a set of 73 open

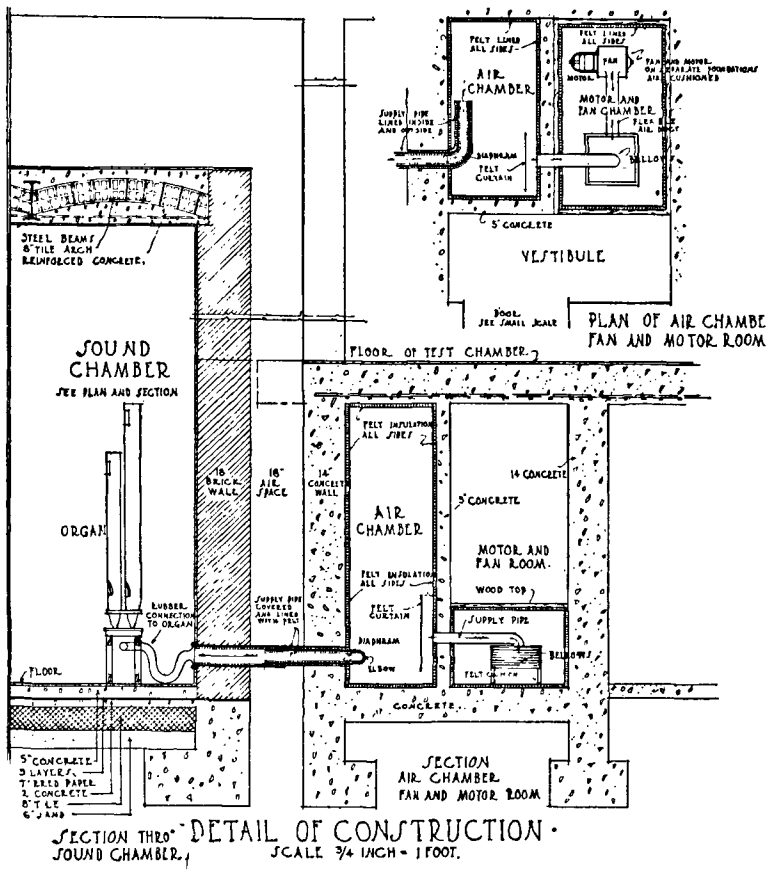
FIG. 1.

View of Sound Chamber.

diapason organ pipes ranging in pitch from 64 to 4,096 vibrations per second is employed. Air pressure for blowing these

pipes is supplied by means of an organ blower located outside the Sound Chamber. Figure 2 shows in plan and section the arrangement by which the noise of the blower is prevented from passing through the air passage into the Sound Chamber.

FIG. 2.



Detail showing Sound Insulation between Blower Room and Sound Chamber

The dimensions of the room are 8.25 x 5.80 x 6.05 meters, giving a volume of 289 cubic meters. From this a deduction of 3 cubic meters is made for the observer's cabinet and the organ, giving an effective volume of 286 cubic meters. Large steel reflectors, mounted on a vertical axis in the center of the

room and extending through the ceiling, are rotated noiselessly at the rate of 1 revolution in two minutes, for the purpose of constantly shifting the standing wave system during the course of the measurements. The walls are of brick, 18″ thick, plastered and finished with hard plaster, and painted with enamel paint.

During the course of the observations, the observer is seated in a wooden cabinet with only his head exposed. This cabinet is mounted on rollers, and a standard set of observations includes five observations for each tone, with the observer's cabinet in each of five standard positions—twenty-five observations in all. Due to the fact that the Sound Chamber is partially below ground and is not heated directly by artificial means, the temperature is fairly constant—58° to 68° F. the year round. The acoustic conditions in the room remain fairly constant. Thus for the tone C_4, 512 vibs./sec., the mean of twenty sets of observations taken by one observer over a period of six years, and selected at random from the notes covering the period was 14.36 seconds with a maximum deviation from the mean of. 6 secs. and an average deviation of .31 secs. The room is naturally sensitive to slight changes of absorbing power. The presence of a single person in the room outside the cabinet will lower the reverberation time at C_4 approximately 1.5 seconds. Changes in the relative humidity produce a marked effect upon the dissipation of sound of frequencies higher than 2,000 vibs./sec. Lowering the relative humidity increases the rate of absorption. No adequate explanation has yet been found for this phenomenon.

Three openings, one 1.83 x 2.44 meters, the other two .91 x 2.44 meters, are provided with solid steel doors. Test samples can be mounted on partitions built into these openings, so that by closing these doors, the Sound Chamber is reduced to its standard conditions. Portable samples are carried into the Sound Chamber and laid on the floor for tests.

The actual procedure in the measurement of absorption coefficients of a material is as follows: With the observer's cabinet in one of the five positions, a set of five observations of the time of reverberation for each of the test tones used is made, with the room in its standard condition. Immediately afterwards, the room is modified exposing test sample and the

set of observations is repeated. The observer's cabinet is moved, and the two sets of observations are repeated, in the reverse order. Alternating in this way, to eliminate any possible effects of fatigue, the reverberation times for the two conditions with the observer in each of the five positions in turn are measured. It may be said that effects of ear fatigue as shown by reduced times are seldom observed, except after prolonged work of this kind. The change in absorbing power of the room due to the introduction of the test panels is computed from the measured reverberation times by means of equation (7).

Calibration of the Sound Chamber.

The use of equation (7) necessitates first the evaluation by some means of a, the total absorbing power of the Sound Chamber in its standard condition, or of E/i for a given source and observer. Given a, $\log E/i$ for any given source and observer, is determined directly by the equation and the observed value of t_1, which being known, enables us in turn to compute a', the absorbing power for the observed value of the time under any other condition of the room. An alternative method of evaluating a is by means of a sound source whose acoustic output can be varied in known ratios. Let t_1 = time for a source of output E_1 and t_2 = time for an output E_2. Then

$$a = \frac{9.2\,V}{v}\left(\frac{\log E_1/E_2}{t_1 - t_2}\right). \tag{8}$$

This method of determining absolutely the total sound absorbing power of a room is originally due to Wallace Sabine in what he called the "Four Organ Pipe Experiment." Reverberation times with one, two, three, and four organ pipes in turn were determined. The total acoustic output of the four pipes was assumed to be four times that of one pipe. The value of a was thus determined for the large lecture room of the Jefferson Physical Laboratory for each of the seven C's from 64 to 4,096 vibs./sec. In his subsequent work, the rates of emission of his organ pipes were measured in terms of his own threshold of hearing by timing in this particular room.[3]

[3] It is worth noting in passing that the method here outlined is not based upon an assumed absorption coefficient of unity for an "open window," and that

It was assumed that these pipes when blown at the same pressure in another room would emit sound at the same rate, and thus afford a means of determining the total absorbing power of the second room. This method of calibration was employed for the Constant Temperature room at Harvard, in which most of Professor Sabine's measurements of absorption coefficients were made.

This method was employed in the original calibration of the Sound Chamber in this laboratory. With the view of making this calibration, two sets of open diapason pipes had been timed by Professor Sabine and by his assistant, Mr. John Connors, in the large lecture room of Jefferson Laboratory. The times for these same pipes by the same observers, in the new Sound Chamber were to be used in determining its absorbing power, to serve as the basis of future work here. Unfortunately the chain of complete continuity with the earlier work at Harvard was broken by Professor Sabine's untimely death in January, 1919. In taking up the work here, the two sets of pipes for which the acoustic outputs in terms of Professor Sabine's threshold intensity were known, were used in determining the absorbing power of the Riverbank Sound Chamber. This procedure neglected the difference that may exist in the values of the threshold intensities for normal hearing persons. At the time, the fact that this variation is appreciable had not been established. Fortunately, the form of the reverberation equation, involving as it does the logarithm of the absolute sensitivity, is such that only rather extreme variations in the sensitivity of the observer's hearing will effect any marked difference in the measured values of absorption coefficients. Thus under the experimental conditions prevailing here, if a variation in sensitivity by a factor of 10 between two observers be ignored, the difference in the absorption coefficient of a given material as computed from careful observation by the two will differ by about 10 per cent. of itself.

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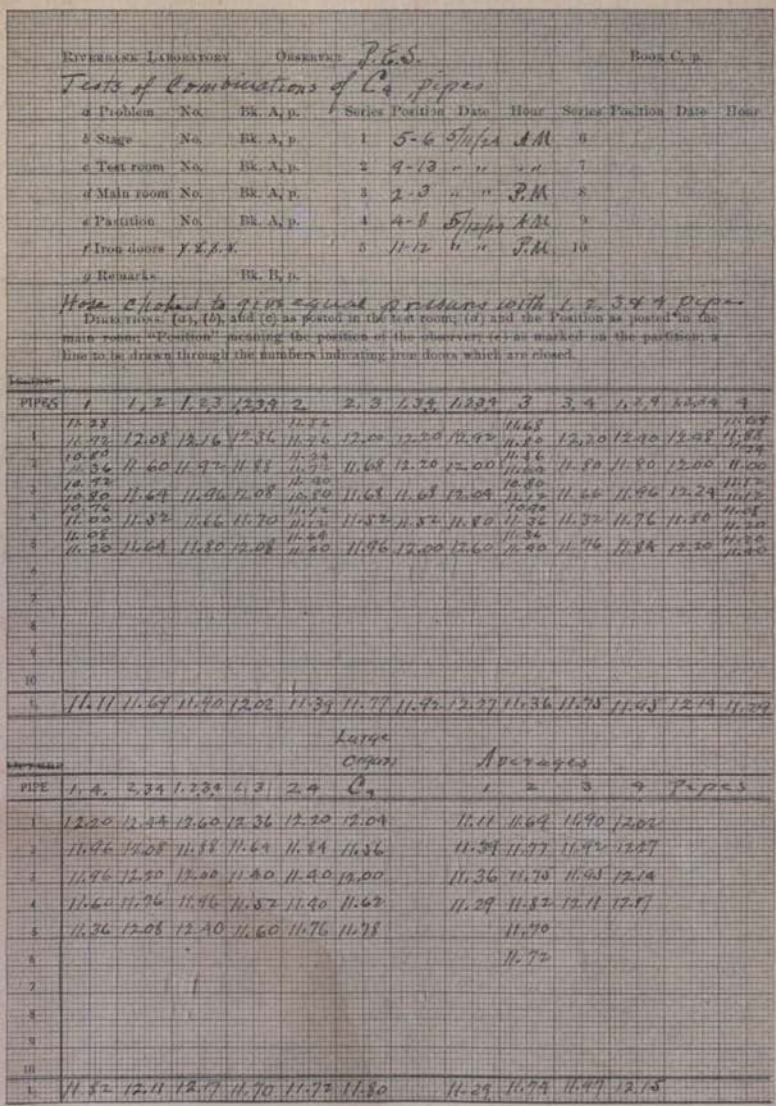
the coefficients given in Sabine's later papers are not determined on this assumption. As will appear later, the phenomenon of diffraction complicates the passage of sound out through an opening, so that it is only under special conditions as to size and dimensions that an open window may be treated as an ideal absorber with a coefficient of unity.

FIG. 3.



RIVERBANK LABORATORY. OBSERVER P.E.S. BOOK C, p.

Tests of Combinations of C_4 pipes

a Problem	No.	Bk. A, p.	Series	Position	Date	Hour	Series	Position	Date	Hour
b Stage	No.	Bk. A, p.	1	5-6	5/11/24	A.M.	6			
c Test room	No.	Bk. A, p.	2	9-13	" "	" "	7			
d Main room	No.	Bk. A, p.	3	2-3	" "	P.M.	8			
e Partition	No.	Bk. A, p.	4	4-8	5/12/24	A.M.	9			
f Iron doors	X.X.X.X.		5	11-12	" "	P.M.	10			
g Remarks		Bk. B, p.								

Hose choked to give equal pressures with 1, 2, 3 & 4 pipes

DIRECTIONS: (a), (b), and (c) as posted in the test room; (d) and the Position as posted in the main room; "Position" meaning the position of the observer; (e) as marked on the partition; a line to be drawn through the numbers indicating iron doors which are closed.

PIPES	1	1, 2	1, 2, 3	1, 2, 3, 4	2	2, 3	1, 3, 4	1, 2, 3, 4	3	3, 4	1, 2, 4	1, 2, 3, 4	4
1	11.28 11.72	12.08	12.16	12.36	11.52 11.76	12.00	12.20	12.92	11.68 11.80	12.20	12.40	12.48	11.08 11.88
2	10.80 11.36	11.60	11.92	11.88	11.24 11.72	11.68	12.20	12.00	11.56 11.60	11.80	11.80	12.00	11.24 11.00
3	10.72 10.80	11.64	11.96	12.08	11.40 10.80	11.68	11.68	12.04	10.80 11.12	11.66	11.96	12.24	11.12 11.12
4	10.76 11.00	11.52	11.66	11.70	11.12 11.12	11.52	11.52	11.80	10.90 11.36	11.32	11.76	11.80	11.08 11.20
5	11.08 11.20	11.64	11.80	12.08	11.24 11.40	11.96	12.00	12.60	11.36 11.40	11.76	11.84	12.20	11.20 11.40
6													
7													
8													
9													
10													
L	11.11	11.69	11.90	12.02	11.39	11.77	11.92	12.27	11.36	11.75	11.95	12.14	11.29

PIPE	1, 4	2, 3, 4	1, 2, 3, 4	1, 3	2, 4	Large Organ C_4	Averages 1	2	3	4	Pipes
1	12.20	12.44	12.60	12.36	12.20	12.04	11.11	11.69	11.90	12.02	
2	11.96	12.08	11.88	11.64	11.84	11.56	11.39	11.77	11.92	12.27	
3	11.96	12.20	12.00	11.40	11.40	12.00	11.36	11.75	11.95	12.14	
4	11.60	11.76	11.96	11.52	11.40	11.62	11.29	11.82	12.11	12.17	
5	11.36	12.08	12.40	11.60	11.76	11.78		11.70			
6								11.72			
7											
8											
9											
10											
L	11.82	12.11	12.17	11.70	11.72	11.80	11.29	11.74	11.97	12.15	

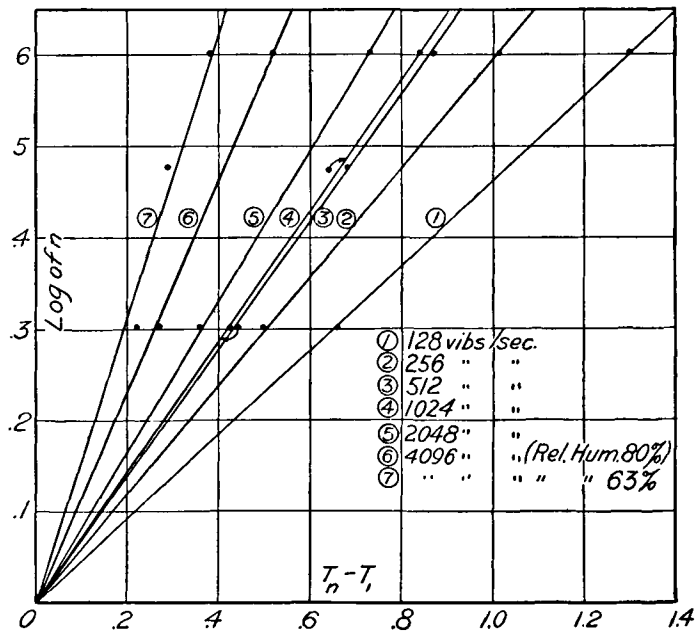
Sample data sheet for Four Organ Calibration.

Comparison of timings by the writer and by Mr. Connors under the same conditions in the Constant Temperature room at Harvard, in 1919, gave no evidence of a marked difference in sensitivity. so that professor Sabine's values of E/i for the two sets of pipes were adopted and the absorbing power of the Sound Chamber for the seven tones at octave intervals from 64 to 4,096 vibs./sec. was determined.

The work on the absolute sensitivity of normal ears, done in this laboratory by Kranz, and by Fletcher and others at the Bell Telephone Laboratories, showed that the assumption of a negligible difference in acuity of hearing between normal hearing persons might be a source of appreciable error, particularly in a room of low absorbing power and long reverberation. Accordingly in 1925, an independent calibration of the total absorbing power of the Sound Chamber by the multiple source method was made. Four small organs, each provided with six C pipes, 128 to 4,096 vibs./sec., were set up in the Sound Chamber with a special keyboard in the observer's cabinet by means of which the four pipes of each pitch could be made to speak singly and in all possible combinations. Air pressure was supplied from the blower for the large organ, the pressure being controlled by throttling so that the speaking pressure was the same whether one or four pipes were speaking. The Sound Chamber was in its standard condition, save for the presence of the four small organs and their equipment. The organs were arranged at the four corners of a square 2.8 meters on a side. The steel reflectors were kept moving during the measurements. Each combination for each pitch was timed with the observer in each of the five standard positions. The four pipes were carefully timed to unison before each set of observations. Preliminary experiments showed that only a slight difference in pitch between the pipes of a given combination produced a marked difference in the measured reverberation time. Figure 3 shows a typical data sheet. Each numerical value represents the mean of five separate observations, giving a total of 575 timings. Obviously, the method calls for a large number of observations made under carefully controlled conditions, in order to give even a fair degree of precision in the results. Three sets of determinations similar to that shown were made for the tone C_4, to establish the linear

relation between the difference in time and the logarithm of the number of pipes, and from that experience it appeared that a better procedure for the remaining pipes was to concentrate on the timings of one, four, and all the combinations of two pipes. In Fig. 4 the data for the six tones are plotted.

FIG. 4.



Increase in Reverberation Time vs. the logarithm of the number of pipes speaking

At the end of the experiment, using the pipes calibrated at Harvard by Professor Sabine, reverberation times first with the four organ pipe apparatus in, and then out of, the room were carefully determined, in order to calculate the value of the total absorbing power of the Sound Chamber in its standard condition. In Table 1, T and T', a and a', give values of these times and the corresponding absorbing powers without, and with, the four organs in the room. a' is determined from the slopes of the lines shown in Fig. 4-a is computed by the equation

$$a = \frac{7.70}{t}\left(\frac{\log 4E}{a'vi}\log - t'/t\right). \tag{8}$$

In the columns headed "W.C.S." the corresponding values as computed from Professor Sabine's values of E/i are shown. In view of all the circumstances, the agreement is probably as close as could be expected.

TABLE I.

Sound Chamber Calibration, Four Organ Pipe Method.

Tone.	T	T'	a'	a		$\log E/i$		Diff.
				P.E.S.	W.C.S.	P.E.S.	W.C.S.	
C_2	14.70	12.58	3.56	3.09	3.44	8.31	8.85	−10%
C_3	14.50	12.42	4.67	4.03	3.71	10.14	9.33	+8.6%
C_4	14.25	11.94	5.38	4.55	4.17	11.01	10.09	+8.9%
C_5	12.80	11.32	5.58	4.98	4.73	10.91	10.28	+5.3%
C_6	9.86	9.16	6.34	5.92	5.86	10.27	10.05	+1.0%
[1]C_7	5.63	5.29	8.90	8.40	7.83	8.98	8.60	+4.0%
[2]C_7		4.86	12.2			8.84		

[1] Relative humidity 80 per cent.
[2] Relative humidity 63 per cent.

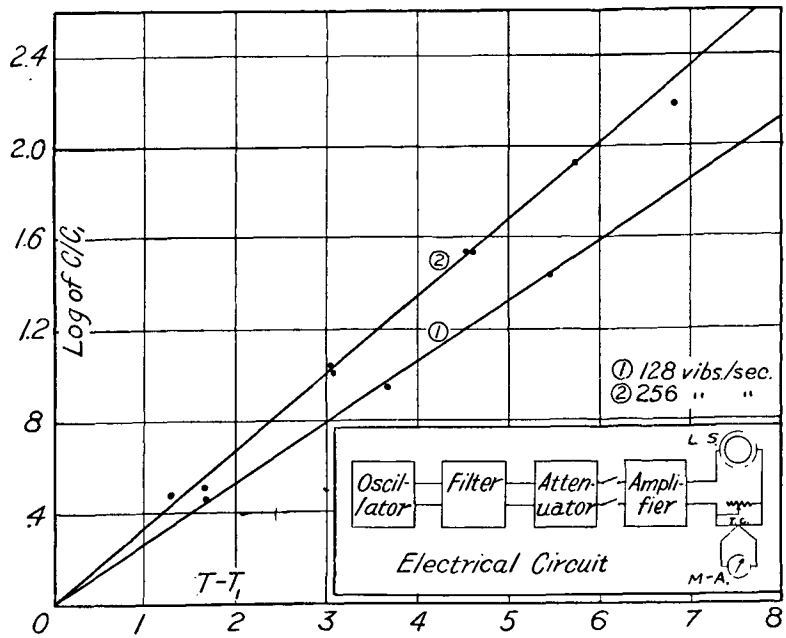
Loud Speaker Calibration.

In October, 1927, a third calibration of the Sound Chamber was undertaken using a vacuum tube oscillator supplying alternating current to a radio loud speaker as a source of sound. The experimental arrangement is shown in Figure 5. All of the electrical apparatus excepting the amplifier and loud speaker was operated and controlled by an assistant outside the Sound Chamber. The key which breaks the current was operated through a relay by means of the mechanism which starts the stop-watch timer. The loud speaker was of the moving coil, free-edge-cone type. The thermo-couple with its shunts was in series with the moving coil. The thermocouple and micro-ammeter with the various shunts used were calibrated with direct current meters for a current range of 0.4 to 250 mil-amperes. For each tone, a low pass electrical filter with a cut-off frequency lower than the first harmonic of the test-tone was used, thus giving a sinusoidal current supply to the loud speaker.

In Figs. 5 and 6, the logarithm of the ratio of the current through the loud speaker to the minimum current is plotted as a function of the difference between the corresponding

reverberation times. Each point represents the mean of 50 observatoins of current and time, under conditions essentially as described under the Four Organ calibration, the loud speaker being substituted for the four organs. Observations for the tones C_3, C_4, C_5, and C_6, (256–2,048 vibs./sec.) were first made in October and November, 1927, and again in

FIG. 5.

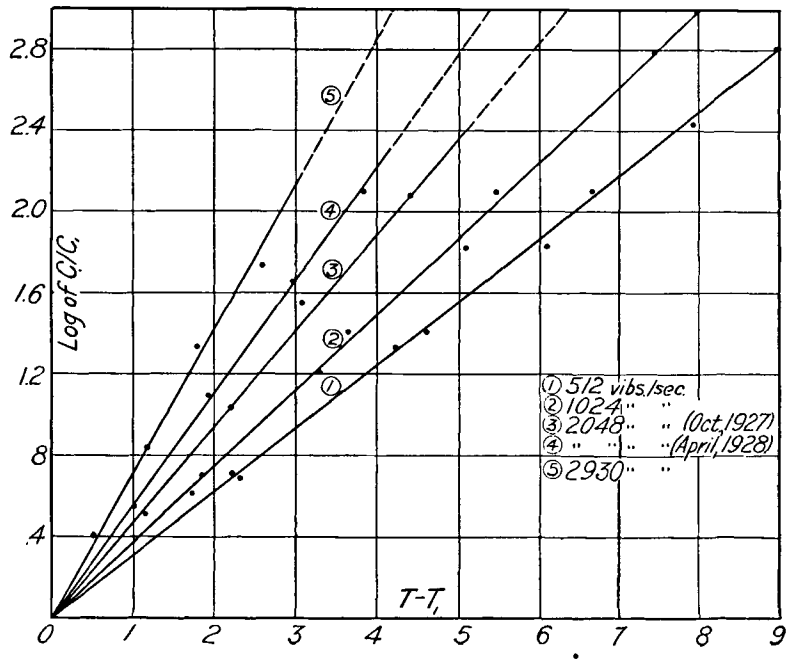


Increase of Reverberation Time vs. the logarithm of loud-speaker current ratio, at 128 and 256 vibs/sec.

April, 1928. With the single exception of C_6, the data in the two experiments were sufficiently congruent that all the points fell very close to a single straight line. In the case of C_6, although each set of points showed a linear relation between the logarithm of the current ratios and the difference in reverberation times, yet the slopes of the two lines for this particular frequency *do not* agree. The later experiment shows an apparent rate of dissipation of sound energy 17.5 per cent. greater than the earlier one, yet the reverberation time using the organ pipe showed only a slight difference and that in the

opposite sense. Since the acoustical conditions were essentially identical in the two experiments, the difference must be ascribed to a difference in the performance of the loud speaker and suggests that as a source of sound for work of this character its constancy can not be relied upon with entire confidence.

FIG. 6.



Increase of Reverberation Time vs. logarithm of current ratio.

We shall for the moment assume that the acoustical output of the loud speaker is proportional to the square of the current. Then

$$E = kC^2$$

$$a' = 2 \times 7.70 \left(\frac{\log C_n/C_1}{t_n - t_1} \right) \tag{9}$$

or a', the absorbing power of the room at any frequency with the loud speaker in is 15.4 times the slope of the corresponding line, (Figs. 5 and 6). In Table 2 are given the results of the loud speaker measurements. Here, as in Table 1, the T and

T' are the values of the reverberation times, using the organ pipes without, and with, the loud-speaker apparatus in the Sound Chamber.

TABLE 2.

Sound Chamber Calibration Loud Speaker Method.

Tone	T	T'	a'	a	Log $E'i$ (Organ pipe)
C_2	14.70	14.58	4.07	4.03	10.26
C_3	14.58	14.24	5.18	5.07	12.22
C_4	14.25	13.43	4.81	4.56	11.01
C_5	12.80	11.70	5.75	5.32	11.35
$^1C_6$	9.80	9.35	8.55	8.15	13.19
$^2C_6$	9.60	9.24	7.28	7.02	11.54
$F\#_6$	7.18	6.57	10.90	9.94	12.30

[1] October, 1927.
[2] April, 1928.

Comparing Tables 1 and 2, it is seen that the loud-speaker method gives uniformly higher values for a than the Four Organ method, and that the latter gives values which are intermediate between those obtained using Professor Sabine's values of E/i, and the loud-speaker values. In view of the fact that fewer assumptions are involved in the Four Organ method than in either of the other two, and that these assumptions are the same as those upon which the earlier measurements at the Jefferson Physical Laboratory were made, the results of the Four Organ measurements are taken as the values of the absorbing power of the Riverbank Sound Chamber, and absorption measurements are based upon these values. At the tones 512 and 1,024, the agreement between the results of the Four Organ and the Loud-speaker methods is fairly close.

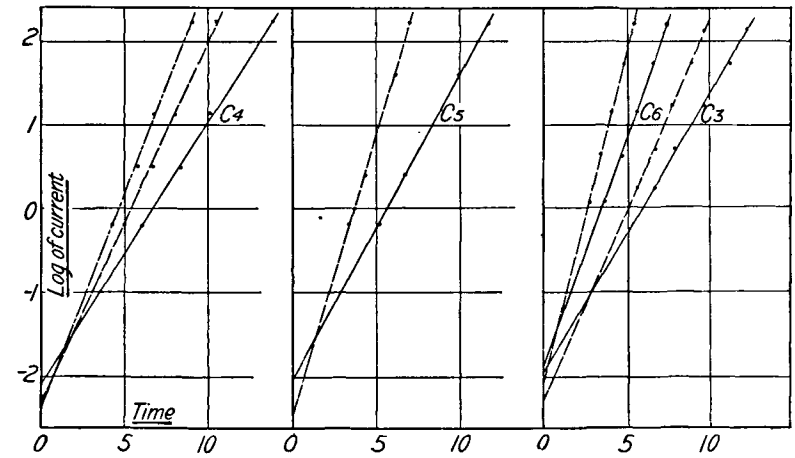
Absorption Measurements Using Loud-Speaker Source.

In Fig. 7*a* are shown the data for the reverberation times at 512 vibs./sec. at different current values in the loud speaker, (*a*) in the empty Sound Chamber, (*b*) with 48 square feet of each of two highly absorbent materials. These data afford two independent means of computing the absorption coefficient of each of the two materials.

(1) Assuming that under fixed room conditions the acoustic output of the loud speaker, and hence the average intensity

in the steady state, is proportional to the square of the loud speaker current, the values of a', a'', and a''', total absorbing power of the room in each of the three conditions may be computed by the equation.

FIG. 7a, 7b, 7c.



Logarithm of loud-speaker current vs. Reverberation time at different frequencies with and without absorbing material in the room.

$$(10) \qquad a' = 2 \times 7.7 \log_{10} \frac{C_1'/C_2'}{t_1' - t_2'} = 2 \times 7.7m,$$

where m is the slope of the straight line representing the experimental points.

(2) Assuming equal acoustic outputs at equal currents under two room conditions, a'' may be computed from a', t', and t'', t' and t'' being the times at a given current value, before and after the introduction of the absorbent material, respectively

$$a't' = 7.70 \log\left(\frac{4E}{a'vi}\right),$$

$$a''t'' = 7.70 \log\left(\frac{4E}{a''vi}\right).$$

Eliminating E/vi, we have

$$(11) \qquad a'' = \frac{1}{t''}\left(a't' - 7.7 \log \frac{a''}{a'}\right).$$

Equation 11 may be solved by a series of approximations starting with the approximation $a't' = a''t''$ and putting successively the approximate values of a'' in the logarithmic correction term.

TABLE 3.

Computed by	a'.	a''.	a'''.
Equation (10)	4.83	6.56	7.80
" (11) log C = 2.2	"	6.21	7.26
" (11) " " = 1.0	"	6.05	7.12
" (11) " " = .0	"	5.91	6.82
" (11) Organ pipe	"	6.63	7.69

Table 3 gives the values of a', a'', and a''' at 512 vibs./sec. computed, in the ways and from the data, indicated:

Figures 7*b* and 7*c* show the results of similar experiments at other frequencies made on the more absorbent of the two materials. Using equations (10) and (11) one may compute the absorption coefficients for this material at various frequencies. Table 4 gives the results of this computation, using the values for the times for different loud-speaker currents, and also for the organ pipes under the same room conditions. The values of a' for the empty room are taken from the slopes of the straight lines shown.

TABLE 4.

Source.	Computed from	Coefficients.			
		C_3.	C_4.	C_5.	C_6.
Loud Speaker	Equation 10	.44	.68	1.06	.85
Loud Speaker (log c = 2.2)	" 11	.28	.58	.79	.70
Loud Speaker (log c = 0)	" 11	.17	.44	.58	.56
Organ Pipes	" 11	.35	.65	.79	.75

The coefficients computed from these data and in the manners indicated do not agree, so that we see here an example of the discrepancies already referred to as existing between the values obtained by different observers. Moreover, the computed value of the absorption coefficient at any frequency is less for small than for large loud-speaker currents. Obviously there is something wrong either with our fundamental theory or with some of the assumptions made in applying it, using the loud speaker as a source of sound.

Inspecting Figs. 7*a*, 7*b*, and 7*c*, one notes that, if the straight line representation of the experimental points be extrapolated, the line for the deadened room cuts the axis of zero time at a lower value of the current than does the line for the bare room in every case. Now a zero reverberation time means that the initial average intensity is the minimum audible intensity. We thus arrive at the conclusion that, granting all the assumptions made, less current is required to produce a sound of threshold intensity in the deadened room than in the reverberant room. This conclusion contradicts the implication of Equation 10, for putting t' and t'' each equal to zero, we have

$$\log \frac{E'}{va'i} = \log \frac{E''}{va''i} = 0,$$

whence

$$\frac{E'}{a'} = \frac{E''}{a''}$$

or, for the threshold intensity the acoustic output of the source should be proportional to the total absorbing power of the room, which would call for larger current with greater absorbing power.

Two solutions of the difficulty suggest themselves. The first, is that the absorption coefficient of the material, considered as a function of the intensity of the sound is not a constant, but a quantity which increases with increasing intensity. The figures in Tables 3 and 4 would appear to justify this explanation. This would lead to the conclusion that the experimental points of Figs. 7*a*, 7*b*, and 7*c* should be represented not by straight lines but by curves slightly concave upward. Such an hypothesis while explaining the results of the present experiments, would also necessitate a modification of the whole Reverberation Theory, which, as we have seen, is based upon the assumption of an absorption coefficient independent of intensity. In the present state of our knowledge as to the mechanics of the absorptive process, the possibility of absorption being a function of intensity must be admitted. On this hypothesis, reverberation measurements based on the variation of the acoustical output of the source would always, as in the present instance, lead to

higher values of the absorption coefficient than those based on the application of Equation 11, with a source of fixed acoustic output. Likewise, coefficients obtained using a powerful source will be greater than when a weak source is employed.

The second point calling for examination is the assumption that equal loud-speaker currents represent equal sound outputs under altered room conditions.[4] Take the extreme case, and consider that the walls are perfectly reflecting and that no sound energy is dissipated in the room. One remembers at the same time that the standing wave system is being continually shifted by the motion of the large reflectors. In the steady state, under these conditions we must consider the loud speaker and the room as a single closed system from which no energy is radiated, and hence one in which no part of the electric power input would be used in generating sound. The acoustic impedance at the diaphragm is zero, and the power required to drive it is the same as though it were driven in vacuo. Now suppose an opening to be made in the Sound Chamber from which sound is radiated, or what is equivalent, that an absorbing surface be introduced. The sound energy of the system is no longer conserved, total impedance at the diaphragm is increased, and to maintain the same current, a higher driving voltage is needed than in the perfectly reflecting room. As a matter of fact, these considerations were not in mind when the experiments were performed. Voltage measurements were not made, the attenuator being adjusted only to give equal currents with and without the absorbent material present. Quantitatively, the importance of this effect depends upon relative magnitudes of the added acoustical impedance due to the presence of the absorbing material, and the sum of the motional impedance of the diaphragm and the electrical impedance of the driving circuit. In the type of loud speaker used in these experiments, the whole acoustical impedance is very probably less than 5 per cent. of the total impedance, so that while qualitatively the effect here considered may be adduced to explain the results, yet quantitatively it would seem quite insufficient.

Summed up, it is to be said that the present experiments

[4] See W. C. Sabine, "Collected Papers on Acoustics," page 278. Crandall, "Theory of Vibrating Systems and Sound," page 222.

with the loud speaker leaves us in a state of uncertainty either as to the unqualified validity of the Reverberation Theory or as to the propriety of the assumptions usually made in the use of the loud speaker as a sound source, while serving the very useful purpose of showing a possible source of discrepancy between the results obtained by different observers. The whole subject of the application of telephonic electrical measurements to problems of architectural acoustics calls for very careful fundamental research.

The inconclusive results of the investigation with the loud speaker up to this point are therefore presented in the nature of a "progress report" with the hope that the facts brought out may be useful to other workers in the field of acoustical measurements. The subject is being still farther studied in this laboratory. Meanwhile, adhering to the methods and assumptions made by W. C. Sabine, the Four Organ Calibration of the Sound Chamber, and organ pipe sources are retained as giving results that are congruent with those now used in practice.

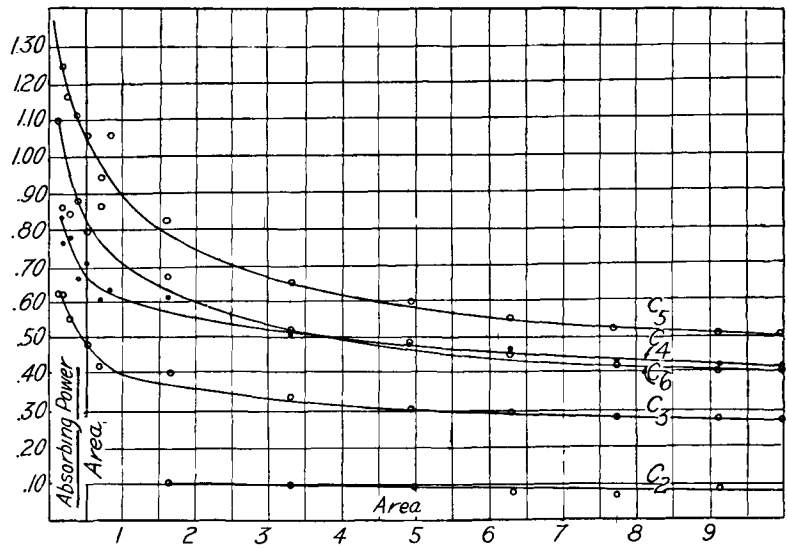
AREA EFFECTS IN ABSORPTION MEASUREMENTS.

In an investigation conducted in 1922, upon the absorption of impact sounds, it appeared that the increase of absorption of sounds of this character was not strictly proportional to the area of the absorbent surface, introduced into the Sound Chamber, and that small samples showed markedly greater absorption per unit area than large samples of the same material. The investigation was extended, using sustained tones, and the same phenomenon was observed as in the case of short impact sounds. In Fig. 8 are shown the apparent absorbing powers per unit area of a highly absorbent hair felt plotted against the area of the test sample. These values were computed by means of equation (11) from observations using organ pipes as sources of sound.

The same effect under somewhat less ideal conditions was observed in the case of the absorbing power of (transmission through) an opening. A large window in an empty room, $30 \times 30 \times 10$ ft. was fitted with a series of frames so that the area of the opening could be varied, the ratio of dimensions being kept constant. The absorption coefficient for these

openings at 512 vibs./sec. varied from 1.10 to .80 as the size was increased from 3.68 to 30.2 square feet. A doorway 8 × 9 ft. in a room 30 × 30 × 9 ft. showed an apparent coefficient as low as .65. (In this case, conditions were complicated by reflection of sound from the ground outside.)

FIG. 8.



Effect of area test sample upon measured value of the absorption coefficient.

In view of the fact that the earlier measurements of Professor Wallace Sabine were based on the open window as an ideal absorber with an assumed coefficient of 1.00, it was of interest to recompute from his data the values of the absorption coefficients of openings. These data were used by him, assuming a coefficient of 1.00 in the determination of the constant K of the simple reverberation formula. Fortunately the data necessary for these computations are found in his original notes, but unfortunately for the present purpose, only the total open window area is given and not the dimensions of the individual openings. The figures are shown in Table 5.

In table 5 w is the total area of the open w indows, and a and a' are computed from the equation

$$at = \frac{9.2\,V}{v}\left(\log_{10}\frac{4E}{avi}\right).$$

TABLE 5.

Room.	V.	t.	t'.	a.	w.	a'.	$\frac{a'-a}{w}$
Fogg Art Museum							
1 pipe	96	4.59	3.00	3.99	1.86	5.85	1.00
16 pipes (7.6×1 pipe)	"	5.26	3.43	3.86	1.86	5.80	1.04
J.P.L.							
Room 15	202	1.78	1.66	19.15	1.34	20.5	1.01
" "	202	1.78	1.56	19.15	2.67	21.7	.96
" "	202	1.78	1.41	19.15	5.28	23.7	.86
Room 1	1,630	3.88	3.26	65.0	12.2	76.2	.92
Room 41	1,960	3.41	2.88	87.0	14.9	101.2	.88

It will be noted that there is a marked variation in the values of the coefficients for the open window. The data for Room 15 show a decrease in the apparent absorbing power as the area of the individual openings is increased, quite in agreement with the results obtained in this laboratory.

One finds an explanation of these facts in the phenomenon of diffraction, and the screening effect of an absorbent area upon adjacent areas. In the Reverberation Theory, we assume a random distribution of the direction of propagation of sound energy. Thus on the average, two thirds of the energy is being propagated parallel to the surface of the absorbent material. Neglecting diffraction, in such a distribution, only that portion travelling at right angles to the absorbent surface would be absorbed by a very large area of a perfectly absorbent material. Due to diffraction, however, the portion travelling parallel to the surface is also absorbed over the entire area, but more strongly at the edges.

The following experiment illustrates this "edge effect." Strips of felt 12" wide were laid on the Sound Chamber floor, forming a hollow rectangle 8 × 5 ft. (2.44 × 1.53 meters). The increase in the absorbing power due to the sample was measured. The space inside was then filled and the increase produced by the solid rectangle measured. The absorbing power per unit area of the peripheral and central portions are given below:

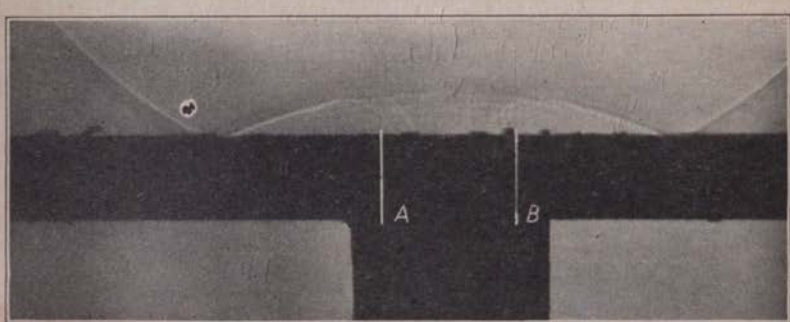
	C_2.	C_3.	C_4.	C_5.	C_6.
Peripheral area	.12	.42	.69	.88	.68
Central area	.05	.27	.24	.29	.30

The screening effect of the edges is obvious, and serves to explain the decrease in absorbing power per unit area shown in Fig. 8. In a similar manner, long narrow samples show more effective absorbing power than equal areas in square form, as shown below:

Area (sq. m.).	Dimensions.	Coefficient.			
		C_3.	C_4.	C_5.	C_6.
1	330 × 33 cms.	.62	.78	1.01	.73
1	100 × 100 "	.42	.62	.86	.70
2	666 × 30	.53	.76	.93	.71
2	141 × 141	.37	.55	.73	.59

The fact that small samples may show an apparent absorption coefficient greater than unity calls for notice. It is to be remembered that we are here dealing with linear dimensions

FIG. 9.



Sound pulse incident upon a barrier with an opening. A–B marks the edges of the opening.

that are of the order of or even less than the wave length of the sound. The mathematics of the problem involves the same considerations as that of radiation of sound from the open end of an organ pipe or the amplification by a spherical resonator. Diffraction plays an important role, and just as in the case of the resonator energy is drawn from the sound field around the resonator to be re-radiated, so the presence of the small absorbent sample or small opening affects a portion of the wave front greater than its own area. Qualitatively,

the effect is shown by the sound photograph of Fig. (9). which shows a sound pulse reflected from the surface of a barrier with an opening. The cross section of the portion cut out of the reflected pulse is clearly greater than the cross section of the opening.

Effect of Tone Quality.

The effect of tone quality upon the measured value of the absorption coefficient is shown in Table 6.

TABLE 6.

Coefficient at 512 vibs./sec.

Sample.	Open Diapason.	Stopped Flute.	Loud Speaker.
1	.36	.24	
2	.43	.31	.31
3	.18	.11	
4	.19	.13	
5	.63	.53	.54

Under the same room conditions, and with very nearly equal initial intensities of sound, as measured by the reverberation times in the empty room, the absorption coefficients of different materials were measured at 512 vibs./sec. using in turn an open diapason pipe, and a stopped flute pipe. For comparison, measurements were made on two of the samples using the loud speaker as a source. The open pipe gave of course the full series of harmonics, the overtones being marked, but not persisting in the reverberant sound longer than the fundamental. The odd harmonics were present in the stopped pipe, but much less prominent, so that the tone approximated a pure tone. Due to the electrical filters, the tone of the loud speaker was practically free from harmonics. The uniformly higher values of the absorption coefficients, shown by the measurement with the open diapason pipe, and the close agreement between stopped pipe and loud speaker values indicate that complex tones are more strongly absorbed than pure tones of the same pitch and intensity.

Cffiicoeents of Materials.

In the light of the foregoing, it is apparent that in giving the absorption coefficients of any material, one must specify carefully the test conditions that involve the various factors

that have been shown to affect the measured values of these coefficients. Further, in comparing the relative efficiencies of two materials as means of controlling reverberation, it is essential that the values of the absorption coefficients should have been determined under identical conditions. The following are the conditions of test that have been adopted at the Riverbank Laboratories and maintained in all measurements of absorption coefficients.

1. Volume of Sound Chamber—286 cubic meters.
2. Absorbing power of empty room determined by Four Organ Pipe Calibration.
3. Sound sources—open diapason organ pipes speaking at 4 inch pressure.
4. Test samples, 1.83 × 2.43 meters, 4.46 sq. meters (6×8 ft.).
5. The powers of the pipes and initial intensities of test tones are, approximately, as follows:

	Frequency.					
	128.	256.	512.	1,024.	2,048.	4,096.
Power, ergs/sec.....	20,000	5,000	4,500	10,000	1,500	1,000
$\frac{\text{Initial intensity}}{\text{Threshold Intensity}}$ ..	10^6	4×10^7	2.7×10^8	1.8×10^8	3.8×10^7	12×10^6

The precise value of the ratios of initial to threshold intensity of course are functions of the absorbing power of the room, of the power of the source and the acuity of hearing of the observer. They simply serve to give the order of magnitude of initial sound intensities, measured in threshold of audibility, of the test tones used.

Table 7 gives the values of the coefficients of various materials that have been measured under the above test conditions. Many of these tests have been made for the manufacturers of the respective materials, and grateful acknowledgment is made in each case for permission to include these figures in the present table. In the use of these figures, it should be remembered that slight variations in the physical properties may cause considerable variation in the absorption coefficients. In some cases it is impossible to specify these properties with exactness. Thus, for example, in felt ma-

terials slight variation in thickness will cause a considerable variation in the coefficient over the range from 256–1,024 vibs./sec. For this reason, as well as for the fact that extreme precision can not be claimed for absorption measurements even under the best conditions, a tolerance of about ±3 per cent. of the value given is allowable. For practical purposes of acoustical correction, this degree of precision is amply sufficient.

TABLE 7.

Material.	Pitch.					
	128.	256.	512.	1,024.	2,048.	4,096.
	Coefficient.					
1. Acoustic-Celotex, Type A perforated fiber board, $\frac{13}{16}$" thick, 441 holes per sq. ft., $\frac{3}{16}$" diameter $\frac{1}{2}$" deep, plain side exposed		.20	.21	.19	.17	.24
2. Acousti-Celotex, Type B same as Type A, but with perforations exposed		.27	.40	.46	.42	.42
3. Acousti-Celotex, Type BB, $1\frac{15}{16}$" thick, 441 holes per sq. ft. $\frac{3}{16}$" diameter $1\frac{1}{2}$" deep		.39	.63	.73	.55	.46
4. Acousti-Celotex, (tested 1924) 1" thick, 400 holes per sq. ft. $\frac{3}{4}$" deep		.32	.46	.66	.53	.47
5. Akoustolith Tile, $\frac{7}{8}$" thick, fine texture cemented to clay tile	.06	.22	.28	.48	.50	.31
6. Balsam Wool, soft wood fiber, paper backing, scrim facing, 1" thick .254 pounds/sq. ft.	.10	.27	.50	.68	.56	.48
7. Same covered with perforated metal cover, $\frac{1}{16}$" holes, 64 per sq. in.	.09	.25	.48	.66	.57	.47
8. Standard Celotex $\frac{7}{16}$" thick, on 1" Furring		.16	.22	.20	.16	.15
9. The same on 2 × 4" studs	.19	.14	.13	.14	.14	.16
10. Draperies, hung straight, in contact with wall, cotton fabric, 10 oz. per sq. yd.	.03	.04	.11	.17	.24	.35
11. The same, cotton fabric 14 oz. per sq. yd.	.04	.07	.13	.22	.32	.35
12. The same, velour, 18 oz./sq. yd.	.05	.12	.35	.45	.38	.36
13. The same as No. 12, hung 4" from wall	.06	.27	.44	.50	.40	.35
14. The same as No. 12 hung 8" from wall	.08	.29	.44	.50	.40	.35
15. Cotton fabric 14 oz./sq. yd. draped to $\frac{7}{8}$ its area	.03	.12	.15	.27	.37	.42
16. The same as No. 15, draped to $\frac{1}{4}$ area	.04	.23	.40	.57	.53	.40

TABLE 7.—*Continued.*

Material.	Pitch.					
	128.	256.	512.	1,024.	2,048.	4,096.
	Coefficient.					
17. Same as No. 15 draped to ½ area	.07	.31	.49	.81	.66	.54
18. Felt, Standard 1", all hair	.09	.34	.55	.66	.52	.39
19. Felt, Asbestos-Akoustikos, (hair and asbestos fiber) ½" thick	.07	.14	.31	.51	.51	.43
20. The same ¾" thick	.08	.23	.45	.65	.56	.46
21. " " 1" thick	.11	.31	.59	.68	.58	.46
22. " " 1½" thick	.13	.41	.73	.73	.58	.46
23. " " 2" thick	.21	.46	.79	.75	.58	.46
24. " " 3" thick	.33	.56	.79	.77	.58	.46
25. Flax-linum, semi-stiff flax fiber board ½" thick	.09	.15	.34	.57	.51	.47
26. Masonite, Standard ½" board (Pressed wood fiber), laid on 1" furring 18" O.C.	.09	.30	.33	.32	.30	.37
27. The same nailed to 2 × 4" studs, 16" O.C.	.16	.26	.34	.36	.30	.25
28. The same, nailed to 1 × 2" furring 16" O.C.	.15	.26	.31	.32	.30	.28
29. 1" Nashkote AAX, 1" felt, with cotton fabric cemented on surface two coats special paint	.11	.25	.34	.46	.48	.36
30. Nashkote B-332, 1" felt with perforated oil-cloth, $\frac{3}{32}$" perforation, 10 per sq. in.	.11	.31	.67	.81	.64	.50
31. Plaster, gypsum on wood lath on wood studs, rough finish	.016	.032	.039	.050	.030	.028
32. The same, with smooth finish ("lime putty")	.020	.022	.032	.039	.039	.028
33. Plaster, lime on wood lath on wood studs, rough finish	.027	.046	.060	.085	.043	.056
34. The same, smooth finish	.024	.027	.030	.037	.019	.034
35. Plaster, "Calacoustic" ½" thick	.06	.10	.14	.15	.15	.20
36. Plaster, Sabinite ½" thick	.06	.16	.21	.29	.34	.37
37. Stockade slab, Wood fiber, cemented with magnesite, 1" thick	.11	.13	.27	.50	.63	.40
38. The same 2" thick	.09	.35	.60	.67	.45	.53

STATISTICAL DESCRIPTION OF THE SIZE PROPERTIES OF NON-UNIFORM PARTICULATE SUBSTANCES.[1]

BY

THEODORE HATCH and SARAH P. CHOATE.

INTRODUCTION.

IT is a well recognized fact that the size properties of a non-uniform particulate substance are influenced by the particle size of the material. It has not been possible to state this relationship as a rigid law, however, owing to the absence of a mathematical definition of particle size.

In the present paper an attempt is made to define particle size by means of well-known statistical methods and to indicate the mathematical laws of relationship between particle size, thus defined, and certain size properties of non-uniform particulate substances, namely, the number of particles per gram of material and the specific surface of the substance.

STATISTICAL ASPECTS OF THE PROBLEM.

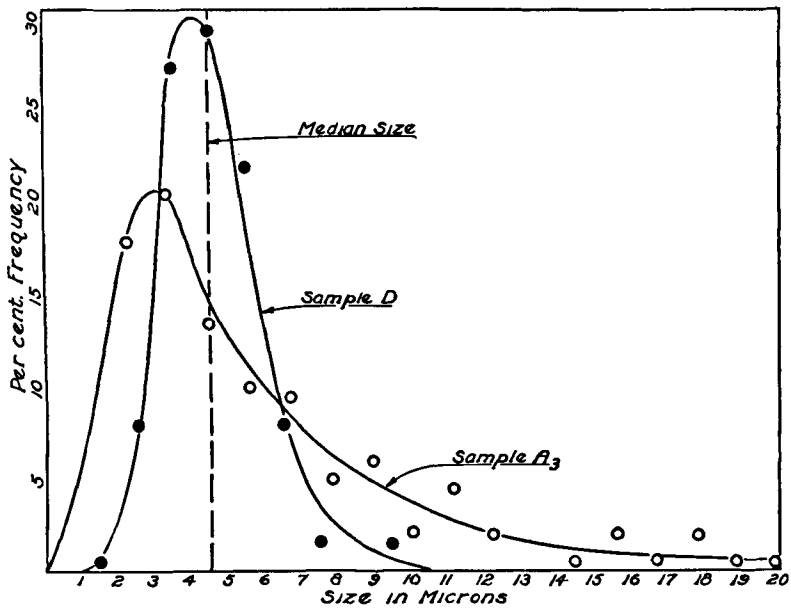
If one measures a large number of particles in a sample of solid particulate matter, the particles will be found to vary in size. If these measurements are arranged in groups of equal size intervals, each group will include a certain percentage of the entire number of measurements, and if the percentage frequency of each group is plotted against the mid-value of sizes included, a "size-frequency" distribution curve for the material will result. The shape of this curve varies according to the size complex of the material. In general, however, curves obtained in this manner may be said to resemble a cocked hat, since they have a definite modal value and approach zero toward the ends. The curves are commonly asymmetrical, or "skewed." (See Fig. 1.)

The particle size of a non-uniform particulate substance is influenced by the shape of its size-frequency curve. This fact

[1] From the Department of Ventilation and Illumination, Harvard School of Public Health, Boston, Mass., and the Department of Sanitary Engineering, Harvard Engineering School, Cambridge, Mass.

is clearly indicated by the curves in Fig. 1 and the microphotographs in Fig. 2. The median size [1] is 4.5 μ in each

FIG. 1.



Size-frequency curves of samples A_3 and D.

case, yet sample A_3 appears to be larger than sample D on account of the greater number of large particles in the former. It will be observed also that the size-frequency curves differ markedly in shape.

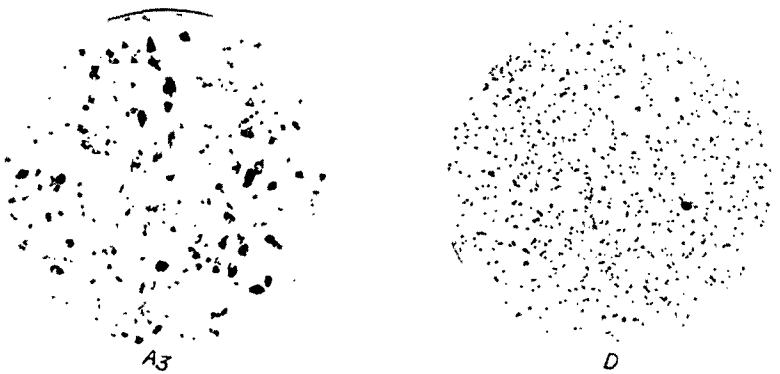
It is evident from the foregoing that particle size must be defined in terms of the shape of the size-frequency curves. Hence the shape characteristics, expressed in mathematical form, of the distribution curves of finely divided materials are needed.

It has been pointed out by Martin (1), Green (2), and others that the curves follow the law of probability, but, since they are generally asymmetrical or "skewed," it is evident that the normal probability equation, or ordinary "law of chance," does not apply, this law requiring, among other

[1] The median is that value than which 50 per cent. of the observations are smaller.

things, that the curve be symmetrical about a vertical axis. Recent investigations by Drinker (3) and by Loveland and Trivelli (4), however, have shown that the asymmetrical or "skewed" frequency curves of non-uniform particulate materials can, in general, be transformed into symmetrical curves

FIG. 2.



Microphotographs of samples A_3 and D at the same magnification.

following the ordinary "law of chance," or normal probability curve, when the logarithms of the sizes are substituted for the sizes themselves. Thus, the asymmetrical curves in Figure 1 give normal logarithmic curves, as in Fig. 3.

This fact is of great value in connection with the present problem, because it enables one to use the well-known parameters of the normal probability curve as "yard-sticks" for measuring the shape of the size-frequency distribution curves and hence the particle size of finely divided materials.

The mathematics of the probability curve is well-known and will not be discussed in this paper. The equation of the curve has the following form:

$$n = \frac{\Sigma n}{\sigma\sqrt{2\pi}} e^{-(X-M)^2/2\sigma^2},$$

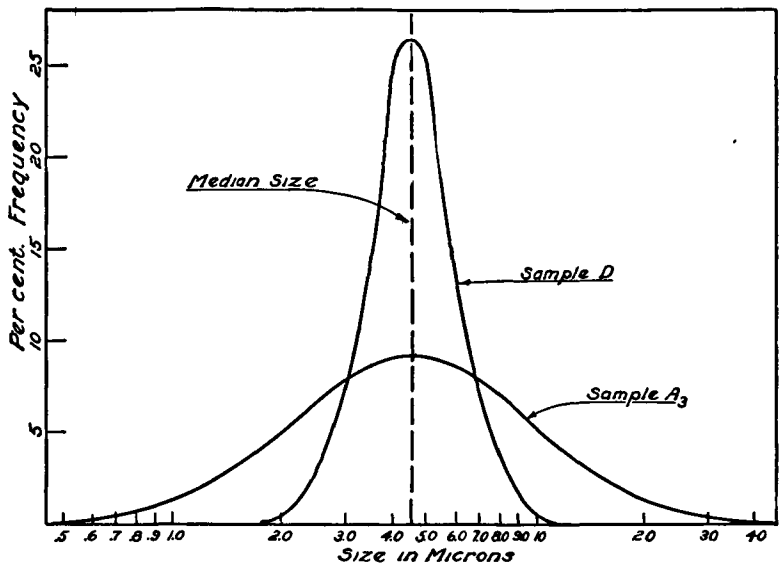
where n is the frequency of observations of value, X; Σn is the total number of observations; and M and σ are constants that determine the position and shape of the curve. The

constant, M, determines the position of the vertical axis of symmetry of the curve and is defined as follows:

$$M = \frac{\Sigma(nX)}{\Sigma n}.$$

It will be noted that it is also the arithmetic mean of the observations. The constant, σ, called the standard deviation,

FIG. 3.



Logarithmic size-frequency curves of samples A_3 and D.

is a measure of the dispersion of the observations from the mean. It is given by the following expression:

$$\sigma = \sqrt{\frac{\Sigma[n(X - M)^2]}{\Sigma n}}.$$

These two constants rigidly define the frequency distribution of a series of observations that vary according to the law of probability (5).

As stated above, a normal probability curve is given by the size-frequency distributions of non-uniform particulate substances when the logarithms of the sizes are substituted for the

measures themselves. Hence the parameters, M and σ, must be determined from the logarithms of the measures. Under this condition, they may be defined as follows:

$$\log M_g = \text{mean of logarithms} = \frac{\Sigma(n \log X)}{\Sigma n}$$

and

$$\log \sigma_g = \text{standard deviation of logarithms} = \sqrt{\frac{\Sigma[n(\log X - \log M_g)^2]}{\Sigma n}}$$

and are known as the log-geometric mean and log-standard geometric deviation, respectively.

The labor involved in the calculation of these two parameters can be considerably reduced by utilizing logarithmic probability paper (6), which allows a graphic solution of the problem. On this paper, the summation curve, or the curve obtained by integration of the equation of the logarithmic probability law, plots as a straight line. Thus, by plotting the percentage of particles below a certain size against that size, a straight line of best fit can be drawn through the plotted points.

The geometric mean is that value obtained by reading the 50 per cent. size.[2] It follows from the probability integral that

$$\sigma_g = \frac{84.13 \text{ per cent. size}}{50 \text{ per cent. size}} = \frac{50 \text{ per cent. size}}{15.87 \text{ per cent. size}}.$$

Thus, by determining the 84.13 per cent. and the 50 per cent. sizes from the fitted line, the geometric mean and the standard geometric deviation can be determined with considerable saving of labor.

INFLUENCE OF THE SHAPE OF THE SIZE-FREQUENCY CURVES UPON THE SIZE PROPERTIES OF NON-UNIFORM PARTICULATE MATTER.

It has been shown by Perrott and Kinney (7) and by Green (2) that the size properties of non-uniform particulate matter can be measured in terms of certain average diameters. It is assumed that the surface area of a single particle is a function

[2] The mean bisects the symmetrical frequency curve.

of the square of its diameter and the volume a function of the diameter cubed. Hence, for a non-uniform material, the diameter of the hypothetical particle having average surface area is given by

$$\Delta = \sqrt{\frac{\Sigma(nd^2)}{\Sigma n}},$$

when d is the diameter of each particle and n the number of particles having a diameter, d. In the same way, the diameter of the hypothetical particle having average volume is given by

$$D = \sqrt[3]{\frac{\Sigma(nd^3)}{\Sigma n}}.$$

It follows that the average surface area, $\bar{s}$, will be given by the product of Δ^2 and a constant depending upon the geometric shape of the particles. Hence we may write:

$$\bar{s} = \alpha\Delta^2,$$

where α is the area shape factor.[3]

Similarly, the average volume is given by

$$\bar{v} = \nu D^3$$

when ν is the volume shape factor.

But

$$\bar{v} = \frac{1}{\rho N}$$

when ρ is the density of the material and N the number of particles per gram. Hence

$$\nu D^3 = \frac{1}{\rho N}$$

The specific surface, S, of a substance is defined as the surface area per gram of material. Therefore we may write:

$$S = \bar{s}\,N = \frac{\alpha\Delta^2}{\nu\rho D^3}.$$

[3] The authors have used the symbol α to denote area shape factor in preference to σ, as suggested by Green (2), in order to avoid confusion with the standard deviation, which is usually denoted by σ.

In the present problem we are interested in the relation between the statistical parameters M_g and σ_g of the size frequency curves and the area and volumetric properties of non-uniform particulate materials.

Since

$$n = \frac{\Sigma n}{\log \sigma_g \sqrt{2\pi}} e^{-(\log d - \log M_g)^2/2 \log^2 \sigma_g}$$

it follows that

$$\frac{\Sigma(nd^2)}{\Sigma n} = \frac{\frac{\Sigma n}{\log \sigma_g \sqrt{2\pi}} \int_0^\infty d^2 \cdot e^{-(\log d - \log M_g)^2/2 \log^2 \sigma_g} \cdot d \log d}{\Sigma n}$$

and

$$\frac{\Sigma(nd^3)}{\Sigma n} = \frac{\frac{\Sigma n}{\log \sigma_g \sqrt{2\pi}} \int_0^\infty d^3 \cdot e^{-(\log d - \log M_g)^2/2 \log^2 \sigma_g} \cdot d \log d}{\Sigma n}.$$

It can be shown that these expressions take the following form: [4]

$$\frac{\Sigma(nd^2)}{\Sigma n} = e^{2 \log M_g + 2.0 \log^2 \sigma_g},$$

$$\frac{\Sigma(nd^3)}{\Sigma n} = e^{3 \log M_g + 4.5 \log^2 \sigma_g}.$$

Reduced to logarithms to the base 10, these expressions become

$$\log \Delta^2 = \log M_g^2 + 4.6052 \log^2 \sigma_g,$$
$$\log D^3 = \log M_g^3 + 10.3617 \log^2 \sigma_g.$$

Thus we may write

$$\log N = \log \frac{\mathrm{I}}{\rho \nu} - \log D^3,$$

or

$$\log N = \log \frac{\mathrm{I}}{\rho \nu} - \log M_g^3 - 10.3617 \log^2 \sigma_g \qquad \text{(I)}$$

and

[4] The authors are indebted to Professor E. B. Wilson and Professor C. R. Doering of the Department of Vital Statistics, Harvard School of Public Health, for assistance in the mathematics of this study.

$$\log S = \log \frac{\alpha}{\rho v} + \log \Delta^2 - \log D^3,$$

or

$$\log S = \log \frac{\alpha}{\rho v} - \log M_g - 5.7565 \log^2 \sigma_g. \qquad (II)$$

It is seen, therefore, that the size properties—surface area and volume—of non-uniform particulate substances are functions of the statistical parameters M_g and σ_g; that is to say, functions of the shape of the size-frequency curves.

EXPERIMENTAL.

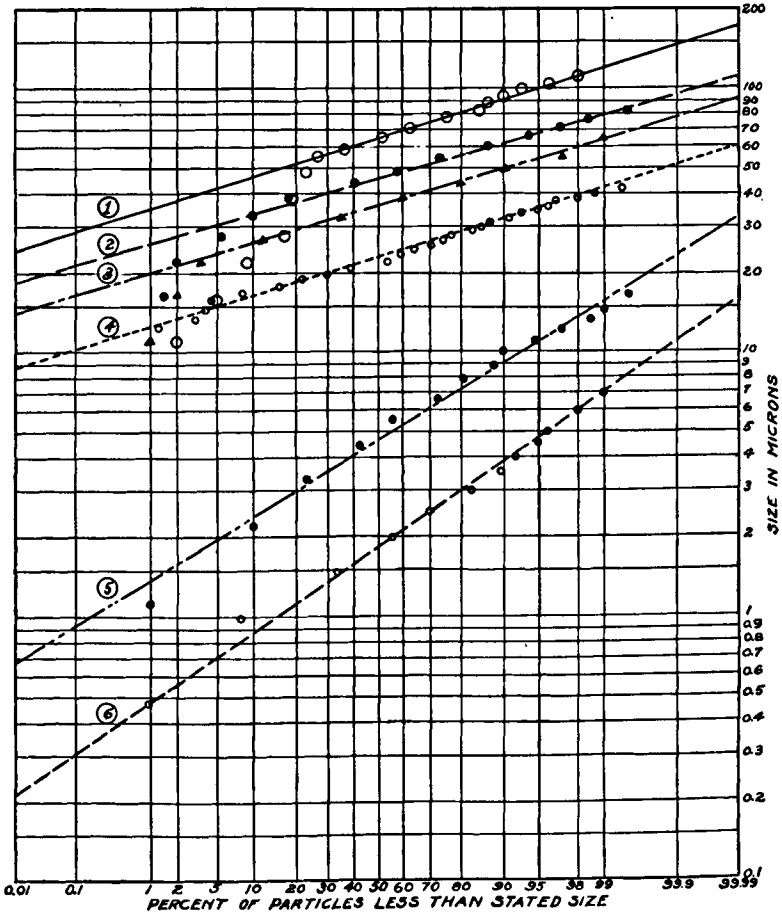
The following study was carried out in an effort to determine experimentally the law of variation between the parameters of the logarithmic probability curves and the volumetric and surface area properties of certain finely divided materials. For this work, three materials were selected: quartz (99.7 per cent. SiO_2), ground crystals of calcite (Iceland Spar) and granite dust obtained from the bag filters of a stone cutting shed.

These materials were screened through a 325-mesh sieve and the residue retained on the sieve was discarded. The dusts were then divided into fractions by elutriation with distilled water in a standard 1,000 c.c. graduate, according to the following time intervals: 0 to 2 minutes, 2 to 5 minutes, 5 to 10 minutes, 10 to 30 minutes, 30 minutes to 24 hours, 24 hours to 4 days. Each fraction was washed until the supernatant liquid showed no particles in suspension at the end of the time interval. Some of the silica dust was also fractionated in a centrifuge for varying lengths of time. The effect of this treatment was to give greater ranges of dispersion than was possible by ordinary elutriation methods. Twenty-six fractions were obtained.

Microscopic slides of the fractions were prepared according to Green's turpentine method (8) and particles measured by means of a filar micrometer. The horizontal diameter only was measured. In some cases this diameter was the major axis of the particle and in others it represented the minor axis, or some measure between the two. It was felt that the diameters measured uniformly in one direction would give a

representative picture of the size-distribution of an entire sample (9). In most cases, 200 particles were measured on each slide. The ordinates of the summation curves of the

FIG. 4.



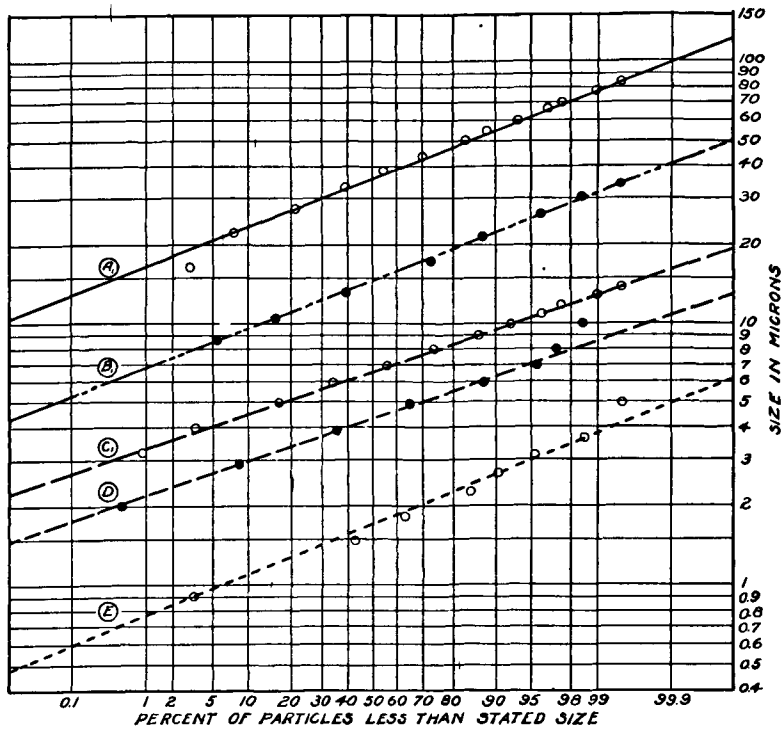
Size-frequency summation curves. Silica.

size-frequency distributions were then calculated according to the following schedule and the values from columns 2 and 5 were plotted on logarithmic probability paper. A standard class-interval of one filar unit was used.

Schedule for Computing Ordinates of Summation Curve.

1	2	3	4	5
Upper limit of group in filar units.	Upper limit of group in microns.	Frequency of group.	Total frequency < upper limit of group size.	Per cent. frequency < upper limit of group.

FIG. 5.



Size-frequency summation curves. Silica.

Figures 4, 5, 6, 7, and 8 show the summation curves obtained. In each case a straight line of best fit was drawn in by eye.[5] The geometric mean values were determined by reading the 50 per cent. sizes from the curves and the standard geometric deviations were computed from the 84.13 and 50

[5] This method of curve fitting is open to question; but, in most cases, an excellent fit was obtained.

per cent. sizes, as described above. Table 1, columns 4 and 5, give these data for the various systems studied.

TABLE 1.

Parameters of Log-Probability Curves, Number of Particles per Gram, and Tyndallmeter Readings of Non-Uniform Particulate Systems of Silica, Granite, and Calcite.

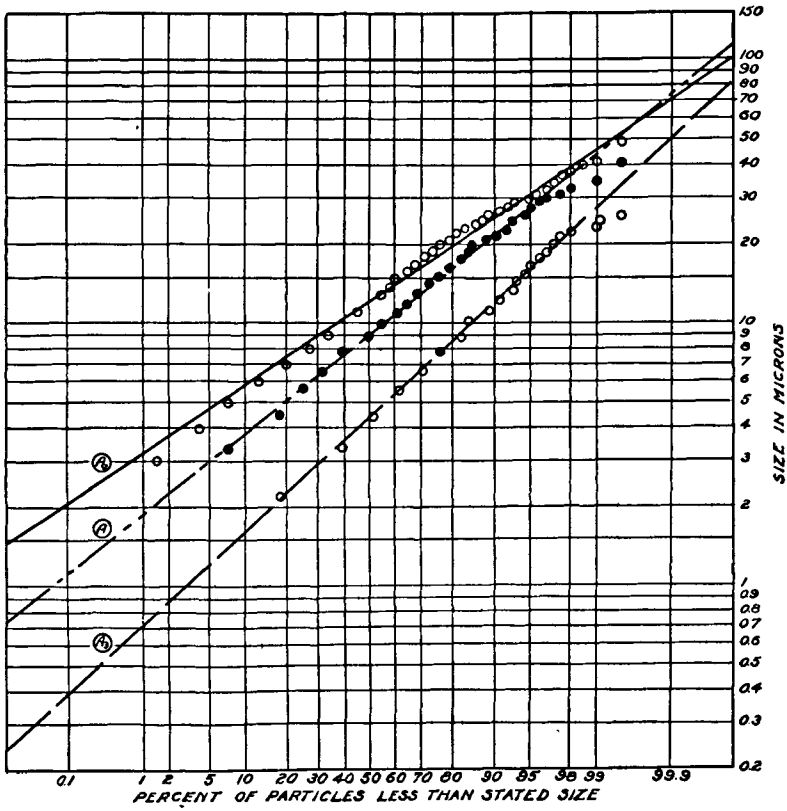
1. Sample Number	2. Material	3. Density ρ	4. Geometric Mean M_g	5. Standard Geometric Deviation σ_g	6. Number of Particles Per Gram in Millions N	7. Tyndall Reading at 50 Mg./Liter T_{50}
1	Silica	2.65	64.2	1.29	8.49	1.7
2	"	"	45.0	1.27	24.13	1.7
3	"	"	36.0	1.28	56.37	1.8
4	"	"	23.0	1.30	173.60	3.55
5	"	"	4.6	1.63	7,357.60	9.70
6	"	"	1.85	1.77		16.5
A	"	"	9.2	2.00	317.96	2.2
A_1	"	"	36.0	1.38	50.40	1.8
A_2	"	"	12.0	1.80	263.32	2.8
A_3	"	"	4.5	2.17	216.00	2.0
B_1	"	"	14.5	1.40	569.84	3.6
C_1	"	"	6.6	1.33	5,397.90	9.0
D	"	"	4.4	1.34	25,288.00	15.5
E	"	"	1.73	1.42		
7	Granite	"	72.0	1.35	8.80	1.10
8	"	"	46.0	1.46	18.05	1.35
9	"	"	28.3	1.42	55.79	2.9
10	"	"	16.5	1.55	210.00	3.75
11	"	"	4.5	1.73	6,480.00	6.5
12	"	"	1.45	1.69		
13	Calcite	2.75	69.0	1.26	9.39	1.8
14	"	"	47.0	1.32	19.91	1.9
15	"	"	37.5	1.30	38.92	2.0
16	"	"	19.3	1.39	163.70	4.0
17	"	"	6.1	1.60	9,332.00	11.3
18	"	"	1.15	1.35	647,000.00	25.0

Two other size measurements were determined in addition to the parameters of the size-frequency curves, viz. the value of N, or the number of particles per gram, and a relative measure of surface area.

The procedure for determining N was as follows: A portion of the dust was accurately weighed and placed in a flask together with a known volume of a suitable dispersing medium. The flask was then agitated until the dust particles were evenly dispersed throughout the liquid. This was easily accomplished in the case of large particles, but the

smaller sizes often flocculated badly and even prolonged agitation failed to disperse them completely. As the size of the dust increased, it became necessary to use dispersing media of higher viscosities, in order to keep the particles in suspension

FIG. 6.

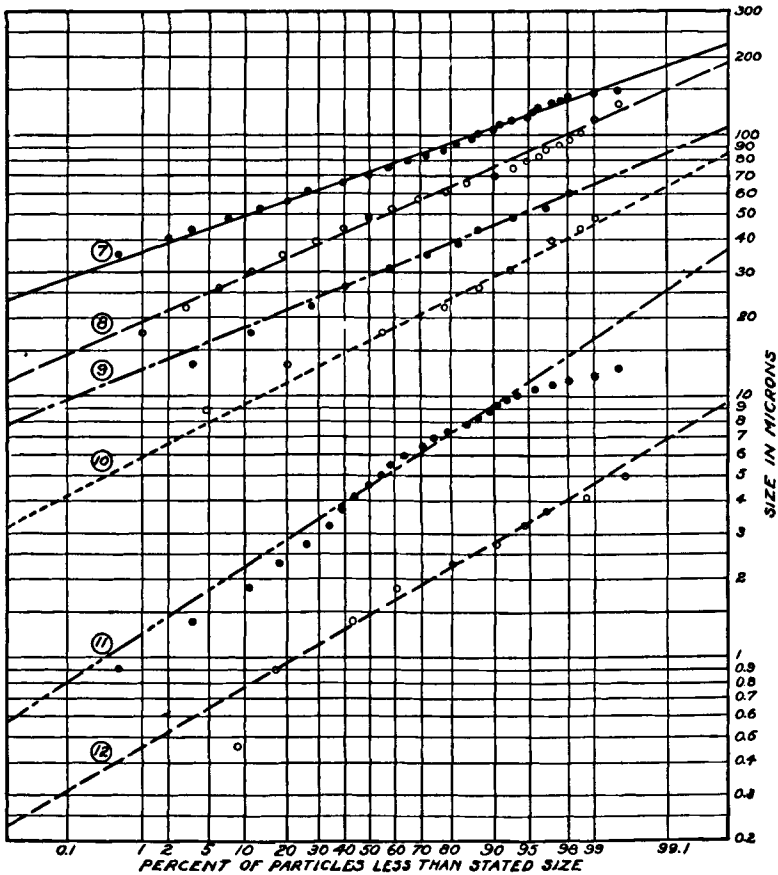


Size-frequency summation curves. Silica.

until a representative portion could be poured into the counting cell. Gelatin was found to be the most satisfactory dispersing medium. By mixing it with various percentages of water, liquids of varying viscosities were obtained, the desired viscosity for dispersion of a particular material being determined experimentally.

Counting cells of differing thickness were required for use under different magnifications. Thus with the low power it was possible to use the standard Sedgwick-Rafter cell which has a depth of 1.0 mm.; for the high power, a cell of 0.5 mm.

FIG. 7.



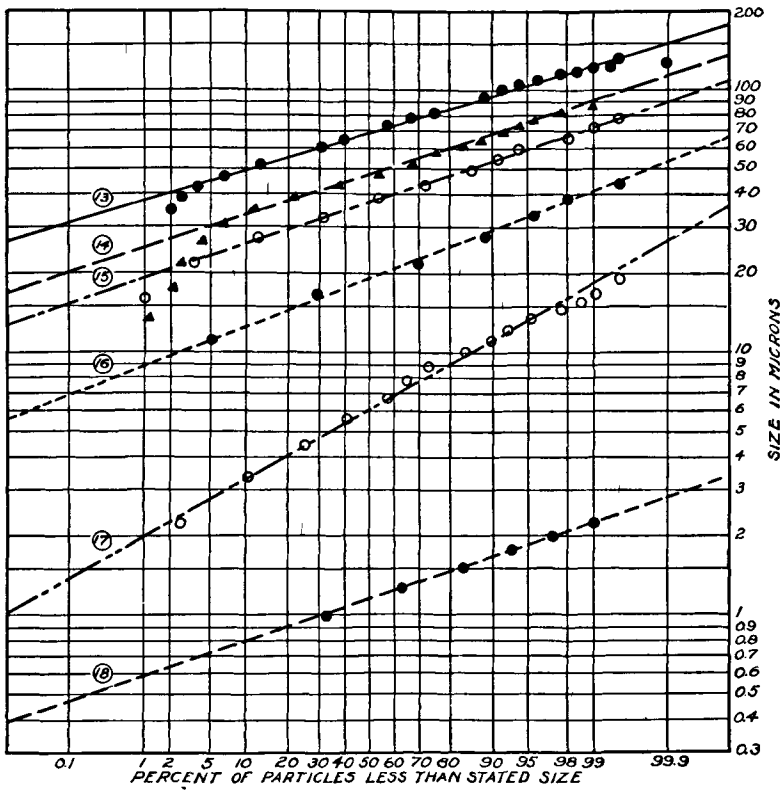
Size-frequency summation curves. Granite.

depth was required; and with the oil immersion objective, the blood cell (0.1 mm. deep) was used. Two cells of each material were prepared and ten fields counted in each cell. In all cases, the materials were prepared in dilution to give about 50 particles in a field.

Values of N for the materials under consideration are given in column 6 of Table 1.

The specific surface of a non-uniform particulate substance cannot be determined by any simple means such as was used in the determination of N.[6] It was necessary, therefore, to

FIG. 8.



Size-frequency summation curves. Calcite.

make use of one of the many indirect methods that have been developed. The Tyndallmeter (12, 13, 14) is of value here, because it is simple and rapid in its operation and gives fairly accurate results.

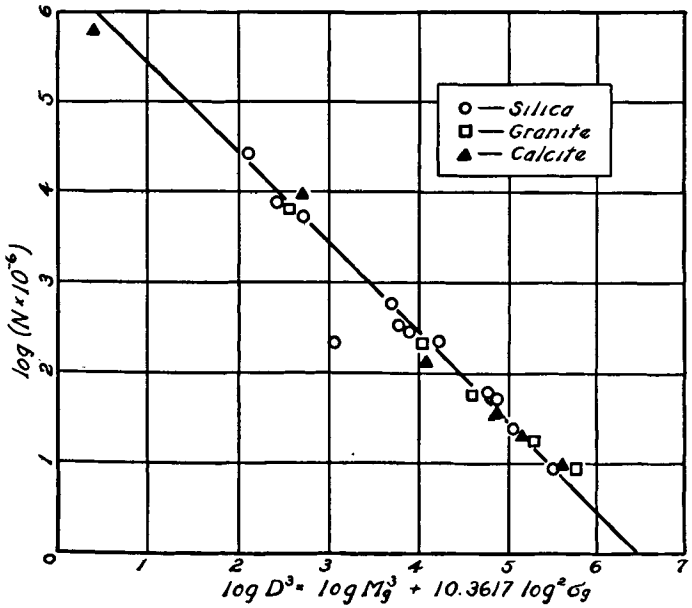
[6] Direct determinations of the surface area of quartz particles have been made by Martin (10) and more recently by Gross and Zimmerley (11), but the methods involved are complicated and could not be followed in the present investigation on account of lack of time.

The Tyndallmeter is an instrument which measures photometrically the strength of the Tyndall beam created by light passing through liquid or gaseous suspensions of particles. Within the particle size range found in the materials studied, the strength of the beam is proportional to the surface area of the suspended matter (15). Thus if suspensions of equal concentration of the same material are used, the Tyndall readings will give relative measures of the surface area in each case. Tyndallmeter readings for suspensions in distilled water at concentrations of 50 milligrams per liter were obtained for the twenty-six fractions and are tabulated in Table I, column 7.

DISCUSSION.

The data in Table I have been plotted in the manner suggested in equations I and II and are shown in Figs. 9 and

FIG. 9.

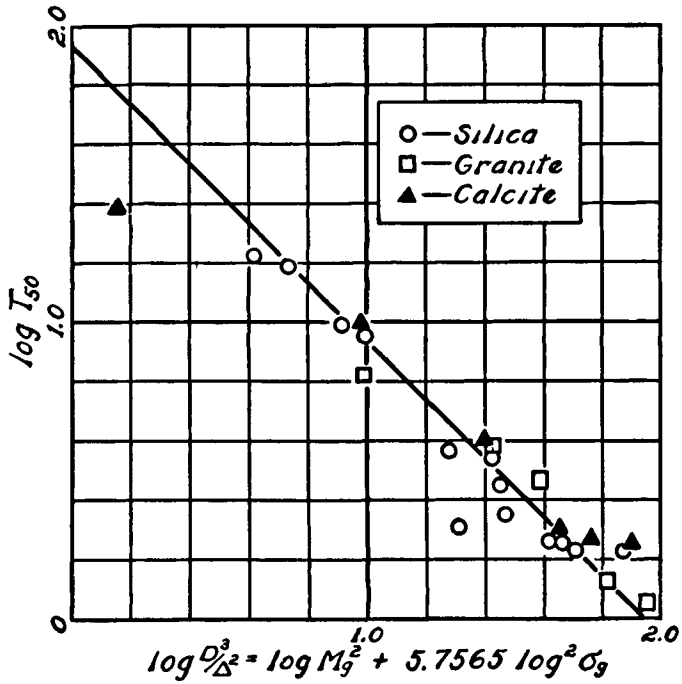


Relation between number of particles per gram and particle size expressed in terms of M_g and σ_g.

10. The plotted points follow the theoretical curves in a satisfactory manner and indicate that the theoretical assumptions made at the outset are in close agreement with the actual

relationship existing between size properties and the shape characteristics of the size-frequency distribution curves of non-uniform particulate substances.

FIG. 10.



Relation between Tyndallmeter reading and particle size expressed in terms of M_g and σ_g.

The equations of the experimental curves are:

$$\log (N \times 10^{-12}) = \log 2.69 - \log M_g^3 - 10.3617 \log^2 \sigma_g, \quad \text{(III)}$$

$$\log T_{50} = \log 85.0 - \log M_g - 5.7565 \log^2 \sigma_g. \quad \text{(IV)}$$

Taking the densities of the three materials to be: silica, 2.65; calcite, 2.75; granite, 2.65, the following values are obtained for the volume shape factor:

$$\text{silica} = 0.140,$$

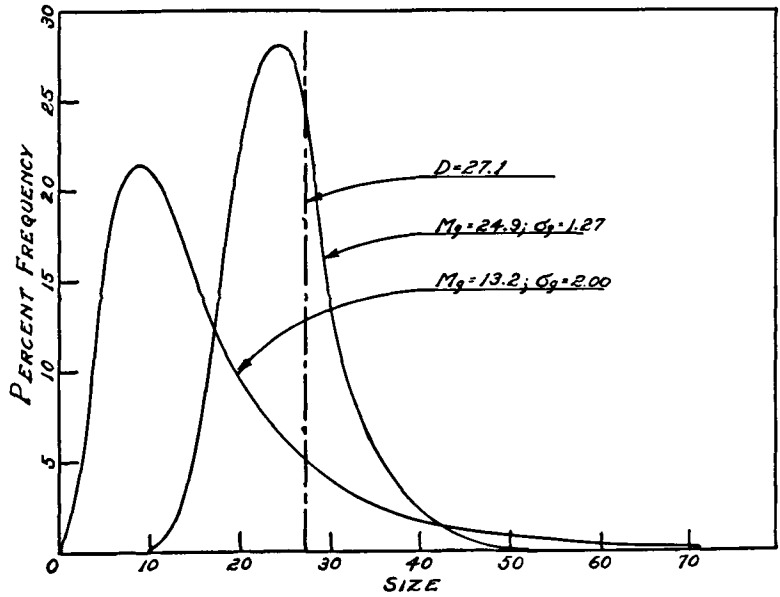
$$\text{calcite} = 0.135,$$

$$\text{granite} = 0.140.$$

These values differ markedly from that of a sphere ($\pi/6 = 0.5286$) and indicate that the frequent assumption that particulate materials are spherical in shape may be in error by a considerable amount.

Since the Tyndall effect gives merely a relative measure of surface area, it is not possible to determine the true value of

FIG. 11.



Non-uniform size-distribution curves having the same value of D but differing in their geometric means and standard geometric deviations.

the area shape factor for the three materials. A relative value, however, may be obtained. Turning to equations II and IV, we may write

$$\kappa\left(\frac{\alpha}{\rho\nu}\right) = 85.0,$$

where κ is a factor representing the relation between the relative and the true area shape factors. Since $\rho\nu$ has a constant value, 2.69, the relative area shape factor is the same for all three materials, namely

$$\kappa\alpha = 228.65.$$

Definition of particle size in terms of M_g and σ_g possesses certain advantages over the use of the average diameters Δ and D. It has been shown that Δ and D are functions of M_g and σ_g. When calculated in the forms $\frac{\Sigma(nd^2)}{\Sigma n}$ and $\frac{\Sigma(nd^3)}{\Sigma n}$, considerable labor is involved. On the other hand, it has been shown that M_g and σ_g can be obtained graphically with considerable ease. Of greater importance, however, is the fact that M_g and σ_g give immediate information concerning the distribution or range in size of particles. With the aid of probability tables or the summation curve on log-probability paper, one can determine immediately the percentage of particles below or above a certain size or within certain size limits. No such information can be obtained from Δ or D. Moreover the geometric mean and standard geometric deviation define one and only one size-frequency curve. It is possible for many different curves to have the same value of Δ or D. For example, the two materials represented by the size-frequency curves in Fig. 11 have the same value of D^3: 20,000; yet they differ markedly both in position and in size distribution. Thus the geometric mean and standard geometric deviation define particle size in greater detail than is possible by means of the average diameters Δ and D.

SUMMARY.

Particle size is defined in terms of the shape of the size-frequency curves of non-uniform particulate substances. It is shown theoretically and experimentally that the size properties are closely correlated with the statistical parameters M_g and σ_g of the log-probability curves followed by the size-frequency distributions of such materials. It is further noted that this manner of defining particle size possesses certain advantages over the use of the average diameters Δ and D. These advantages are as follows:

1. The statistical parameters M_g and σ_g define one and only one size-frequency distribution curve.

2. Information concerning size-distribution and relative frequency of certain sizes can be obtained from M_g and σ_g.

3. The average diameters Δ and D are functions of M_g and σ_g. Thus M_g and σ_g give all information concerning particle size that can be obtained from Δ and D.

4. M_g and σ_g are determined graphically from the summation curves on log-probability paper, thus effecting considerable saving in labor over the calculation of $\frac{\Sigma(nd^2)}{\Sigma n}$ and $\frac{\Sigma(nd^3)}{\Sigma n}$.

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The Separation of Krypton and Xenon from the Air. A. LEPAPE. (*Comptes Rendus*, July 23, 1928.) The constant ratio between xenon and argon and between krypton and argon in air and in subterranean gases shows that there is no hope of finding in nature any mixture richer in xenon and krypton than the air itself which contains about 1 cc. of krypton and .09 cc. of xenon per cubic meter. Because both of these gases are soluble in liquid oxygen and have such low vapor tensions that they do not escape by evaporation they accumulate in the liquid oxygen left from the fractional distillation of the air. It is from this source that the rather few investigators who have prepared these rare gases have derived them. A table gives data for Ramsay, 1903, Moore, 1908, Antropoff, 1919 and for Dewar and Aston, 1923. Though Ramsay obtained only 7.5 cc. of krypton he got 5% of all that was present in his source. Dewar and Aston succeeded in getting 130 cc. of the gas with an efficiency percentage of .03. For xenon also Ramsay has the highest percentage, 6, while Dewar and Aston collected 575 cc. with efficiency of 1.8%. Lepape studied the recovery of the gases from liquid oxygen in a Claude apparatus for liquefying air. He found that at the beginning of the evaporation of the liquid oxygen the resulting gas contains about one part in a million of krypton while at the end one-thousandth consists of the two gases desired.

To separate the two gases from gaseous oxygen he caused the vapor coming from the liquid to traverse a tube holding 72 grams of cocoanut charcoal. The tube was in a Dewar bulb containing the evaporating liquid. Hence the charcoal was at the temperature of liquid oxygen. The charcoal adsorbs the krypton and xenon. Later these gases are driven off by heat and are freed of impurities by chemical means. The resulting mixture of the two gases is fractionated by the use of cocoanut charcoal at appropriate temperatures. With this method on the average 86% of the krypton present was recovered and 96% of the xenon. Several liters of krypton and one liter of xenon have been prepared and are being examined for industrial applications. Silica gel acts as well as charcoal in removing the two gases. G. F. S.

NOTE ON THE VISCOSITY OF GASES.

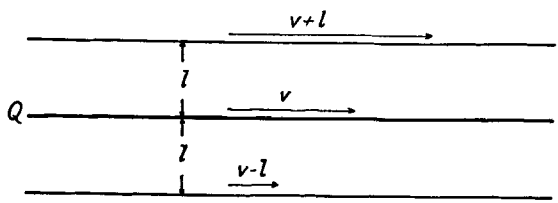
BY

S. S. KISTLER.

A careful examination of the customary derivations of the equation for the viscosity of gases reveals a rather surprising error in the chain of reasoning. Viscosity is generally conceived to be due to the transfer of momentum through the gas, and it is in the method of arrival at the value for the quantity of momentum transferred that the error occurs. It has apparently remained hidden because the equation obtained is doubtless correct.

In most methods of derivation the gas is assumed to be confined between two parallel planes of indefinite extent, whose distance apart remains constant, that are moving at a constant rate with respect to each other. The mass-velocity of the molecules in any plane parallel to the given plane is assumed constant. According to the kinetic theory, the number of molecules crossing unit area of a plane in a gas from one side in one second is $\frac{1}{6}Nc$ where N is the number of molecules per unit volume and c is the velocity of the molecule with average kinetic energy. The simpler derivations assume that the velocity gradient is unity.

FIG. 1.



Consider the plane Q, Fig. 1, parallel to the confining planes of the gas. From the principle of equipartition of energy it is permissible to assume that one third of the molecules are moving in a direction perpendicular to the plane with the velocity c. On the average, the molecules whose

paths terminate in the plane will come from a distance above and below it equal to the mean free path, l. There will be coming down to the plane from above $\frac{1}{6}Nc$ molecules per second and likewise the same number will be coming up from below. If the velocity gradient is unity, the molecules coming from the plane l distance below Q will come with l less velocity than the mass-velocity of Q and will therefore impart to it $\frac{1}{6}Ncm(v - l)$ momentum, while the molecules coming from the plane l distance above will possess on the average l greater velocity than the plane and will therefore impart to Q $\frac{1}{6}Ncm(v + l)$ momentum. The universal conclusion follows that consequently the momentum transfered across the plane is

$$\eta = \tfrac{1}{6}Ncm[(v + l) - (v - l)]$$
$$= \tfrac{1}{3}Nmcl,$$

where m is the mass of a single molecule and v is the mass-velocity of Q.

The above derivation has been copied many times with apparently little critical review. The mechanism of the transfer of momentum looks simple but upon examination it is seen that an assumption is tacitly introduced that is quite contrary to the fundamental assumptions of the kinetic theory. The molecule proceeding from a distance toward the plane is assumed to possess the mean mass-velocity of the plane from which it originated, and this assumption is probably correct, at least as a first approximation, but when the path of the molecule terminates in the plane under consideration something peculiar is supposed to happen in such a way that it acquires the mass-velocity of that plane and delivers to it a quantity of momentum only equal to the product of its mass and the difference in velocity between the initial and terminal planes. Such a collision would be inelastic, and since one of the fundamental assumptions of the kinetic theory is that the molecules are perfectly elastic it is difficult to see how this error has passed notice so long. Evidently the fact that the average horizontal velocity of the molecules leaving a plane is very nearly equal to the mass-velocity of the plane has led men to assume that each indi-

vidual molecule suffering a collision in a plane can be treated as though it left the plane with the mass-velocity superimposed upon its linear velocity.

I wish to call attention at this point to the method of adding momenta used in the customary derivations. As many molecules are moving from the plane Q to the upper plane as are coming down, so that the result is a transfer of $\frac{1}{6}Nmcl$ momentum to Q per second. The same thing is occurring between Q and the plane below and the result is the transference of $\frac{1}{6}Nmcl$ momentum per second from Q to the lower plane, usually expressed as a transfer of $-\frac{1}{6}Nmcl$ momentum from the lower plane to Q. By some turn of the imagination these two momenta add up to a transfer of $\frac{2}{6}Nmcl$ across the plane. As a matter of fact the molecules from above merely pass on their momentum to those below, and if the above mechanism for the transfer of momentum could be accepted the value for the viscosity of a gas would be half what it is found to be. The accepted derivation halves the momentum transferred by assuming inelastic impact in the plane, but compensates for this error by counting twice the momentum transferred. The final equation obtained, that is

$$\eta = \tfrac{1}{3}Nmcl$$

is unquestionably correct.

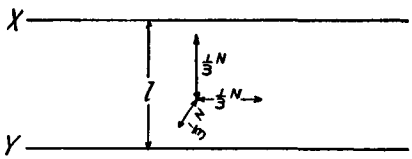
The above fallacy in counting momentum is of the same order as placing a coulometer on both sides of an electrolytic cell and adding the two values obtained in order to get the total current passed through the cell.

As O. E. Meyer pointed out,[1] there is a very close similarity between viscosity and pressure in gases. Let us turn for a moment to a consideration of pressure and its bearing on viscosity. Fig. 2 represents two imaginary parallel planes with the same mass-velocity situated in a gas. For our purpose the planes may be assumed to be stationary. As already stated, the pressure upon these planes may be assumed without loss of rigor to be equal to the pressure that one third of the molecules between the planes would produce

[1] Kinetic Theory of Gases, 1889.

if they were moving normal to the planes with velocities equal to that of the molecule with mean kinetic energy. In one second the number of such molecules producing impacts with one square centimeter of either of the planes is $\frac{1}{6}Nc$ and the force exerted upon that area by each impact is $2mc$,

FIG. 2.



because elastic impacts are assumed. The total force per unit area, pressure, will then be

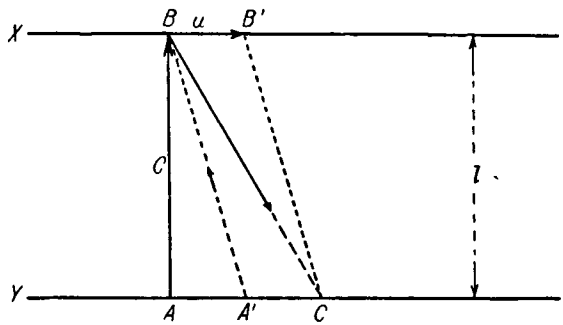
$$p = \tfrac{1}{3}Nmc^2.$$

Let us assume for simplicity that the distance between the planes X and Y is one mean free path and that all of the molecules traverse the intervening space without collision. Of course in the actual case the collisions will be distributed throughout the space and the chance that any one molecule will continue to move back and forth in a direction normal to the planes for even a fraction of a second is insignificant, nevertheless the validity of our assumptions is not vitiated by assuming that one third of the molecules are moving normal to the planes and experiencing impacts only in the planes. Although each plane is merely an imaginary locus of impacts between the molecules under consideration and those coming from beyond, for our purpose it may be imagined as a rigid elastic continuous surface, infinitely rough, so that regardless of the angle at which a molecule impinges, it is returned along the same path from which it approached.

Let us imagine now that the plane X is moving with a velocity u with respect to Y. Represent by the vectors c and u, Fig. 3, the velocities and directions of motion of a molecule leaving plane Y at A, and the plane X respectively. Assume u to be very small compared to c. The path of the molecule will be from A to B but its motion relative to the plane X will be along the path $A'B$. The distance AA' will be equal

to BB' and is so small compared to AB that the difference between the paths AB and $A'B$ is entirely negligible at low rates of shear. Since a molecule impinging on plane X will rebound along the same path, relative to the plane from which it approached, it will follow BC so that when the point of impact B has moved to B' the molecule will have moved to C, which has the same relative position with respect to the

FIG. 3.



plane as A' had when the molecule was at B. The actual path of the molecule will be ABC. We must not carry our artificial assumptions too far, however, as it is evident that the molecule impinging at C no longer is moving normal to Y nor has it the average velocity assumed for all molecules making impact. After the impact at B, this individual molecule loses itself in the chaotic motion of the aggregate and divides its added energy among many. The temperature of the mass is prevented from rising by conduction to the walls of the containing vessel.

It is plain to see that the pressure exerted upon X when it is in motion with respect to Y will not be directed normal to it but will be exerted along the direction $A'A$ so that there will be a component of force tangent to the plane whose magnitude per square centimeter will be u/c times the pressure, or

$$F = \frac{u}{c} \cdot \frac{1}{3} Nmc^2 = \frac{1}{3} Nmcu.$$

If now the rate of shear be taken as unity, $u = l$, and we have

$$\eta = \tfrac{1}{3}Nmcl,$$

which is the accepted equation for viscosity.

Of course it is immaterial which plane we take as moving with respect to the other so that we could just as well have considered the molecules moving down from the upper plane producing a tangential force on the lower plane equal to that we just calculated but opposite in direction.

The picture just drawn of the origin of viscosity in a gas lends itself well to a visualization of the transformation of work into heat. Each molecule as it impacts in a plane moving with respect to the plane from which it came, is given kinetic energy, and if this is not lost by conduction to the container the whole gas must rise in temperature.

An indication of the validity of the assumptions made lies in the calculation of the increase in internal energy of a monatomic gas under shear. Assume as above the existence of imaginary planes in the gas perpendicular to the velocity gradient, which is constant. These planes are one mean free path apart. The number of times each molecule strikes a particular plane will be $\frac{1}{6}\cdot c/l$. From Fig. 3 it is seen that the velocity of a molecule after impact with a moving plane is $\sqrt{c^2 + 4u^2}$ and consequently its kinetic energy is $\frac{1}{2}m(c^2 + 4u^2)$. Its increase in energy is therefore $2mu^2$. It follows that the increase in energy of each molecule in a second is

$$2mu^2 \cdot \frac{1}{6} \cdot \frac{c}{l} = \frac{1}{3} \cdot \frac{mcu^2}{l}.$$

The energy added to each cubic centimeter of gas under shear is force per square centimeter times distance per second. The distance one plane will move in a second with respect to a plane one centimeter distance will be u/l so that

$$\text{energy/cm.}^3\text{/sec.} = F \cdot \frac{u}{l} = \frac{1}{3} Nmcu \cdot \frac{u}{l} = \frac{1}{3} \cdot \frac{Nmcu^2}{l}.$$

Dividing by the number of molecules per cubic centimeter we get

$$\text{Energy per molecule} = \frac{1}{3} \cdot \frac{mcu^2}{l},$$

which checks the above calculation.

No such simple calculation of the increase in energy of translation of the molecules in a gas during shear can be made using the customary mechanism depicting viscosity, and unless one introduces assumptions not at all obvious about the way the molecule entering a new plane acquires the velocity of that plane, the calculation leads one far astray.

COLLEGE OF THE PACIFIC,
DEPARTMENT OF CHEMISTRY,
December 15, 1928.

Available Energy. ROBERT A. MILLIKAN. (*Science*, Sept. 28, 1928.) This address was delivered before the Society of Chemical Industry when the author received the Messel Medal for his work in relation to atoms.

Dr. Millikan directed attention to four recent achievements in pure science that are likely to have industrial importance—"1. the discovery of the relation between mass and energy; 2. the development of methods of making very exact atomic-weight determinations; 3. the discovery of the cosmic rays; 4. the development of relativity-quantum mechanics." The first of these, the quantitative equivalence of mass and energy was stated by Einstein. The energy in ergs that is equivalent to a mass stated in grams is found found by multiplying the number of grams by the square of the velocity of light. By combining No. 1 with No. 2, the determination of accurate atomic weights by Aston, the conclusion is reached that "the process of radioactive disintegration with the ejection of an alpha particle is a process that can take place only in the case of a very few of the very heavy and rare elements." This destroys the dream of replacing coal by the atomic energy of common elements, since "for the great majority of the elements, such as constitute the bulk of our world, are in their state of maximum stability already."

With regard to the cosmic rays they "are in fact the birth-cries of the infant atoms of helium, oxygen and silicon. We think that the atom building processes that give rise to the observed cosmic rays can take place only under the extreme conditions of temperature and pressure existing in interstellar, or intergalactic, space." Modern conceptions of matter lead "to the picture of a continuous atom-destroying process taking place under the extreme conditions existing in the interior of stars, and an atom-creating process continually taking place under the equally extreme conditions of just the opposite sort existing in interstellar space.

A limited amount of available energy can be obtained by the disintegration of atoms. An almost unlimited quantity of energy would be set free by the building up of atoms of common elements from the hydrogen contained in the waters of the earth but at the present time the conditions for such combination do not exist on the earth. The sun will continue to supply energy to the earth for a billion years to come. "When the matter of the sun has all been stoked into his furnaces and they are gone altogether out another sun will probably have been formed, so that on this earth or on some other earth—it matters not which some billions of years hence—the development of man may still be going on."

G. F. S.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

PRESENT STATUS OF COMMERCIAL AIRPLANE ENGINE TESTING.

IN March 1928, the Bureau of Standards began testing new commercial airplane engines as a basis for the granting of approved type certificates by the aeronautics branch of the Department of Commerce. The type certificate authorizes the use in licensed aircraft of any engine conforming to the specifications of the engine submitted for type test, and specifies the speed and power rating of such engines. To date, 21 type tests have been undertaken at the bureau and of this number 4 engines have been withdrawn, 11 have failed, and 6 have completed the test successfully. The results show that the average manufacturer should do more development work before going into production, and indicate the importance of type testing as a protection to the public.

The following engines have been approved and rated as a result of type tests at this bureau: Kinner (5-cylinder, radial, aircooled), 90 rated horsepower at 1810 revolutions per minute; Velie (5-cylinder, radial, aircooled), 55 at 1815; Comet (7-cylinder, radial, aircooled), 130 at 1825; Axelson (7-cylinder, radial, aircooled), 115 at 1800; LeBlond (7-cylinder, radial, aircooled), 90 at 1975; Harris (8-cylinder, vee, watercooled), 90 at 1400. The Warner "Scarab" engine was approved and rated 110 horsepower at 1850 r.p.m. on the basis of a test at Detroit, observed by bureau engineers, before the present regulations went into effect. Thirteen other engines —including for example, the Curtiss "Challenger," the Wright "Whirlwind" and the Pratt & Whitney "Wasp"—have been approved on the basis of Army or Navy tests and have received commercial ratings recommended by the bureau.

The number of airplane engines intended primarily for commercial use which are under development at the present time exceeds all estimates made last year, and the bureau is planning to install additional testing facilities in the near

* Communicated by the Director.

future. A 9-cylinder radial aircooled engine made by the Aeromarine Plane & Motor Company is now under test. The following manufacturers are ready to submit engines for test: Hallett Aero Motors Corporation, Inglewood, Calif.; Velie Motors Corporation, Moline, Ill.; Rocky Mountain Steel Products, Inc., Los Angeles, Calif.; LeBlond Aircraft Engine Corporation, Cincinnati, Ohio; Brownback Motor Laboratories, Inc., New York City, N. Y.; Hurricane Motor Company, Inc., Houston, Texas; Menasco Motors Company, Los Angeles, Calif.; Alco Oil Tool Company, Compton, Calif.; MacClatchie Manufacturing Company, Compton, Calif.; and the Western Enterprise Engine Company, Los Angeles, Calif.

The airplane engine test requirements are stated on pages 40 to 42 of Aeronautics Bulletin No. 14. Applications for commercial airplane engine type tests may be addressed to the Bureau of Standards and should be accompanied by general information regarding the design of the engine. No fee is charged the manufacturer but he is required to furnish the gasoline and oil for use during the official test. His representatives are allowed to adjust the engine to their satisfaction before the test starts and to make any necessary adjustments between test periods.

RADIO EQUIPPED AIRPLANE.

THE Department of Commerce assigned to the bureau a cabin airplane for use in the development of radio aids to air navigation. This airplane was originally designed to carry four passengers and a pilot. In order to convert it to a flying laboratory two seats were removed and a permanent radio installation made. A set of complete transmitting and receiving equipment having a top which forms a desk was installed in front of the observers' seats. The essential navigation instruments are duplicated on an instrument board above this desk. On this instrument board and on the one in front of the pilot, visual reed indicators for the directive radio beacon and marker beacons are provided. Two persons may observe the operation of the radio equipment and at the same time be completely informed as to the operation of the air-

plane without disturbing the pilot. The top of the desk is shock mounted and available for use in testing experimental equipment which it may be desirable to try out in the air.

In order to insure satisfactory operation of the receiving and transmitting equipment the entire airplane structure was bonded and the engine ignition system completely shielded. The shielding is so successful that a very sensitive aircraft receiving set may be operated at full sensitivity without interference from the engine ignition. Power for the operation of the transmitting set is obtained from a generator driven by the airplane engine.

This airplane, which is in fact a flying radio laboratory, is being used in the study of the operation of the radio beacon system and other uses of radio under conditions of flight.

REPORT ON ORIFICE METERS FOR LARGE VOLUMES OF GAS.

THE March issue of the Bureau of Standards *Journal of Research* will contain a paper describing the orifice meter investigation carried out at Edgewood Arsenal in cooperation with the Chemical Warfare Service, and giving the principal results obtained with square-edged orifices such as are commonly employed in the commercial metering of large volumes of gas.

Absolute rates of discharge were measured by means of flow nozzles. This method has been investigated both experimentally and theoretically, and it appears to be the most convenient and accurate one now available for measuring high rates of flow. The discussion of this subject is an important part of the paper.

Of the 48 orifices tested, 26 were borrowed from several commercial firms and the remainder were made at the bureau. They were installed in 4, 6, and 8 inch pipes, with long straight runs in each direction, and with various combinations of pressure taps. They were tested with air at static pressures up to 275 lbs. per sq. inch and with differentials up to 70 per cent. of the upstream pressure.

The average results obtained are expressed in the usual manner by numerical tables of discharge coefficients applicable to the three most common locations of the pressure taps and

to various formulas for computing the rate of flow. Aside from these quantitative results, the following conclusions may be stated:

a. The discharge coefficient of a square-edged orifice for air, as defined in any of the usual ways, varies with the ratio of the differential to the static pressure, but it does not vary appreciably with the absolute value of the static pressure. The variations mentioned are negligible in most commercial practice, but at low static pressures, where the differential is sometimes a considerable fraction of the upstream pressure, they may be of considerable importance.

b. To produce duplicate orifice plates which shall all have the same discharge coefficient within one half of one per cent. requires very good mechanical work, even when the diameter of the orifice is one inch or more. With a few exceptions, the plates tested were all well made and in good condition; but the coefficients of individual plates often varied by as much as one half of one per cent. from the average of all, and sometimes by one per cent. without there being any obvious reason in the appearance of the plate.

c. With a given ratio of orifice to pipe diameter, the tests in the 6 and 8 inch pipes did not show any certain effect of absolute size. Similar plates in the 4 inch pipe gave, on the average, somewhat higher coefficients, but the experiments were not adequate to establishing any general relation between the coefficient and the absolute diameter.

In the autumn of 1924 the Bureau of Standards cooperated with a committee of the American Gas Association, under the Chairmanship of M. E. Benesh of the Peoples Gas Light and Coke Company of Chicago, in an investigation of several types of large gas meters. The experiments were carried out at one of the stations of the company just named and the absolute measurement of the rates of flow of air was effected by the use of a large gas holder. Among the metering devices tested were three very carefully made orifices installed in a 24 inch pipe and it is of interest to note that the discharge coefficients found for these orifices agree with the average results obtained or 6 and 8 inch pipes at Edgewood, well within the errors of measurement.

PRESENT STATUS OF COOPERATIVE ORIFICE METER TESTS.

EXTENSIVE tests of orifice meters are being conducted by the measurement committee of the natural gas department of the American Gas Association in cooperation with the Bureaus of Mines and Standards of the Department of Commerce.

The first group of tests was made at Cleveland, Ohio, in the fall of 1925, and consisted of a number of tests on three four-inch orifices in series with the first lift of a covered gas holder.

The second group of tests was made during the summer and fall of 1926 at Buffalo, N. Y., where some of the same equipment was utilized. The object was to determine the effects, upon orifice meter indications, of various kinds and combinations of pipe fittings placed close to an orifice meter flange. Based upon the results of these tests the committee prepared and issued a tentative code on the installation of orifice meters.

The tests made during 1927 may be divided into two groups. The first group was a supplement to the 1926 tests, and included a few check tests on the 1926 work and a short series of "disturbance fitting" tests using a 4 instead of an 8 inch pipe line. The second group of tests designated as "rate of flow tests," was made with orifices in an 8 inch line, ranging in orifice diameters from one to 6½ inches. The rates of gas flow through these orifices, as indicated by the pressure drop across the orifice, ranged from about 10 to over 400 inches of water. From these tests it was possible to determine a series of orifice coefficients for the range of sizes used.

The 1928 tests may be divided into three groups. The first group was a supplement to the 1927 rate of flow tests, being a short series of rate of flow tests with orifices in a 4 inch pipe. The second group of tests was designated as "high pressure" tests. The orifice under test was used in a line in which the pressure ranged from about 100 to about 300 lbs./in.^{2}. The third group of tests may be designated as "flange design" tests, and was made to determine what effect, if any, the interior form of an orifice flange has upon orifice measurements. In one case, the flanges were screwed on to the pipe until the ends of the pipe were flush with the faces of the flanges. In the other set, there was a considerable recess

within the flanges between the ends of the pipe and the flange faces. It was convenient to make many of the tests for the first and third groups together. A part of the second group or high pressure tests was made at Hastings, W. Va., where higher pressures were available than at Buffalo. The results of the 1928 tests are now being carefully studied.

During 1927 and 1928, H. S. Bean, Chief of the Gas Measuring Instruments Section of the Bureau of Standards, supervised the tests for the committee. Mr. Bean is now in southern California where he is carrying on the review of the 1928 data. He has discussed with Wm. Moeller, Jr., Chairman of the National Gas Bureau, F. A. Hough, Chairman of the Committee on Natural Gas Measurement of the Pacific Coast Gas Association, and other members of these organizations, the question of making some further high pressure tests. Hearty cooperation and support have been promised in the event such tests are made. Because of the favorable conditions found in southern California, plans for making further high pressure tests there are being considered. The desirability of making additional rate of flow tests on a large size line, possibly as large as 16 inches has also been discussed. In the event that the high pressure tests are made, it is probable that the cooperation of the Bureau of Mines will be sought. This bureau is also interested in methods for determining the so-called "open flow" of a gas well without resorting to the wasteful practice of "blowing" the well. Such tests could be made to advantage at the same time the high pressure tests are made, and have been considered in the discussion between Mr. Bean and the committee chairman named above.

STANDARD SAMPLES.

NEW standard samples that have been prepared recently are the standard sample of Fluorspar No. 79 which is priced at $2.50 per 60 grams and Ferrophosphorus No. 90 which is priced at $2.00 per 75 grams. The former contains 94.9 per cent. of calcium fluoride, 1.9 per cent. of silica, 1.0 per cent. of carbon dioxide, 0.14 per cent. of sulphur and small amounts of zinc, lead, copper, iron, aluminum, phosphorus, titanium, potassium, sodium, magnesium, barium, and manganese.

Sample No. 90 contains 26.17 per cent. of phosphorus. A sample of opal glass containing 68 per cent. of silica, 6 per cent. of fluorine, in addition to iron. aluminum, titanium, zirconium, calcium, magnesium, sodium, potassium, phosphorus, manganese, chlorine, and arsenic has been prepared and will be available about March 1. Two new standard samples of glasses containing boron, one 12 per cent. and the other 0.75 per cent. of B_2O_3 are being prepared.

Recently renewed standard samples are: (1) Cast Iron No. 4d which contains total carbon 2.66 per cent., graphite 2.00, combined carbon 0.66, manganese 0.894, phosphorus 0. 081, sulphur 0.075, silicon 1.27, and copper 0.24; (2) Cast Iron No. 5f which contains: total carbon 2.25 per cent., graphite 1.52, combined carbon 0.72, manganese 0.755, phosphorus 0.243, sulphur 0.090, silicon 2.31, and copper 0.55; (3) Acid Open Hearth Steel No. 34a which contains: carbon 0.763 per cent., manganese 0.499, phosphorus 0.029, sulphur 0.026, silicon 0.276, copper 0.222, nickel 0.232, and chromium 0.275; and (4) Nickel Steel No. 33a which contains: carbon 0.366, manganese 0.700, phosphorus 0.037, sulphur 0.032, silicon 0.233, copper 0.114, nickel 3.48, and chromium 0.029.

In ordering standard samples, money orders, etc., should be made payable to the Bureau of Standards. If remittance does not accompany order, samples will be sent under Government frank by parcel post C.O.D. to addresses in the United States and its possessions. With the exception of Canada and Mexico, 20 cents postage must be added for every 300 grams of samples or fraction thereof. Payment for foreign orders should be by an international money order or by a check payable through the New York Clearing House or a bank in the United States.

A list of the standard samples issued to date may be had free on application to the Bureau of Standards by asking for "Supplement to Circular No. 25."

LIQUID SHRINKAGE OF ALLOYS.

THE American Foundrymen's Association, operating through its committee on cast iron and sub-committee on cast iron research, has established a Research Associateship at the

bureau, to continue for one year, for the study of methods of determining the liquid shrinkage of alloys.

The soundness of a casting depends, among other things, on its freedom from concealed piping or internal voids due to shrinkage, from trapped gas bubbles, and from shrinkage cracks. Three main factors are concerned in respect to the metal itself, as distinguished from the foundry practice, in securing freedom from these defects, i.e., fluidity, liquid shrinkage or change of volume between the liquid and the solid state, and solid shrinkage or change of volume between the freezing point and room temperature. Methods for the determination of the last property, solid shrinkage, are in fairly satisfactory shape, but those for the first two are not, as yet.

The bureau is already studying methods for determination of fluidity, and the establishment of this Research Associateship will allow an extension of the work to the study of liquid shrinkage.

The subject is of especial importance in the case of cast iron where the amount of liquid shrinkage seems to have a bearing on the suitability of different compositions of cast iron for different uses, but the American Foundrymen's Association is interested in the problem from the point of view of all other foundry alloys as well.

The object of the work is therefore not to collect masses of data on the liquid shrinkage of all the many alloys in use in the foundry, but rather to establish the suitability of some one or more methods for its determination in the various types of alloys. With reliable methods available the various branches of the foundry industry would be able to amass reliable and comparable data on the alloys in which they are interested.

IMPORTANCE OF TEMPERATURE AND HUMIDITY CONTROL IN RUBBER TESTING.

THE physical testing committee of the rubber division, American Chemical Society, through its research associate, F. E. Rupert, has conducted a series of tests at the bureau to determine the effect of temperature and humidity control in rubber testing.

This investigation involved several thousand tests, made

upon five rubber compounds under conditions in which temperature and humidity were varied during both preparation and testing. The compounds represent types of formulas in general use in the tire industry—a laboratory test compound, two high-grade gum tire carcass compounds, and two high-grade tread compounds. These formulas give an opportunity to observe the effect of humidity and temperature upon compounds of high and low rubber content and upon compounds containing two widely different types of organic accelerators, as well as varying amounts of common softeners.

The first part of the report deals with the effect of temperature and relative humidity subsequent to vulcanization but prior to testing. The temperatures used were 5, 15, 25, 35, and 45° C., and the relative humidities used at each of the above temperatures were 0, 20, 60, 80, and 100 per cent. Tests were made at each of these relative humidities at each temperature with each of five cures of the five compounds, thus providing for the effect of these variables upon both under and over cured materials.

This part of the investigation proved that variations in temperature which may occur from day to day in an uncontrolled testing room, may affect the physical tests to the same extent as a 25 to 40 per cent. change in time of cure. The relative humidity affects the results in only a minor degree.

The second part of the investigation, in which the same five compounds were used, dealt with the effect of temperature and relative humidity after mixing but prior to vulcanization. Only three cures of each compound were used, while the temperatures employed were 15, 25, and 35° C., and the relative humidities 10, 40, 70, and 100 per cent.

The results of this part of the investigation showed that variations in the absolute humidity of the room in which the unvulcanized rubber is stored between the time of mixing and the time of curing may affect the tensile strength and modulus of rubber compounds to as great a degree as does the temperature after vulcanization.

The effects upon the physical properties of vulcanized compounds produced by storing the raw ingredients under various humidities previous to mixing were also studied. No appreciable differences in the properties of the vulcanized

rubber were found, from which it was concluded that ordinary storage conditions are satisfactory for crude rubber compounding ingredients.

In an extension of this investigation, now in progress at the bureau, the effect of temperature and humidity, both prior to and subsequent to vulcanization, upon the resistance to abrasion, is being studied.

DIATOMACEOUS SILICA AS ADMIXTURE IN CONCRETE.

A PREVIOUS item dealing with this investigation appeared in Technical News Bulletin No. 141 (January, 1929). Other properties of the diatomaceous silicas are being studied as an aid in the development of specifications. Settling tests have been made on the representative samples according to one particular method of procedure which has been proposed. In this method, 2 gram samples of the different materials are placed in graduated cylinders, and sufficient water is added to make a total volume of 100 ml. at 20–30° C. The mixture is then shaken thoroughly, after which the cylinders are allowed to stand on a level space for 20 minutes and for 5 hours, respectively. The percentage of material in suspension in the 80 ml. nearest the top of the cylinders is then determined. The material in suspension in the top 80 ml. of water varied from 13.2 to 60.4 per cent. at the end of 20 minutes, and from 2.4 to 25.3 per cent. at the end of 5 hours.

It was noted that the diatomaceous silica swells when placed in lime water. A study of the volume of the resulting floc from the various representative samples may be an index of the fineness of subdivision and purity of the diatomaceous silica. It has been found that 2 grams of the various materials form flocs varying from about 12 to 35 ml. when placed in 100 ml. of saturated calcium hydroxide and allowed to settle 24 hours after an initial shaking of about 30 seconds. If the same procedure is followed, the measurements of the volume of the resulting floc from each diatomaceous silica may be checked to within 1 or 2 ml.

It has also been observed that diatomaceous silica will remove lime from a calcium hydroxide solution, from 1.15 g. CaO per liter (the approximate concentration of a saturated

solution at 30° C.) to about 0.07 g. CaO per liter. Increasing amounts of silica apparently do not further decrease this latter value. A study of the rate of this reaction has shown that different kinds of silicious materials decrease the concentration of the calcium hydroxide at different rates. The rate of the reaction has been measured by both chemical analyses of the resulting solutions and pH determinations, with very good agreement. It has been found possible to distinguish the purer grades of diatomaceous silicas (as determined from petrographic examination) from those containing considerable impurities, by this method, but to date it has not been found possible to differentiate the purer grades since the rate of decrease of concentration of the calcium hydroxide is so nearly the same with these latter materials. The time involved in obtaining these rates is too long to be of any value as a method for specifications.

CONSISTENCY MEASURING INSTRUMENTS FOR CALCINED GYPSUM.

AT the request of committee C–11 on gypsum of the American Society for Testing Materials, a study of the instruments for determining the standard testing consistency of calcined gypsum was undertaken. The instrument used at the present time is the Southard viscosimeter. It consists essentially of a circular plate 12 inches in diameter with a well 2 inches in diameter in the center, fitted with a false bottom. The well is filled with the mix to a depth of 2.5 inches and the mix forced out at the top by means of a screw under the false bottom. The mix is said to be of standard testing consistency when the spread of the pat is 9.7 cm. as read on the circular plate. The other instruments which were compared with the Southard viscosimeter were the Hammond and the modified Vicat apparatus. The Hammond apparatus consists essentially of a 2 × 4 inch cylinder which is filled with the mix and then raised vertically by means of a weight, through lever action, allowing the mix to spread out on a glass plate. The diameter of the pat is read on a scale beneath the glass plate. The modified Vicat instrument consists of a plunger of a fixed weight made of aluminum tubing. The end that penetrates the mix is 0.75 inches in diameter and 1.75 inches in length. The plunger is allowed to drop vertically into the mix con-

tained in a rubber ring mold, 7 cm. in diameter at the base, 6 cm. in diameter at the top and 4 cm. in depth. The penetration is determined by the drop of the plunger as read on a millimeter scale. The weight of the plunger may be varied by loading with shot.

A mix of gauging plaster was made up to standard testing consistency as determined by the Southard viscosimeter, and after many tests it was found that the most suitable penetration of the plunger of the Vicat was 30 mm., and this penetration was obtained when the plunger weighed 51 grams. A mix of gauging plaster of standard testing consistency when tested on the Hammond machine gave a spread of 4.1 in. It was found necessary to add retarder before making any test, or reproducible results could not be obtained.

The three instruments were then used to determine the standard testing consistency of 6 types of plaster and to determine which instrument was the most satisfactory. It was found that the three instruments did not check for all types of materials and that reproducible results could not be obtained for some types on any one instrument. The modified Vicat apparatus was found to be more sensitive to changes in consistency, easier to operate, and the penetration as shown by the drop of the plunger was more nearly proportional to the consistency than was the spread as shown by the Southard viscosimeter and Hammond machine.

FUNDAMENTAL STUDY OF GLAZE FIT.

As previously reported in Technical News Bulletins Nos. 125 and 128 (September and December, 1927) an investigation is being conducted at the Columbus branch of the bureau to determine the effect of the various oxides on the tensile strength, modulus of elasticity, and coefficient of expansion of glazes. Factors have been worked out for the various properties, based on the composition of the glazes as determined by chemical analysis.

The coefficient of expansion of the glazes increases slightly with temperature, but for the present work it is thought best to use the mean coefficient and express the effect of the various oxides as simple constants rather than as expressions such as

$a + bT$, where a and b are constants and T is the temperature in degrees C. The following tentative values for the oxides apply fairly well for the average linear coefficients over the range 20° C. to the lower critical temperature: Na_2O, 380; K_2O, 300; CaO, 150; BaO, 120; ZnO, 100; PbO, 75; Al_2O_3, 50; B_2O_3, 20; MgO, 20; all these values to be multiplied by 10^{-9}.

The factor for SiO_2 varies with the amount of SiO_2 present in the glaze, from 51 with 30 per cent. SiO_2 to 19 with 90 per cent. SiO_2. The first part of this work will be published in the near future.

Some work on the application of the data that have been collected on the physical properties has been started. The following subjects will be investigated: (1) resistance to thermal shock; (2) the effect on glaze fit of the upper critical temperature of the glaze and its position with respect to the alpha-beta inversion; and (3) the variation with temperatures of strain in a glaze attached to a body and its effect on crazing and shivering. The last subject will be investigated with the aid of a thermal microscope that has been built at the Columbus station.

A Study of the Fluctuations in the Amount of Ozone in the Atmosphere at Night. D. CHALONGE. (*Comptes Rendus*, June 25, 1928.) By photographing the ultra-violet spectrum of the sun and of the moon the author has determined the thickness of the layer of ozone in the upper atmosphere as often as possible on consecutive nights and days from October, 1927 to April, 1928. The measurements were made on the spectral region from 3100 to 3250 Å., where the absorption of ozone begins to be appreciable without being strong. Here unfortunately the absorption coefficient is less accurately known than for wave-lengths near 3000 Å. where others have worked in measuring the quantity of ozone. The feebleness of moonlight, however, prevented the use of the better known region.

The calculated depth of the ozone layer is never less at night than during neighboring days, indeed it is generally greater than by day, and it shows no seasonal variation, departing not greatly from its mean value of 3.35 mm. This conclusion is not as free of significance as might at first appear, for "From this it would follow that to explain the formation and fluctuations of ozone at high altitudes it is necessary to invoke other causes than solar ultra-violet radiation alone."

G. F. S.

Auroral Observations at the Sitka Magnetic Observatory, August, 1927 to June, 1928. F. P. ULRICH. (*Ter. Mag.*, Sept., 1928.) The station at Sitka got its time signals by radio from Annapolis, Mare Island and Honolulu. Reports were received by radio from several stations in Alaska and from Fort Yukon. Occurrences of the aurora were reported on 58 occasions and characterized thus, 34 faint, 21 bright and 3 brilliant. "The observations for this year seem to indicate that good (radio) reception is very much more apt to occur than poor reception during a bright or faint aurora. Aurora causes poor reception in cable-transmission, while in radio reception no difficulties are experienced in the majority of cases." For the past year the aurora was not accompanied by pulsations, draperies or coronas. Rays were seen 16 times. As the result of 901 observations on the relation of the state of the earth's magnetic field to the reception of radio messages it is stated that "in general the condition of the earth's magnetic field is no index of the way radio reception is received." G. F. S.

NOTES FROM THE RESEARCH LABORATORY, EASTMAN KODAK COMPANY.*

EFFECT OF ENVIRONMENT ON PHOTOGRAPHIC SENSITIVITY. II. EFFECT ON CERTAIN SALTS.[1]

S. E. Sheppard and E. P. Wightman.

THE action of certain neutral salts on the formation of the latent image has been studied. Potassium citrate had no detectable effect, potassium chloride a slight effect upon developability, potassium bromide a very considerable depressing effect on the formation of the latent image. Contrary to the behavior with the visible image, acid silver nitrate was not found to accelerate latent image formation, at any rate with fairly high speed plates. The results are discussed in terms of the transfer of electrons from bromide ions to silver ions of the lattice, and it is suggested that the electrostatic field of the adsorbed double layer is a modifying factor of differing importance for visible and latent image formation.

THE ACTION OF FATTY ACIDS ON CELLULOSE.[2]

C. J. Malm and H. T. Clarke.

WHEN native cellulose is boiled with a lower fatty acid, esterification takes place which tends to a limit expressed by the formula: $C_{24}H_{39}O_{20}(CO.R)$. When hydrated or mercerized cellulose is treated in the same way, the limit is $C_6H_9O_5(CO.R)$. From this it is concluded that the constitution of cellulose must be expressed as a multiple of the unit $C_{24}H_{39}O_{20}$.

* Communicated by the Director.

[1] Communication No. 349 from the Kodak Research Laboratories and published in *Phot. J.*, 69: 22. 1929.

[2] Communication No. 367 from the Kodak Research Laboratories and published in *J. Amer. Chem. Soc.* 51: 274. 1929.

Obsolescence vs. Depreciation. In a pamphlet issued by the American Institute of Steel Construction the results are presented of extensive studies of the loss in earning power of large modern constructions such as office buildings, hotels, apartment houses, and department stores. Through special facilities, many details were obtained of the vicissitudes that such buildings suffer in the feverish development of modern American cities. Shift of business centers is a not infrequent cause, but a more frequent one is the change in the details of construction and equipment. Every few years new methods of heating, lighting, refrigeration, ventilation, elevator service, fireproofing, and other accessorial features suffer modification. Tenants of the older types of building are more or less dissatisfied with their occupancy. The pamphlet, of course, distinguishes between obsolescence and depreciation. the latter being an inevitable result in all constructions. It is the fact that so many constructions from toys to railroad terminals get out-of-date long before wearing out that places so serious an economic burden on our modern life. The pamphlet deals only with large buildings, but it is worthy of note that in the matter of naval armament a similar and very costly condition arises and is constantly operative. Expensive ships become useless long before they wear out and they represent in their "scrapped" condition a mere trifle of the original cost. H. L.

High-Frequency Fluctuations in the Intensity of a Beam of Light. (*Proc. Phys. Soc.*, London, Vol. 40, Pt. 1.) A beam of light traversed the first of a pair of crossed nicol prisms but before reaching the second it passed through a piezo-electric crystal. When the crystal was unstressed no light emerged from the second nicol, but when the crystal was set in vibration by sending the current from a valve through a winding surrounding it light emerged from the system. This light was reflected from a polygonal drum and caught upon a screen. The variations in intensity due to the vibrations of the crystal manifested themselves by the appearance of bands 1.5 mm. apart. From this it was calculated that the frequency of the light beam was 168,800 cycles per sec., or about double the frequency of the crystal. G. F. S.

NOTES FROM THE U. S. BUREAU OF MINES.*

CARBON MONOXIDE CONTENT OF AUTOMOBILE ENGINE EXHAUST GASES NOT INCREASED BY USE OF ETHYL GASOLINE AS MOTOR FUEL.

THE use of gasoline containing ethyl fluid as an automobile engine fuel does not affect materially the percentage of carbon monoxide contained in the exhaust gases, the United States Bureau of Mines concludes, as the result of a series of tests conducted at its Pittsburgh Experiment Station.

Public interest in atmospheric pollution by automobile exhaust gas and in the ventilation of vehicular tunnels makes it desirable to ascertain whether the use of modern automobile fuels is tending to change the amount and composition of the products of combustion. The United States Bureau of Mines, in coöperation with the Ethyl Gasoline Corporation, has completed a series of tests to determine whether any significant difference exists in the carbon monoxide content of the exhaust gas produced by an internal-combustion engine when its fuel is changed from straight gasoline to the same gasoline containing ethyl fluid whose active ingredient is tetraethyl lead.

In the tests data was sought relative to the composition and amount of the gas produced by ordinary comparatively low-compression motors as well as by higher-compression motors in which distinct detonation occurred with straight gasoline, but which operated without detonation when using the same gasoline containing tetraethyl lead.

The gasoline used for these tests was all taken from the same refinery run to insure similar composition and characteristics. One portion was used as received from the refinery; to a second portion, ethyl fluid in the proportion of $2\frac{1}{4}$ c.c. of tetraethyl lead per gallon was added (this represented the antiknock value of standard ethyl gasoline as marketed); to a third portion was added ethyl fluid in the proportion of

* Published by permission of the Director, United States Bureau of Mines.

3 c.c. of tetraethyl lead per gallon. Tests were made with engine operating at full load and at three quarter load with various carburetor adjustments.

When the engine was operated at a fixed adjustment no significant change was found in the carbon monoxide content, nor in the content of any other constituent of the exhaust gas, upon changing the fuel supply of the engine from straight gasoline to gasoline containing tetraethyl lead (ethyl gasoline). This was true of both the tests in which a detonation was evident and those in which no detonation was audible. Also no significant difference in the amount of carbon monoxide per horse-power hour was noted.

However, if the spark was retarded to alleviate detonation during operation on straight gasoline the amount of carbon monoxide per horse-power hour was approximately 5 to 7 per cent. less for ethyl gasoline than for straight gasoline.

As to the effect of ethyl gasoline on health and safety the amounts of carbon monoxide produced by the engine, under any comparable operating condition, were the same for ethyl gasoline and straight gasoline.

PRODUCTION OF FERROMANGANESE FROM MANGANIFEROUS IRON ORES.

THE degree of dependence of the United States upon importations of manganese will be influenced to a great extent by the success achieved in obtaining ferromanganese from domestic low-grade reserves. For several years the United States Bureau of Mines has been investigating, at its North Central Experiment Station, Minneapolis, Minn., the possibilities of obtaining ferromanganese from manganiferous iron ores occurring as large deposits in Minnesota, New Mexico, and other localities. The main objective of this investigation is to determine the feasibility of recovering from these ores the manganese now used largely for its indirect effects. Although the indirect results from the use of these ores, such as aiding desulphurization in the blast furnace and producing more fluid slags in the open hearth, are useful functions, only a small portion of the manganese reaches the steel ingot, the greater part of it passing into irretrievable slag dumps.

The Bureau of Mines, in coöperation with the Minnesota School of Mines Experiment Station, demonstrated with a small 6-ton blast furnace and a small open-hearth furnace that it is feasible to produce from manganiferous iron ores, containing 5 to 10 per cent. manganese and averaging 7.5 per cent., a slag containing 40 to 50 per cent. manganese and permissible amounts of iron and phosphorus. Slags containing less than 10 per cent. silica have been made, but the time required is greater due to increased slag viscosity accompanying lower silica content. A larger experimental open-hearth furnace of the tilting type is nearing completion.

ABNORMALITY IN CASE-CARBURIZED STEEL.

DIFFERENT results on depth of carbon penetration, grain size and uniformity of surface hardness are often found in carburized steels of essentially the same chemical analysis, and the term "abnormality of case-carburized steel" has been used to denote certain of these variations.

A study of this perplexing problem is being conducted at the Pittsburgh Experiment Station of the United States Bureau of Mines, Department of Commerce. It has been found that steels may show two distinct types of abnormality on carburization; grain size abnormality and structural abnormality. The first is usually due to the addition of certain elements to the steel and may be regulated by the elements added in the manufacture of the steel. For example, an increase in manganese tends to increase the grain size and the addition of vanadium refines the grain and the carburized piece will show grain size abnormality, i.e. smaller grains than are usually found. Heat treatments considerably affect this type of abnormality.

Structural abnormality has been found to be associated with either a high FeO content or with the presence of very small inclusions in the experimental steels made by the Bureau of Mines. Apparently as the inclusions become larger they affect structural abnormality less and less. Heat treatments do not affect this type of abnormality.

Further data on grain size and structural abnormality are being collected on samples prepared in a vacuum fusion

furnace under closely controlled conditions, and on steels deoxidized with various deoxidizers which give different types of inclusions.

A NEW TREND IN ORE DRESSING.

CRUSHING is the most costly step in all ore-dressing processes, observes the Department of Commerce. It is done in order to free the minerals comprising the ore from each other and to produce grains of a size within the recovering power of the process to be used. Costly fine grinding has heretofore been considered essential to flotation. Since there is considerable mineral loss in the slimes produced in such crushing, coarse sand flotation is being studied at the Moscow, Idaho, field station of the United States Bureau of Mines as a possible means of effecting economy. There are two main reasons why coarse flotation has been thought to hold little promise: First, there is a limit to the size of grains that flotation will lift. This limit is not exactly known, but has often been placed at 65 mesh. Coarser grains will float under favorable conditions. A second reason for favoring fine grinding is the difficulty of circulating coarse sand. The gravity-flow type of machine has overcome this objection. There is no need to float the largest mineral grains in a coarsely crushed pulp. They are ordinarily best recovered by gravity concentration.

The coarse sand flotation problem has been solved in the Michigan copper district. It embodies flotation of coarse sand followed by classification and table concentration. Flotation removes fine and intermediate-size mineral grains; the classifier overflow is tailing; the table recovers coarse mineral and middling, and rejects coarse tailing. A great metallurgical and economic advantage has been gained by this process. Experiments indicate that it applies equally effectively to ores of many other districts.

This process does not require flotation to recover unduly large mineral grains, nor tables to save the slime and fine mineral too small to feel the influence of gravity, nor does it require a classifier to deslime completely. The classifier functions only as a tailing-maker.

STUDY OF DEVICES FOR RECORDING PRESSURES DEVELOPED IN EXPLOSIONS.

SEVERAL different types of manometers have been used for recording the pressures developed during the course of mine explosions, says the United States Bureau of Mines, Department of Commerce. It is important that the results obtained with the different manometers be compared. Accordingly, the Pittsburgh Experiment Station of the Bureau of Mines undertook to determine the relative accuracy or degree of agreement of several commonly used types of manometers. The final comparison included the British Coal Dust (B.C.D.), Crosby, Illinois, Diederichs, and the Bureau of Mines instruments. The comparisons were made by connecting the manometers to a spherical explosion bomb in which mixtures of methane and air or hydrogen and air were exploded. The manometers were connected to the bomb in pairs, the Bureau of Mines instrument being common to all comparisons.

Although analysis and interpretation of the experimental data are not yet complete, it appears as a result of the intercomparisons that in general the Bureau of Mines manometer indicates a higher maximum pressure than any of the other instruments tested. This was not true in all tests with the B.C.D. instrument but was uniformly true for all tests with all the other manometers. The Bureau of Mines instrument attained maximum pressure in a shorter time after ignition of the gas than did any of the other manometers except the B.C.D. For the latter, the time to maximum pressure was sometimes greater and sometimes less than for the Bureau of Mines instrument.

The Bureau of Mines instrument showed more favorable characteristics as regards elastic hysteresis than did the other instruments. In the test using hydrogen-air mixtures, pronounced vibrations appear in the time-pressure records. These are believed to be vibrations of the diaphragms caused by the suddenness of the explosions, rather than by stationary waves set up in the explosion bomb, although the latter appears as a possibility. Initially the vibrations were of approximately the same frequency as that of the experimentally determined free-period vibrations of the respective instruments; but the frequency invariably decreased as the pressure decreased and the vibration died away. The frequency finally reached about one-half the original frequency.

The Index of Refraction of Glass for X-Rays of Long Wave-length. J. THIBAUD. (*Comptes Rendus*, July 23, 1928.) In previous papers reporting their spectroscopic investigations of very soft X-rays Thibaud and Soltan directed attention to the seeming dependence of the wave-lengths of the same rays upon the method used to measure them. The length got by the use of ruled diffraction gratings was not the same as that obtained from diffraction in crystals. They explained the difference as due to refraction in the interior of the crystal and drew the conclusion that there is a real index of refraction of value less than unity, when X-rays of long wave-length pass from a vacuum into a material medium. Their measurements further indicated, in accordance with the Drude-Lorentz theory of dispersion, that the quantity by which the index is less than unity increases with the square of the wave-length. The object of the present investigation is to calculate the index of refraction of glass from the measured angle of total reflection. When an X-ray of considerable wave-length falls on a reflecting surface of glass, it is totally reflected provided that the angle between the direction of the ray and the surface does not exceed in value a limiting angle, θ. When θ is known the index of refraction μ is calculated by the formula, $Q = \sqrt{2(1-\mu)}$.

To determine the limiting angle θ a vacuum grating spectrograph was utilized. Different monochromatic X-rays fell on a glass grating and photographic registration of the diffracted rays showed that their intensity diminished progressively as the angle between the rays and the plane of the grating approached the angle θ. Visual perception of the varying intensities was aided by the fact that the spectra sometimes extended to the seventh order.

In the following table of results λ, the wave-length is given in Angstrom units; θ, the limiting angle, in radians and δ is the number that must be subtracted from 1 to get the refractive index.

	Ordinary X-Rays	X-Rays of Great Wave-Length			
λ	1.5	20.	44.9	54.9	65.
θ	.004	.031	.051	.061	.078
δ	.000007	.00048	.0013	.00186	.00305

The limiting angle for the wave-length 1.5 is 21 minutes, for 45 Å. it is 3° and for 65 Å. it is 4° 30′. For this last wave-length the index of refraction is 1 − .00305 or .99695. The observed value of the limiting angle does not grow with increase of wave-length as rapidly as theory requires it to grow. G. F. S.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, FEBRUARY 20, 1929.

The stated monthly meeting was called to order at eight-fifteen by Mr. C. C. Tutwiler, Second Vice-President, in the absence of the President.

The Secretary announced that the minutes of the January meeting had been published in full in the Journal of The Franklin Institute for February, and moved that the minutes be approved as printed. The motion was adopted unanimously.

For the Committee on Membership the Secretary reported that since the beginning of the new year, the following additions had been made to our membership: two Life members; eight Resident members; five Non-Resident members; 1 Student member. He announced with great regret the deaths of eight members during the same period.

The Secretary stated that this was the meeting at which the Institute Committees for the year 1929 should be announced and said that the President of the Institute, Mr. Hayward, had appointed, in accordance with the By-laws, the following Committees of the Institute:

Committee on Library

C. W. Bates, Chairman
H. J. M. Creighton
W. H. Fulweiler
Clarence A. Hall
Joseph S. Hepburn
George A. Hoadley
Lionel F. Levy
Malcolm Lloyd, Jr.
M. M. Price
George F. Stradling

Committee on Meetings

Howard McClenahan, Chairman
James Barnes
George S. Crampton
T. G. Delbridge
Arthur W. Goodspeed
Charles Penrose
George D. Rosengarten
Coleman Sellers, 3d
W. C. Wagner
W. Chattin Wetherill

Committee on Museum

Hugo Bilgrim, Chairman
G. H. Benzon, Jr.
Arthur L. Church
Theobald F. Clark
George A. Hoadley
John J. L. Houston
Henry Leffmann
Frederic Palmer, Jr.
H. H. Quimby
William O. Sawtelle

The Secretary stated further that the following persons had been elected to membership in the Committee on Science and the Arts, for a period of three years:

Henry B. Allen
Clifford W. Bates
Hugo Bilgram
William L. Brown, 3d
Edward L. Clark
Theobald F. Clark
Theodore K. Cleveland
Thomas D. Cope
William G. Ellis
W. H. Kavanaugh
C. H. Masland, 2d
Frederic Palmer, Jr.
Charles Penrose
M. M. Price
James S. Rogers
Samuel Shoemaker
F. R. Wadleigh
Robert L. Wood
Winthrop R. Wright

He also announced that Dr. Thomas D. Cope, Department of Physics, University of Pennsylvania, had been elected Chairman of the Committee on Science and the Arts for the present year.

There being no further business, the Chairman of the meeting introduced as the speaker of the evening Mr. A. R. Stevenson, Jr., of the General Electric Company, Schenectady, N. Y., who spoke on "Refrigeration in Theory and Practice."

The paper was most interesting and admirably delivered. It was followed by some discussion, after which the meeting closed with a vote of hearty thanks to Mr. Stevenson for his valuable lecture.

HOWARD MCCLENAHAN,
Secretary

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, February 6, 1929.)

HALL OF THE COMMITTEE,
PHILADELPHIA, February 6, 1929.

DOCTOR GEORGE S. CRAMPTON *in the Chair.*

The Chairman announced the reelection by the Board of Managers of seventeen members of the Committee to serve for three years and the election of the following new members to serve for three years:

Doctor Theodore K. Cleveland
Mr. Samuel Shoemaker

The Special Committee on Nomination of a Chairman for the ensuing year reported as its choice Doctor Thomas D. Cope. Doctor Cope was unanimously elected and took the Chair.

The following reports were presented for final action:

No. 2840: Klydonograph.

This device may be called a wave writer since it automatically makes a graphic record of information concerning the electrical surges that take place on extended transmission lines. This information includes the polarity of the surge, its magnitude and the steepness of its wave front, the interpretation of the record giving the desired information.

The fundamental experiments that have led to the development of the klydonograph were made by Doctor Lichtenburg in 1777 by passing a condenser discharge across a spark gap onto a terminal in contact with an insulating plate resting upon a grounded metal plate. A powder, sifted upon the upper plate, would show figures characteristic of the discharge and these were known as Lichtenburg figures.

If the discharges take place across the film surface of a photographic plate or film the record is made on its surface and can be used for reproduction and study.

Mr. J. F. Peters of the Transformer Engineering Department of the Westinghouse Company has standardized the form of instrument used and by controlling the conditions under which the figures are obtained, making a comparative study of these figures and interpreting their forms, has made a valuable addition to the knowledge of these surges and their results.

The most recent form of instrument is provided with a clock mechanism by which a roll of sensitized film is moved at a known rate, thus providing means to secure permanent records for future study.

The award of the Edward Longstreth Medal was recommended to Mr. John F. Peters, of Swissvale, Pennsylvania, "In consideration of his successful development of a practical instrument for the recording of voltage surges on transmission lines."

No. 2879: Temperature Control for Gas Furnaces.

The purpose of this device is to enable a gas furnace to control its own temperature automatically. This is done by utilizing the minute current generated in a thermocouple as the first step in operating the gas and air valves that supply its fuel.

The current from the thermocouple passes around a pivoted rotating coil which carries a needle and controls its position, the reading of the needle indicating the temperature.

A vertically moving contacting device that moves up and down in the path of the needle at a predetermined temperature is provided and when this makes contact with the needle, mechanical devices are set in motion that operate the valves controlling the feeding of the mixed air and gas into the furnace thus obtaining the desired temperature. The award of the Certificate of Merit to Mr. George Machlet, of the American Gas Furnace Company, of Elizabeth, New Jersey, was recommended by the Sub-Committee, "In consideration of his pioneer work in perfecting the automatic temperature control for gas fired furnaces."

No. 2885: Work of Sir James C. Irvine in Chemistry.

The few years that have elapsed since the beginning of the present century have seen a notable advance in the science of chemistry and in the practical application of the results of these advances.

Among the brilliant workers in an especial field is Sir James C. Irvine, the Principal of the University of St. Andrews, Scotland, in carbohydrate chemistry. His work on the structural chemistry of the carbohydrates involved the difficult task of separating from each other a series of substances that were closely allied and that had resisted many attempts at separation.

In the beginning of his work he undertook the methylation of various mem-

bers of the sugar group and from a study of the simpler methylated derivatives he was able to determine the structure of the parent compound.

"In 1910, an entirely fresh field was entered upon in Irvine's laboratory, where, by means of a new method of methylation, methyl radicals were introduced into cellulose and starch, and Irvine succeeded in attaining a deeper insight into the structure of the molecules of these very complex substances."

"Irvine's work on the methylation of carbohydrates resulted in the gradual accumulation of a mass of evidence which led to the discovery of a new class of sugars—the gamma sugars, in which the oxygen ring is displaced from the normal, stable position.

"In consideration of his brilliant researches in the field of carbohydrate chemistry, the award of the Elliott Cresson Medal was recommended to Sir James C. Irvine, of St. Andrews, Scotland."

The Franklin Medal—Doctor James Barnes, Chairman.

No. 2903: To Mr. Emile Berliner, Inventor, of Washington, D. C.

"In recognition of a life-work in successful invention: the discovery of the electrical characteristics of the loose contact and application of them to telephony in the microphone, which he originated; development of the talking machine which uses a disc upon which the sound is recorded by a laterally cut groove; and, in recent years, his contributions to the production of proper acoustical qualities in architectural structures."

No. 2904: To Doctor Charles Thomson Rees Wilson, Jacksonian Professor of Natural Philosophy, Cambridge University, England.

"In recognition of his numerous pioneer contributions to atomic physics, especially for his studies of condensation nuclei and his development of a method of photographing their tracks and for the vital importance of these studies to our knowledge of atomic structure and quantum mechanics."

The following report was presented for first reading:

No. 2901: Self-Aligning Coupling.

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(*Stated Meeting, Board of Managers, February 13, 1929.*)

LIFE.

MR. MALCOLM LLOYD HAYWARD, Student, Adirondack-Florida School, Onchiota, N. Y.

MR. NATHAN HAYWARD, JR., Student, E 31 McKindock, Harvard University, Cambridge, Mass.

RESIDENT.

MR. SOLOMON D. BENOLIEL, Electro-Chemical Engineer, International Chemical Company, Philadelphia, Pa. For mailing: c/o International Chemical Company, 2628 North Mascher Street, Philadelphia, Pa.

MR. LOUIS S. CLARKE, Consulting Engineer, c/o The Autocar Company, Ardmore, Pa.

MR. EDWARD S. W. FARNUM, JR., Attorney, 1528 Land Title Building, Philadelphia, Pa. For mailing: 101 West Graver's Lane, Philadelphia, Pa.

MR. OTHMAR KARL MARTI, Chief Engineer, American Brown Boveri Electrical Corporation, Camden, N. J.

MR. JACKSON BORNMAN RAIGUEL, Chemical Engineer, Franklin Sugar Refining Company, Philadelphia, Pa. For mailing: 105 Walnut Street, Jenkintown, Pa.

DR. OWEN LOUIS SHINN, Professor of Applied Chemistry, Acting Director of Laboratory, Harrison Laboratory, University of Pennsylvania, Philadelphia, Pa.

DR. ENOS E. WITMER, Assistant Professor of Physics, Randal Morgan Laboratory of Physics, University of Pennsylvania, Philadelphia, Pa.

NON-RESIDENT.

DR. HARVEY NATHANIEL DAVIS, Teacher, President, Stevens Institute of Technology, Hoboken, N. J.

MR. JOSEPH E. KUCHER, Consulting Engineer, 10 West 170th Street, New York City. For mailing: 314 Claremont Avenue, Jersey City, N. J.

MR. KELVIN SMITH, Engineer, The France Manufacturing Company, Cleveland, Ohio. For mailing: 3077 Huntington Road, Cleveland, Ohio.

CHANGES OF ADDRESS.

MR. W. R. ARMSTRONG, c/o Merck and Company, Inc., 916 Parrish Street, Philadelphia, Pa.

MR. WILLIAM L. AUSTIN, Room 2607, 123 South Broad Street, Philadelphia, Pa.

MR. ALFRED E. BLAKE, 1511 Dickinson Road, West Brookline, Upper Darby P.O., Pa.

MR. BEAUVEAU BORIE, JR., Bell's Mill Road, Chestnut Hill, Philadelphia, Pa.

CAPTAIN HIRAM B. ELY, Room 1051, Bell Telephone Laboratories, 463 West Street, New York City.

MR. H. LINWOOD GODFREY, Packard Building, Fifteenth and Chestnut Streets, Philadelphia, Pa. c/o C. N. Anderson, Patent Attorney.

MR. JOHN M. HAMMOND, The Phototechnical Laboratory, Inc., 6 "N" Street, Northeast, Washington, D. C.

MR. DANIEL O. LANDIS, 612 North Thirty-fourth Street, Philadelphia, Pa.

MR. H. W. LILLEBRIDGE, 5015 Copley Road, Germantown, Philadelphia, Pa.

MR. PERCY S. LYON, c/o Hall Electric Heating Company, 1429 Walnut Street, Philadelphia, Pa.

MR. CHARLES W. MOORE, 350 High Street, Sharon Hill, Pa.

MR. GEORGE A. ORROK, 52 Vanderbilt Avenue, New York City.

MR. RODOLPHO ORTENBLAD, The Rio de Janeiro Tramway Light and Power Company, Ltd., Caixa de Correio 571, Rio de Janeiro, Brazil.

DR. A. D. POWER, Research Laboratory, Westinghouse Lamp Company, Bloomfield, N. J.

MR. BENJAMIN WILEY SANDS, 411 South Pioneer Street, Lyons, Kansas.

MR. BIRCHARD TAYLOR, Harper and Taylor, Inc., 225 South Fifteenth Street, Philadelphia, Pa.

MR. WARREN P. VALENTINE, 154 North 11th Street, Philadelphia, Pa.

NECROLOGY.

Mr. A. J. Bradley, New York City.
Mr. Thomas Duncan, Lafayette, Indiana.
Mr. James Elverson, Jr., Philadelphia, Pa.
Mr. Harold A. Freeman, Philadelphia, Pa.
Dr. C. C. Harrison, Philadelphia, Pa.
Mr. A. M. Randolph, Warrenton, Va.
Mr. J. Bunford Samuel, Philadelphia, Pa.

LIBRARY NOTES.

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BOOK REVIEWS

NATIONAL ADVISORY COMMITTEE FOR AËRONAUTICS.

Report No. 293. Two Practical Methods for the Calculation of the Horizontal Tail Area Necessary for a Statically Stable Airplane, by Walter S. Diehl. 20 pages, illustrations, quarto. Washington, Government Printing Office, 1928. Price, ten cents.

This report is concerned with the problem of calculation of the horizontal tail area necessary to give a statically stable airplane. Two entirely different methods are developed, and reduced to simple formulas easily applied to any design combination. Detailed instructions are given for use of the formulas,

and all calculations are illustrated by examples. The relative importance of the factors influencing stability is also shown.

Report No. 295. The Variation in Engine Power with Altitude Determined from Measurements in Flight with a Hub Dynamometer, by W. D. Gove. 12pages, illustrations, quarto. Washington, Government Printing Office, 1928. Price ten cents.

The rate of change in power of aircraft engines with altitude has been the subject of considerable discussion. Only a small amount of data from direct measurements of the power delivered by airplane engines during flight, however, has been published. This report presents the results of direct measurements of the power delivered by a Liberty 12 airplane engine taken with a hub dynanometer at standard altitudes from zero to 13,000 feet. Six flights were made with the engine installed in a modified DH-4 airplane. The tests were conducted at the Langley Memorial Aëronautical Laboratory of the National Advisory Committee for Aëronautics.

The experimental relation of brake horsepower to altitude is compared with two theoretical relations and with the experimental results, for a second Liberty 12 engine, given in N. A. C. A. Technical Report No. 252. The rate of change in power with altitude of a third Liberty engine, measured with a calibrated propeller, is also given for comparison.

The data presented substantiate the theoretical relation of brake horsepower to altitude based on the correction of ground level indicated horsepower for changes in atmospheric temperature and pressure with the subsequent deduction of friction horsepower corrected for altitude. The equation for this relation is

$$B\cdot\mathrm{HP}_a = B\cdot\mathrm{HP}_o\left[\left(\frac{P_o}{P_a}\right)\left(\frac{T_o}{T_a}\right)^{1/2}\left(1+\frac{\lambda-\lambda n}{n}\right)-\left(\frac{\lambda-\lambda n}{n}\right)\right],$$

where P is the absolute atmospheric pressure, T is the absolute temperature, n is the mechanical efficiency of the engine at sea level and λ is the ratio of mechanical friction to friction horsepower at sea level. The subscripts o and a denote sea level and altitude conditions, respectively. R.

THE RADIO MANUAL. For radio engineers, inspectors, students, operators and radio fans. By George E. Sterling, Radio Inspector, Radio Division, U. S. Department of Commerce, member, Institute of Radio Engineers. Edited by Robert S. Kruse, B.S., formerly Technical Editor Q. S. T., Magazine of American Radio Relay League; consultant for development of short wave devices, technical editor and writer. vii–666 pages, illustrations, 8vo, fabricoid. New York, D. Van Nostrand Company, Inc., 1928. Price, $6.

If radio has yet far to go with the realization in usable form of the present aspirations of research laboratories it has certainly already attained an important position in electrical science and has given rise to a large and thriving industry. The literature of the subject has naturally had an equally rapid growth. The present volume which is written by authors, whose activities in the field are administrative and authoritative as well as technical, is for that reason of particular interest to the reader in search of definite information. The work is intended to serve as a guide and text-book to those who expect to enter the radio profession as engineer, inspector, commercial or amateur operator. Besides the purely

physical features of the subject which are treated arithmetically, formidable mathematical formulas being conspicuously absent, code matters issued by the International Radiotelegraphic Convention and by the Federal Radio Commission and the like are included. Particular attention is directed to the authoritative sources of information included. A bibliography of such references, which appears to have been omitted, would be decidedly helpful in a work of reference of this kind. It is of interest to note that the work appeared in October 1928 and reached its third printing in December. The full and well-illustrated treatment and the authoritative sources of the subject-matter should insure for the work a continuation of a large initial demand.

LUCIEN E. PICOLET.

AN INTRODUCTORY TEXTBOOK OF ELECTRICAL ENGINEERING. By John Robert Benton, Professor of Physics and Electrical Engineering in the University of Florida. xi–347 pages, illustrations, 8vo, cloth. Boston, Ginn and Company, 1928. Price, $3.60.

Between the royal road of superficiality and an exhaustive study of a subject, there must be some middle course of procedure which permits a satisfactory survey within the time limits imposed by harmonious relation to the other subjects of the college course. This text, which covers practically the entire field of electric generators and motors, is designed for a purpose of that kind. The mechanical specialist will there find in compact form a resumé of the principles of electrodynamics, their development and application to well-known types of power appliances in common use. The electrical specialist may cover the same ground with fewer omissions and then proceed to other works to which this one will serve as an introduction. As is proper in a text, a copious collection of examples accompanies each chapter. Some knowledge of college physics and elementary calculus are essential. Numerous clear and effective diagrams and half-tones of actual machines add materially to this satisfactory presentation of the principles of electrical machinery. The volume is one of the Engineering Series edited under the supervision of Andrey A. Potter, Eng. D., Dean of the School of Engineering and Director Engineering Experiment Station, Purdue University. L. E. P.

ELECTRICAL ENGINEERING LABORATORY PRACTICE. By Oskar E. Edison, E.E., Associate Professor of Electrical Engineering in the University of Nebraska, and Ferris W. Norris, E.E., Associate Professor of Electrical Engineering in the University of Nebraska. ix—276 pages, illustrations, 8vo, cloth. Boston, Ginn and Company, 1928. Price, $2.80.

Of the same series as the preceding textbook, this work, which deals with the laboratory operations of electrical measurements and testing of electrical machinery, may properly be regarded a companion volume. The experiments adequately cover the subject-matter of the preceding work and of course any other work covering the same ground, and each one is described under the headings of "object," "reference," "apparatus," and "method" with such additional directions as may be demanded by the nature of the experiment. These directions are definite enough to properly guide the student without such amplification as might deprive him of the educational value of thinking for himself in carrying out the details of an experiment. Numerous well-executed diagrams of circuits, apparatus, and performance curves accompany the text.

LUCIEN E. PICOLET.

THE ENGINEER, HIS WORKS AND HIS EDUCATION. By ROBERT LEMUEL SACKETT, C.E., Dean of Engineering, The Pennsylvania State College. vii–196 pages, illustrations, 20 x 13.5 cm., cloth. Boston, Ginn and Company, 1928. Price, $1.40.

The accelerating pace of achievement in science and industry during the last three decades has brought about an enormous expansion of activities in every phase of economic effort. Whether the rapid progression of this change is for the better or the worse, the new conditions which accompany it must be met. Engineering education has been notably affected by this expansion and the task of the professional schools to provide facilities to meet a corresponding demand for technical education is one of considerable magnitude, and not the least of the problems encountered are those of a pedagogic character incident to large-scale operation. The training of large numbers of individuals in technical subjects demands careful planning to insure an efficient utilization of the time allotted to a college course. Again technical students are recruited from widely diversified individual types. One student, prior to taking an engineering course, may have had contact with an environment of engineering activity and so possesses an adequate conception of what he is undertaking; another in a different environment may have an altogether faulty notion of the real objectives of his course of study.

Professor Sackett provides a deeply interesting and informative account of what a prospective candidate for entrance into a school of engineering should know concerning the profession apart from the purely academic entrance requirements as well as the personal characteristics which should determine the expediency of the undertaking. All such questions are presented and discussed in a convincing manner. Modern courses and their characteristics are described at length from which constructive information may be derived by a reader whose conceptions of their objects and functioning may be faulty. An appendix contains short biographies of famous engineers.

A college course cannot be undone if it proves ill-advised nor, generally, can it be done over. This book provides the stitch in time.

LUCIEN E. PICOLET.

INTRODUCTION A LA PHYSIQUE DES RAYONS X ET GAMMA. Par MM. Maurice deBroglie, Membre de l'Institut, et Louis deBroglie, Docteur ès Sciences. v–201 pages, illustrations, 8vo, paper. Paris, Gauthier-Villars et Cie, 1928. Price, 50 francs.

The mode of conquest of any new principle in physical science may be likened to a succession of assaults upon a stronghold each one of which discloses a better method of attack. Ever since the discovery of x-rays, which greatly stimulated the efforts of physicists in the study of radiation phenomena, the keenest minds of the realm of physical science have unremittingly pursued the quest of a solution of the characteristics of these rays and others of recent discovery. The deBroglies have been active contributors in this field, and with the accumulated wealth of data from many other important sources, they describe at length in this work the salient properties of x-rays and gamma-rays more particularly in their relation to a profound modification of the classic conceptions of the structure of light.

Before making a detailed examination of x-rays and gamma rays, the authors

make a general survey of their nature and point to experimental evidence of their periodic character, again to other indications of their corpuscular nature and then to general views of their probable actual nature. The detailed examinations begin with a consideration of the atomic theory of Bohr and then proceeds to its application to the x-ray spectrum. Among the topics which follow are: the photo-electric effect, rays of very short and very long wave-length, diffraction by the use of crystals, physical optics of x-rays, and thermodynamic and astrophysical considerations. Two postscripts are added, the one on the possible existence of a phenomenon analogous to that of Laue of electrons impinging upon a crystal, the other, on the magnetic electron and the formula of Sommerfeld. The views there expressed if verified, the authors believe, will effect a rational coördination of conflicting views on matter and radiation.

The obvious value of a discussion of x- and gamma-radiations in harmony with modern ideas of the ultimate structure of matter by these celebrated authors will surely bespeak for the work a wide circulation.

L. E. P.

THYROXINE. By Edward C. Kendall, M.C., Ph.D., D.Sc. 265 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1929. Price, $5.50.

The nature and function of the ductless glands were for many years the bugbear of physiology. Naturally, physicians were strongly inclined to the view that such a definite series of organs must have positive functions but information was not forthcoming. The obscurity gave rise to a reaction by which the functions were the subject of humor. The incident, which is claimed to have happened in every class in every medical school and happened in none, typified the hopeless, ness of the case. "What is the function of the spleen?" said the Professor- "Doctor," said the student, "I knew that last night and I have forgotten it." "The most unfortunate incident in the history of physiology. Only one man knew the function of the spleen and he forgot it."

Seriously, however, the data that have been accumulated by patient investigation during the last quarter century have been of tremendous importance in the realms of physiology, pathology and therapeutics. The volume in hand (the 47th of the series of American Chemical Society Monographs) is devoted entirely to the important proximate principle of the thyroid gland, and is the result of long and careful laboratory work by the author with comprehensive search of the extensive literature. The subject is presented from the view-point of the chemist, especially directed to correlating the activity of the thyroid gland. Much still remains to be elucidated, but the author shows that the clinical and physiologic results are clear and quantitive. Naturally, in the midst of the early ignorance and the primary investigations many theories were suggested, but most of these have lost value and the few remaining are restricted in application.

Physical chemistry has been brought into action, so that while the problems of the gland are far from being wholly solved, the investigations from the clinical and physiologic points of view are strictly rational and guided by satisfactory principles.

To the trained investigator in this field the problems are attractive; thyroxine, the author says, bridges the gap between living protoplasm and temperament, or what we may more definitely call personality. The study of the substance is carried into the most difficult of all departments of medicine, mental aberrations.

It is not yet possible to say if the action is catalytic. It is even suggested that it may be a still more remote action, that is, controlling catalytic (enzymic) action. The complexity of chemistry increases steadily. One discovery opens up the duty and opportunity of making more. We might modify Milton's language and say: "In the lowest deep, a lower deep still threatening to devour us opens wide."

The author concludes the preface with these words: "There are many questions left unanswered; time will bring many additions and possibly many alterations." HENRY LEFFMANN.

A TEXTBOOK OF INORGANIC CHEMISTRY. Edited by J. Newton Friend, D.S., Ph.D., F.I.C., Carnegie Gold Medallist. Volume XI, Organo-metallic Compounds, Part 1, Derivatives of the Elements of Groups I to IV, by Archibald Edwin Goddard, M.Sc. (B'ham), A.I.C., and Dorothy Goddard, M.Sc. (B'ham). 417 pages, illustrations, 8vo. London, Charles Griffin & Company, Ltd., and Philadelphia, J. B. Lippincott Company, 1928. Price, $14.

The list of elements included within this volume stops short of the element from which the first organo-metallic compound was obtained. Arsenic was the first element to be comprised in an organic group. It was about the time of the American revolution that Louis Claude Cadet de Gassicourt, a French apothecary, distilled a mixture of an alkali acetate and arsenous oxide and obtained the fuming liquid still known by his name. This is a mixture of several substances, principally dimethylarsin oxide, commonly called kakodylic (cacodylic) oxide. The composition of the principal substance and of many of its derivatives was elucidated by Bunsen in a comprehensive and painstaking research, which is a model of procedure in chemistry. He lost an eye by an explosion during the work. Arsenic belongs to the fifth group of the periodic system and, therefore, the kakodylic derivatives do not appear in the book. A future volume will cover the subject, but very little is available in supplement of Bunsen's work. Kakodylic acid has found some applications in medicine from the fact that the arsenic is in a position in the molecule that renders it less toxic than the familiar forms.

The volume in hand contains many compounds that are more familiar than the kakodyls, especially those of mercury, tin and lead. About one hundred and fifty pages are devoted to mercury compounds. Lead receives notice for about a score of pages, among which is, of course, noted the lead tetrethyl, now so well known and so much discussed as an "anti-knock for motors."

Methods for the preparation of *key* compounds are given in detail so that the book may be used as a treatise on practical chemistry. The substances described are limited to those in which the second element is linked directly to carbon, excluding those in which it is linked through oxygen or nitrogen as well as double compounds of organic substances with inorganic salts. In spite of these limitations about 2,300 compounds are described. In view of the wide field covered no systematic attempt has been made to discuss theoretical aspects, but the large number of substances described and formulated will materially assist those who desire to make extensive comparisons. Physical constants of a large number of compounds have been set forth in an appendix. Throughout the book greater attention has been paid to physical properties generally than is usual in this type of book.

A large proportion of the organo-metallic substances are highly poisonous.

Many of them are volatile which increases the danger. Reference has been made above to the poisonous fumes of arsendimethyl oxide. Bunsen found kakodyl cyanide to be one of the most poisonous compounds then known. Mercuric dimethyl and mercuric diethyl caused several deaths among those working with them many years ago, and gave rise to very severe criticism of Dr. Frankland in connection with the case. In the early days of the use of tetrethyl lead, numerous cases of poisoning occurred.

The book represents an enormous amount of patient labor and searching the literature and compiling the vast amount of data already accumulated and also embodies the authors' own extensive investigations. Women authors have appeared more frequently in this series than is usual in the literature of chemistry. A great deal of praise is due to the authors for their work, but the reviewer cannot refrain from objecting to the form of the subject-index. This is arranged under the individual metals as they occur in the periodic system. This seems to be classification "run amuck." An index is a purely mechanical aid to finding text relating to a given subject. No attempt should be made to adapt it as either a table of contents (as the French do so frequently) or to serve the purpose of key to any theoretical or practical relationship. The man who catalogued "Lead compounds" and followed this entry by "Ditto, Kindly Light" was on the right track.

HENRY LEFFMANN.

SPEECH AND HEARING. By Harvey Fletcher, Ph.D., Acoustical Research Director, Bell Telephone Laboratories, Inc., with an introduction by H. D. Arnold, Ph.D., Director of Research, Bell Telephone Laboratories, Inc. 331 pages, illustrations, 8vo. New York, D. Van Nostrand Company, Inc., 1929. Price, $5.50.

Darwin stated in the "Descent of Man" that he had collected many definitions of the distinctions between man and the lower animals. He seems never to have published the collection. The distinction most likely to occur to man is that he is the only animal that possesses articulate speech and means of recording it, yet some animals have perfect articulation and some races of men no written language. It is indeed worthy of remark that among the highly artistic paintings that have been found in caves, no indication of inscriptions is seen. In later periods, when written and spoken language was highly developed, inscriptions of all kinds are abundant, even to excess, for the "scribblers" have left evidence of their tendencies even on the sacred figures of catacombs of Rome.

Naturalists who have given special attention to the course of evolution of the human species are in great part of the opinion that it is not the development of the speech center, but of the visual center that has been the main cause of man rising so far above his animal relatives. Speech and hearing are important means of communication, but sight is the most precious of our special senses. There is a French proverb "that in the country of the blind, the one-eyed man is king," but a British author wrote a story to show that this is false for a one-eyed who went to the country of the blind and told the people about the sights around them was adjudged insane.

However, the object of this article is to discuss the merits of the work of which the title is given above. It proceeds from workers in the laboratories of the Bell Telephone Company, and is, therefore, one more reminder of the benefit to science

that has accrued of late from the public spiritedness of the great corporations, which have been devoting much time and money to the advancement of American research. The investigations began about fifteen years ago, and have been zealously pursued in the interval. Special methods have been devised for several phases of the work. Electrical apparatus has been largely employed, and many graphs are given showing the characters of vibrations in human speech. Detailed descriptions of the anatomy of the organs of speech and hearing are given. The necessity for special apparatus was early felt and in the course of constructing and operating these, their application to the development of better service in telephony and phonograph engineering became evident and thus important practical results came out of the theoretical studies. In addition to such practical results, the investigation brought out a wealth of information, new and useful on the fundamentals of speech and hearing.

Though a great deal of new information is presented in the book, the studies, of which it is a report, are far from finished, and Doctor Fletcher tells us that there are fields already in view that will occupy the workers for years to come. An important part of the topics presented in the book has been to determine the degree of precision with which the mind can differentiate and interpret sounds that are very nearly alike. Common experience shows that human beings differ greatly in this respect. Owing to the mingling of many languages in this country, a tendency to slurring speech is all too much in evidence. Teachers of elocution, trainers of children, especially of those who have charge of instruction of those children who come from homes of foreign parents will find the book of much aid.

It is liberally illustrated with graphs, forms of apparatus and tables.

HENRY LEFFMANN.

CASTLES IN THE ETHER. By John Wills Cloud. Revised edition with two appendices. 210 pages, 14 figures, no index, 8vo. Printed for the author by Charles Francis Press, New York, 1928. Price, $5.

In this day and age the term "castle" holds a dual significance. One meaning is that recorded in history, where, in mediaeval times a castle was a concrete (as apposed to abstract) structure and possessed a value intensely practical. It was in truth a fortress and stronghold built of rock on foundations of rock and meant to withstand the attacks of the elements and batterings of the foe. Today we are wont to consider a castle in the abstract sense, it is a figment of the imagination, intangible and romantic. The author has used the term in its second sense and has constructed "castles in the ether" which, compared with "dream castles" and "castles in the air" seem to be of sterner stuff.

Science in its usual meaning concerns itself with the study and correlation of facts so as to produce a system logical in arrangement and structure. Philosophy goes a step further, beyond the facts, and seeks the fundamental causes of the observed phenomena. This is the realm which has attracted the author. His "Castles in the Ether" are founded upon the assumption that ether is a "tangible" substance composed of extremely small but finite particles called ethrons arranged in a definite geometrical way and possessing a rigidity by virtue of vibrations of very high frequency. This is the primary substance of the universe, the stuff of which matter is made, the transmission medium for impulses traveling with the speed of light. With such as a major premise, the author starts out to

interpret the causes of various natural phenomena and show them to be corrollaries of the general theorem of which it is possible to include here only a brief sketch. The ethrons are spherical in shape, packed in the closest possible manner, each sphere in contact with twelve other spheres. Matter is formed by the sudden collapse of the ethrons contained in spaces of a definite magnitude and the particles of matter occupy the vacant spaces or interstices between the ethron spheres. Vibrations due to electromagnetic, heat, light, X-ray and cosmic waves are modifications or modulations of the fundamental vibrations which are an integral part of the ether. Further, it is probable that the formation of matter (collapse of ethrons) occurs at some point in the ether where the frequency is nil (absolute zero) while the transformation of matter to ether occurs at extremely high temperatures where the frequency approaches a maximum. From these assumptions concerning ether and matter as well as the influence of each upon the other, the author evolves an explanation for various natural phenomena such as gravitation, rotation and mutation, inertia and mass, cohesion, polarity and chemical action, matter and energy, molecular motion in gases, electricity, magnetism, surface tension and the spiral rotation of the earth. The fundamental ideas of relativity also seem to be included in the author's scheme of things. Such manifestations as the motion of bodies, their attraction for each other (gravitation), electric and magnetic forces are caused through pressure inequalities in the ether acting in and around objects possessing mass.

It would be a miracle indeed if every reader of this book agreed with the author's explanations of how the ether functioned in bringing about various processes of nature. For, while many will welcome more or less such seemingly rational explanations for processes in general, strenuous objections will be raised over the attempts to explain in more detail certain chemical, physiological and psychological processes.

To anyone at all interested in a philosophy of the universe this book should prove to be mentally stimulating. It is evident that the author's general knowledge extends to practically all branches of science and as a result he has spent much time and thought striving to show a common course and origin for all these apparently unrelated manifestations of natural forces. How well he has succeeded readers of the book may judge for themselves.

The revised edition contains two appendices, A and B. A is a reprint and treats of the genesis of matter in considerable detail. B deals with surface tension as viewed from the standpoint of differences existing between ethereal pressures internal and external to a body.

T. K. CLEVELAND.

STORIES IN STONE: TELLING OF SOME OF THE WONDERLANDS OF WESTERN AMERICA AND SOME OF THE CURIOUS INCIDENTS IN THE HISTORY OF GEOLOGY. By Willis T. Lee, Geologist, United States Geological Survey, Washington, D. C. 226 pages, 49 illustrations, 3 figures, 8vo. New York, D. Van Nostrand Company, 1926. Price, $3.

Here is the opportunity to view the scenic wonders of America from the depths of some comfortable chair. Here, in the realms of the imagination, the reader, accompanied by the spirit of the author, may stand on the brink of the Grand Canyon surveying with mixed feelings of awe and exaltation the kaleido-

scopic panorama of majestic splendor. This would be followed by a trek along the windswept road of the Painted Desert where the rich blue of the skies throws into silhouettes, mesas, buttes, towers and ridges which, in richness and variety of shades would bid fair to rival Jacob's coat of many colors. Not far distant we come upon the Petrified Forest, a natural wonder indeed. Here are found the prostrate trunks of arboreal giants belonging to an age long past. As if at the wave of a great magician's hand they have been transformed from perishable wood to the time-resisting agate, chalcedony, and jasper. If your thirst for rugged splendor is still unappeased you may next go to a wonder child belonging to the Jurassic Age, Zion Canyon. Here, we wend our way along the base of the Vermillion Cliffs flanked by towering buttresses and broad escarpments to come finally upon a scene no mortal might hope to describe adequately, the Temples and Towers of the Virgin, two enormous masses with bases of red sandstone surmounted with towers and domes of white stone soaring upward to the clouds. We are not done yet, in rapid succession the spirit presents to view the Gates of Zion flanked by almost sheer cliffs over half a mile in height; Rainbow Bridge, Mesa Verde National Park, scene of the ancient cliff dwellers, Glacier, Yosemite and Rocky Mountain National Parks, Mammoth Cave and Carlsbad Cavern. Each one unique and marvelously formed.

The Fates were indeed unkind for the author died in June, 1926 before this book was yet in type. A study of geological formations and a correct interpretation of their formative processes have been a source of joy and satisfaction to him. Every page of the book reflects enthusiasm for his subject. Consequently the reader will find that "Stories in Stone" has been written in a style most interesting and a manner most engaging.

The first two chapters are occupied with the question—"What's the Use" (of studying geology) and the answer to the same. Some excellent examples exemplifying the utility of such study also are given. In the chapters following the periods of formation of the various types of landscapes are taken up in chronological order. The author, visualizing the process of rock-strata deposition and the flora and fauna existent at that time, draws freely upon the imagination of the reader yet does not tax it. In these chapters the reader is given the "low-down" on those prehistoric animals which have intrigued so greatly the present-day popular fancy. Judging from the size of the skeletons unearthed they did things in a big way in those early days. Here, also is traced the gradual speeding up of life upon our planet. Beginning with sluggish lizards and ponderous mastodons we see them gradually superseded by winged reptiles (birds) and a fleeter and more agile race of animals such as the progenitors of the modern horse.

A chapter on a new method of observation sketches the adaptation of the aeroplane and aerial photography to geologic exploration. This is followed by an interesting chapter on the legends of creation. The two chief present-day theories of the earth's origin, namely, the nebular and planetesimal are clearly outlined and discussed in an interesting manner. In the final chapter on "Facts, Fancies, and Nebulous Thoughts" there is traced up through the centuries the seemingly eternal conflict between science and dogma. Fossils were believed to be spontaneously generated or the imperfect works of an experimenting Creator. Founders of experimental and natural science were ostracized, forced to retract their teachings or be put to death.

The volume is liberally illustrated with photographs of natural scenery, many of them taken from aeroplanes and possessing a perfection difficult to surpass.

T. K. CLEVELAND.

PUBLICATIONS RECEIVED.

Vibration Problems in Engineering, by S. Timoshenko. 351 pages, illustrations, 8vo. New York, D. Van Nostrand Company, Inc., 1928, price $4.50.

Fundamentals of Quantitative Analysis, by Walter C. Blasdale, Ph.D. Third edition. 403 pages, illustrations, 8vo. New York, D. Van Nostrand Company, Inc., 1928, price $3.75.

Introduction to Theoretical Physics, by Leigh Page, Ph.D. 587 pages, illustrations, 8vo. New York, D. Van Nostrand Company, 1928, price $6.50.

Industrial Carbon, by C. L. Mantell, Ph.D. Industrial Chemical Monographs. 410 pages, illustrations, 8vo. New York, D. Van Nostrand Company, Inc., 1928, price $4.50.

Enzyme Actions and Properties, by Ernst Waldschmidt-Leitz. Translated and extended by Robert P. Walton. 255 pages, illustrations, 8vo. New York, John Wiley & Sons, Inc., 1929, price $4.00.

The Radio Manual for Radio Engineers, Inspectors, Students, Operators and Radio Fans, by George E. Sterling, edited by Robert S. Kruse. Third printing. 666 pages, illustrations, 12mo. New York, D. Van Nostrand Company, Inc., 1928, price $6.00.

Thyroxine, by Edward C. Kendall, M.S., Ph.D., D.Sc. 265 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1929, price $5.50.

A Textbook of Inorganic Chemistry, edited by J. Newton Friend, D.Sc., Vol. XI. Organo-metallic Compounds. Part 1, Derivatives of the Elements of Groups I to IV, by Archibald Edwin Goddard, M.Sc., and Dorothy Goddard, M.Sc. 418 pages, illustrations, 8vo. London, Charles Griffin and Company, Ltd., Philadelphia, J. B. Lippincott Company, 1928, price $14.00.

Steam, Air and Gas Power, by William H. Severns, M.S. and Howard E. Degler, M.E., M.S. 425 pages illustrations, 8vo. New York, John Wiley & Sons, Inc., 1929, price $4.00.

Steam Turbines. A practical and theoretical treatise for engineers and students including a discussion of mercury and gas turbines by James Ambrose Moyer, S.B., A.M. Sixth Edition, Revised. 557 pages, illustrations, diagram, 8vo. New York, John Wiley & Sons, Inc., 1929, price $4.50.

Speech and Hearing, by Harvey Fletcher, Ph.D., with an introduction by H. D. Arnold, Ph.D. 331 pages, illustrations, quarto. New York, D. Van Nostrand Company, Inc., 1929, price $5.50.

CURRENT TOPICS.

Cathode-Ray Oscillographs and Their Uses. E. S. LEE. (*General Electric Rev.*, Aug., 1928.) In the electromagnetic oscillograph a beam of light is made to move over a screen or photographic plate by the motion of a current-carrying element in a magnetic field, the current being in proportion to the effect under examination. Since the element carries a single current such an instrument can record the variation with time of a single quantity. Furthermore, the inertia of the current-carrying element limits the range of response of this instrument to phenomena not having more than about 10,000 cycles per sec. In the cathode-ray oscillograph the moving part that sweeps over screen or plate is a beam of cathode rays that is deflected by the action of an electrostatic or electromagnetic field in proportion to the effect under examination. Since more than one deflecting field can be applied at the same time to the beam and in different planes, records are obtainable that connect two quantities such as volts and amperes. Furthermore, the mass of a beam of cathode rays is negligible and thus inertia ceases to limit the range of application.

There are two general kinds of cathode-ray oscillographs, viz., the low-voltage hot-cathode and the high-voltage cold-cathode types. The rays must be deflected and they must produce an effect on the photographic plate on which they fall, but maximum deflection is antagonistic to maximum photographic effect. The rays have mass, small though it be, and hence their deflection from their rectilinear course while they are traversing the electrostatic or the electromagnetic field is greater the longer they are under the action of the field, that is, the more slowly they move through the field. On the other hand the larger the speed of the rays the greater the effect on the photographic plate. High speed rays produced by high voltages between the anode and the cathode of the tube where the rays are developed are little deflected in the field but strongly affect the plate, while rays due to low voltages on the contrary are much deflected but affect the plate only slightly. From this opposition arise the two types of instruments. In the low-voltage type a heated filament at the cathode is used to facilitate the production of cathode rays at relatively low differences of potential such as 300 to 3000 volts. In the high-voltage cold-cathode type, however, the rays are caused by no auxiliary means but are due to the action of the high potential itself, this being

roughly from 30 to 60 kilovolts. Thus the photographic sensitivity is exalted so that transient electric phenomena can be photographed.

A newly developed oscillograph is described. The parts are mounted on two trucks. The two units weigh 700 lbs. and need a maximum of 4 kw. at 110 volts, 60 cycles. The instrument is for work in the field and the factory during daylight. Records illustrating its results are given among which is one of a voltage wave at a frequency of 1,000,000 cycles per sec. Another shows the building up of current in a resonance circuit at the same frequency. In this the change of current in a millionth sec. can readily be followed.

This high voltage instrument lends itself to investigations of time-voltage characteristics of spark gaps and insulation, of transmission disturbances, of lightning arrester phenomena, of conditions in circuits at radio frequencies, of telephone protection problems, of mechanical vibrations, of pressures in guns and gasoline engines, of circuit breaker and switching results and of discharge phenomena.

G. F. S.

The Large Electromagnet of the Academy of Sciences. A. COTTON. (*Comptes Rendus*, July 9, 1928.) Of course the design of a powerful electromagnet is determined by the purposes it is to serve. The one planned by Kapitza at Cambridge produces fields of several hundred thousand gauss that exist only inside of small coils and for about 1/100 sec. To be generally useful for researches, however, the space where the strong field is produced must be as large as possible. This new French model sacrifices strength of field to general usefulness. As long ago as 1912 Cotton obtained the reservation of a part of a legacy for the building of a large electromagnet, and a committee developed a preliminary sketch of an instrument with iron cores a meter in diameter. Then came the invasion of France by Germany. After the War the project was resumed and a million francs were granted from the funds of Pasteur Day. New information was then available and a fresh study was needed. Accordingly a committee was again appointed. On it were such men as Picard, Appell, d'Arsonval, Branly, Blondel, Weiss, Brillouin, Ferrié, de Broglie, Claude and Cotton. After the general design had been selected a quarter-size model was constructed upon which experiments were made that led to modifications of the first design. By the beginning of 1927 the final plans were ready but prices had gone up so that pecuniary troubles were frequent. These were overcome and the electromagnet is now installed at Bellevue at the National Office of Scientific and Industrial Researches and of Inventions. The

cores are in the form of truncated cones, tapering in diameter from 121 to 75 cm. Each core weighs 3.5 tons and is 65 cm. long. Of electrolytic copper 8.6 tons are used, the frame weighs 50 tons; altogether 105 tons of iron enter into the structure whose total weight is 120 tons. The windings are hollow tubes of copper through which water flows to remove the heat. 400 amperes flow ordinarily under 240 volts pressure, the power being thus 100 kw. As there are 1250 turns, the number of ampere-turns is 500,000. Ferro-cobalt is used in the central part of the pole pieces. The measured strength of field between the pole pieces is 46,400 gauss through a cylindrical space 4 cm. in diameter and 2 cm. long. The rise of temperature is 25° C. The entire electromagnet is about 6.3 m. long with a square cross-section 2.75 m. on a side. Connected with the electromagnet is a vertical spectrograph installed in a well 9 m. deep, by which very delicate optical researches can be conducted.

It is claimed that this instrument surpasses any other hitherto available in size, power and facilities for operation. G. F. S.

The Luminescence of Water and of Carbon Disulphide Due to Gamma Radiation. L. MALLET. (*Comptes Rendus*, July 23, 1928.) Two years ago the author stated that water and certain organic substances luminesce with white light when acted on by γ-rays from radioactive substances. On account of its feebleness this white light could not be resolved spectroscopically by ordinary apparatus but the author has the use of a special instrument designed by Fabry that gave adequate images on Lumiere plates with exposures of from two to four days. The rays emanated from 250 grams of radium sulphate. In the case of water as well as of carbon disulphide the substance acted on by the rays emitted a continuous spectrum covering the entire visible field and all of the ultraviolet that was not absorbed in the apparatus. The luminescent radiation was richer in the short wave-lengths than the light of a half-watt lamp. It appears that the emission of visible and ultra-violet radiations by bodies acted on by gamma rays is a general phenomenon. G. F. S.

The Element of Time in the Photoelectric Effect. E. O. LAWRENCE AND J. W. BEAMS. (*Phys. Rev.*, Sept., 1928.) "How soon are electrons emitted from a metal surface after it is irradiated by light and in what manner does the electron emission persist after cessation of the illumination? The experiments described below have given an answer to this fundamental query." A method was needed of illuminating a metal surface for time intervals

of about a ten-millionth of a sec. and then of finding exactly when in respect to the illumination electrons were ejected. By the use of crossed Nicol prisms between which were two Kerr cells with the normals to their surfaces at rght angles to each other it was arranged to let the light from an electric spark pass through the combination for a length of time depending upon the interval between the instants when the two cells, connected by wires of different lengths with the spark gap, began their discharge. This interval was controlled by varying the length of these wires. The flashes of light coming through when the spark gap was connected to the secondary of a transformer irradiated a potassium hydride surface. The emitted electrons made their way to a collecting electrode joined to an electrometer. "Determining what length of wire was necessary to obtain an appreciable photoelectric current to the electrometer was therefore a measure of the instant at which the electrons first were emitted." The wire here mentioned was connected to the spark gap. Its length determined the time of arrival from the spark gap of the electric potential wave which in turn governed the relative potentials of the surface emitting electrons and of a grid surrounding the collecting electrode. It was only while the plate was at a lower potential than the grid that electrons passed through it to the collecting electrode.

It is thus possible to take measurements and construct curves connecting the strength of the electron current with the time after the beginning of the spark. From one such curve it is seen that the light flash and the photoelectric emission began only one-two hundred millionth of a sec. apart. It is true, however, that according to the curve the spark was the later to occur. A careful consideration of this and of other possible errors in the measurements leads the authors to state that the light flash and the beginning of electron emission are instantaneous with a possible difference of not more than three billionths sec., and that the emission lasts not more than 10^{-8} sec. after the light has been cut off. G. F. S.

The Velocity of Sound in Liquids at High Frequencies by the Sonic Interferometer. J. C. HUBBARD AND A. L. LOOMIS. (*Phil. Mag.*, June, 1928.) The method hereafter described makes possible the measurement of the velocity of sound waves in relatively small quantities of liquids. In addition the accuracy attained is such that there seems reason to hope that the method may be used for purposes of analysis. Direct measurements of sound velocities require large bodies of the substance for the experiment with multiplied difficulties in finding the temperature and other physical data of the liquid

under experiment. Even resonance methods for audible frequencies have demanded apparatus of large dimensions so that only the commoner liquids could be examined. In addition it was necessary to correct for the influence of the material and dimensions of the apparatus upon the observed wave-length and different observers used different correction formulæ so that their results do not agree even when they are comparable.

The present paper summarizes the results obtained in the Loomis Laboratory during the summer of 1927. In a liquid contained in a cylindrical chamber compressional waves were set up by the vibration of a quartz plate 50 mm. in diameter. This piezoelectric plate was placed between two electrodes one of which was grounded while the other was joined to a point "on the coil of a high-frequency oscillating electricai circuit." The plate was horizontal and at the bottom of the cylinder of liquid. A horizontal piston above the plate was adjustable by a micrometer screw. An essential prerequisite for satisfactory operation was the absence of gas bubbles between the plate and the liquid above it. The sound waves, about 3 mm. in length, passed through the liquid to the piston and, after reflection there, returned toward their source, interfering with the oncoming waves. The tuning of the interval between piston and plate for exact resonance could be done by any one of several electrical methods of which a double heterodyne method was found most suitable. "In practice, the reaction is so sharp at the node that settings may be made by ear within a few thousandths of a millimeter. Under usual conditions the precision obtainable in the value of the half wave-length from such readings is about one part in five thousand." For example, with mercury at 20° C. seven determinations of the half wave-length show the same value to the third decimal place, viz., 0.165 cm. and differ only in the next decimal place. The smallest value found was .16570 and the largest .16583. The mean velocity is 1451.0 m./sec.

The velocity of sound was obtained in water from zero to 40° C. It increases linearly up to about 15°; after this the increase is less rapid. The velocities at 0°, 20° and 40° are respectively 1407, 1484.2 and 1530.3 m./sec. In the case of mercury the velocity decreased linearly with increase of temperature, the velocities at 0° and 70° being 1460.2 and 1427.7 m./sec. Five different aqueous solutions of NaCl were examined. The greater the salt content the higher the velocity. Similar results were obtained for NaI. At all the temperatures used the NaCl solutions showed higher velocities than water, but a solution of 30.5 per cent. NaI at 25° C. gave the same velocity as water at that temperature, while for lower temperatures the solution had higher velocities and for higher temperatures smaller velocities than water at corresponding temperatures.

This method makes it possible to get the velocity of sound without the admixture of effects due to the size and materials of the containing vessel and gives results of such accuracy that several thermodynamic quantities can be calculated from them.

G. F. S.

The Normal Photo-Electric Effect. P. LUKIRSKY and S. PRILEZAEV. (*Zeit. für Physik*, Vol. 49, Nos. 3 and 4.) When quanta of radiant energy impinge on bodies electrons are torn away from them. In this action two distinct cases are to be distinguished. When the electrons are bound by merely weak forces while the energy of the incident quanta is large, the Compton Effect manifests itself, only a part of the energy of the incident quantum being transferred to the electron and the remainder going off as a quantum of longer wave-length. In the other case, when the energy of the quantum falling on the body is not much greater than the energy requisite to free an electron from the body. The quantum of radiation is then expended in full on an electron. The electron receives a velocity upon leaving the body, but before this can occur it must be separated from its atom and a little later must make its way through the surface of the body into outer space. Einstein's equation relating to this series of exchanges is $\frac{1}{2}mv^2 = h\nu - p_1 - p_2$.

The left side of the equation is the kinetic energy of the emerging electron, p_1 is the energy spent in separating the electron from the atom and p_2 is the work done in getting the electron through the surface of the body. This equation is known to apply to all frequencies of radiation from those of the visible spectrum to those of gamma radiation.

In the present experiment a spherical condenser was used. The inner sphere was the body on which radiation fell and from which electrons were emitted. The two spheres had respective diameters of 11 and 1.5 cm. What becomes of the issuing electrons depends on the strength and direction of the electrical field between the two plates of the condenser. If the direction of the field is such as to retard the electrons it is possible to adjust its strength so that no electrons at all will pass from the inner to the surrounding sphere. In this case an electrometer joined to the inner sphere would show no current in spite of its ability to indicate 10^{-13} ampere. In getting the retarding potential, account had to be taken of the contact difference of potential between the inner sphere and the silvered sphere surrounding it. From the curve connecting the applied difference of potential and the current flowing from the inner sphere considerable interesting information can be deduced. The

value of Planck's constant, h, is calculated to have the following values in 10^{-27} ergs per sec. for the metals named,—Al, 6.539; Zn, 6.556; Ni, 6.546. The mean of all the determinations is 6.543×10^{-27} ergs/sec., whereas optical methods give 6.545×10^{-27} ergs. sec.

As stated above the energy of the incident quantum gives kinetic energy to the ejected electron, frees it from its atom and forces it through the surface of the body. If the quantum is equal to the sum of the energies needed for these two last acts, no energy is left over to provide velocity for the electron. Let radiation of longer and longer wave-length be made to fall in succession on the body. The value of the quantum grows progressively smaller, because it equals the product of Planck's constant by the frequency of the radiation. Thus eventually a wave-length will be found for which the quantum is completely expended in freeing the electron and getting it through the surface. The electron cannot leave the body as there is no energy to put in it motion. Again from the curves this limiting wave-length can be obtained. For platinum it is 3018 Å. and for aluminium 4132 Å, Besides this the contact difference of potential between the metal of the inner sphere and the material of the outer sphere can be computed, and also that between any two metals used for inner spheres.

An important theoretical conclusion drawn from the results is that "In the photo-electric effect due to optical wave-lengths the same electrons are torn from the metal that are responsible for its electrical conductivity." A study of thin films in comparison with massive pieces of metal indicates that the electrons set free in the interior of the metal lose some of their velocity there.

G. F. S.

Evidence that the Cosmic Rays Originate in Interstellar Space. ROBERT A. MILLIKAN AND G. H. CAMERON. (*Proc. Nat. Acad. Sciences*, August, 1928.) On the assumption that the cosmic rays are due to the formation of common elements from positive and negative electrons, the next question is "where are these creative processes going on?" There are two possible localities, in the stars where temperatures, pressures and densities are very high or in interstellar space where these three quantities are very small. Two reasons are given for eliminating the stars from consideration: (1) If atom-building with the consequent production of cosmic rays is favored by the conditions that hold inside of stars, then the sun, the star nearest to us, should send to the earth more rays than any other star. It is, however, true that the intensity of these rays is not affected by the change from noon to midnight. "This can

only mean that the conditions existing in and about the sun, and presumably also in and about other stars as well, are unfavorable to the atom building processes which give rise to these rays. Since, however, the rays do come to us at all times, day and night, and, according to all observers, at least very nearly equally from all directions, there is scarcely any escape from the conclusion that the atom-building processes giving rise to the cosmic rays are favored by the conditions existing in interstellar space." (2) The hardest cosmic rays are completely absorbed by passing through about 70 meters of water. If such rays were produced inside of stars they would be absorbed on their way out to the surface unless they were developed in the outer layer of the star so near to the surface that the absorption between their origin and the surface would be not greater than that of 70 m. of water. Measurements have shown that the energy of the cosmic rays reaching the earth amounts to about one-tenth of the energy coming to our globe as light and heat from all the stars except the sun. "This last fact means that if the cosmic rays have their origins within the stars, they cannot, even at the points of their origin, have an intensity more than ten times that which they have when they reach the earth's atmosphere, for if they had then the cosmic-ray energy transformed into heat by absorption on the way out would yield a total heat outflow from the stars larger than the observed ten to one ratio."

Jeans and Eddington have sought the origin of the heat of the stars in the process of annihilation of atoms occurring within them, with the transformation of mass into radiation. The authors have, however, been unable to find among the cosmic rays the frequencies that correspond to this process. This failure in no way counts against the possibility of the process occurring inside of stars where the radiation produced would be absorbed before it could escape into outer space. By combining the idea of the formation of matter in interstellar space with that of the transformation of matter into radiation within stars the following incomplete cycle is suggested:

"1. Positive and negative electrons exist in great abundance in interstellar space. (See the evidence of the spectroscope).

"2. These electrons condense into atoms under the influence of the conditions existing in outer space, viz., absence of temperature and high dispersion. (See evidence of the cosmic rays.)

"3. These atoms aggregate under their gravitational forces into stars. (See evidence of the telescope.)

"4. In the interior of stars, under the influence of the enormous pressures, densities and temperatures existing there, an occasional positive electron, presumably in the nucleus of a heavy atom, falls

into complete coincidence with a negative, i.e., transforms its entire mass into an ether pulse the energy of which, when frittered away in heat, maintains the temperature of the star and furnishes most of the supply of light and heat which it pours out. (See evidence of the lifetimes of the stars.—Eddington-Jeans.)"

The authors direct attention to the absence of any evidence for the formation of hydrogen from radiation at least so far as their experiments go. G. F. S.

Resolving into Separate Drops the Cloud Paths Caused by Alpha Rays. O. KLEMPERER. (*Z. f. Physik*, Vol. 45, Page 225.) When a volume containing water vapor is sufficiently expanded a cloud is formed provided that proper nuclei are present on which the water may condense. When beta rays pass through air the resulting ions serve as such nuclei. About 100 pairs of these ions are produced per cm. of path and they can be counted by moderate magnification. Since alpha rays develop 100 times as many per cm. the resolution of the resulting path into individual droplets has not hitherto been accomplished. To attain the desired separation three courses were followed, 1. the moist gas, hydrogen, was used at as low a pressure as possible so that as small a number as possible of ions would be produced in a unit of length. The final pressure used was about 7 cm. of mercury. 2. In photographing the cloud path the highest possible magnification was used, and 3. The photograph obtained was in its turn magnified to the limit. A reprint of a photograph is shown in which the distances between drops are magnified 260 times. From it the author derives the formation of about 16,000 pairs of ions per cm. of path of alpha rays in normal air. The mean diameter of the cloud path comes out as equal to 4×10^{-3} cm.

G. F. S.

Selective Adsorption from Gaseous Mixtures by a Mercury Surface formed in the Mixture. M. L. OLIPHANT. (*Phil. Mag.*, Sept., 1928.) When a mercury surface increases in area molecules recently in the interior of the liquid are brought to the surface. Their orientation with respect to their fellow molecules probably determines the forces to which they are subject. Moreover it is very unlikely that all the fresh molecules arrive at the surface in any definite alignment. It is therefore to be expected that from the moment of the formation of the new surface onward there will proceed a rearrangement of the molecules that will result in the establishment of a surface having minimum energy. The consequent rapid fall of surface-tension will be difficult to follow by measurement. Let the expansion of the area of the mercury surface

take place, however, in contact with a gas. It may occur that a molecule reach the surface with an end toward the gas then grasps a molecule of the latter, i.e., produces adsorption. This union delays the attaining of the condition of minimum energy for the surface, and renders it easier to trace the concomitant changes in surface-tension "Thus Popesco has shown that five seconds after the formation of a large drop of mercury in contact with a gas the surface tension is above 500 dynes/cm., and that this falls exponentially to a final value of about 400 dynes/cm., the time taken varying with the gas. On the other hand, Harkins using the 'drop-weight' method for measuring surface-tension, does not observe any large difference between the surface-tension of a drop formed *in vacuo* and one formed in a gas. This would indicate either that the gas is not adsorbed at the surface so as to interfere with its orientation, or that the assumed abnormally high initial surface-tension due to the random orientation in the fresh surface does not actually exist. One of the methods for measuring the surface-tension must give the wrong result, and the following experiments have been carried out in order to test whether the adsorption of gas does take place at an expanding mercury surface."

Mercury issued from a nozzle having 25 holes at the rate of 20 drops per sec. through each hole. The drops fell through a mixture of carbon dioxide with argon or hydrogen and collected in a pool at the foot of the tube. If one of the gases in the mixture is more readily adsorbed than the other, then, where the drops form molecules of this gas will become attached to them and will be conveyed to the pool of liquid below where they will be disengaged. Thus the composition of the mixture changes with the falling of the drops. The adsorbed gas disappears from the vicinity of the nozzle and is present in greater proportion at the lower end of the tube. Following the procedure of Schofield a stream of the mixed gases is supplied to the middle of the tube, where it divides into an upward and a downward stream of equal velocities. The two streams are conducted to the two tubes of a Rayleigh interferometer, capable of detecting a change of .001 per cent. in the CO_2 content of a mixture of hydrogen with 2 per cent. of carbon dioxide. The shift of fringes in the interferometer enabled the change of composition to be calculated. From the number of drops per sec., the volume of mercury falling per sec. and the change in composition the number of carbon dioxide molecules attached to each sq. cm. of mercury drop is calculated to be $.7 \times 10^{15}$. From the already known diameter of the mercury molecule this is interpreted to mean that the adsorbed CO_2 layer is one molecule thick. The experiment clearly shows that fresh mercury surfaces do adsorb CO_2 and that

the forces concerned cause the gas to remain attached during a fall through a distance of 50 cm. Moreover adsorption is selective as is demonstrated by the change in the composition of the gas. The experiment speaks against Harkins' method of measuring surface-tension. G. F. S.

The Foundation of Electromagnetism. (*Isis*, June, 1928.) To Hans Christian Oersted we owe the discovery of the magnetic effects of the electric current. He was born on a Danish island in 1777, became professor in the University of Copenhagen and died in 1851. "The experiments which he began in April 1820 are among the most memorable in the whole history of science. They enabled him to correlate two departments of natural philosophy, hitherto separate, electricity and magnetism—and thus to carry one step further the unification of knowledge."

There is given a facsimile of a lithographic portrait of Oersted made by his compatriot Bearentzen who in making his likenesses used to draw directly on the stone. Two papers of Oersted are presented in facsimile, one his account in Latin of his discoveries and the other the English translation of the same paper, sent to the Annals of Philosophy. Both bear the date of July 21, 1820, and both describe the author as "Knight of the Order of Danneborg, Professor of Natural Philosophy, and Secretary to the Royal Society of Copenhagen." The title in English is "Experiments on the Effect of a Current of Electricity on the Magnetic Needle." This, however, is a free translation of the Latin heading "Experimenta circa Effectum Conflictus Electrici in Acum Magneticam." What the Latin words for "conflict of electricity" meant is explained in the English text where in regard to a wire connected to the ends of a galvanic battery it is said "To the effect which takes place in this conductor and in the surrounding space, we shall give the name of the *conflict of electricity*." The 'electric conflict' then includes both the electric current and its magnetic field. For example it is said "It is sufficiently evident from the preceding facts that the electric conflict is not confined to the conductor, but dispersed pretty widely in the circumjacent space." It is indeed surprising to note how well Oersted gets along without any equivalent for the modern expression 'the electric current.' He is definite by stating which end of the wire receives positive or negative electricity, though references to negative electricity preponderate.

It seems that others had previously sought in vain for an effect of open galvanic circuits upon the magnetic needle. Oersted in the winter of 1820 found an effect to occur provided the circuit be closed, or, as he quaintly phrases it, "the galvanic circle must be

complete." He repeated the experiments using a stronger battery and gives the names of six men who were witnesses. This method of attesting the credibility of discoveries has its merits. Had Blondlot and Charpentier followed the example of the Danish investigator perhaps the myth of the *N* rays would not have arisen a quarter of a century ago.

The science of electrical measurements was not yet born for we read "A smaller apparatus will answer provided it be strong enough to heat a metallic wire red hot." A straight piece of wire conveying the electric current was tried in various positions in relation to the needle and the deflections noted. The needle was seen to be deflected in a horizontal plane or a vertical plane according to the location of the wire and very acutely the conclusion is drawn that "The effect cannot be ascribed to attraction" and later "This conflict performes circles; for without this condition, it seems impossible that the one part of the uniting wire, when placed below the magnetic pole, should drive it toward the east, and when placed above it toward the west; for it is the nature of a circle that the motions in opposite parts should have an opposite direction." The nature of the wire made no difference for "Wires of platinum, gold, silver, brass, iron, ribbons of lead and tin, a mass of mercury were employed with equal success." Many different substances were interposed between the current and the needle but all tried were without effect. No ferromagnetic material was tested. A brass needle and needles of gum lac and of glass in turn replaced the magnetic needle but all remained undeflected by the current.

Oersted considers but rejects as valueless for explanatory purposes "a motion in circles, joined with a progressive motion, according to the length of the conductor." This, however is surprisingly reminiscent of one modern mnemonic device for connecting the deflection of the needle with the direction of the current.

G. F. S.

An Attempt to Polarize Electron Waves by Reflection. C. J. DAVISSON AND L. H. GERMER. (*Nature*, Nov. 24, 1928.) Now that certain striking resemblances between electrons and light waves have been shown to exist it is quite in order that search should be made in the endeavor to find additional similarities. The authors who proved electrons to be reflected from crystals much as light waves are reflected from mirrors have looked for evidence that upon the reflection of electrons from a crystal some change occurs that is analogous to the polarization imposed upon light when it is reflected from glass. They report "So far as our observations go, there is no polarization of electron waves by reflection." G. F. S.

Detecting Leaks in Glass Vacuum Apparatus. P. SELENYI. (*Zeit. für Physik*, Vol. 48, page 743.) Glass tubes and vessels in which a vacuum is produced and maintained are likely to develop leaks that are difficult to locate. The author presents a method of detection which he devised in connection with the manufacture of incandescent lamps. By Tesla currents the entire vacuous space is made to glow with a color characteristic of the contained gas. Now if a different gas is leaking in its entrance will manifest itself by the appearance of a different color. For rapid recognition the two gases should give sharply contrasting colors. Thus to the red light given by air at low pressure the bluish color of carbon dioxide provides an easily visible contrast. In practice it was found convenient to have the CO_2 inside the apparatus and the air outside. When the air found its way into the vessel a red patch showed itself at once surrounding the place of entrance. This was effective in locating even the slightest leak especially when an artifice discovered by the women workers was used namely that of touching with the hand the part of the apparatus to be examined. If it is inconvenient to have the apparatus filled with carbon dioxide the difference of colors may be got by using air inside and running the CO_2 over the outside from a tube. G. F. S.

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EDITORS' NOTE: The following fourteen papers were read at a Symposium on Quantum Mechanics, held under the auspices of the American Physical Society in New York City, December 31, 1928. It is much regretted that the editors have not been able to procure two other papers which were read on that occasion.

PHYSICAL MEANING OF WAVE MECHANICS.

BY

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Harvard University.

AN understanding of the physical meaning of wave mechanics is essential if one is going to do useful work in the subject; even purely mathematical research in quantum theory is of small value unless it is carried out with the proper understanding of the physics behind it. For that reason, it seemed well to start this discussion with a brief treatment of physical interpretation rather than mathematical details. Of course, only a small portion of so large a subject can be taken up; and I have chosen to speak about the statistical side of wave mechanics, both because this is one of its most fundamental sides, and because it is a topic that will fit in well with the other papers.

Wave mechanics is an extension, not of ordinary Newtonian mechanics, but of statistical mechanics; and this simple observation is enough to explain many of its otherwise puz-

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zling features. Since ordinary statistical mechanics is a subject about which there is much misunderstanding, it will be well to start with a little description of it. Being statistical, it must deal with a great many similar objects or events; but it is necessary at the outset to understand that this by no means limits us to problems where, as in the theory of gases, there are a great many identical physical systems. One can perfectly well treat statistically a problem with even one single degree of freedom, as for example an oscillator. The statistics comes in in the fact that one works, not with single observations, but with ensembles of observations. By an ensemble, one means a set of repetitions of the same experiment—just such a set of repetitions as one actually makes in working a real physical experiment. One can set up a simple mathematical picture of an ensemble of observations. Suppose we have a space with a number of dimensions to coincide with the number of measurements we make; for example, if our system is a mechanical one with n degrees of freedom, there will be $2n$ dimensions, n for the coördinates of the system, the other n for its velocities or momenta. Each time we make our measurements, we can represent the result by a single point in this space. Then if we repeat the experiment many times, we get as many different points as there are repetitions, and this swarm of points is the mathematical picture of the ensemble. In most cases, we wish to suppose the number of trials to increase without limit, so that the swarm approaches a continuous distribution, and we can define a function representing the density of points in our space. This distribution function then will also serve as a mathematical representation of the ensemble.

The particular sort of ensemble to be used in any case depends on the physical conditions. We must judge, as closely as possible, what sort of distribution of the quantities we are measuring will really occur in our experiment, and set up an ensemble accordingly. Here, as in every case, the purpose of our mathematics is to run, so to speak, parallel to the physics; one can imagine them written in parallel columns, with a definite correspondence between, such that wherever we can make a physical observation, it can be matched with some feature of the mathematics. Then it is plain why we

need ensembles: to match the physical process of repetition. And the task of setting up the proper one must definitely be one of finding a parallel to the physics.

For example, suppose that our system were a linear oscillator; that we first took it, without energy; at an arbitrary time gave it a definite energy, the same in each repetition of the experiment; and then at a later time measured its coördinate and momentum, plotting them in a two-dimensional space. Then the ensemble of repetitions of this measurement will be easy to describe; the points will all correspond to the same energy, and will all lie on a curve in the two-dimensional space whose equation is energy = constant (an ellipse, since our system is an oscillator). But, on account of the arbitrary time when the energy was communicated, the phase will be different each time the measurement is made, and the points of the swarm will be distributed in a uniform fashion about the ellipse, according to a law which could be readily found. This ensemble is well known in statistical mechanics; it is the microcanonical ensemble. It is useful in thermodynamics, when we wish to work with systems of constant energy.

As another example, we might take our same oscillator, but expose it to different external circumstances. We now couple it, with small forces of interaction, to a system with a great many degrees of freedom, and a definite energy. At a definite time, we measure its coördinate and momentum. In this case, the oscillator can acquire any energy by interaction, and sometimes it will have one value, sometimes another, but the relative chances of the different values will be distributed according to a law which can be calculated. As before, the phases will be arbitrary. The swarm of particles in the ensemble will then cover the whole of this two-dimensional space, with a density that can be definitely found. This is the canonical ensemble. It is particularly useful in thermodynamics, because it represents what one means by a system at constant temperature; that is, a system capable of interchanging energy with a much larger system—temperature bath—of fixed properties.

The two ensembles we have mentioned are the most useful in the application of statistical mechanics to thermo-

dynamic problems; but statistical mechanics is a much wider subject in its possibilities, and an indefinite number of other ensembles may be useful. Suppose we still take our oscillator; but let us now give it approximately a definite energy, with approximately a definite phase, the precise values varying from one repetition of the experiment to another. Then, if we measure the coördinate and momentum at a definite time, all the points in our swarm will be clustered together in a small region of the two-dimensional space. This is the sort of ensemble that could be used to describe a measurement, almost definite, that was subject to small errors. The amount of spreading of the swarm would be directly connected with ideas of probable error.

Wave mechanics, being a form of statistical theory, operates with ensembles. They are represented mathematically by giving the distribution function representing the swarm of points. Thus, the familiar $\psi\psi^*$ gives the density of points in a space in which the various coördinates of the system are plotted. This is, it is true, different from the space usually used in ordinary statistical mechanics, where coördinates and momenta both are plotted. But the difference is not essential, from the present point of view, and it is easy to show the relation of the two methods. There are some other differences; we can not, for example, restrict all the points of the ensemble to too small a region, or we meet the difficulty of the principle of indeterminateness. But the essential principles are the same. Just as in ordinary statistical mechanics, we must here choose the ensemble, determined by ψ, by considering the sort of statistical distributions actually present in the repetitions of the experiment being performed.

An ensemble, in statistical mechanics, takes the place of a single set of measurements in ordinary mechanics; and as such it is but the beginning of the problem. Our real task is, for example, to trace what happens to the system as time goes on. Given an initial ensemble, we wish the final ensemble. This is where mechanics enters the problem; and here a characteristic and important difference in method between classical mechanics and wave mechanics appears. In classical statistical mechanics, we consider each separate

point of the original ensemble, representing a system with definite initial conditions; find its behavior according to the principles of Newtonian mechanics; derive in this way a final point from each initial one, and so a final ensemble from the initial one. The result, of course, depends on the sort of external forces to which the system is subjected. For example, if we have the microcanonical ensemble, and expose the system only to the internal forces natural to the oscillator, then it can be easily shown that, although the individual points of the swarm move, the ensemble as a whole remains unchanged. This would not be true if an external force were present. With a canonical ensemble, we are most interested in finding what happens when the system interacts with the temperature bath. Here each point moves so as to change not only its phase, but also its energy, with time; yet the ensemble is so chosen that it remains unchanged as a whole. Both these ensembles, since they remain constant under the action of forces, are useful in the theory of steady states. The third ensemble we have mentioned, in which all the points were concentrated close together, naturally behaves differently; as these systems are exposed to forces, all the points of the ensemble travel together, so that after the lapse of an interval of time, the ensemble will consist of a concentrated set of points in some new parts of the space.

The method by which the wave mechanics investigates the change of the ensemble with time is essentially different. The problem, briefly, is this: given one ensemble, find another; given one function, find another. The wave mechanics achieves this directly, without intermediate steps. A thing which converts one function into another is an operator; hence the importance of operator theory in wave mechanics. Functions are conveniently considered in a space of an infinite number of dimensions where each point represents a function; in this space, an operator changes one point into another. All the operators we are interested in are linear operators in function space, and this explains the importance of linear transformations and matrix theory, which is bound up directly with them, in wave mechancs. As is well known, it is not the distribution functions with which we work directly, but the wave function ψ. For

example, in the particular case of the change of the ensemble with time, we find the time rate of change of ψ as a certain operator acting on ψ:

$$-\frac{h}{2\pi i}\frac{\partial \psi}{\partial t} = H\psi.$$

Thus the progress of the system is shown by the path of a point in function space with time, just as in ordinary mechanics it is shown by the path of a point in ordinary space with time. If our problem is as simple as, given the distribution function at one time, to calculate it at a later time, we need merely take the point representing the initial ψ; follow its motion by Schrödinger's equation until the later time; and the final point will give the distribution function of the ensemble at the final time.

But more often we have a more complicated problem where our final distribution function is not to be merely a function of the same variable as the initial one, only at a later time, but is to be a function of different variables. For example, we may observe the distribution of positions of particles to start with; our final measurement may be the distribution in energy. In a case like this, it proves that the process of finding the distribution with respect to a new quantity means really finding the representation of the same function ψ with respect to new coördinates in function space. If, for example, the system is a quantized one, and we wish to find the distribution in energy, we refer to new "coordinates" in which the "unit vectors" are the orthogonal functions found in Schrödinger's theory. The distribution function, as referred to energy, is a discrete one: it consists of a value $c_n c_n^*$ giving the probability that the energy is that of the nth stationary state. The set of $c_n c_n^*$'s as a function of n is as truly a distribution curve, describing an ensemble, as is $\psi\psi^*$ as a function of x, and one form depends directly on the other, as we can see from the relation

$$\psi = \Sigma(n) c_n u_n,$$

where the u_n's are the orthogonal functions mentioned above. The general theory of changing from one set of variables to another, in this way, is the transformation theory of Dirac

and Jordan, and we see that it is just part of the general scheme of wave mechanics, in finding a distribution function for the final ensemble from that of the original one.

It is plain from what has been said that the statistical side of wave mechanics is a very important side. To get full advantage of the methods of the subject, the statistical feature should be constantly kept in mind. Problems should be followed through mathematically as they are worked physically; by looking at them in this broad way, many errors and uncertainties will be avoided.

The Magnetic Work of the United States Coast and Geodetic Survey. D. H. HAZARD. (*Ter. Mag.*, Sept., 1928.) In connection with its proper work the Coast and Geodetic Survey has taken observations of the magnetic elements so that now there are available for 5,000 places in the United States the values of the declination, inclination and horizon-intensity. Besides this at intervals of five years the observations have been repeated at a part of the stations and the results have been charted for 1905, 1915 and 1925.

The Survey operates also five widely separated magnetic observatories. "On inspecting the photographic records from an observatory the first impression is that irregular change is the predominant feature, one day's record differing from the next, sometimes to a marked degree. If the records for the less disturbed days are compared, however, it will be seen that there are certain general features which repeat themselves with considerable regularity, maximum and minimum values of fairly definite amounts occurring at fairly definite times of the day. By combining month by month the hourly values of those less disturbed days, diurnal-variation tables may be prepared showing how great a departure from the daily mean may be expected at any hour of the day on days of that type." It is thus made manifest that in different latitudes diurnal variations differ both in amplitude and phase, and that, depending on the season of the year, there is a further difference that affects the amplitude of the daily variation as well as the time when the maxima and minima occur. The declination curves are much alike all having an easterly extreme a couple of hours before noon and a westerly extreme one or two hours after noon. Sitka, the most northerly station, shows the greatest summer-range and the least winter-range. This, coupled with the fact that in general the main part of the variation occurs during daylight, seems to indicate some connection between the earth's magnetism and sunshine.

The dip curves change from station to station not only in details but in general character. "At Honolulu the predominant feature is a minimum about noon, whereas at Sitka it is a maximum about noon. The amplitude of the diurnal variation based on the less-disturbed days is not large, of the order of ten minutes of arc in declination, two minutes of arc in dip and twenty-five gammas (one gamma = 0.00001 c.g.s. unit) in horizontal intensity and vertical intensity." At times of magnetic storms the departure from the mean may amount to two degrees in declination, and to 500 gammas in the intensities. G. F. S.

WAVE MECHANICS.

BY

W. F. G. SWANN, D.Sc.,

Director of the Bartol Research Foundation of The Franklin Institute.

I TAKE it that, in its ideal form, the fundamental problem of atomic physics, in the spirit of the wave mechanics at any rate, is this: We have a number of entities, such as electrons, hydrogen atoms, water molecules, and so forth. Each of these is specified by a certain number of coördinates. For example, one may be specified by coördinates $q_1, q_2, \cdots q_{n_1}$, another by coördinates $q_{n_1+1}, \cdots q_{n_2}$, a third by $q_{n_2+1}, \cdots q_{n_3}$, and so on, and we wish to formulate the laws of nature as regards all the experiments we do and things we observe, in the following way: We wish to discover as a function of the q's, a quantity

$$\psi = \psi(q_1, q_2 \cdots t)$$

such that the probability that the coördinates of any one of the entities specified by, say, $q_{n_p}, q_{n_p+1}, \cdots q_{n_s}$ shall lie between $q_{n_p}, q_{n_p+1} \cdots q_{n_s}$, and $q_{n_p} + dq_{n_p}, q_{n_p+1} + dq_{n_p+1}, \cdots q_{n_s} + dq_{n_s}$, irrespective of the values of the other q's, is

$$(dq_{n_p}, dq_{n_p+1}, \cdots dq_{n_s}) \int\int\int \cdots \int\int \psi\bar{\psi} dq_1 dq_2 \cdots,$$

where the variables with respect to which the integral is taken include all of those whose differentials do not occur in the factor in parenthesis outside of the integral, where the integral itself is taken for all positive and negative values of the variables of integration, and where $\bar{\psi}$ is the conjugate to

[1] We may well raise the question as to what distinguishes one entity from another. How does the mathematics know that certain of the coördinates are to be associated with one entity and certain others with another entity? Perhaps the most logical view to take is one in which we postulate that the coördinates may be divided off into groups for which the statement in the text is true. These groups then serve to *define* the separate entities. Our mental picture of the distinctness of one hydrogen molecule from another is so vivid that we are apt to forget that the analysis requires any criterion for that distinctness.

ψ.[1] Moreover, we wish to discover the function ψ as a solution of a differential equation.

$$\text{Expression involving } \psi, \text{ its derivatives, the coördinates, and the time } t = 0,$$

of which the initial, or appropriate boundary conditions shall be the things whose assignment represents the expression of the particular experiment which is being carried out with the entities, or the particular postulated condition of the entities whose consequences we are interested in tracing.

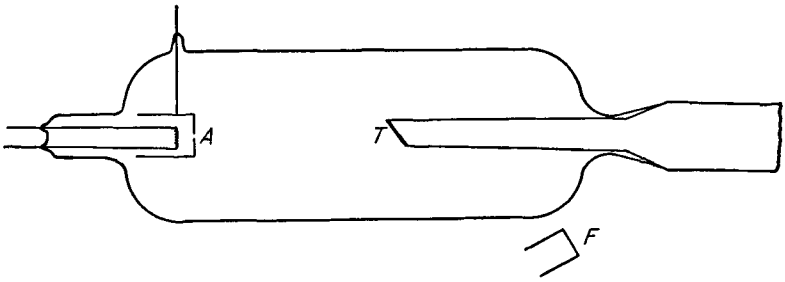
The foregoing statement represents the whole problem. There are, in the last analysis, no light quanta, no X-rays, etc., except in so far as it may be convenient to introduce these concepts as an intermediary scaffolding in the process of the analysis. The only thing that anyone ever *measures* is the presence of such an entity as an electron, or a molecule, within some specified limits of the coördinates. For example, we observe the entry of an electron into a Faraday cylinder, or the darkening of a photographic plate which we associate with the arrival of certain atoms at the plate. Even in the case where the darkening of the plate is produced by what the physicist would term a beam of light, we may, and must, ignore the consideration of the light from the true wave mechanics standpoint, and recognize instead, the chemical changes produced in the silver salt as symbolized by the growth from practically zero to a finite value of the probability of certain associations of values of the coördinates identified with the silver salt. Strictly speaking, a more refined logical sensitivity would drive us still further into the realms of the abstract. We should not leave the electron at the stage where it was endowed with a large probability of being in the Faraday cylinder, we should trace, via wave mechanics, the theory of the Faraday cylinder, of the electrometer to which it is connected, of the spot of light on the scale, and we should even trace the matter through to our brain, and represent in true wave mechanics fashion, the act of writing the ultimate result down in our notebook. In this, we should encounter much complexity, and should have to pause on the way to settle certain subtle questions concerned with the wave mechanics theory of vital processes. We decide there-

fore, not to be too pedantic, but to leave the wave mechanics story of our electron, for example, when it has been calculated as having a large probability of being found in the Faraday cylinder; and, stifling our conscience in the matter of its not having appealed to any of our senses in that position, we are content to carry on the remainder of the story by old classical methods, and take the indication of the electrometer as a criterion for the presence or absence of the electron in the cylinder. In the recognition of all the things which we really do observe, the deflection of the electrometer needle, the darkening of the photographic plate, etc., there becomes involved an association between the actual phenomena observed, and the probability of occurrence of certain of the coördinates of our system, within certain specified limits, an association which permits us to complete the story by classical, physical or chemical means in those parts in which we have no doubt as to the approximate applicability of those means.

It is worthy of notice that, in the spirit of the foregoing attitude towards wave mechanics, there remain none of those difficulties concerned with the propagation of waves in a multidimensional space, difficulties which really do not belong to the subject at all, and which find their origin in an attempt to retain the idea of a wave propagation in a medium—that old medium, the æther—invented by man for the purpose of propagating his misconceptions from one place to another—the æther which of all the subtle fluids invented for the stimulation of the imagination, is the only one which has so far not been prohibited.

In order to illustrate the significance of the foregoing remarks, let us suppose that we are presented with a problem, which in the language of the experimental physicist, would be described as follows: A beam of 100,000 volt electrons comes through the hole A, and strikes the target T, producing X-rays. Some of the X-rays fall upon the plate P from which they eject electrons. some of which are caught in the Faraday cylinder F. We may further complicate the experiment in imagination by supposing that the heating of the plate P by the X-rays is so great that some of the atoms evaporate from the plate and condense on the X-ray tube. It is desired to inquire of wave mechanics as to the story of

the influx of electrons to the Faraday cylinder, and the evaporation of the substance of the plate P on to the glass of the X-ray tube.



Our first step must be to write down a differential equation for ψ. In the stage to which wave mechanics seems so far to have crystallized itself in the matter of writing down this equation the plan is to seek the Hamiltonian function H which would be appropriate to the discussion of the electrons, X-ray target, plate P, etc., as one system on the basis of the old mechanics and then after replacing any momentum p_r by $\frac{h}{2\pi i}\frac{\partial}{\partial q_r}$, treat the resulting expression for H as an operator which operates on ψ, and write for our differential equation

$$-\frac{h}{2\pi i}\frac{\partial \psi}{\partial t} = H\left(\frac{h}{2\pi i}\frac{\partial}{\partial q_1}, \frac{h}{2\pi i}\frac{\partial}{\partial q_2}, \cdots q_1, q_2, \cdots t\right)\psi.$$

We shall, for a moment, defer any criticism of the significance of this procedure, and rest our attention on the supposition that we have *some* differential equation for ψ. It is now our business to solve it subject to specified initial conditions. Now here we encounter a logical difficulty. Unless we know the form of our differential equation, we are unable to specify the elements which must be assigned in order to determine its solution. Our physical experience tells us that, as regards the conditions at A, at any rate, that which ought to be assigned must provide the equivalent of the assignment

of the density and the velocity of the electron stream coming out of the hole A. However, we have no guarantee, without specification of the equation, that the equation itself will be content with this. Waiving a logically complete answer to this difficulty, we may suppose that at $t = 0$, for example, ψ must assume in the vicinity of the target, of the plate P and of the Faraday cylinder, etc., the values appropriate to some normal states of the atoms contained in those elements of the system, while in the vicinity of A, the initial value must be such that the resulting value of $\psi\bar{\psi}$ in that vicinity will have a finite value over only a limited region, and a value of such amount that the integrated value of $\psi\bar{\psi}$ corresponding to the probability that an electron shall be in that limited region shall have value which corresponds to a definitely assigned amount of electricity in the electron beam which may be supposed to have just emerged from A. This is not all, however, for we have to introduce the idea that the electrons have the assigned velocity of, say 100,000 volts. Presumably we must do this by analyzing our ψ in the neighborhood of the hole A into plane waves, and then requiring of it that it shall consist of such an admixture of plane waves, that if we associate with anyone of them a vector quantity p, defined by $\lambda = h/p$, the amplitude of the wave of length λ shall be such that if we make the probability of momenta lying between p and $p + dp$ depend upon its square, the distribution of momenta p among the electrons will be such as to give a large enough value to the probability of the momentum p corresponding to the velocity 100,000 volts to satisfy our experimentalist that our mathematical representation of the problem corresponds to his facts. The initial conditions of the problem having been expressed, the remainder of the story is to be told by the solution of the differential equation. It must result that the distribution of ψ in the vicinity of the target will remain sensibly what it has always been until such time as is represented by the time of arrival of the electrons there, when it should begin to change rapidly. The experimentalist will want to talk about the emission of X-rays. However, we must not listen to him, but must let ψ tell its own story by gradually propagating its alterations until the regions at P begin to experience such

alterations. The time rates of change of $\psi\bar{\psi}$ should then contain in itself the story of the passage of the electrons from the plate and of their appearance in the Faraday cylinder. It must tell this by the integrated value of $\psi\bar{\psi}$ with respect to all the coördinates, other than those which were inserted in our original differential equation to represent the electrons in the beam, becoming finite at points within the cylinder, whereas, at $t = 0$, and for some time subsequently, the integrated value of $\psi\bar{\psi}$ in question was zero at all points within the cylinder. The story of the evaporation of the metal from the plate A must be told in similar manner by the integrated value of $\psi\bar{\psi}$ with regard to all coördinates other than those introduced to represent the atoms on the plate P becoming finite at points in the vicinity of the glass whereas at $t = 0$ and for sometime subsequently, it was zero.

If the experimentalist should place a stopping potential V on the Faraday cylinder and should seek to measure the velocities of the electrons by adjusting V so as to just keep them out, the wave mechanics story must be that of including V in the original differential equation, and subsequently determining for it a limiting value which just insures that the integrated value of $\psi\bar{\psi}$ referred to above is zero for all time for points inside the Faraday cylinder. If, in the experiment, the glass tube of the X-ray tube were replaced by lead, the lead would also figure in the Hamiltonian function, and in such a way as to cause the solution for ψ outside of the tube to be sensibly independent of all boundary conditions expressed for the regions inside the tube. In this manner the story of the shielding action of the lead would be told.

Now, I am perfectly aware of the fact that the procedure usually adopted in wave mechanics problems is not that outlined above. The procedure actually adopted in such problems is to consider the quantum transitions created in the atoms of the X-ray target by the arrival of the electrons, and then, by treating $\psi\bar{\psi}$, or something closely allied to it, as density of electric charge, determine by its oscillations as produced by a suitably defined velocity associated with it, an electromagnetic wave which falls upon the plate P. The electromagnetic wave then exerts its influence, mathematically, by contributing, through its scalar and vector potentials,

to the Hamiltonian function of the atoms in P, and in this way determines the subsequent probabilities of transitions associated with the photoelectric effect. This procedure, involving as it does, the association of $\psi\bar{\psi}$, or something like it, with charge density, involves ideas not necessarily contained in the more fundamental method of attack; and, if it is to be right, it remains to show that the two procedures are the analytical equivalents of each other. It is no excuse to say that the more fundamental method is too difficult to carry out and that, therefore, we are permitted to adopt a simpler method.

Of course, in the more fundamental method, it is obvious that those features of the electromagnetic theory which provide for propagation with velocity c must, in some way, be provided for in the fundamental operator H, or at any rate in the differential equation which governs ψ. Insofar as classical procedure provides any precedent, the suggestion is to start with a Lagrangian function

$$L = L(q_1, q_2, \cdots \dot{q}_1, \dot{q}_2, \cdots t) \tag{1}$$

and build up a Hamiltonian

$$H = \sum p_r \dot{q}_r - L,$$

expressed, however, as a function of the p's and q's, with p_r defined as

$$p_r \equiv \frac{\partial L}{\partial \dot{q}_r}. \tag{2}$$

Unfortunately, even classical electrodynamic theory does not reveal that part of its content which is concerned with the finite rate of propagation of effects by starting with a Lagrangian function as simple as (1). There occur the vector and scalar potentials, which cannot be expressed in terms of instantaneous velocities and positions; and, indeed, the approximation to such a representation robs the whole scheme of the properties of finite rate of propagation. If, however, the Lagrangian function is to contain higher time derivatives than the first, even should we arbitrarily define a set of momenta by relations of the type (2), there remains the impossibility of defining along the conventional lines of

procedure a Hamiltonian function H, as a function of the p's, q's and t, to form a basis for constructing a Schrödinger equation by the replacement of p_r by $\frac{h}{2\pi i}\frac{\partial}{\partial q_r}$.[1] It would seem that we are here confronted with a situation analogous to that prevailing before Maxwell introduced the displacement current into electromagnetic theory; and, just as the attempt to retain the word momentum in the specification of the phenomena necessitated a wider concept of that quantity than could be expressed in terms of the motion of electricity directly, a concept in which the field played a non-eliminatable part, so in wave mechanics it would seem that we must ultimately come to a way of formulating the fundamental equation for ψ which is less naïve, and more widely embracing than that exemplified in the well-known equation

$$-\frac{h}{2\pi i}\frac{\partial \psi}{\partial t} = H\left(\frac{h}{2\pi i}\frac{\partial}{\partial q_1}, \frac{h}{2\pi i}\frac{\partial}{\partial q_2}, \cdots q_1, q_2, \cdots t\right)\psi.$$

Finally, I would close with this observation. We say that a theory such as the electromagnetic theory is complete when, given unlimited mathematical ability, at any rate, there exists a method of solving any problem in its field when that problem is completely stated, i.e., when the necessary boundary conditions are given, I would suggest that, in this sense, a theory of wave mechanics does not exist today.

[1] At best, we should obtain an equation of high, and possibly infinite order. Concerning such equations there is doubt as to whether they say very much. Thus, confining oneself, for illustration, to non-partial differential equations in t, an equation of the 100th order requires, to complete its solution, the specification of the time derivatives up to the 99th order at $t = 0$; and as we proceed to the limit, the numbers of the derivatives to be expressed tends to infinity. If, however, we express all the time derivatives at $t = 0$, the function is completely determined by McLaurin's expansion, without the differential equation, over the range in which it is analytic; and, if it ceases to be analytic, the whole mathematical procedure requires revision in any case. Speaking rather crudely, we may say that the higher the order of the differential equation, the less it says; and, while in certain cases where the coefficients of the higher derivatives are small, these considerations may be trivial, they may mount to fundamental importance when we come to force out of our expression for ψ the anticipation of events which may be long delayed in such a sense, for example, as that concerned in our imaginary experiment cited earlier, where, long before the arrival of the electrons at P, the ψ function anticipates their arrival, in its structure.

THE OPERATOR H IN SCHRODINGER'S EQUATION.

BY

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IN the short time at my disposal I wish to say a few words about the nature of the operator H which occurs in Schrödinger's equation

$$\frac{ih}{2\pi}\frac{\partial\psi}{\partial t} = H(\psi)$$

and which dominates the wave-theory. The accepted point of view is to proceed on the particle theory of matter setting up the expression for the energy of the system and obtaining the Hamilton-Jacobi partial differential equation. If

$$L = T - V$$

is the kinetic potential and if

$$F = Lt',$$

where primes denote differentiation with respect to an arbitrary independent variable θ of which the coördinates q of the system and t are functions, then

$$H = -\frac{\partial F}{\partial t'}.$$

(If

$$T = \frac{1}{2}g_{\alpha\beta}\dot{q}^{\alpha}\dot{q}^{\beta} = \frac{\frac{1}{2}g_{\alpha\beta}q^{\alpha'}q^{\beta'}}{t'^2}$$

then

$$F = \frac{\frac{1}{2}g_{\alpha\beta}q^{\alpha'}q^{\beta'}}{t'} - Vt'$$

and

$$H = -\frac{\partial F}{\partial t'} = \frac{1}{2}\frac{g_{\alpha\beta}q^{\alpha'}q^{\beta'}}{t'^2} + V = \frac{1}{2}g_{\alpha\beta}\dot{q}_{\alpha}\dot{q}_{\beta} + = VT + V).$$

The Hamilton-Jacobi equation is obtained by eliminating the $\dot{q}^r$ from the equations

$$p^r = \frac{\partial L}{\partial \dot{q}^r}; \qquad H = T + V.$$

If, then, we wish to construct a wholehearted wave theory we must find a function F which is homogeneous of the first degree in the $n + 1$ derivatives q', t' (n being the number of degrees of freedom of the system); the $n + 1$ derivatives $\frac{\partial F}{\partial q'}, \frac{\partial F}{\partial t'}$ are, then, homogeneous functions of degree zero in the derivatives q', t' or, in other words, they are functions of the n ratios of these derivatives. Eliminating these n ratios from the $(n + 1)$ functions we get a relation

$$\varphi\left(\frac{\partial F}{\partial q'}, \frac{\partial F}{\partial t'}\right) = 0.$$

The Hamilton-Jacobi equation follows on setting

$$\frac{\partial F}{\partial q'} = \frac{\partial S}{\partial q}, \qquad \frac{\partial F}{\partial t'} = \frac{\partial S}{\partial t}$$

where S is a function of (q, t) obtained by integrating F from a fixed point along an extremal curve for the integral $\int F d\theta$. The wave equation follows by considering those wave equations which have the surfaces $S =$ constant as wave fronts. The problem is, therefore, essentially that known as the inverse problem of the Calculus of Variations; the surfaces S being regarded as of physical significance one must start with them and work back to F.

In conclusion we would like to point out that there are certain advantages in general dynamical theory in the introduction of a state-energy space of $2n + 2$ dimensions instead of the usual state-space of $2n + 1$ dimensions. In this state-energy space the coördinates are (q, t, p, H) and a spread of $2n + 1$ dimensions $\phi(q, t, p, H) = 0$ defines a dynamical system. The dynamical theory of contact transformations becomes identical with Lie's theory of geometrical contact transformations and the symmetry given to the situation seems to confer a real simplicity.

QUANTUM MECHANICS OF MOMENTUM SPACE.

BY

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IN Schrödinger's work on quantum mechanics he starts from the Hamiltonian function of the mechanical system under investigation and makes from it an operator by the process of replacing each momentum, p_σ, by the differentiation operator $\frac{h}{2\pi i}\frac{\partial}{\partial q_\sigma}$. A wave function $\psi(q_1 \cdots q_n)$ is then introduced which is required to satisfy the equation

$$H\left(\frac{h}{2\pi i}\frac{\partial}{\partial q}, q\right)\psi = W\psi, \tag{1}$$

where W is a constant. That is, the energy operator, operating on ψ must give simply a constant into the same function. Only such ψ's as are finite and continuous throughout the range of the q's are allowed. It is then found that such ψ's exist only when W has certain characteristic values depending on the form of the operator H. These values of W are then taken as the allowed values of the energy. The interpretation given by Schrödinger for his function ψ however is not tenable and must be given up in favor of one due to Born that the quantity $\psi(W, q)$ is related to the probability of finding the system in a little volume element of the coördinate space, dq, when it is known that the system is in the energy state W. This probability is in fact taken equal to

$$\psi(W, q)\bar{\psi}(W, q)dq, \tag{2}$$

where $\bar{\psi}$ is the conjugate complex function to ψ. This method of finding energy levels and this interpretation of ψ are at the heart of many of the successes of the new quantum mechanics.

As Professor Slater has pointed out, this statistical feature runs throughout the whole of quantum mechanics. The

questions which it attempts to answer are to be phrased in the following form: (*A*) What are the numerical values which a physical quantity may assume, and (*B*), In a given set of circumstances what is the relative probability that it does assume any one of these allowed values? In terms of this general formulation one sees that the equation of Schrödinger (1) and the expression (2) form answers to two very special questions of this type. Equation (1) is the means employed for finding what values the particular physical quantity, total energy of the system, may have. It answers a question of the first type. Expression (2) is the answer to a question of the second type. We are supposed to know that the configuration may assume any value represented by the allowed ranges of the q's. The given set of circumstances is the certain information that the system is in the state of energy W and then the expression gives relative probabilities that the system be found in various elements of the configuration space.

It has been the service of Dirac and Jordan to point the way to the generalization of equations (1) and (2) so that the means is at hand to answer any questions of the type (*A*) and (*B*) rather than simply the special cases of them covered by the Schrödinger equation. The purpose of these remarks is to illustrate the ideas by the presentation of a few simple cases in which one works with the momenta as independent variables, i.e. in which an equation analogous to (1) is utilized to find a $\psi(W, p)$ which can be used in an expression analogous to (2) to get the relative probability that the momenta will lie in the element of momentum space defined by p_1 between the values p_1 and $p_1 + dp_1$, p_2 between the values p_2 and $p_2 + dp_2$ and so on. It is believed that the results, while containing nothing new of importance to atomic physics will nevertheless be useful in facilitating the study of the general theory.

To each physical quantity can be associated an operator and the laws of association of operators with quantities constitute an important branch of the subject. There are different representations of a physical quantity and the letter denoting the quantity has to be used in two senses. Where written in bold type it will be meant as the quantity in its

capacity as an element of the algebra of quantum mechanics. That is, as a matrix in many cases, or more generally as what Dirac calls the q-number ($q \sim$ quantum). In the language of Weyl's recent book,[1] each physical quantity is associated with an operator in the unitary function space. Where ordinary type is used the ordinary real variable which is the arithmetic value of the quantity is meant. The guiding principle in associating operators with the physical quantities is that this must be done consistently with the matrix mechanics of Heisenberg. Thus between coördinate and conjugate momentum one has the rule

$$\mathbf{pq} - \mathbf{qp} = \frac{h}{2\pi i}. \tag{3}$$

In getting Schrödinger's equation (1) we associate each $\mathbf{q}$ with the operation: Multiplication by the arithmetic value q and each $\mathbf{p}$ with the operation: Multiplication by $h/2\pi i$ and partial differentiation with respect to the conjugate q. One sees that this is in fact consistent with (3) since

$$\frac{h}{2\pi i}\frac{\partial}{\partial q}(q\psi) - q\frac{h}{2\pi i}\frac{\partial \psi}{\partial q} \equiv \frac{h}{2\pi i}\psi. \tag{3a}$$

But another way [2] of associating operators with $\mathbf{p}$ and $\mathbf{q}$ which is just as consistent with (3) as the method leading to Schrödinger's equation is the following:

$\mathbf{p}$: Multiplication by p.

$\mathbf{q}$: Operation $-\frac{h}{2\pi i}\frac{\partial}{\partial p}$.

for this gives

$$p\left(-\frac{h}{2\pi i}\frac{\partial \psi}{\partial p}\right) + \frac{h}{2\pi i}\frac{\partial}{\partial p}(\psi p) \equiv \frac{h}{2\pi i}\psi. \tag{3b}$$

so that this association is consistent with (3).

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[1] Weyl, "Gruppentheorie und Quantenmechanik." Leipzig, Hirzel, 1928.

[2] This method of introducing the operator $-\frac{h}{2\pi i}\frac{\partial}{p}$ for q when p is multiplication by p was suggested to me after the symposium by my colleague, Dr. H. P. Robertson. It is so neat, that I have taken the liberty of using it rather than the method used in my remarks.

Before passing to the use of this method in determining energy values it is essential to consider the question: What values may each $\mathbf{p}$ assume? Because of the kinematical connection between $\mathbf{p}$ and $\mathbf{q}$ the allowed values of $\mathbf{p}$ are determined by the allowed values of $\mathbf{q}$, as Jordan [3] has shown. As to the allowed values of $\mathbf{q}$ these are supposed to be known almost intuitively from the nature of the system. Thus, the allowed values of a Cartesian coördinate are all real values from $-\infty$ to $+\infty$; of a radius vector in polar coördinates are from 0 to $+\infty$; of the azimuthal angle in spherical or cylindrical coördinates, 0 to 2π; of the co-latitude in spherical coördinates, 0 to π. From these allowed ranges in $\mathbf{q}$ certain results for the allowed values of the associated $\mathbf{p}$ are found. Thus for a Cartesian coördinate or a radius vector whose range is infinite the $\mathbf{p}$ may take on all values from $-\infty$ to $+\infty$. But for an azimuthal angle like φ the allowed values of the associated $\mathbf{p}$ are the discrete range given by $n(h/2\pi)$ where n takes on all *integral* values from $-\infty$ to $+\infty$.

Corresponding to this second mode of associating operators with p and q one finds as the equation analogous to (1),

$$H\left(p, -\frac{h}{2\pi i}\frac{\partial}{\partial p}\right)\psi = W\psi, \tag{4}$$

which will now be considered in some simple instances:

The Free Particle.—For the motion of a particle of mass μ in one rectilinear coördinate, x, under no forces the Hamiltonian function is $\mathbf{H} = (1/2\mu)\mathbf{p}^2$. As it is independent of $\mathbf{q}$, (4) takes the simple algebraic form

$$\left(\frac{1}{2\mu}p^2 - W\right)\psi = 0. \tag{4a}$$

It is at once apparent that if p has a value differing from $\pm\sqrt{2\mu W}$ then ψ must be zero. As the range of p is confined to real values one has $W > 0$. But here one strikes a characteristic feature of the theory in that if one adheres to the notion that ψ is to be normalized with respect to the complete allowed range of p, i.e.,

[3] Jordan, Zeitschrift für Physik, 44, 1 (1927).

$$\int_{-\infty}^{+\infty} \psi(W, p)\bar{\psi}(W, p)dp = 1$$

then ψ must be strongly infinite at $p = \sqrt{2\mu W}$ or $-\sqrt{2\mu W}$ or both in order that the integral shall not vanish. Thus ψ is a function of the type called a δ-function by Dirac:

$$\delta(x) = 0 \text{ for } x \neq 0 \quad \text{and} \int_{-\infty}^{+\infty} \delta(x)dx = 1.$$

Plainly one does not deal here with a simple requirement of finiteness and continuity for $\psi(W, p)$ as seems usually to be the case with the $\psi(W, q)$ of Schrödinger.

The analogue of (2) is that this $\psi(W, p)\cdot\bar{\psi}(W, p)dp$ is equal to the probability that p have a value between p and $p + dp$ when W is known to have the value W. The equation (4a) shows that this vanishes except for

$$\frac{1}{2\mu}p^2 = W,$$

so that we recognize in this an equation of classical mechanics which retains its validity in quantum mechanics.

All values of W are on an equal footing in regard to the nature of the $\psi(W, p)$ associated with them so (4a) would show that all values of W greater than zero are allowed.

The Freely Falling Particle.—This is the same as the preceding except that now the Hamiltonian is $(1/2\mu)\mathbf{p}^2 - \kappa\mathbf{x}$ so (4) becomes

$$\left(\frac{1}{2\mu}p^2 + \frac{\kappa h}{2\pi i}\frac{\partial}{\partial p}\right)\psi = W\psi \tag{4b}$$

of which the solution is

$$\psi = \exp.\frac{2\pi i}{\kappa h}\left(Wp - \frac{1}{6\mu}p^3\right).$$

This solution is finite throughout the range of values of p no matter what the value of W, therefore all values of W are allowed, both positive and negative, agreeing with the classical mechanics of uniformly accelerated motion. The value of $\psi\bar{\psi}$ is here independent of p which means that all

values of p are equally probable. This is in accord with the classical motion if we understand that all values of the time are to be taken as equally probable in reckoning the relative probabilities for the momentum. That this is the correct analogue with classical mechanics is consistent with quantum mechanics for exact specification of the energy W, makes the conjugate quantity, the time, to be wholly indeterminate, as is well known.[4]

The Harmonic Oscillator.—Here the Hamiltonian is

$$\frac{1}{2\mu}\mathbf{p}^2 + \frac{1}{2}\kappa\mathbf{x}^2$$

and because it is quadratic in both p and x one gets an equation of the same form here (from (4)) as by Schrödinger's method, (1). It is:

$$-\frac{h^2\kappa}{8\pi^2}\frac{\partial^2\psi}{\partial p^2} + \frac{1}{2\mu}p^2\psi = W\psi. \qquad (4c)$$

Requiring that ψ be finite at $p = \pm\infty$ this gives the energy levels

$$W_n = \left(n + \frac{1}{2}\right)h\nu, \qquad n = 0, 1, 2 \cdots, \qquad \nu = \frac{1}{2\pi}\sqrt{\frac{\kappa}{\mu}};$$

and

$$\psi_n(p) = \exp.\ (-p^2/p_0^2)H_n(p/p_0), \qquad p_0^2 = \frac{h}{2\pi}\sqrt{\kappa\mu}.$$

Here $H_n(x)$ is the nth Hermitian polynomial as with Schrödinger. One sees that p_0 is the maximum value of the momentum in classical oscillations of energy $\frac{1}{2}h\nu$ just as the characteristic length, a, occurring in Schrödinger's functions was the classical amplitude of oscillation in the classical motion of this energy. For various values of n therefore these ψ functions bear the same relation to the momentum values associated with the classical motions of the same energy as they do with Schrödinger when they give the coördinate probabilities instead of momentum probabilities.

[4] Heisenberg, Zeitschrift für Physik, 43, 172 (1927) and Bohr, Nature, April 14, 1928.

Free Rotator with Fixed Axis.—The Hamiltonian is $(1/2\mu)\mathbf{p}^2$ as with the free particle moving in a straight line. Here μ is the moment of inertia. But as the coördinate now is one restricted to values from 0 to 2π, it follows that p is restricted to values $n(h/2\pi)$ with n integral.. Therefore the allowed values of W are

$$W_n = \frac{n^2h^2}{8\pi^2\mu}$$

with ψ functions of the same type as with the free particle.

The Physical Pendulum.[5]—This offers an interesting variation of the preceding, the Hamiltonian being $(1/2\mu)\mathbf{p}^2 - \kappa \cos \theta$. To form equation (4) calls for interpretation of the operator $\cos \theta$. This is clearly

$$\cos\frac{h}{2\pi i}\frac{\partial}{\partial p} = \frac{1}{2}\left[\exp.\left(\frac{h}{2\pi i}\frac{\partial}{\partial p}\right) + \exp.\left(-\frac{h}{2\pi i}\frac{\partial}{\partial p}\right)\right].$$

Recalling that the terms on the right are a symbolic form of Taylor's theorem it is plain that the operation on $\psi(p)$ gives

$$\cos\theta\psi(p) = \frac{1}{2}\left[\psi\left(p + \frac{h}{2\pi}\right) + \psi\left(p - \frac{h}{2\pi}\right)\right]$$

which remains consistent with the general result that the allowed values of p differ by $h/(2\pi)$ for the momentum conjugate to the coördinate θ of period 2π. Introducing $p = n(h/2\pi)$ the equation (4) for $\psi(W, n)$ becomes

$$\frac{\kappa}{2}[\psi(n+1) + \psi(n-1)] = \left(W - \frac{n^2h^2}{8\pi^2\mu}\right)\psi(n) \quad (4d)$$

The allowed energy levels for W are those for which this difference equation for ψ has solutions which remain finite as n takes on integral values between $-\infty$ and $+\infty$. But there seems not to be a very useful mathematical literature on boundary value problems for difference equations.

In these examples we see cases where the equation (4), which is analogous to (1), in momentum space may be alge-

[5] For the treatment of this problem by Schrödinger's method see Condon, Physical Review, 31, 891 (1928).

braic or may be a finite difference equation. In coördinate space, since the Hamiltonian always includes a quadratic form in the momenta a second order differential operator is always generated. This makes (1) always resemble the partial differential equations of wave propagation from elasticity theory. In fact this analogy is fundamental in Schrödinger's approach to quantum mechanics. From the more general standpoint this resemblance to a wave equation is somewhat accidental.

PALMER PHYSICAL LABORATORY,
PRINCETON UNIVERSITY,
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THE STATISTICAL INTERPRETATION OF VARIOUS FORMULATIONS OF QUANTUM MECHANICS.

BY

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THE various formulations of the quantum mechanics, although to a large extent fundamentally equivalent, are so numerous and varied in superficial structure that they are apt to confuse the casual student. Some of the more important mathematical forms are the following:

1. Born and Heisenberg's matrix theory.[1]
2. Schroedinger's wave mechanics.[2]
3. Dirac's theory of q-numbers.[3]
4. Born and Wiener's operator theory.[4]
5. The transformation theory of Dirac, Jordan, and Pauli.[5]

Numbers 1 and 2 are the commonest, and those which will be discussed primarily in this paper. The fifth is, however, the most comprehensive and far reaching, as it includes the others as special cases. The formulation of the quantum mechanics given by Professor Weyl in the first part of his recent book [6] can to a great extent be regarded as a very beautiful geometrical interpretation of the transformation theory in the Hilbert space, and stands in roughly the same relation to this as Minkowski's four dimensional world to special relativity or non-Euclidean geometry to general

[1] Born, Heisenberg, and Jordan, *Zeits. f. Physik*, **34**, 858 and **35**, 557 (1925–26).

[2] E. Schroedinger, "Abhandlungen zur Wellenmechanik" (Leipzig, 1927; English translation, London, 1928).

[3] P. A. M. Dirac, *Proc. Roy. Soc.*, **109A**, 642; **110A**, 561; **111A**, 281 (1925–6).

[4] Born and Wiener, *Jour. of Math. and Phys., Mass. Inst. of Tech.*, **5**, 84 (1926) or *Zeits. f. Physik*, **36**, 174 (1926).

[5] P. A. M. Dirac, *Proc. Roy. Soc.*, **113A**, 621 (1927); Jordan, *Zeits f. Physik*, **40**, 809, **41**, 797, **44**, 1 (1926–7); (cf. also London, *Zeits. f. Phys.*, **40**, 193 (1926)).

[6] H. Weyl, "Gruppentheorie und Quantenmechanik" (Leipzig, 1928).

relativity. The appeal depends largely on whether one is geometrically or analytically inclined.

The *mathematical* relations of equivalence between these various formulations are fairly well known. What I wish to emphasize is how the statistical interpretation of quantum mechanics, which has been so nicely described in Professor Slater's paper in this symposium, serves as a focal *physical* viewpoint and unifying influence which blends together these different formulations and makes them no longer seem distinct mathematical entities. There is one fundamental physical lemma which epitomizes the spirit of the whole statistical outlook, just as the invariance of the velocity of light does the spirit of relativity or the second law that "heat doesn't flow uphill" does the spirit of thermodynamics. This lemma is "Heisenberg's indeterminism principle." [7]

Heisenberg's Indeterminism Principle.—"It is impossible to specify or measure accurately and simultaneously both a coördinate q and its conjugate momentum p." The experiment itself inherently spoils the possibility of such a simultaneous determination. Instead the product of the errors Δq, Δp in determining q and p respectively is of the order of Planck's h, so that

$$\Delta q \Delta p \sim h.$$

An important special case is where q is a Cartesian coördinate; then $p = mv$ and consequently it is impossible to specify simultaneously the position and velocity of an electron. Another important special case is where the p's are "action variables" J_i which take on quantized values $n_i h$ and which thus specify the stationary state or energy level. In this instance the conjugate coördinates are phase angles ("angle variables"). Thus when we specify a stationary state we cannot specify the phase of an electron in its orbit. We shall soon see that this item furnishes the clue to the physical interpretation of the Heisenberg matrices.

A situation which is of particular importance for our uses is that which arises when one variable of a canonical pair is specified with complete precision, an ideal state which is

[7] W. Heisenberg, *Zeits. f. Physik*, 43, 172 (1927); N. Bohr, *Nature*, 121, 580 (1928).

perhaps never reached completely but which nevertheless is fulfilled to all intents and purposes. If q be thus accurately specified, there is an infinite error in its conjugate momentum p. More precisely, this means that all values are equally probable for p. All we can then measure about a function $f(p, q)$ is its average with respect to p, which in classical theory would be proportional to the expression $\int f(p, q)dp$.

Various simple illustrations of the indeterminism principle, such as the "gamma ray microscope" are already becoming so hackneyed that I shall not attempt to repeat them. At first thought this principle appears contrary to our experience. The main problem, for instance, of driving an automobile is that it certainly does have simultaneously both position and velocity. However, such "large scale" phenomena, unlike occurrences inside the atom, involve such large amounts of action compared to Planck's h, that the limits of precision fixed by the indeterminism principle become negligible.

We must now apply the indeterminism principle to the interpretation of the various formulations of the quantum mechanics.

Matrix Theory.—Historically the first form of the quantum mechanics to make its appearance was the matrix theory. The indices of the Heisenberg matrices refer to definite stationary states or energy levels. By the indeterminism principle the phase or epoch angles are indeterminate as soon as we specify the quantum numbers. Hence the only observable physical quantities belonging to a given stationary state are those which are averages over all phases or positions of the electron in its orbit, and so we cannot localize an electron at a definite position at a definite time. Thus one can no longer speak of an aphelion or perihelion distance in the atomic orbit. One has, in fact, no way of measuring these distances experimentally, and it is the essence of the philosophy of the new mechanics that it should deal with only observable quantities. One can, however, still retain the concept of a mean square radius. This, of course, cannot be determined experimentally by ordinary means of measuring distances such as meter sticks, but indirectly is really observable. It can be shown, for instance, that the diamagnetic susceptibility is proportional to this mean value of r^2, which

is a diagonal Heisenberg matrix element $r^2(n;\ n)$. In general, *the diagonal elements of a Heisenberg matrix representing a function f give the average value of f when the quantum numbers are specified and all phases are equally probable.*

As to the non-diagonal elements, it is my opinion that they do not have any direct physical reality, but are only mathematical aids ("Hilfsmittel") to the calculation of the diagonal elements of other matrices. Thus as by the matrix law of multiplication $r^2(n;\ n)$ equals $\Sigma_k r(nk)r(kn)$, it is evident that in order to know the diagonal elements of r^2, it is not sufficient to know the diagonal elements $r(nn)$ of the first power of r, but we must also know the off-diagonal elements $r(nk)$. One may urge that in the existing quantum theory a certain amount of reality is given to the non-diagonal elements by interpreting them as transition probabilities, and it is true that the proper formulas for the rate of radiation can be obtained by substituting the off-diagonal matrix elements for the Fourier coefficients in the classical radiation formulas. Such a procedure, however, is only an extrapolation from the classical theory, like the correspondence principle in the old Bohr quantum theory, rather than something self-contained in the quantum mechanics. When we are really able to calculate from purely quantum considerations the rate of radiation we will doubtless find that these off-diagonal elements in a conservative system are important because they give the average rate or diagonal term of the radiation when the system is made non-conservative by considering external radiation fields and also the force on an electron due to its own radiation. Calculations by Dirac, Slater and Born [8] have already verified this for absorption and forced emission, but our theory of spontaneous emission is at present less complete. The law of conservation of energy in the statistical viewpoint means that there is no difference between the mean square and the square of the mean of the energy function H, and from this one readily sees with the matrix law of multiplication that H must be a diagonal matrix.

Wave Mechanics.—We must now turn to the Schroe-

[8] P. A. M. Dirac, *Proc. Roy. Soc.*, 112A, 661; 114A, 243 (1926–7); Slater, *Proc. Nat. Acad. Sci.*, 13, 7 and 104 (1927); M. Born, *Zeits. f. Phys.*, 40, 167 (1926).

dinger wave mechanics, which is probably the most popular and simplest way of presenting elementary quantum theory. Here the all important question is the physical interpretation of the wave function ψ, and in this one must steer between two extremes. One extreme is to say that the electron is a fluid-like substance with a density proportional to $|\psi|^2$. In this way one can make considerable progress towards constructing what I may term a "hydrodynamical" model of the atom which retains a simultaneously determinate time and position. This idea appeals to many physicists who are rather loath to accept the revolutionary philosophy of Heisenberg's indeterminism principle. It is, I believe, the opinion of the majority of theoretical physicists, including my own, that this hydrodynamical, non-statistical viewpoint is not tenable when pushed too far. I shall mention only three difficulties: first, the hydrodynamical theory is rather difficult to extend to a system with more than one electron because more than three dimensions are required; second, point potentials are used in the equations despite a continuous charge distribution, and third, perhaps the most serious difficulty, the rate of spontaneous radiation comes out proportional to the product of the concentrations of electrons in the initial and final states rather than to just the concentration in the initial state.[9] It is to be clearly understood

[9] This argument, which seems to have caused some misunderstanding at the time this paper was presented, may be elaborated as follows: The general Schroedinger equation which has the operator $h\partial\cdots/2\pi i\partial t$ in place of the energy constant W has the solution $\psi = \Sigma c_n\psi_n \exp.\ (2\pi i W_n t/h)$, where the c's are arbitrary constants and $\psi_n(x, y, z)$ is a solution of the restricted Schroedinger equation having the characteristic value W_n. If we regard $|\psi|^2$ as proportional to a fluid charge density, the term which results from the multiplication of $c_n\psi_n \exp.\ (2\pi i W_n t/h)$ and $c_m^*\psi_m^* \exp.\ (-2\pi i W_m t/h)$ in the vector electrical moment $\mathbf{p} = -e\int\int\int \mathbf{r}|\psi|^2 dv$ has a time factor exp. $[2\pi i(W_n - W_m)t/h]$ in accord with the Bohr frequency condition, forming an attractive argument for the "field" view formerly adopted by Gordon, Madelung, Schroedinger, etc. (*Zeits. f. Physik*, **40**, 117 and 322, 1926; *Annalen der Physik*, **81**, 109; **82**, 265, 1926–7). However, such a term in the moment when squared to give the rate of radiation according to the classical formula $2\ddot{\mathfrak{p}}^2/3c^3$, will have a factor $|c_n|^2|c_m|^2$. Now the square of the absolute value of a coefficient c_i is ordinarily interpreted as giving the statistical concentration of atoms in the state i. Dirac, Slater, and Born have shown that this interpretation, which is in complete accord with transformation theory, gives just the right value for the Einstein

that these objections are not raised against the proper use of the wave theory, but only against the extreme hydrodynamical interpretations (the so-called "field theories"), now largely abandoned, which do not in some way or other embody the statistical outlook involved in the indeterminism principle.

The other extreme is to regard the wave function as only a mathematical tool for calculating the matrix elements, for it is well known that the Heisenberg matrix elements for any function $f(p_1, \cdots, p_f; q_1, \cdots, q_f)$ are given by the expression [10]

$$f(nm) = \int \cdots \int \psi_n^* f\left(\frac{h}{2\pi i}\frac{\partial}{\partial q_1}, \cdots, \frac{h}{2\pi i}\frac{\partial}{\partial q_f}; q_1, \cdots, q_f\right)\psi_m dq_1 \cdots dq_f,$$

where as usual the asterisk denotes the conjugate imaginary and ψ_n means the normalized wave function for the state n. This is usually by all odds the easiest method of calculating the matrix elements. It is my feeling that in most applications of the quantum mechanics to specific problems, the computer will do best to have a firm grasp of both the matrix and differential equation methods. It is true that the infinite chains of difference equations furnished by the pure matrix theory would be almost impossible to solve without the above Schroedinger-Eckart formula, but it is likewise true that the solution of Schroedinger's equation when there is a small perturbing force is greatly facilitated by using the matrix language. It is my feeling that the latter feature is not properly appreciated by some writers who go through complicated manipulations of a differential equation and think they have found something radically new when all they are really

absorption coefficients. Either one must sacrifice this very satisfactory interpretation of the c's, as does Klein (*Zeits. f. Physik*, 41, 420, 1927), or else assume that the radiation is proportional to both the initial and final concentrations, which, as we might surmise, is contrary to experiment (see Gaviola, *Nature*, Nov. 17, 1928). An escape from this dilemma is, of course, that the mechanism of radiation is not as simple as supposed in the too naïve hydrodynamical or field theory.

[10] E. Schroedinger, *Annalen der Physik*, 79, 734 (1926); C. Eckart, *Phys. Rev.*, 28, 711 (1926); Dirac., ref.[8]

doing is just to sum a few matrix elements often already calculated by others.

The wave function, however, is surely entitled to more physical interpretation than as just a mathematical tool. It is perhaps best described by stating that the square of its absolute value is proportional to what may be termed the "statistical" or average charge density. This can be seen from the above matrix formula as follows. Suppose we take f to be a function which is zero everywhere except in a small element of volume $dv = dq_1 \cdots dq_f$ and which has the value unity there. Then with $n = m$ the integral clearly reduces to

$$\psi_n^* \mathrm{I} \psi_n dv = |\psi_n|^2 dv$$

as only in dv is there a non-vanishing integrand. This expression records the fractional amount of time the electron spends in dv just as a stop watch which is set in operation only when one is in a certain room of a building will, when divided by the total elapsed time, record the fractional amount of time which one spends in that particular room. It may seem rather paradoxical that while one can never say when an electron is in a certain position, one can nevertheless evaluate the fraction of time it is in this configuration.

One may elucidate this concept of a statistical charge density by a rather foolish illustration. If one tried to photograph a firefly on a dark night, it might be necessary to take a time exposure rather than a snapshot. If the firefly is active and flutters all around, the photographic plate would then record a nebulous cloud, and from this a person seeing only the photograph might argue that a firefly is a fluid-like substance pervading all space rather than a discrete bug. It seems to me that the same fallacy is committed in the hydrodynamical model of the electron in wave mechanics. It is the essence of Heisenberg's indeterminism principle that all the information which we can get about the motion of an electron in a stationary state is the time exposure, as when we specify the energy all values of the conjugate variable, namely the time, become equally probable. The electron is not a wave pervading all space, but instead it is a discrete entity or "bug" whose probability of location often shows wave-like maxima and minima. Instead of taking the time

average for one atom, it is, of course, equally legitimate to calculate the instantaneous average over an ensemble of atoms all in the same stationary state. The result will be the same in either case, just as a snapshot of a multitudinous swarm of fireflies will resemble the time exposure for the single fly.

Transformation Theory.—A great service of Dirac and Jordan [5] was to interpret the quantum mechanics as a transformation from one coördinate system to another. Space will not permit me to discuss the mathematics of the transformation theory, but I do want to intimate how it includes the Heisenberg and Schroedinger theories as special cases. In a classical transformation from one set of canonical coördinates α, β to another p, q the values of the new coordinates p, q are determined as soon as we know α, β. In quantum mechanics we cannot specify both α and β simultaneously, but we can nevertheless determine the probability that any function $f(p, q)$ will take on a given value when say α is given accurately and all values of β are supposed equally probable. According to Dirac and Jordan each transformation from one coördinate system to another is characterized by a certain "probability amplitude" $\Phi(q, \alpha)$ such that [11] $|\Phi(q, \alpha)|^2 dq$ is the probability that q will fall in the interval dq when α is given and all values of β are equally probable. The Schroedinger wave function is a special case of such a probability amplitude or transformation function, while the Schroedinger wave equation is a special case of the differential equation for determining the probability amplitude. The specialization which yields the Schroedinger theory is the restriction that the α, β coördinate system be the quantum analog of a set of action and angle variables which make the energy a function only of α, i.e., a diagonal matrix. The Schroedinger wave function can thus be regarded as associated with a transformation from positional coördinates to a set of action and angle variables. It involves two sets of arguments, viz., the positional coördinates q_i and the α_i or quantum numbers, as α_i equals $n_i h$ if it is an action variable. Similarly the Heisenberg matrices are special cases

[11] We restrict ourselves to "real" or "Hermitianized" systems, so that $\Phi(\alpha, q)$ is the conjugate imaginary of $\Phi(q, \alpha)$. See ref.[5]

of the matrices used in transformation theory, as the former are indexed ("labeled") with respect to the quantum numbers or action variables, whereas the latter can be indexed with respect to any set of coördinates α. The most common and important transformations are, of course, those built on the energy function, yielding the Schroedinger and Heisenberg theories, just as in classical theory the most important contact transformations are those connected with the Hamilton-Jacobi equation or with action and angle variables. However, the ability to use more general transformations in quantum theory, corresponding to the same degree of generality as the complete group of contact transformations in classical theory, is occasionally useful as well as philosophically satisfying. Space will not permit a discussion of the group property of these transformations, how they can be regarded as rotations in the Hilbert space, or how probabilities are compounded. In the latter we encounter the rather curious "interference of probabilities" whereby probability amplitudes, i.e., square roots of probabilities are superposed rather than probabilities themselves, but which bears some resemblance to the superposition principles in classical electromagnetic theory as discussed in Professor Wiener's paper.

Asymptotic Connection with Classical Theory.—As already stated by Professor Slater in this symposium, quantum mechanics is an extension not of ordinary mechanics but rather of statistical mechanics. This fact comes to light clearly in the nature of the asymptotic connection between the classical and quantum theories, and most of the balance of the present article will be devoted to showing how the quantum expressions for probabilities and averages merge asymptotically for large quantum numbers into classical expressions if, and only if, the classical mechanics is supplemented by statistical assumptions involved in integrating over certain variables, usually phase angles. To my mind this comparison with classical theory is very instructive in acquainting one with the true physical nature of quantum mechanics.

First let us take a very simple case, a particle of mass m moving in one dimension subject to a potential $V(x)$, and confined to a stationary state of energy W. The wave equation is then

$$h^2(d^2\psi/dx^2) + 8\pi^2 m(W - V)\psi = 0. \tag{1}$$

According to quantum theory the expression

$$|\psi|^2 dx \tag{2}$$

measures the probability that a particle in this state be in the interval dx. What is the analogous classical expression? The variable canonically conjugate to the energy is the time, and we must therefore supplement the straight analytical mechanics by the assumption that all values of the time, i.e., all phases or epochs in a system with just one degree of freedom, are equally probable if the energy is given, so that the probability that a particle be in an interval dx is to be taken proportional to the time dt spent in this interval. The places where it moves slowest are therefore those where it is most likely to be. As by the law of conservation of energy $\frac{1}{2}m\dot{x}^2 + V = W$, the probability of being in dx is therefore

$$Cdt = C\frac{dt}{dx}dx = C\left[\frac{m}{2(W - V)}\right]^{1/2} dx, \tag{3}$$

where C is a constant. We must now show that there is an asymptotic agreement between the expressions (2) and (3). To do this we shall adopt the artifice of solving the Schroedinger equation (1) as a power series in Planck's constant h. The convergence of this procedure should be most satisfactory when dealing with large quantum numbers.[12] Let us change the dependent variable in (1) from ψ to S by the substitution

$$\psi = Ae^{2\pi iS/h} \quad \text{with} \quad S = S_0 + hS_1 + h^2S_2 + \cdots, \tag{4}$$

where A is a constant. Then on removal of the exponential factor after differentiation, Eq. (1) becomes [13]

$$\left[\frac{1}{2m}\left(\frac{\partial S_0}{\partial x}\right)^2 + V(x) - W\right] + h\left[\frac{1}{4\pi i}\frac{\partial^2 S_0}{\partial x^2} + \frac{\partial S_0}{\partial x}\frac{\partial S_1}{\partial x}\right] + h^2[\cdots] + \cdots = 0. \tag{5}$$

[12] The convergence of this development is open to grave question, though the series nevertheless seem to give significant results at least for the first two terms. Cf. H. A. Kramers, ref.,[18] also C. Eckart, *Zeits. f. Physik*, 48, 295 (1928).

[13] We do not develop W as a power series in h, as the necessity for this can be avoided without loss of generality by comparing the quantum system with a classical one of exactly the same energy. See ref.[20]

The values of S_0, S_1, S_2 $\cdots$ are determined by equating to zero the terms in (5) which are respectively constant and proportional to h, h^2, $\cdots$, etc. The differential equations so obtained for S_0 and S_1 are immediately integrable, yielding

$$S_0 = \int [2m(W - V)]^{1/2} dx;$$
$$4\pi i S_1 = -\log(\partial S_0/\partial x) = -\log [2m(W - V)]^{1/2}. \quad (6)$$

One immediately recognizes S_0 as the classical action function, and the differential equation by which it is determined, viz., the part of (5) which does not disappear in the limit $h = 0$, is the classical Hamilton-Jacobi equation. Hence the logarithm of the Schroedinger wave function is by (4) roughly proportional to the classical action function. The requirement that the wave function be single valued means that the exponent in (4) must increase by an integral multiple n of $2\pi i$ when the radical in the integrand of (6) traverses a complete cycle of values. In the first approximation, where $S = S_0$, this means that

$$\oint [2m(W - V)]^{1/2} dx = \oint p dx = nh,$$

which is nothing other than the Sommerfeld-Wilson quantum condition. Hence we see why the old Bohr theory was a good approximation in many cases.

This derivation of the Sommerfeld-Wilson quantum conditions as a first approximation by means of the substitution (4) is due to Wentzel and Brillouin.[14] To obtain the asymptotic identity of the classical and quantum probabilities it is, however, necessary to go a step further to the second approximation S_1. We must now distinguish between two regions I and II of values of x:

I. One region is that where the radical $[W - V]^{1/2}$ is real. This is the region in which the potential energy is less than the total energy, and is therefore the interval inclosed between the classical libration or oscillation limits. Here S_0 is real and the absolute value of the factor $e^{2\pi i S_0/h}$ is unity. Hence by (4) and (6)

$$|\psi|^2 dx = A^2 |e^{2\pi i S_1}|^2 dx = [A^4/2m(W - V)]^{1/2} dx. \quad (7)$$

[14] G. Wentzel, *Zeits. f. Physik*, 38, 518 (1926); L. Brillouin, *Jour. de Physique*, 7, 353 (1926).

This is the same as (3), thus giving the desired asymptotic agreement between the classical and quantum results. The constants of proportionality $Cm^{1/2}$ and $A^2/m^{1/2}$ in (3) and (7) respectively, which would otherwise be arbitrary, become equal when account is taken of the normalization, both classical and quantum, that the total integrated probability $\int_0^{\tau/2} Cdt$ or $\int_{-\infty}^{\infty} |\psi|^2 dx$ must equal unity.[15]

II. Let us now turn to the other region in which the radical and S_0 are imaginary, and which is exterior to the classical region of motion as it would demand more potential energy than the total energy W. Here the exponent in $e^{2\pi i S_0/h}$ is a positive or negative real number which increases monotonely in absolute value as we go further and further away from the classical libration limits. If the exponent is positive the wave function increases without limit and is not a characteristic function ("Eigenfunktion") of the differential equation. If it is negative the wave function will by (4) drop off in a rapid exponential fashion beyond the classical libration limits, and so the statistical density can be considered as virtually zero outside these limits. In general only certain particular values of the energy constant W may give this rapid diminution instead of unlimited increase beyond the classical limits (unless at the expense of continuity at the classical libration limits or of finiteness at some singular point) and these values are the "characteristic values" to be used in the Bohr frequency condition. The sharpness of dropping off beyond the classical limits is greater the larger the quantum number or the smaller h (which for mathematical purposes need not be restricted to 6.55×10^{-27}), and becomes infinitely abrupt in the limit $h = 0$ which should correspond

[15] We take the classical normalizing integral over a half rather than complete period τ in order to have the particle pass through dx only once. We could, of course, take the normalizing integral over a complete period, but then the value of C would be halved and twice the expression (3) would give the probability of being in dx, as the particle might be there either in moving from left to right or from right to left, supposing the system oscillatory. When the variable of integration is changed to x the classical integral is between libration limits whereas the quantum one is from $-\infty$ to $+\infty$, but this distinction is immaterial because of the rapid exponential drop of the quantum probability beyond the classical libration limits if the quantum numbers are large.

to classical theory. The presence of a finite, though rapidly diminishing density beyond the classical libration limits when $h \neq 0$ gives rise to many interesting phenomena, as it shows that there is a probability, though small, of a particle passing beyond the classical libration limits and even surmounting a peak of potential energy greater than the total energy of the particle. This effect underlies the interesting quantum theory of nuclear disintegration given by Gurney and Condon and also simultaneously by Gamow,[16] and also the theory developed by Oppenheimer, Fowler and Nordheim [17] of the pulling of electrons out of cold metals by electric fields. This quantum probability of passage beyond the classical limits may be likened to the diffraction of light beyond the geometric shadow of a straight edge—wave mechanics and the undulatory theory of light respectively enable us to understand these two phenomena which would be totally incomprehensible with classical dynamics or geometrical optics.

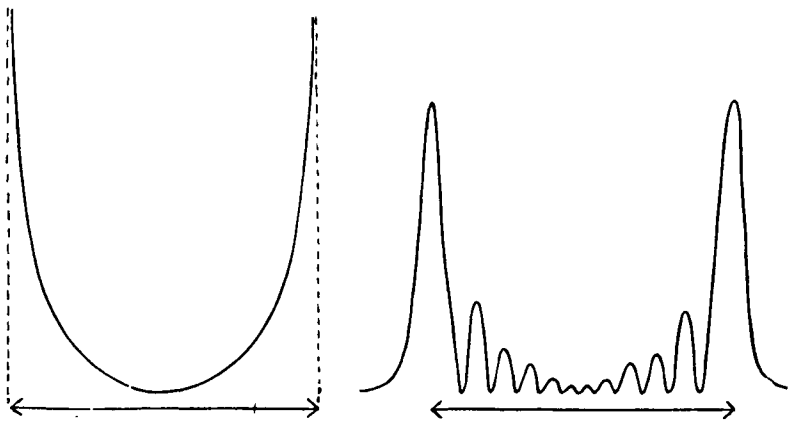
One point which must be mentioned is that we have taken only the solution corresponding to $+ i$. There is also clearly another solution obtained by reversing the sign of i, and the most general solution is, of course, a linear combination of these two solutions. The proper linear combination is determined by requirements of continuity, etc., at the classical libration limits, and has been shown by Kramers [18] to involve coefficients of equal absolute value for the solutions corresponding to $+ i$ and $- i$. This means that superposed on $Ae^{2\pi i S_1} = [A^2/2m(W - V)]^{1/4}$ we no longer have a factor $e^{2\pi i S_0/h}$ which can be disregarded between the classical limits because of its unit absolute value, and instead we have a periodic factor $\cos(2\pi S_0/h)$, whose square should be multiplied into the right side of (7). The quantum probability function then shows wave-like maxima and minima within the classical libration limits. It is the envelope of these waves, which is obtained by disregarding the cosine factor, which corresponds

[16] R. W. Gurney and E. U. Condon, *Nature*, Sept. 22, 1928; G. Gamow, *Zeits. f. Physik*, **51**, 204 (1928).

[17] J. R. Oppenheimer, *Proc. Nat. Acad. Sci.*, **14**, 363 (1928); Fowler and Nordheim, *Proc. Roy. Soc.*, **119**, 173 (1928); also Richardson, *ibid.*, **117**, 719 (1928).

[18] H. A. Kramers, *Zeits. f. Physik*, **39**, 829 (1928).

asymptotically with classical theory, but this modification need not worry us, as the successive maxima come closer and closer together as the quantum numbers become large or h small, making the irregularities due to the wave-like structure less and less important just as the distinction between physical and geometrical optics usually becomes less important with shorter wave-lengths. A graphical comparison of the classical and quantum distribution functions, in which the probabilities are taken as ordinates and the positions x as abscissa, would be somewhat as follows:



CLASSICAL. QUANTUM.

The classical libration limits are indicated by arrows. The classical probability becomes infinite, though integrable, at these limits as the particle moves infinitely slowly here. The quantum probability will not become infinite but will be large in this vicinity if the quantum numbers are large. The quantum curve is drawn for a state of quantum number 11, and is somewhat idealized as the agreement with classical theory might not be as perfect as indicated with so low a quantum number, since the terms $S_2, \cdots$ in the expansion (4) might have a perturbing effect. That the quantum number is 11 can be read directly from the figure because of the mathematical theorem [19] that the number of nodes of the wave function is the quantum number, i.e., index number of

[19] Courant-Hilbert, "Methoden der Mathematischen Physik," pp. 363–367.

the characteristic value with lowest value numbered zero. Without giving the rigorous proof, this is plausible in view of the fact that there was a factor $\cos [2\pi \int p dx/h]$ in our approximate expression for ψ.

This proof of the asymptotic identity given for a one-dimensional conservative system can be extended to systems which have any number of degrees of freedom, and which are non-conservative, so that W is replaced by the operator $h\partial \cdots /2\pi i \partial t$. The results are summarized in the following table, in which it is supposed for concreteness that the system is conservative and that the transformation is from a set of positional coördinates and momenta q, p to a set of action and angle variables J, w, making the Jordan-Dirac transformation theory reduce essentially to the Heisenberg and Schroedinger procedures.

CLASSICAL TRANSFORMATIONS.	QUANTUM ANALOG.
Given a contact transformation connecting two sets p, q and J, w of canonical variables. Then $p_k = \partial S/\partial q_k$, $w_k = \partial S/\partial J_k$ with	Let $\psi = \psi(q_1 \cdots q_f;\ J_1 \cdots J_k)$ be the wave function with $J_k = n_k h$ $(k = 1 \cdots f)$
$S = S(q;\ J)$	$\longleftrightarrow (h/2\pi i) \log \psi + \text{const.}$
If the energy W is a function only of the J's,	The wave equation for ψ is
$H(\partial S/\partial q;\ q) = W(J)$	$\longleftrightarrow H(h\partial \cdots /2\pi i \partial q;\ q)\psi = W(J)\psi$

(Asymptotic connection since

$$\frac{h}{2\pi i}\frac{\partial}{\partial q}[f(q)e^{2\pi i S/h}] = \frac{\partial S}{\partial q} f(q) e^{2\pi i S/h}[1 + h(\cdots) + \cdots])$$

Assume the J's given and all values of the w's equally probable. Then the probability that the system be in any given configuration is proportional to the "lapse in phase"	Probability that the system be in the configuration $dq_1 \cdots dq_f$ is
$dw_1 \cdots dw_f = \Delta dq_1 \cdots dq_f$	$\longleftrightarrow \lvert\psi\rvert^2 dq_1 \cdots dq_f$

(Asymptotic connection since [20] $\psi = \Delta^{1/2} e^{2\pi i S/h}[1 + h(\cdots) + \cdots]$)

Average value of any function $f(p(J, w);\ q(J, w))$ is	Corresponding average is a diagonal matrix element
$\overline{f(p, q)} = A \int \cdots \int f(p;\ q) dw_1 \cdots dw_f$	
$= A \int \cdots \int f(\partial S/\partial q;\ q) \Delta dq$	$\longleftrightarrow f(JJ) = \int \cdots \int \psi^* f(h\partial \cdots /2\pi i \partial q;\ q)\psi dq$

In this table expressions which correspond asymptotically are connected by arrows, and Δ is used as an abbreviation

[20] J. H. Van Vleck, *Proc. Nat. Acad. Sci.*, 14, 178 (1928).

for the functional determinant

$$\Delta = \frac{\partial(w_1, \cdots, w_f)}{\partial(q_1, \cdots, q_f)} = \left| \frac{\partial^2 S}{\partial q_i \partial J_k} \right|$$

also (p, q) and dq are abbreviations for $(p_1, \cdots, p_f; q_1, \cdots, q_f)$ and $dq_1 \cdots dq_f$, while S is similar to our previous S_0. For proof that the wave function does contain asymptotically the factor $\Delta^{1/2}$ as stated in the table the reader is referred to another paper in which I show that in the general case the first term in the series development (4) is the classical action function while $e^{2\pi i S_1}$ is proportional to $\Delta^{1/2}$. (In the simple one-dimensional example previously considered Δ was proportional to the radical $(W - V)^{-1/2}$.) It is to be particularly noted that it is necessary to include the second approximation S_1 in order to obtain the factor involving the functional determinant and thus establish the asymptotic identity of probabilities. This second term, unlike the still higher terms $S_2 \cdots$, does not give an effect which disappears in the limit $h = 0$ as the h contained in hS_1 just cancels the h in the denominator of the exponent in (4).

The last line of the above table deals with averages over all configurations rather than with the probability of being in one particular configuration. Most of our previous discussion of the asymptotic identity, especially in the one-dimensional example, has been primarily relative to the probabilities rather than averages, but the asymptotic connection of the classical statistical averages with the diagonal matrix elements is shown in virtually the same fashion, the proof hinging on the fact that the operator $h\partial \cdots / 2\pi i \partial q$ applied to $f(q)e^{2\pi i S/h}$ has the same effect in the limit $h = 0$ as multiplication by $\partial S/\partial q$, the classical value of p. In order to bring out more clearly the role of coördinates in the transformation theory the indices of the matrix in the last line of the table are taken as the action variables J rather than the quantum numbers n from which they differ by a factor h.

Although the above table is ostensibly for a set J, w of action and angle variables it applies equally well with any set α, β of canonical coördinates in place of J, w except that ψ becomes a more general "probability amplitude" or "transformation function" in place of the Schroedinger one,

and the differential equations for determining S and ψ are no longer necessarily the Hamilton-Jacobi and wave equations connected with the energy. It can be shown that the passage from the instantaneous values of variables specifying a dynamical system at a given time to their values at subsequent times can be regarded as a contact transformation. We have said that contact transformations have an analog in quantum mechanics but are associated with distribution or probability functions rather than with precise correlations. This means that the future of a particle is only statistically determined. The reason for the ambiguity is not necessarily to be imputed to breakdown of the "causality principle" but rather to the fact that the initial conditions are inevitably indeterminate because of Heisenberg's principle. This is indicated in the following diagram:

CLASSICAL DETERMINISM.	QUANTUM INDETERMINISM.
Given initially $x, y, z; v_x, v_y, v_z$.	Given v_x, v_y, v_z accurately; then x, y, z are undetermined (i.e., all values are equally probable for x, y, z); or vice versa x, y, z given and v_x, v_y, v_z undetermined.
↓ (Arrow indicates calculation by classical dynamics)	↓ (Arrow indicates calculation by quantum dynamics)
Accurate determination of x, y, z, v_x, v_y, v_z at all subsequent times.	Only statistical determination of x, y, z, v_x, v_y, v_z at subsequent times (i.e., probability of different values predicted).

These considerations may be illustrated by the scattering of alpha particles. Classically an alpha particle of given velocity is deflected through a certain angle whose magnitude depends on the closest distance of approach to the scattering center. On the other hand in quantum mechanics if the alpha particles have a definite vector velocity their positions are indeterminate by Heisenberg's principle, so that we cannot say how close a given alpha particle comes to the scattering center. However, the quantum mechanics does furnish directly formulas for the "distribution in angle" of the scattered alpha particles, which is what is observed experimentally; whereas the classical theory furnishes such formulas only with the intermediary of the non-observable distance of closest approach and with the supplementary statistical as-

sumption that all distances of approach are equally probable. Both the classical and quantum formulas for this distribution are virtually the same, luckily for the classical theory, which means that it is a tolerably good approximation to stop with the second term S_1 in the series (4).[21] On the other hand the inadequacy of classical theory in the Davisson Germer effect is due to the fact that free electrons, unlike alpha particles, have such long wave-lengths h/mv that quantum modifications and diffraction effects become important.

Difficulties of Existing Theory.—In closing I shall mention briefly what seem to be some of the outstanding difficulties and problems of quantum mechanics.

1. The first of these, probably not the most fundamental, is the choice of coördinates in the wave equation or Hamiltonian function. In the case of the hydrogen atom, for instance, one obtains the correct equation by substituting $h\partial \cdots/2\pi i\partial q$ for each momentum p in the classical Hamiltonian if one uses Cartesian coördinates but not if one uses polar.[22] Instead if one desires to use polar coördinates one must first set up the differential equation in Cartesian and then transform to polar. We have as yet no adequate explanation of why the extrapolation from classical coördinates to operators should be successful with Cartesian coordinates, or, more broadly, no adequate theory of the choice of Hamiltonian functions.

2. A second difficulty is that of the structure of the nucleus.[23] Certain nuclei, for instance, nitrogen, are known from atomic weights to contain probably an odd number of electrons, which according to ordinary spectroscopic theory

[21] Cf. J. R. Oppenheimer, *Phys. Rev.*, **31**, 72 (1928).

[22] If one uses polar coördinates, p_r^2 must be replaced by

$$(h/2\pi i)^2(d^2\cdots/dr^2 + 2r^{-1}d\cdots/dr)$$

with the ordinary choice of weight function in the definition of orthogonality. In an interesting paper (*Phys. Rev.*, **32**, 812, 1928) Podolsky points out that the reason a correct extrapolation to quantum mechanics cannot be obtained by a promiscuous substitution of momenta by operators is the ambiguity due to the non-commutativeness of quantum multiplications, but this, of course, does not explain the outstanding role of Cartesian coördinates.

[23] R. de L. Kronig, *Naturwissenschaften*, **16**, 335 (1928); Ornstein and van Wijk, *Zeits f. Physik*, **49**, 315 (1928).

would imply a paramagnetic behavior, whereas actually the nitrogen molecule is diamagnetic.[24] A somewhat related difficulty also pointed out by Kronig [23] is that Ornstein and van Wijk's measurements on alternating intensities in the N_2 band spectrum give a different intensity ratio between succeeding lines than would be predicted by the ordinary theory of statistical weights and number of particles in the nitrogen nucleus.

3. A third difficulty is with the relativity corrections, which are inextricably tied up with the subject of electron spin. Dirac [25] has very brilliantly devised a system of simultaneous first order linear equations which are tantamount to including both electron spin and relativity and which yield the desired Sommerfeld fine-structure formula to all degrees of approximation. Dirac's theory, however, has four wave functions per electron, which appears to be twice too many. Pauli [26] was able to interpret very nicely two wave functions per electron as the two values of a single function one of whose arguments could assume only two discrete values, one corresponding to an axial component $+\frac{1}{2}(h/2\pi)$ of spin angular momentum, and the other to $-\frac{1}{2}(h/2\pi)$. There is apparently no room for four wave functions per electron unless one interprets two of these as corresponding to a negative mass or positive charge. If the four wave functions were separable into two pairs which did not "combine" with each other, the difficulty would not be so perplexing, but actually this is not the case, so that there is a finite probability of the mass or charge of the electron changing sign, a most disquieting state of affairs!

4. Another, and perhaps related difficulty is that of spontaneous radiation. The only theory of this is Dirac's, which treats the ether as an ensemble of oscillators in a rather formal fashion suggestive of Jean's method of handling black body radiations. Undoubtedly Dirac's [27] procedure

[24] In this connection it is significant to note that Breit finds that Dirac's theory predicts an abnormally low magnetic moment at velocities comparable with that of light, such as might exist inside the nucleus. (*Nature*, **122**, 649, 1928.)

[25] P. A. M. Dirac, *Proc. Roy. Soc.*, **117A**, 610; **118A**, 351 (1928).

[26] W. Pauli, Jr., *Zeits. f. Physik*, **43**, 601 (1927).

[27] P. A. M. Dirac, *Proc. Roy. Soc.*, **114A**, 243, 710 (1927).

gives the correct method of attacking the problem from one angle, but presumably it should also be possible to give a treatment more nearly paralleling the classical theory of radiation associated with retarded potentials. Inherently connected with this is the question of the meaning of electromagnetic mass in quantum mechanics, as in classical theory the force of inertia due to electromagnetic mass and the force on an electron due to its own radiation are inextricably connected, being two succeeding terms in the same series development for the interaction between different parts of the electron.[28] Until we are able to transcribe into quantum language the classical analysis which made the mass of the electron inversely proportional to its radius it can scarcely be said that we have really solved the radiation problem in quantum mechanics. It is not impossible that an answer to this and especially to the preceding question of the physical interpretation of the Dirac relativity theory of the electron will not be obtained until we have a theory that explains the time-honored puzzle of "why is an electron," that is why is $e = 4.774 \times 10^{-10}$ and particularly why does a proton have a mass about 2,000 times greater than that of the electron.

5. A final difficulty is the cause underlying Pauli's exclusion principle. Why does nature have such a peculiar preference for antisymmetric wave functions?

I hope that mention of these five difficulties has not inculcated too much pessimism. The transformation theory and its statistical interpretation of quantum mechanics are, in my opinion, very perfect in their sphere of application, and most of the difficulties relate to phenomena beyond their present scope. The degree of perfection of the present quantum mechanics may perhaps be likened to that of classical analytical mechanics before electromagnetic theory and relativity.

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[28] For slightly fuller elucidation of this statement and references to Lorentz's calculation, see footnote 5 of my "Quantum Principles and Line Spectra."

USES AND DEFECTS OF THE WAVE EQUATION.

BY

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THE masterly survey of the various formulations of quantum mechanics which Professor Van Vleck has given us seems to suggest that, if physicists in general are to understand the theory, some simplification in its formulation will be needed. After working with both transformation and wave theory, I have become convinced personally that much can be said in favor of adopting the de Broglie-Schrödinger-Heisenberg wave-theory as the fundamental formulation and regarding the matrix and transformation theories as special mathematical methods to be learned and used only by specialists in the mathematical theory. Apparently all results that possess physical significance can be obtained, and in most cases about as easily, in terms of the wave-theory.

A formulation in which the wave-theory is made fundamental has an advantage of formal character in that a much narrower fundamental postulate is required as to the physical interpretation. Consider, for instance, a particle in a conservative force-field, for which the wave equation can be written in either of two forms applicable to any conservative system:

$$\frac{h}{2\pi i}\frac{\partial \psi}{\partial t} = H\left(-\frac{h}{2\pi i}\frac{\partial}{\partial q}, q\right)\psi, \tag{1}$$

or

$$\frac{h}{2\pi i}\frac{\partial \psi}{\partial t} + H\left(\frac{h}{2\pi i}\frac{\partial}{\partial q}, q\right)\psi = 0,$$

the values of ψ in the two cases being complex-conjugates of each other. The fundamental postulate as to physical interpretation consists in the statement that $\psi\psi^* dq$ is the probability that an exact observation of the position of the particle (for instance with cosmic-ray microscopes) would reveal it as having values of the coördinates within dq. The

fundamental rôle thus assigned to position fits in well with the fact that "position at a certain time" (the "event" of Relativity) is nearly always the thing actually observed in practice.

To obtain the probability for any other variable we then merely analyse a method of measuring that variable. Examples of such a procedure are to be found in papers by Darwin (e.g. *Roy. Soc. Proc.*, 117, 258, 1927) and Kennard (*Phys. Rev.*, 31, 876, 1928); Born's treatment of collision problems can be regarded as another. Let us consider, for instance, the treatment from this point of view of momentum. Defining it as $p = mv$, we may turn to the astronomer's method of measuring the velocity v. At any initial time, t, suppose that the probability amplitude ψ for a particle is sensibly zero outside of a certain region of diameter D; let us allow the particle to run until a time t' and then let us observe its position with precision. The distance traversed divided by the interval, $t' - t$, can then be taken as an observed value of v; the indefiniteness in the initial position introduces an uncertainty only of order $D/(t' - t)$, which can be reduced ad libitum by increasing t'. The result of such an observation is, of course, known only statistically at the instant t; the detailed analysis shows that we can take as the probability amplitude for the momentum of a free particle, as measured in this way, the usual expression of which the one-dimensional form is

$$M(p) = \frac{1}{\sqrt{h}} \int \psi(q) e^{(2\pi i/h)pq} dq.$$

If the particle is not free, difficulties arise. Perhaps we can, with Heisenberg and Bohr, imagine the forces to be suddenly removed for the purpose of an experimental observation; if this cannot be done, the correct conclusion would seem to be that momentum cannot in such cases be made the object of precise physical observation and so does not possess direct physical significance.

A similar analysis of an apparently plausible method of measuring the energy of a molecule leads to the result that the wave packet becomes spread out, in the space of the

center of mass of the molecule, into an "energy spectrum"; if the molecule is then brought to light experimentally at a certain point in this spectrum, its displacement from the zero position constitutes a mechanically observed value of the energy. The part of the original packet that comes to the point in the spectrum corresponding to the energy W_n of the nth quantum state contains the internal coördinates in the familiar harmonic standing-wave form, viz.:

$$\psi = \psi_1(x_c, y_c, z_c)e^{(2\pi i/h)W_n t}u_n(q),$$
$$H\left(-\frac{h}{2\pi i}\frac{\partial}{\partial q}, q\right)u_n(q) = W_n u_n(q),$$

x_c, y_c, z_c being the coördinates of the center of mass. We have thus a mechanical interpretation of W_n as a physically observable value of the energy.

The second point to which I wish to call attention is the peculiar nature of the imperfection which still exists in the wave-theory, just as it does in all other forms; for no form of quantum mechanics can yet deal quite satisfactorily with spontaneous transitions. Presumably the difficulty will disappear when the quantum theory of the electromagnetic field shall have been elaborated, and we shall then find that the completed wave-equation contains an additional term.

The new term must, however, be of a rather remarkable sort. Suppose, for instance, that we isolate an atom in a metastable state, in which it remains for a considerable interval until an instant t_j, when it jumps spontaneously into the normal state. By watching for the emitted photon we can actually determine experimentally the time at which the jump occurs. We know, therefore, that for quite a while ψ had the form, $\psi = \psi_1(q, t)$, ψ_1 being that solution of (1) which represents the metastable state; then, at time t_j, ψ suddenly ceases to obey this differential equation and changes quickly (perhaps instantaneously) into $\psi_2(q, t)$, the solution of (1) that is proper to the normal state. This statement about the change in ψ has physical meaning, for the values of $\psi_1\psi_1^*$ and $\psi_2\psi_2^*$ might be determined statistically by making many repetitions of the necessary observations; in each repetition we should make one determination of the

values of the q's, and in each case we should know whether we were testing ψ_1 or ψ_2 because we should know whether the photon had already been emitted or not. Obviously something is missing from the wave-equation as stated in (1); and of course the missing thing is simply the analogue of the radiation damping in classical theory. The forms so far proposed for this term produce, however, a steady change in ψ, beginning immediately; this is satisfactory for an ensemble of atoms, but does not cover the case of one atom as here described; nor is the latter case covered by the emission theory of Dirac. It would seem as if the missing term must be one whose values can be assigned only statistically; it must be a term which is usually zero but which has the power to assume temporarily very large values and so to cause a rapid change in ψ, at a time which is determined only by a probability law. It would seem that an element of probability must thus enter into the structure of the completed wave-equation itself.

STATISTICAL INTERPRETATION OF VARIOUS FORMULATIONS OF QUANTUM MECHANICS.

DISCUSSION BY

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AT the conclusion of his valuable paper Professor Van Vleck has concisely pointed out some of the outstanding difficulties in the present formulation of quantum mechanics.

To his first comment we would like to add that the main question does not seem to be so much the choice of the coördinates in the Hamiltonian as the "symmetrization" difficulty of Schrödinger. Cartesian coördinates enjoy a privileged position, just as in elementary geometry, because then the square of the distance between two points is a quadratic expression in diagonal form involving only the sum of squares. One can always find the wave equation in arbitrary coördinates by making use of the variational principle of Schrödinger. The ultimate reason for the symmetrization difficulty remains however unknown. (Cf. Schrödinger, *Annalen der Physik*, 79, p. 747, 1926.)

The problems raised by Dirac's theory of the electron spin are many and varied. It is well known that this theory starts out from the postulates, (*a*) that the wave equation must satisfy simultaneously the requirements of restricted relativity (invariance with respect to the Lorentz transformation) and (*b*) of the transformation theory (the wave equation should involve first order derivative with respect to time, i.e., the solutions of the wave equation form a normal orthogonal set). In addition Dirac requires (*c*) that for a vanishing field his iterated wave operator reduce to Schrödinger's relativistic wave equation. Thus the electronic spin in his theory appears as a consequence of relativity, or, alternately, as a consequence of combining relativity with the transformation theory; and it can be readily shown that the spin disappears in the limit for $c \to \infty$; more precisely the magnetic axis of the electron becomes unobservable.

Pauli has shown (*Zeits. für Phys.*, 43, p. 601, 1927) that, because of the fact that in a field only two orientations of the electron axis are possible, parallel and antiparallel to the field, two and no more than two functions are required to describe the spin. The fact that Dirac's equations involve twice as many functions is due to their undesirable characteristic that they apply equally well to an electron with a negative as with a positive charge—if there were such electrons. Further, owing to the fact that in general these four functions coexist, the probability for a change of sign of the electronic charge is finite and of the order of magnitude $(v/c)^4$ (v = velocity of the electron, c = velocity of light). In the case of the motion of an electron in a central field these four functions coalesce into two, which however refer just as well to the positive as to the negative electron. Under the circumstances it is rather unexpected to find that this theory is able to give Sommerfeld's fine structure formula exactly.

Darwin (*Proc. Royal Soc.*, 118, p. 658, 1928) has pointed out that, because of the lack of an adequate formulation of quantum electrodynamics, there is no quantum analogue to the classical calculation of electromagnetic mass. In fact the electronic mass appears in Dirac's theory as a consequence of his third requirement (*c*) (see above). Possibly this is connected with the fundamental question raised by Professor Van Vleck on the difference between the mass of the electron and that of the proton; certainly the absence of quantum electrodynamics has to do with the puzzles of spontaneous radiation. However, it can be readily shown that in the limiting case of vanishing electronic mass, and taking one of Dirac's functions constant, his equations go over, by applying a suitable canonical transformation, into Maxwell's equations in free space, just the same as Schrödinger's relativistic equation goes over into the light equation. In the same limit, and applying the same unitary (canonical) transformation, it can be shown that Dirac's electric density and electric current become the energy and the Poynting vector of Maxwell's theory. The statistical interpretation of these quantities, in agreement with that given to the corresponding ones in Dirac's theory, is then the probability that a light

quantum be in a certain region at a given time, and the probability that it cross a given boundary in a given direction, respectively. Further the change in the probability of its being within a region is equal to the probability of its having crossed the boundary of the region. It will be noted that while the energy and the Poynting vector satisfy the same continuity equation as the electric density and current, the former transform under the Lorentz group, not like a four-vector, but like the space-time components of the symmetrical energy-impulse tensor. While the algebra of the matrix operators involved in Maxwell's equations is that of quaternions, the Dirac operators involve an algebra connected with sedenions. (Cf. Felix Klein's "Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert," Vol. II, pp. 82, 85.)

The present transformation theory of quantum mechanics is based on the existence of a normal orthogonal set of (complex) functions and its related unitary transformations studied by Professor Weyl, and is also closely connected with Professor Wiener's coherency-matrix theory. Some of the conclusions arrived at by Professor Van Vleck in his paper find their adequate mathematical expression in the theory of coherency matrices, for example, the lack of *direct* physical reality of the non-diagonal elements of the matrices. There are great difficulties in formulating the transformation theory, which after all is molded on classical theory, in general relativity, for the transformations dealt with in the latter are not unitary transformations. Consequently the theory of orthogonal functions loses its privileged position. A generalization of Einstein's equivalence principle, or, mathematically, a generalization to function space of the theory of parallel displacement might remove this difficulty. A suggestion in this direction is contained in Einstein's paper on parallelism (*Berliner Sitzungsber.*, 1928, p. 217) where he introduces a system of four orthogonal axes h_a^ν at every point of a general four-dimensional manifold X_4 and obtains the gravitational potentials $g^{\nu\mu}$ (ν, μ = 1, 2, 3, 4) by summing on a (a = 1, 2, 3, 4). If to each point of the X_4 corresponds, not the tangential four-dimensional neighborhood, but a function space, we can obtain the $g^{\nu\mu}$ in an analogous way by taking four

complex functions $h^{\nu}(a)$ of a single variable in this space and integrating

$$g_{\nu\mu} = \int h_{\nu}(a)h_{\mu}(a)da.$$

This idea, which came to light during a conversation of the present authors with Professor Wiener, can also be considered as a generalization of certain considerations of Cartan ("Sur les variétés à connection projective," *Bull. de la Soc. Math. de France*, 52, p. 205, 1924). The $g_{\nu\mu}$ in this case are, however, positive, definite, and only a modification of the law of their formation can have a physical meaning. The application of Wiener's coherency matrices to the present case can be carried through without difficulty.

One has to distinguish between two kinds of statistical mechanics: that of an assemblage of point systems, in Gibbs' sense, and that of waves. Professor Van Vleck has pointed out one way of going over from one theory to the other. In either case one has to do with the study of certain averages, that is, with an integration problem over a certain range. One of the main problems here is the discovery of integral invariants, for example the $|\psi|^2$ of wave mechanics, or the number of particles in a given volume in classical statistical mechanics. Both are positive integral invariants.

THE PENDULUM PROBLEM IN THE WAVE MECHANICS.

BY

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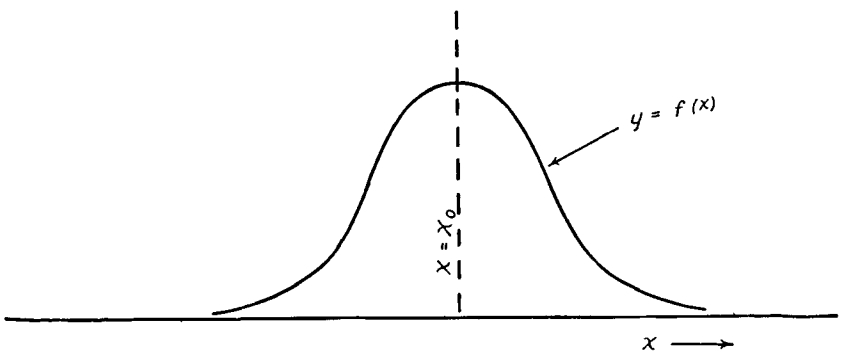
MY remarks have to do with an application and extension of the discussion of the one-dimensional oscillator problem given by Professor Van Vleck. They contain nothing particularly new but take up a simple illustrative problem which may be of interest.

Let us assume that we have a vibrating particle such as a pendulum with a known mass and known potential energy $V(x)$. We wish to predict the future position of the particle from an initial observation of position and corresponding

FIG. 1.



velocity. Assuming that the dimensions of the "particle" are large compared with the wave-length of light we may obtain the desired simultaneous initial observations from a photograph on a moving plate which will yield a graph of position as a function of time in the neighborhood of the time $t = 0$ at which the particle crossed the fiducial mark $x = x_0$. The slope of the trace at $x = x_0$ will give the initial speed and momentum of the particle. From the sharpness of the photograph, the uncertainty in the speed of the plate and other relevant information we may estimate

the probable errors x_1 and p_1 in the measured values of the initial position and momentum. We may even draw an error curve $y = f(x)$ with a maximum at $x = x_0$ to represent the probabilities of various actual initial positions or to represent the statistical distribution of initial positions in the case of a large number of experiments. With the drawing of a similar error curve for the momentum our information is complete.

We wish to determine the error curve for position at some future time and we do so with the aid of a function $\psi(x, t)$ called a probability amplitude. ψ is a solution of the Schrödinger differential equation and if properly adjusted to the initial conditions it has the property that $\psi\bar{\psi}dx$ is the probability that the particle has the coördinate x at the time t. ($\bar{\psi}$ is the complex conjugate to ψ.) Clearly $\psi(x, t_0)\bar{\psi}(x, t_0)$ must be identified with $f(x)$ to satisfy initial conditions.

But this one condition is insufficient to fix the form of the complex function $\psi(x, t_0)$. The momentum of the particle is linked with the wave-length λ of the ψ function in accordance with the famous formula $\lambda = h/p$. If we analyse ψ into a spectrum of monochromatic waves with the aid of the Fourier integral

$$\psi(x, t) = \int_{-\infty}^{+\infty} \varphi(\lambda, t) e^{2\pi i x/\lambda} d\left(\frac{1}{\lambda}\right) \tag{1}$$

$$\varphi(\lambda, t) = \int_{-\infty}^{+\infty} \psi(x, t) e^{-2\pi i x/\lambda} dx \tag{2}$$

the amplitude $\varphi(\lambda, t)$ plays the same rôle in determining the probable momentum as $\psi(x, t)$ plays in determining the probable position. If the momentum of the particle is sharply defined $\psi(x, t)$ will be approximately monochromatic and $\varphi(\lambda, t)$ will vanish except in a very narrow range of wave-lengths.

No general technique has been developed for fitting $\psi(x, t)$ to the initial experimental data for the momentum and hence we content ourselves with discussing the ideal case where the accuracy of the determination of the momentum is the maximum consistent with the wave picture or with the Heisenberg principle of uncertainty. We then make $\psi(x, 0)$

as nearly monochromatic as possible by giving it the value

$$\psi(x, 0) = \sqrt{f(x)}e^{-2\pi i x/\lambda_0} = \sqrt{f(x)}e^{-2\pi i p_0 x/h}. \tag{3}$$

In order to follow the variation of $\psi(x, t)$ in time it is convenient to resolve it into a series of periodic solutions or characteristic functions.

$$\psi(x, t) = \sum_n c_n u_n(x) e^{2\pi i(W_n t/h + \epsilon_n)}. \tag{4}$$

Here the u's are the wave-like functions of x drawn by Professor Van Vleck and representable by the Kramers approximation formula

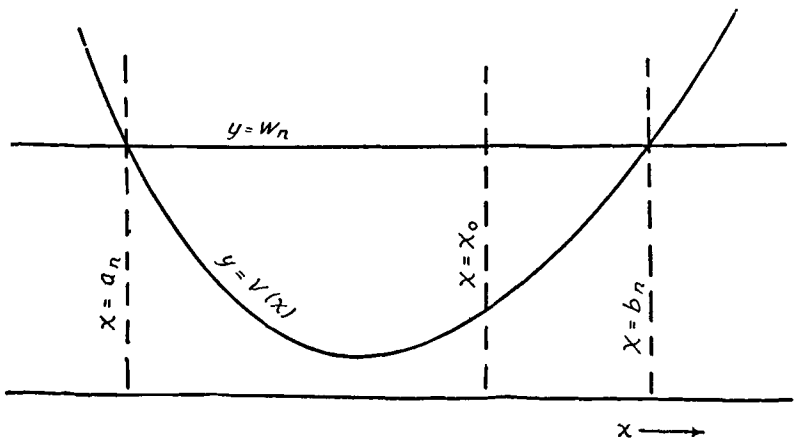
$$u_n(x) = A_n \sqrt{\lambda_n} \cos 2\pi \left[\int_{a_n}^{x} \frac{d\xi}{\lambda_n(\xi)} - \frac{1}{8} \right], \tag{5}$$

where

$$\lambda_n(x) = \frac{h}{|p_n(x)|} = \frac{h}{\sqrt{2m[W_n - V(x)]}}.$$

In (5) the action function S_0 of Professor Van Vleck's notation is replaced by its equivalent $\int_{a_n}^{x} h/\lambda_n d\xi$, where a_n is the lower of the two classical libration limits for the energy W_n. (Cf. Figure 2.)

FIG. 2.



Introducing the approximation function (5) into (4) and replacing the cosine by the sum of two exponentials we resolve ψ into two parts each representing a set of complex progressive waves, one moving to the right, the other to the left. Thus

$$\psi(x, t) = R(x, t) + L(x, t), \tag{6}$$

where

$$\left.\begin{aligned} R(x, t) &= \sum_n \frac{c_n A_n}{2} \sqrt{\lambda_n} e^{2\pi i \left\{ \frac{W_n}{h} t - \int_{a_n}^{x} \frac{d\xi}{\lambda_n} + \frac{1}{8} + \epsilon_n \right\}}, \\ L(x, t) &= \sum_n \frac{c_n A_n}{2} \sqrt{\lambda_n} e^{2\pi i \left\{ \frac{W_n}{h} t + \int_{a_n}^{x} \frac{d\xi}{\lambda_n} - \frac{1}{8} + \epsilon_n \right\}}. \end{aligned}\right\} \tag{7}$$

The constants c_n, ϵ_n are to be determined with the aid of the initial condition (3). By the usual rule for the expansion of a given function in terms of a series of orthogonal functions we have

$$\begin{aligned} c_n &= \int_{-\infty}^{+\infty} \psi(x, 0) u_n(x) e^{i\epsilon_n} dx \\ &= \frac{A_n}{2} \int_{-\infty}^{+\infty} \sqrt{\lambda_n} f e^{2\pi i \left[\frac{x}{\lambda_0} - \int_a^x \frac{d\xi}{\lambda_n} + \frac{1}{8} + \epsilon_n \right]} dx \\ &\quad + \frac{A_n}{2} \int_{-\infty}^{+\infty} \sqrt{\lambda_n} f e^{2\pi i \left[\frac{x}{\lambda_0} + \int_a^x \frac{d\xi}{\lambda_n} - \frac{1}{8} + \epsilon_n \right]} dx. \end{aligned} \tag{8}$$

Now the entire theory of diffraction is based on the fact that if we multiply a function of x having the general form of an error curve by a cosine function whose wave-length is short compared with the "spread" of the error curve and integrate over all values of x we get a result sensibly equal to zero. By a slight extension of this principle we see that the second term of the right hand member of (8), where the equivalent wave-length of the exponential factor (i.e., $\lambda_0\lambda_n(x)/[\lambda_0 + \lambda_n(x)]$) is very short, must reduce to zero.* On the other hand the equivalent wave-length of the first term of the right hand member of (8) becomes very large in

* It is assumed that $f(x)$ is spread over many wave-lengths (λ_0) as required for an accurate determination of the initial momentum of the particle.

the neighborhood of the initial position of the particle x_0 if $\lambda_n(x_0)$ is close to λ_0. Hence we may expect c_n to differ appreciably from zero only for a narrow range of n values which approximately satisfy the equation

$$\lambda_n(x_0) \equiv \frac{h}{\sqrt{2m[W_n - V(x_0)]}} = \frac{h}{|p_0|} \equiv \lambda_0. \tag{9}$$

The phase angles ϵ_n are determined to make the c_n's real which is accomplished by making the phases of all the constituents of $R(x, 0)$ agree with the phase factor in equation (3) at the starting point $x = x_0$. Analytically

$$\frac{x_0}{\lambda_0} = \int_{a_n}^{x_0} \frac{d\xi}{\lambda_n(\xi)} + \frac{1}{8} + \epsilon_n.$$

With the aid of this explicit formula for ϵ_n the expressions for the forward and backward traveling wave packets become

$$R(x, t) = \sum_n \frac{c_n A_n}{2} \sqrt{\lambda_n} e^{2\pi i \left\{ \frac{W_n}{h} t - \int_{x_0}^{x} \frac{d\xi}{\lambda_n} - \frac{x_0}{\lambda_0} \right\}}, \tag{10}$$

$$L(x, t) = \sum_n \frac{c_n A_n}{2} \sqrt{\lambda_n} e^{2\pi i \left\{ \frac{W_n}{h} t + \int_{a}^{x} \frac{d\xi}{\lambda} + \int_{a}^{x_0} \frac{d\xi}{\lambda} - \frac{1}{4} - \frac{x_0}{\lambda_0} \right\}}. \tag{11}$$

Neither of the two wave groups R and L can differ appreciably from zero except for values of x and t which make the phases of the constituent waves approximately the same. At the initial instant this condition is satisfied by R in the neighborhood of x_0 but by L nowhere. For R the condition is expressed by the equation

$$\frac{d}{dn}\left[\frac{W_n}{h} t - \int_{x_0}^{x} \frac{d\xi}{\lambda_n(\xi)} \right] = 0, \tag{12}$$

where n is to have the value given by (9) and corresponding to the most probable energy of the particle. The differential equation for the values of t and x which satisfy (12) is

$$\frac{dx}{dt} = \frac{dW_n}{dn} \div h \frac{d}{dn}\left(\frac{1}{\lambda_n} \right) = \frac{dW_n}{dp_n}. \tag{13}$$

As this is one of the classical Hamiltonian equations for a particle with energy W_n it is evident that the initial wave packet will move to the right with the classical speed until it reaches the libration limit b. At that point the wavelength becomes infinite and the approximation function breaks down. Moreover the constituent waves of R get hopelessly out of phase as t continues to increase and so R fades out of the picture.

At this point, however, the phases of the backward moving waves come into harmony at b and a backward moving packet travels toward a with the classical speed. It is perhaps sufficient to show that at $x = b$ each forward travelling wave agrees in phase with the corresponding backward wave. In other words

$$\int_a^b \frac{d\xi}{\lambda_n} + \int_a^{x_0} \frac{d\xi}{\lambda_n} - \frac{1}{4} - \frac{x_0}{\lambda_0} + \int_{x_0}^b \frac{d\xi}{\lambda_n} + \frac{x_0}{\lambda_0} = 0.$$

This statement is equivalent to the "Sommerfeld" quantum condition that

$$\oint p_n d\xi = 2h \int_a^b \frac{d\xi}{\lambda_n} = n + \frac{1}{2}$$

and is automatically fulfilled for each of the characteristic energy values W_n.

In closing, I may remark that the foregoing discussion of the one-dimensional wave packet problem lacks the elegance of the virial method of Ehrenfest and Ruark. It has, however, a slight advantage in that it shows that the packet will remain concentrated for a considerable length of time and it makes clear the close connection between this mechanical problem and the elementary theory of optical diffraction.

THE PROBLEM OF SYMMETRY IN QUANTUM MECHANICS.

BY

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AN atom or an ion, whose nucleus is considered as a fixed center of force O, possesses two kinds of symmetry properties: (1) the laws governing it are spherically symmetric, i.e., invariant under an arbitrary rotation about O, (2) it is invariant under permutation of its ν electrons. The first type of symmetry is described by the continuous group γ of orthogonal transformations of the spacial coördinates xyz and the second by the finite symmetric group of order $\nu!$ which consists of all permutations of ν things. Quantum mechanics represents the state of a given physical system by means of the vectors—or rather the rays—of an associated unitary system space $\mathfrak{R}_n$. The dimensionality n of $\mathfrak{R}_n$ is in general infinite, unless we restrict ourselves to states in which the energy, for example, possesses a definite value. The transformations which allow us to pass from one normal coördinate system to another such in unitary vector space are the unitary transformations, under which the fundamental form

$$\bar{x}_1 x_1 + \bar{x}_2 x_2 + \cdots + \bar{x}_n x_n$$

is invariant. I can now indicate how the representations of the symmetry group are introduced into quantum theory; I take the first type of symmetry as an instance. On subjecting the atom to the virtual rotation s an arbitrary state $\mathfrak{x}$ of the atom goes over into a new state $\mathfrak{x}'$; all relations and properties which have an objective significance must remain unaltered under s, and the transition $\mathfrak{x} \to \mathfrak{x}'$ must therefore constitute a unitary transformation $U(s)$ of system space into itself. It is furthermore clear that on compounding two rotations s the corresponding transformations $U(s)$ are likewise compounded

$$U(st) = U(s)\,U(t).$$

And this is exactly what we mean when we say that the correspondence $s \to U(s)$ is a unitary representation of the group γ in system space.

From the physical standpoint there exists an essential difference between the two kinds of symmetry which we have mentioned; the spherical symmetry can be destroyed by external influences, but the intrinsic identity of the electrons can not. If, for example, the atom is in a homogeneous magnetic field the rotational symmetry of the laws to which it is subject is reduced to the symmetry described by the group of rotations about the direction of the magnetic field. We shall in the following be exclusively concerned with that indestructible symmetry which is due to the intrinsic identity of all electrons.

I shall first develop the fundamental concepts of the theory of groups which we need for our present purpose. The vector spaces in which we operate are always unitary; only normal coördinate systems are allowed and all linear transformations are unitary, even though we do not mention it explicitly. The two fundamental operations on vector spaces are *additive* and *multiplicative composition*. If

$$\mathfrak{x} = (x_1, \cdots, x_m) \qquad \mathfrak{y} = (y_1, \cdots, y_n)$$

be vectors in the m- and n-dimensional vector spaces $\mathfrak{R}$, $\mathfrak{S}$ respectively, we consider the pair $(\mathfrak{x}, \mathfrak{y})$ with $m + n$ components $(x_1, \cdots, x_m, y_1, \cdots, y_n)$ as a vector in a new $(m + n)$-dimensional space $\mathfrak{R} + \mathfrak{S}$. A linear transformation $\mathfrak{x} \to \mathfrak{x}' = A\mathfrak{x}$ in $\mathfrak{R}$ and a linear transformation $\mathfrak{y}' = B\mathfrak{y}$ in $\mathfrak{S}$ together constitute a linear transformation A,B in the total space, which is conversely decomposed into the transformations A and B of the linear sub-spaces $\mathfrak{R}$, $\mathfrak{S}$ of $\mathfrak{R} + \mathfrak{S}$. It is in fact, this process of addition which allows us to generate an n-dimensional vector space from the one-dimensional, which is nothing other than the continuum of complex numbers.

On the other hand, we consider the totality of products $x_i y_k = z_l$ as the components of a vector $\mathfrak{x} \times \mathfrak{y}$ in an (mn)-dimensional space $\mathfrak{R} \times \mathfrak{S}$. l is an abbreviation for the pair (i, k) which serves to indicate the various vector components in the product space. Under the influence of linear transformations A and B of x_i and y_k the products z_l undergo a

linear transformation which we denote by $A \times B$. This process is the foundation of tensor analysis; a tensor of third order in the vector space $\mathfrak{R}$ for example, is the same as a vector in $\mathfrak{R} \times \mathfrak{R} \times \mathfrak{R} = \mathfrak{R}^3$ and under the influence of a linear transformation A in $\mathfrak{R}$ the tensor components undergo the corresponding transformation $A \times A \times A$ of $\mathfrak{R}^3$. Multiplicative composition is of utmost importance in quantum theory for the following reason: If $\mathfrak{R}$ be the system space of a physical system P and $\mathfrak{S}$ that of the system Σ, then $\mathfrak{R} \times \mathfrak{S}$ is the system space of the total system composed of the two parts P, Σ. The special case considered above—that of repeated $\times$-multiplication of $\mathfrak{R}$ with itself—is of particular interest for our present discussion: if $\mathfrak{R}$ be the system space of a single electron moving in the field of the nucleus the state of each of the ν electrons of the atom can be represented as a vector in the same system space $\mathfrak{R}$ because of the likeness of the electrons, and the state of the atom is therefore represented by a vector in $\mathfrak{R} \times \mathfrak{R} \times \cdots \times \mathfrak{R}$ (ν factors) or by a tensor of order ν in $\mathfrak{R}$.

But to return to the theory of groups, let γ be a given group and let s run through the elements of γ. If $\mathfrak{a} : s \to A(s)$ be a unitary representation of γ of degree m (i.e. a unitary representation in the m-dimensional vector space $\mathfrak{R}$) and $\mathfrak{b} : s \to B(s)$ a representation in the n-dimensional vector space $\mathfrak{S}$ then $s \to A(s), B(s)$ is a representation in the $(m + n)$-dimensional space $\mathfrak{R} + \mathfrak{S}$. We designate this representation by $\mathfrak{a} + \mathfrak{b}$. Conversely $\mathfrak{a} + \mathfrak{b}$ is decomposed into the two independent constituents $\mathfrak{a}$ and $\mathfrak{b}$. A representation is said to be *primitive* if it cannot be so decomposed by any choice whatever of the normal coördinate system in system space. Two representations which differ only in the coördinates in terms of which they are expressed are said to be *equivalent*. The *first fundamental problem* for the theory of representations is *to construct a complete system of inequivalent primitive representations of the given group* γ. The solution of this problem for the symmetric group is due for the most part to G. Frobenius. Again, the product $\mathfrak{a} \times \mathfrak{b}$ of two representations $\mathfrak{a}$ and $\mathfrak{b}$ is defined by the correspondence $s \to A(s) \times B(s)$. The *second fundamental problem* is: *Given the primitive representations* $\mathfrak{a}$ *and* $\mathfrak{b}$ *of* γ, *to decompose* $\mathfrak{a} \times \mathfrak{b}$ *into its primitive constituents.*

The trace of $A(s)$—the sum of the elements in the principal diagonal of the matrix $A(s)$—is the group-characteristic, or simply the character $\chi(s)$ of the representation $\mathfrak{a}$; equivalent representations have the same characters. The converse of this theorem is valid for unitary representations, and we can consequently employ the character to designate the representation. The character of $\mathfrak{a} + \mathfrak{b}$ is the sum of the characters of $\mathfrak{a}$ and $\mathfrak{b}$, and that of $\mathfrak{a} \times \mathfrak{b}$ is their product. We always have

$$\chi(s^{-1}) = \bar{\chi}(s).$$

The characters of inequivalent primitive representations constitute an orthogonal system

$$\mathfrak{M}\{\bar{\chi}(s)\chi(s)\} = 1, \qquad \mathfrak{M}\{\bar{\chi}(s)\chi'(s)\} = 0$$

where χ, χ' are inequivalent and $\mathfrak{M}$ denotes the mean value. In this formulation I have in mind primarily finite groups—such as the symmetric group—but the theory is also valid for closed continuous groups; the mean value is then obtained by integrating over the group manifold instead of by summation. If the representation X contains the primitive representation χ exactly g times, the primitive representation χ' g' times, etc., then

$$\mathrm{X}(s) = g\chi(s) + g'\chi'(s) + \cdots.$$

The coefficients g are uniquely determined by this equation; e.g. we have

(*) $$g = \mathfrak{M}\{\mathrm{X}(s)\bar{\chi}(s)\}.$$

The decomposition of a representation X into its primitive constituents is consequently unique (in the sense of equivalence) and equation (*) shows how many times the primitive representation χ occurs in X. In particular this equation gives the solution of our second fundamental problem: The number of times that the primitive representation χ occurs in the cross product of the two primitive representations χ_1, χ_2 is given by the mean value

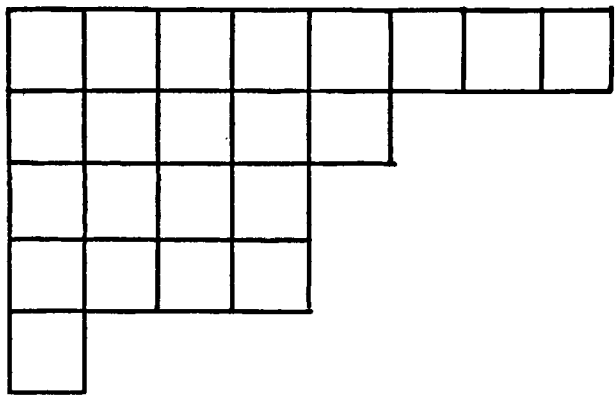
$$g = \mathfrak{M}\{\chi_1(s)\chi_2(s)\bar{\chi}(s)\}.$$

We take as an example the group Γ_n of all unitary trans-

formations in n dimensions, a case of great importance for physics. Since Γ_n itself consists of linear unitary transformations, the correspondence $s \to s$ constitutes a representation — in fact, a primitive representation — i, the field of operation or *substratum* of which is the aggregate of *vectors* of the n-dimensional vector space. All tensors of order ν constitute the substratum of the representation $\mathfrak{i} \times \mathfrak{i} \times \cdots \times \mathfrak{i} = \mathfrak{i}^\nu$ (ν factors) but it is not primitive. Take, for example, the case $\nu = 2$. There are symmetric and anti-symmetric tensors of order 2. Since such symmetry is not destroyed by linear transformation of the vector space, the $\frac{1}{2}n(n-1)$-dimensional linear family of all anti-symmetric tensors of second order constitute the substratum of a representation of Γ_n contained in the representation $\mathfrak{i} \times \mathfrak{i}$. This representation, as well as that obtained from symmetric tensors, is primitive, and $\mathfrak{i} \times \mathfrak{i}$ is decomposed into these two primitive constituents. This simplest case suggests that the substrate of the primitive representations contained in $\mathfrak{i}^\nu$ are *tensors of order ν with certain definite symmetry properties.* We must therefore examine the various types of symmetry possible in tensors of order ν. But this problem is closely related to that of the representations of the group σ_ν of permutations of ν things. This can be shown quite readily, but I shall content myself with giving the result. With each representation $\mathfrak{d}$ of σ_ν there is associated a definite representation $\mathfrak{G}$ of Γ_n of order ν. This correspondence is to be understood in the sense that on both sides equivalent representations are to be taken as indistinguishable. $\mathfrak{G}$ is primitive if $\mathfrak{d}$ is; if $\mathfrak{d}$ be of degree f then $\mathfrak{G}$ occurs exactly f times in $\mathfrak{i}^\nu$. If χ be the group-characteristic of $\mathfrak{d}$, I designate $\mathfrak{G}$ more precisely by $\mathfrak{G}_\chi$. $\mathfrak{R} \times \mathfrak{R} \times \cdots \times \mathfrak{R}$ is decomposed into a number of primitive sub-spaces $\mathfrak{R}_\chi^r$ which are independent of the coördinate system employed, and the group of all unitary transformations in $\mathfrak{R}$ induces in each of these a primitive representation $\mathfrak{G}_\chi$. This correspondence between the representations $\mathfrak{d}$ and $\mathfrak{G}$ of σ and Γ must be expressible by a simple relation between their characters χ and X. I shall not write down this formula, but I shall have occasion to refer to it later and shall call it the "bridge"; that is, the bridge which connects the representations of σ and Γ.

Since continuous groups are much easier to handle than discrete, the "bridge" leads over X to an explicit determination of the primitive character χ of the permutation group σ_ν.

Diagram $\mathfrak{S}$



I have yet to indicate the simple construction which enables us to obtain the tensors of order ν of highest possible symmetry. By a permutation s a tensor F of order ν is changed into sF. On subjecting an arbitrary tensor F to all possible permutations s we can generate the most general symmetric tensor by the formula $\Sigma_s sF$ and the most general antisymmetric one by $\Sigma_s \pm sF$. The sign $\delta_s = \pm$ is $+$ if s be an even permutation and $-$ if it be odd. We call the first process symmetrization and the second alternation. We now write the numbers from 1 to ν in a diagram $\mathfrak{S}$ which consists of non-interrupted rows and columns as in the accompanying figure. An arbitrary tensor is subjected to symmetrization with respect to the figures of each of the rows and then to alternation with respect to the figures of each of the columns. The tensors so found constitute a linear family which is the substratum of a primitive representation $\mathfrak{G}_\chi$ of Γ of order ν, which is determined by the diagram $\mathfrak{S}$, and to this there corresponds a definite primitive representation $\mathfrak{d}$ of σ_ν. Different diagrams give rise to inequivalent representations. If the diagram consists of but one row, we obtain the family of all symmetric tensors; the corresponding

representation of σ_ν is one-dimensional and associates the identity $x' = x$ with each permutation s. If the diagram consists of but one column we obtain antisymmetric tensors; the corresponding representation of σ_ν is again one-dimensional and associates the transformation $x' = \delta_s x$ with the permutation $s : \chi(s) = \delta_s$. On interchanging rows and columns in $\mathfrak{S}$ we obtain the dual diagram $\mathfrak{S}^*$; the corresponding characteristics are related by

$$\chi^*(s^{-1}) = \delta_s \chi(s) \quad \text{or} \quad \bar{\chi}^*(s) = \delta_s \chi(s).$$

The primitive sub-spaces $\mathfrak{R}_\chi^\nu$ of $\mathfrak{R} \times \mathfrak{R} \times \cdots \times \mathfrak{R}$ have the following significance for quantum physics: If the state of an atom is represented by a vector in $\mathfrak{R}_\chi^\nu$ no influence can drive it out of $\mathfrak{R}_\chi^\nu$. To each sub-space there belongs a certain symmetry-class of spectral terms; equivalent sub-spaces—in which Γ_n induces equivalent primitive representations—yield the same terms. Corresponding to the primitive representation χ (of degree f) of the permutation group there is a single class of terms whose members are f-fold, and it is impossible to resolve this hidden multiplicity. The various primitive sub-spaces are, so to speak, worlds which are fully isolated from one another. But such a situation is repugnant to Nature, who wishes to relate everything with everything. She has accordingly avoided this distressing situation by annihilating all these possible worlds except one—or better, she has never allowed them to come into existence! The one which she has spared is that one which is represented by anti-symmetric tensors, and this is the content of Pauli's exclusion principle. The system space of an atom is accordingly not the space $\mathfrak{R}^\nu$ of all tensors of order ν in $\mathfrak{R}$ but only the sub-space of anti-symmetric tensors. Our entire theory thus appears to be quite superfluous for physics—were it not for the spinning electron!

Briefly and vaguely stated, this amounts to the following: the electron is in the abstract composed of two partial systems, the electron translation and the electron spin. Denoting the corresponding system spaces by $\mathfrak{R} = \mathfrak{R}_n$ and $\mathfrak{R}' = \mathfrak{R}_a$ the system space of the electron is $\underline{\mathfrak{R}} = \mathfrak{R}' \times \mathfrak{R}$. The dimensionality n is infinite, but the dimensionality a appears to be 2. But the dynamical laws governing the state vector in $\underline{\mathfrak{R}}$

involve to first approximation (on neglecting the small perturbation due to the spin) only the translational part $\mathfrak{R}$ of $\underline{\mathfrak{R}} = \mathfrak{R}' \times \mathfrak{R}$. Let $\Gamma_a = \Gamma'$, $\Gamma_n = \Gamma$ and $\underline{\Gamma}$ denote the groups of unitary transformations in $\mathfrak{R}_a$, $\mathfrak{R}_n$ and the product space $\mathfrak{R}_a \times \mathfrak{R}_n$ and u', u the elements of the first two. Let $\mathfrak{G}'_{\chi'}$ be a primitive representation of $\Gamma' : u' \to U'$; $\mathfrak{G}_\chi$ of $\Gamma : u \to U$. Then the correspondence

$$(u' \times u) \to (U' \times U)$$

yields a primitive representation of $\Gamma' \times \Gamma$ which we designate by $\mathfrak{G}'_{\chi'} \otimes \mathfrak{G}_\chi$; its character belonging to the element

$$u' \times u \quad \text{is} \quad \chi'(u')\chi(u).$$

The quantum-theoretic symmetry problem is now: $\mathfrak{G}_\xi$ being a primitive representation of $\underline{\Gamma}$, to determine how $\mathfrak{G}_\xi$ is decomposed into primitive constituents of the form $\mathfrak{G}'_{\chi'} \otimes \mathfrak{G}_\chi$ on restricting $\underline{\Gamma}$ to the sub-group $\Gamma' \times \Gamma$. We can add this third problem, formulated generally, to the above mental-funda problems. The "bridge" reduces it to the second fundamental problem for the symmetric group, by means of the following reciprocity: $\mathfrak{G}_\xi$ contains the representation $\mathfrak{G}'_{\chi'} \quad \mathfrak{G}_\chi$ on restricting $\underline{\Gamma}$ to $\Gamma' \times \Gamma$ as many times as the representation of $\mathfrak{d}_{\chi'} \times \mathfrak{d}_\chi$ contains the representation $\mathfrak{d}_\xi$. But in accordance with (*) this multiplicity is given by

$$g = \mathfrak{M}\{\chi'(s)\chi(s)\bar{\zeta}(s)\}.$$

The quantum theory is only interested in the case where $\mathfrak{G}_\xi = \{\mathfrak{G}\}$ is the Pauli representation which is associated with the character $\zeta(s) = \delta_s$. But $\chi(s)\delta_s$ is by the above $\bar{\chi}^*(s)$ and in consequence of the orthogonal properties of $\chi(s)$ mentioned above $g = 1$ or 0 according as χ' is the dual of χ or not. The representation induced by the group $\Gamma' \times \Gamma$ in the Pauli system-space of the atom contains only primitive constituents of the form $\mathfrak{G}'_{\chi^*} \otimes \mathfrak{G}_\chi$ and these but once. Finally, we must replace the element u' (which has so far been arbitrary) of Γ' by the identity. On thus restricting ourselves to the transformations $1 \times u$ of $\underline{\Gamma}$, $\{\mathfrak{G}\}$ becomes a representation of Γ whose decomposition into irreducible constituents is given, in accordance with our development, by

$$\sum_\chi F_a(\chi^*)\mathfrak{G}_\chi,$$

where $F_a(\chi^*)$ means the degree of $\mathfrak{G}'_{\chi^*}$, i.e. the degree of the representation of the a-dimensional linear group which corresponds to the dual character χ^*. Again restricting ourselves to the electron translation, as is natural when the spin perturbation is neglected, the Pauli exclusion principle together with the existence of spin produces the following modifications of our original theory: The multiplicity of the term class χ is no longer $f(\chi)$, the degree of the irreducible representation χ of the permutation group, but is now $F_a(\chi^*)$. In particular, all term classes whose symmetry diagrams possess more than a columns do not occur. The various classes of terms exist side by side, for their isolation is destroyed by the spin-perturbation which gives rise to weak intercombinations; furthermore, the spin-perturbation resolves the "multiplets" of a class into a number of simple components which is given by the degree $F_a(\chi^*)$.

The reciprocity indicated above is concerned with the case in which Γ_{an} is restricted to $\Gamma_a \times \Gamma_n$, but we can set up the analogue for the permutation group. Divide the numbers from 1 to ν into two classes containing ν' and ν'' integers: $\nu' + \nu'' = \nu$. An arbitrary permutation p' of the numbers in the first class and an arbitrary permutation p'' of those in the other class together constitute a permutation of all the numbers which is designated by $p' \times p''$ in accordance with our convention; these special permutations constitute a sub-group $\sigma' \times \sigma''$ of $\sigma = \sigma_\nu$. If $\mathfrak{d}_\xi$ be a primitive representation of σ, then on restricting σ to the sub-group $\sigma' \times \sigma''$, $\mathfrak{d}_\xi$ is decomposed into certain primitive constituents of the form $\mathfrak{d}'_{\chi'} \otimes \mathfrak{d}''_{\chi''}$. The factors $\mathfrak{d}'$, $\mathfrak{d}''$ are representations of σ' and σ'' respectively. And the second reciprocity theorem to which we are immediately led by the "bridge," is: $\mathfrak{d}'_{\chi'} \otimes \mathfrak{d}''_{\chi''}$ appears in $\mathfrak{d}_\xi$, if considered as a representation of the subgroup $\sigma' \times \sigma''$, exactly as many times as the representation $\mathfrak{G}_{\chi'} \times \mathfrak{G}_{\chi''}$ of the unitary group Γ contains the primitive representation $\mathfrak{G}_\xi$ of the same.

This reciprocity law governs the fundamental chemical problem of combining two atoms to obtain a molecule. Let the number of electrons contained in the two atoms and the molecule be ν', ν'' and $\nu = \nu' + \nu''$, and let the atoms be in the symmetry states characterized by the permissible

representations χ', χ'' of σ', σ''. The molecule which is obtained by combining the two atoms will be in one of the symmetry states ζ whose corresponding $\mathfrak{G}_\zeta$ appears in $\mathfrak{G}_{\chi'} \times \mathfrak{G}_{\chi''}$ and the calculation of the associated energy is accomplished with the aid of the characteristics. These circumstances, which cannot be represented by a spacial picture, constitute the basis for the understanding of the homeopolar bond, the attraction (or repulsion) existing between neutral atoms. The second fundamental problem as applied to the unitary group Γ is the quantum-theoretic formulation of that combinatorial problem which the chemists describe by the aid of valence bonds.

Unfortunately it has not been possible to show here how completely the mathematical methods of the theory of representations lend themselves to these physical applications. The connection between mathematical theory and physical application which are revealed in the work of Wigner, v. Neumann, Heitler, London and the speaker is here closer and more complete than in almost any other field. The theory of groups is the appropriate language for the expression of the general qualitative laws which obtain in the atomic world. In particular the reciprocity laws between the representations of the symmetry group σ_ν and the unitary group Γ are the most characteristic feature of the development which I have here indicated; they have not previously come into their own in the physical literature, in spite of the fact that quantum physics leads very naturally to this relation.

THE ROLE OF GROUPS IN A PHYSICAL THEORY

DISCUSSION BY

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PROFESSOR SLATER mentioned this morning that in theoretical physics we have a series of physical facts and a series of mathematical statements that run parallel to the facts. If we want to discuss the particular character of application of the Theory of Groups we have to begin by reformulating and amplifying this statement of Professor Slater.

A physicist, we may take it, is a person who measures according to certain rules. Let us denote by a the number he obtained in a given situation by applying the rule number one, by b the number obtained in the same situation by measuring according to rule number two and so on (a may be e.g. the volume, b the pressure, c the temperature of gas in a given container). The physicist finds further that the results of measurements of the same kind undertaken in different situations satisfy certain relations, we may write for instance

$$r(a, b) = c.$$

A mathematician is busy deducing from some given propositions other propositions; this usually leads to numbers which we may call A, B, $C \cdots$. These numbers also satisfy certain relations, say

$$R(A, B) = C.$$

Then comes, as Professor Weyl says, a messenger, a go-between who may be a mathematician or a physicist, or both, and says: "If you establish a correspondence between the physical quantities and the mathematical quantities, if you assign A to a, B to b, etc., the *same* relations hold for the physical quantities as for the corresponding mathematical quantities so that $R \equiv r$."

In the course of time new procedures of measurement are invented, some physical relations do not find their counterpart in the mathematical theory, the mathematical theory has to be patched up by introducing new quantities till too many quantities appear in it which do not correspond to physical quantities; then comes the phenomenological point of view and sweeps the theory out of applied mathematics—the theory becomes pure mathematics once more, and physicists begin to look around for a new theory. Everybody can find examples for this situation; it is enough to mention the Bohr atom which was not even mentioned today only 15 years after its introduction.

However the theory of groups which is being applied to physics is not just one of many mathematical theories of the character described above; its application is of a far more fundamental nature and we shall be able to indicate what it is by analysing further the scheme outlined above.

It may happen, and in fact it happens often, that the same mathematical theory can be applied to the same physical facts in more than one way; for instance, instead of assigning to the physical quantities $a, b, \cdots$ the mathematical quantities $A, B, \cdots$ we might have assigned to them $A', B', \cdots$ with the same results, that is, the relations for physical quantities are the same as for the mathematical quantities corresponding to them now (think of space considered from the experimental point of view—and of coördinate geometry; different ways of establishing a correspondence result from different choices of coördinate axes). If this happens it means that the mathematical theory possesses a peculiar property, namely, that if A' is substituted for A, B' for B and so on, no relation of the type $R(A, B) = C$ which was correct before the substitution is destroyed; in other words, there are substitutions or transformations for which all relations are invariant. All such transformations constitute what we call a group; the existence and the properties of such a group present a very important characteristic of the mathematical theory. Moreover it is clear that if two different mathematical theories can be applied—in the sense described above—to the same physical theory, the groups of these two theories will be essentially the same, so that the

groups reflect some of the most fundamental properties of physical systems.

In Quantum Mechanics it seems that some groups have been discovered and put to work with considerable success before the complete theories are ready. It is to be hoped, however, that some satisfactory complete theories will be offered soon; because, after all, the application of groups inspite of its fundamental character may be of restricted scope; not all essential features of a mathematical theory need be reflected in the group; even in geometry—the classical example of the application of groups—further refinement has lead to such changes (in the General Relativity Theory) that the group was reduced to nothing. There are indications that in Quantum Mechanics the groups cannot take care of everything: in one case Professor Weyl thought it necessary to invoke the wishes of Nature; it may be thought desirable to substitute for that another type of explanation, and we have to strive to arrive at it.

Preliminary Results of Ocean Magnetic Observations on the Carnegie. J. P. AULT. (*Ter. Mag.*, Sept., 1928.) On May 1, 1928 the *Carnegie* left her wharf in the city of Washington to begin a new series of scientific observations on the oceans of the earth. On the way to England the weather was unfavorable, it being necessary to wait ten days off the entrance to the English Channel on account of gales. The copper sheathing started to peel off at the water-line and both cabin and staterooms were flooded at times.

At 29 stations on the voyage observations of declination were made with the marine collimating compass. Chart corrections ranging from + 1° .7 to − 1° .0 were found for declination, while the largest error for inclination amounted to 0° .9. The vessel was stopped at six stations on the only days when the weather permitted and observations of the temperature of the water were made and samples of the water were collected. Water-bottles at one place failed to operate. "Some animal of the deep had fouled the wire."

From England the *Carnegie* went to Hamburg, where Dr. Kolhoerster delivered an instrument for observing the penetrating radiation associated with his name and that of Millikan in the United States. Thence the voyage led toward the Faroe Islands and on to Iceland. Off Iceland the vessel crossed her track of 1914. Reykjavik was at last reached on July 20.

Thus has begun the long voyage of the *Carnegie* that will furnish fresh data for magnetic charts of the water areas of the globe and throw light upon the secular variation of the magnetic elements of the earth. G. F. S.

QUANTUM MECHANICS.

BY

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THE concept of the atom, which considers the electrons moving in orbits restricted by certain quantum rules, has been, and still is, of extreme importance for the description of atomic properties. Many spectroscopic experiences and various other properties can be expressed in a very simple way with the help of this model.

The discovery and analysis of atomic properties, such as the Compton and photoelectric effects which did not fit into this orbital picture, were not however the only reasons which necessitated the development of the new Mechanics. A close study of the description of spectroscopic laws with the help of the atomic model reveals many unsatisfactory deductions, ad hoc hypotheses and unnecessary complications. The introduction of "half" quantum numbers, the replacement of the square of some quantum number, say j^2, by $j(j+1)$, the existence of orbits with an angular momentum which has to be taken equal to zero, are some examples of this.

The new Quantum Mechanics, though allowing no longer a naïve picture for the atom, has made the great accomplishment of providing us with a completely consistent description of the atomic properties, giving at once the right results wherever the calculation has been possible. It is obvious that such a new theory changes considerably our conceptions of atoms and electrons, though it has been shown at the same time that the orbital picture is a very good approximation to the truth in most cases and the Quantum Mechanics corroborates even some of its results. For instance, it has been very surprising that the old theory gave such excellent results for the spectroscopic properties of alkali atoms, if one replaces the inner electron shell merely by a spherically symmetric charge distribution. This approximation has been completely justified by the Quantum Mechanics.

The introduction of the Theory of Groups into the Quantum Mechanics by Professor Weyl has thrown light upon a special class of atomic properties, namely those connected with symmetry. For instance, the selection rule for complicated spectra, the Laporte rule, which is so important for spectroscopy, has been shown by the methods of Weyl to be closely connected with the fundamentals of the Quantum Mechanics, whereas it did not find its right place in the older conceptions.

The success of this development gives hope that very soon some properties of spectra which still lack a rigorous explanation, may be described with the help of the Quantum Mechanics. Such problems of special interest for spectroscopy are, among others, the interaction between two spins, the interaction between the core of electrons and the magnetic properties of the nucleus. A rigorous solution of these problems will throw light on the laws of multiplet structure and on the properties of hyperfine structure.

HARMONIC ANALYSIS AND THE QUANTUM THEORY.

BY

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THE author desires to open the present paper with an apology. It purports to be not so much a new formal development in the quantum theory as an interpretation of results already obtained and a bridge connecting them with older branches of physics. Mathematical attacks on the quantum theory, except from the side of the professional physicist, have proved disappointing so far, and indeed Professor Weyl's treatment of the group theory is the only contribution from the side of the pure mathematician which has definitely become incorporated into the physics of quantum theory. It therefore seems highly doubtful whether the present paper will lead to any results important to the physicists. On the other hand, there is a large body of results belonging to the older physics which has up to the present time remained formally unconnected with the quantum theory, although its subject matter is the same and its results will have to be brought into connection with present work. Thus the pure mathematician may serve a temporary need, although the final version of the theory may altogether dispense with his help.

One of the remarkable things about light is that the quantities which are primarily introduced in the Maxwell theory are not observed; at any rate, in the case of moderately high frequencies. The Maxwell equations concern the electric and the magnetic vector. Our optical observations deal with the blackening of a photographic plate or a measurement by a photometer, or the determination of the intensity of light with a photoelectric cell, or with the estimation of this intensity by the naked eye, or other measurements of the same general sort. Every optical observation terminates in this manner. The quantities in the Maxwell theory which most nearly correspond with these observations are the

energy density, the Poynting vector, and similar expressions, which depend quadratically on electric and magnetic vectors. We thus find in classical optics the same relation between the linearity of the fundamental quantities entering into the equations and the quadratic character of observable quantities which plays such an important role in the quantum theory. In the latter the Schrödinger $\psi\bar{\psi}$ depends quadratically on the function which satisfies the Schrödinger equation. Similarly, in the revised theory of Dirac the quantities which are given a real physical existence are components of the electric current-density vector which again depends quadratically on the quantities satisfying the linear differential equations of Dirac. Indeed we do not need to go to such remote fields to find this contrast of linearity and quadraticity playing such an important role. If we consider the instruments which we use to measure alternating currents, voltages, power, etc., we see that in almost no case do they yield a linear reading in terms of current and voltage. The electrodynamometer records the attraction of two coils through each of which a current flows, and gives a reading proportional to the product of these currents. The hot-wire ammeter records the energy loss in a heated wire, and this again is proportional to the square of the current. As a matter of fact, the only instruments which give alternating-currens readings which are linear in current or voltage are the variout forms of oscillograph. These carry us a certain distance into the Hertzian region, but even they fail for the shorter Hertzian waves.

In order to understand the meaning of this linear-quadratic duality, it is very useful to see if we can find somewhere in mathematics a model of a similar duality. This we can find in connection with harmonic analysis. The development of a function in a Fourier series is a linear problem, as the coefficients depend linearly on the function expanded. The same thing applies to the expansion of a function as a Fourier integral. On the other hand, there are several important applications of harmonic analysis in which the coefficients of our trigonometrical development, which are complex quantities, possessing a phase as well as an amplitude, are of interest to us only in so far as the absolute value of their amplitude

is concerned. For example, when we break up light by a prism, we know nothing about the phase of the different rays into which the light is split. All that interests us is the absolute value of the amplitude of each component, or indeed the square of this absolute value, which is proportional to the power going into light of that frequency. Another illustration is the periodogram of Sir Arthur Schuster. If we have a phenomenon of Nature which we suspect of containing hidden periods, Schuster's method enables us to determine the squares of the amplitudes of the harmonic terms with the given periods. Again we discard phase. This is not due to the fact that we adopt in these cases an artificial system of analysis; it belongs to the nature of harmonic analysis itself.

Let us start with a function $f(t)$ which we wish to analyze trigonometrically. In the particular case where f is a sum of a finite number of complex exponential functions,

$$f(t) = \sum a_n e^{t\lambda_n t}$$

we can associate with f a related function whose harmonic analysis is derivable from that of f in a very simple manner. Let us form

$$\varphi(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} f(t+\tau)\bar{f}(t)dt.$$

That is, we take the conjugate imaginary function of $f(t)$, multiply it by f of t plus a displacement term τ, and average over all time; i.e. all values of t. This function will be itself the sum of a finite number of complex exponentials, with frequencies the same as those occurring in the harmonic analysis of $f(t)$, but with coefficients which are the squares of the absolute values of the coefficients in f. Thus harmonic analysis of this new function φ will contain all the information concerning f that is yielded by the study of frequencies and amplitudes, but no information concerning phases. If we then submit this function φ to an harmonic analysis we obtain what is substantially the periodogram or spectrum of f.

It is clear that φ is quadratic in comparison with f. What I propose to do in the present paper is to show how the quadratic character of this function is formally related

to the quadraticness of the quantum theory. Of course we need not confine our attention to functions f which are trigonometrical polynomials. We shall indeed consider all functions f for which the definition of φ, which we have given above, yields a value of φ for every τ. The harmonic analysis of φ is then carried out in accordance with the following definition:

$$r(u) = \int_{-\infty}^{\infty} \varphi(\tau) \frac{(e^{iu\tau} - e^{-\tau^2})d\tau}{i\tau}.$$

The function $r(u)$ which we thus obtain may be proved to exist, with an appropriate definition of the infinite integral, and to be definable as a monotone increasing function. $r(u)$ represents the total energy in the spectrum up to frequency "u." In case $r(u)$ has a derivative for a given value of u, $r'(u)$ represents the spectral intensity for this frequency.

It is not enough for the quantum theory for us to develop the spectrum theory of a single function. Our function $f(t)$ is replaced by a function $f(t, n)$, where n represents a parameter which may assume a discrete number of values, or a set of values corresponding to the points in an m-dimensional space, or finally, a set of values corresponding to these points, each with index assuming a discrete set of values. We can proceed to the formation of a function $\varphi(\tau, n)$ and a function $r(u, n)$, just as in the case of a single function. So far there is nothing of new interest. The essential difference between the spectrum theory with a parameter and the spectrum without a parameter only appears when the function $f(t, u)$ is subjected to a linear transformation. The φ and r corresponding to the transformed function cannot then be expressed exclusively in terms of the original φ and r. We are thus led to introduce the new set of φ functions:

$$\varphi(\tau; m, n) = \lim_{T\to\infty} \frac{1}{2T} \int_{-\infty}^{\infty} f(t + \tau, m)\bar{f}(t, n)dt,$$

and the corresponding r functions:

$$r(u; m, n) = \int_{-\infty}^{\infty} \varphi(\tau; m, n) \frac{e^{iu\tau} - e^{-\tau^2}}{i\tau} d\tau.$$

These functions depend upon two independent parameters,

and form what is known as a *matrix* in a finite or infinite number of rows and columns. They are in general complex unless $m = n$. It is a most important fact, and one which is easily proved, that the matrix of the quantities $r(u;\ m,\ n)$ is Hermitian or that

$$r(u;\ m,\ n) = \bar{r}(u;\ n,\ m).$$

We term this matrix a *coherency-matrix*. If we make use of this matrix as representing the spectrum of the set of functions $f(t,\ n)$, we see that a linear transformation no longer changes our spectrum in such a way that it cannot be expressed in terms of the untransformed spectrum. Indeed, if

$$g(t,\ m) = \sum a_{mn} f(t,\ n)$$

and $s(u;\ m,\ n)$ is the analogue of $r(u;\ m,\ n)$ when g is substituted for f,

$$s(u;\ m,\ n) = \sum\sum a_{mp} \bar{a}_{nq} r(u;\ p,\ g).$$

It thus appears that the effect of a linear transformation on the variables $f(t,\ n)$ is to multiply the matrix of the r's on the left by the adjoint of the matrix of this transformation and on the right by the matrix of this transformation itself. This transformation of our matrix has a physical meaning. If we take an optical instrument, its effect is to yield at the plane of the image an electromagnetic disturbance which depends linearly on the electromagnetic disturbance at the object plane. Our most general observation of the image plane now would yield us point by point the spectrum of the light in this plane. (This is a somewhat simplified version of what happens, but is not false in character.) The optical instrument then transforms the matrix representing the electric disturbance at the object plane linearly into the matrix representing the electric disturbance at the image plane, and then yields us the spectrum of this, which consists in the diagonal terms of the transformed matrix.

This stands in exceedingly close relation to the procedure of quantum mechanics. We find an Hermitian matrix exactly analogous to that of quantum mechanics. This matrix is subject to linear transformations in both cases, but once it has been transformed, only the diagonal terms have a physical

meaning. The transformations in each case correspond to the use of definite physical instruments. Once the diagonal terms have been formed, it is no longer true that, if we subject them, or the matrix made up of them alone, to a new linear transformation, we shall get the same matrix or the same diagonal terms which we would have got from our original matrix by the two transformations repeated end on end, as in an intermediate stage we have discarded all non-diagonal terms. This corresponds to the fact often cited in the quantum theory that an instrument always distorts and modifies the atomic phenomenon which it is meant to measure.

The transformations corresponding to the optical instruments at our disposal have the mathematical property of forming a group. If we examine the image produced by one system of lenses through another system of lenses, the combined systems form themselves an optical instrument. This group will in general be more restricted than the group of all possible linear transformations. In the case of light, for example, no optical instrument can transform a solution of Maxwell's equations into anything but a solution of Maxwell's equations. What the precise group of transformations will be in any particular case requires an investigation in that particular case, but in very many instances we shall find some Hermitian expression in the coefficients of the linear transformation invariant, and this invariant will restrict our group to the sort that Weyl calls a group of Unitärtransformationen. It is worth while pointing out, however, that the origin of the group theory is from transformations which may or may not be unitary, and that Weyl's restriction becomes difficult to verify or even to formulate under the assumptions of general relativity.

It is perhaps worth while to give working models of the ideas involved in this type of harmonic analysis. The matrix of the r's plays an important rôle in the statistical analysis of time series, and is indeed the closest analogue for statistical series of the notion of correlation matrix among frequency series. If we have a number of different quantities $x_1^{(n)}$, $x_2^{(n)}$, $x_3^{(n)} \ldots x_m^{(n)}$ corresponding to a single occurrence n, the correlation matrix of these quantities is the one whose terms are:

$$\sum x_i^{(n)} x_j^{(n)}.$$

If now we plot the points $(x_1^{(n)}, \cdots, x_m^{(n)})$ in an m space, as we can do when m is 2, we get a figure known as a scatter-diagram. One interesting example of a scatter-diagram is that in Professor J. L. Coolidge's book on statistics in which he plots in such a manner the Big Bertha shells that fell on Paris. Of the linear transformations to which the coördinates x and y in this diagram are subject the most natural are obviously those consisting of translation and rotation; that is, the orthogonal. Now we can connect with any such diagram an ellipse, which best represents the points of the diagram. The problem of doing this is the same as the problem of reducing the matrix to diagonal form by an orthogonal transformation. This ellipse, that is, will yield us a diagonal matrix if we refer our scatter-diagram to its axes as coördinate axes. It is thus connected with the diagram in a manner which remains invariant under the rotation group.

On the other hand, scatter-diagrams may be used for all sorts of statistical problems where the quantities with which we are dealing have no sort of natural scale of measurement, and where rotation is of no importance whatever. The group of transformations which is significant in such a case is the affine group preserving horizontal and vertical directions. The quantities which interest the statistician are thus invariants of the scatter-diagram with respect to this group. Without going any further it may be said that correlation coefficients and lines of regression are such invariants.

To come to an example more directly allied with quantum theory, let us look into the coherency-matrix representation of polarized light. We here consider two components of the electric vector $(x(t), y(t))$ as functions of the time t. The coherency-matrix may be represented as the sum of the four following matrices each multiplied by an appropriate real coefficient.

$$\left\| \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \right\| \quad \left\| \begin{matrix} 1 & 0 \\ 0 & -1 \end{matrix} \right\| \quad \left\| \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix} \right\| \quad \left\| \begin{matrix} 0 & i \\ -i & 0 \end{matrix} \right\|$$

1 2 3 4

The coefficient of matrix 1 represents the total intensity of

the light in question. The coefficient of matrix 2 represents the excess of light polarized in a horizontal direction over that polarized in a vertical direction, let us say. The coefficient of matrix 3 similarly represents the excess of light polarized at 45 degrees over light polarized at 135 degrees. The coefficient of matrix 4 represents the excess of light polarized circularly clockwise over light polarized circularly in a counterclockwise direction. It will be noted that in the form given matrices 1 and 2 are diagonal. Any unitary transformation leaves 1 unchanged. On the other hand, the matrices 3 and 4 are now free of diagonal terms. The unitary transformation which changes 2 into 3 will transform 3 and 4 into matrices clear of diagonal terms and a similar remark applies to the transformation which transforms 2 into 4. Physically the significance of these comments is the following: There are four optical instruments which together are sufficient to give us all possible information concerning the polarization of a beam of light. The first is a photometer; the second is obtained by combining a photometer with a doubly-refracting crystal in a certain plane orientation and reading the difference in the intensities of the ordinary and extraordinary rays; the third is obtained by rotating this instrument through 45 degrees; and the fourth measures the degree of circular polarization of the light in a similar manner. The light emerging from each of the instruments 2, 3, and 4 appears to each other one of the instruments as if it were unpolarized. The instruments of 2, 3, and 4 determine a group, which has already been discussed by Weyl in connection with the Pauli spinning electron. Clearly in this case the significant group is a sub-group of the unitary group.

It may now be asked, "What has all this to do with quantum theory? How can we interpret the matrices of quantum theory as coherency matrices?" Two alternatives suggest themselves. In the first, the coherency is taken on the fifth unobservable direction of Kaluza, Klein, and Fock. In the second, the coherency is taken on the time. The relativist invariance of the first form of the theory is obvious. The functions whose coherency is taken are solutions of the five-dimensional Schrödinger equation:

$$\Delta\Psi = 0. \tag{1}$$

The coherency matrix of a solution of this equation is taken for one specific x frequency; namely, $h/2\pi$. The problem of reducing this matrix to diagonal form is then the problem of determining the characteristic functions of the Schrödinger equation. It is indeed true that there is no immediate reason why just precisely the characteristic functions with respect to the time should bring this matrix into diagonal form, but if we remember that a single solution of the equation is unobservable to us, and that all we ever can do is to observe the sum of a large number of coherency matrices which are placed at haphazard with respect to the time, it will be seen that the real positive terms on the principal diagonal will ultimately become very large in comparison with the terms away from the principal diagonal, which are distributed at haphazard in phase. Thus if

$$f(x, y, z, t) = \sum a_n e^{i\lambda_n t}\psi_n(x, y, z),$$

where the ψ_n's are the characteristic functions and the λ_n's the characteristic numbers of the Schrödinger equation, the expression

$$\sum |a_n|^2 \psi_n(t;\ x, y, z)\bar{\psi}_n(t;\ x, y, z)$$

represents everything that can be determined from the coherency matrix of the function f. Consequently we see that the Schrödinger theory picks on just the one aspect of the function f which can have physical significance.

Of course the Schrödinger theory in its original form has been supplanted by the Dirac modification. How this Dirac modification can be put into analogous shape depends upon connecting it with a five-dimensional equation analogous to (1). So far, attempts in this direction have met only with moderate success, but it is perhaps too early to say that they are hopeless.

The other alternative method of approach is to make our coherency on the time dimension. This is of course not invariant under relativist transformation, at least in appearance. If, however, we take as our wave equation the ordinary equation in free space, we see that our coherency matrix when reduced to diagonal form yields us just the same

quadratic distribution over all possible plane waves with velocity 1, which we should obtain if we were to extend our spectrum theory into four dimensions and form this four-dimensional spectrum of the solution of the wave equation. As I hint here, it is possible to extend the spectrum theory which I mentioned earlier in the article to cover the case of spectra in time and space simultaneously. This suggests an alternative on which I am at present at work, but which is not yet sufficiently mature to be presented in detail. It is possible to extend this spectrum theory even further and to analyze in terms of a set of functions more general than the complex exponential functions:

$$e^{i(x\xi+y\eta+z\zeta+t\tau)}.$$

This analysis may be carried through in a completely invariantive way. It is probably this invariantive spectrum theory which is to be taken as the true substratum of the coherency-matrix theory, for in the case where the functions in terms of which the analysis is made are trigonometrical in the time, the result of this three-dimensional-spectrum analysis coincides precisely with the array in space Eigenfunktionen and time frequencies which we obtain by reducing to diagonal form the time coherency matrix of the function analyzed.

HARMONIC ANALYSIS AND THE QUANTUM THEORY.

DISCUSSION BY

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I SHOULD like to make a few specific remarks concerning Professor Wiener's interesting paper and the subject—the place of Harmonic Analysis in quantum theory—with which it deals. In the first place, Professor Wiener has shown how one can obtain coherency functions or matrices from solutions $f(x, y, z, t)$ of Schrödinger's equation by taking the coherency with respect to the time t or the Kaluza-Klein fifth dimension, considering the remaining variables as parameters or indices of a three or four parameter family of functions. But we could also take the coherency with respect to any or all of these remaining variables by repeating the process an appropriate number of times. I shall briefly indicate the result of taking the coherency with respect to all four variables occurring in Schrödinger's function and give its quantum-mechanical interpretation for the case of free motion of a particle.

I first associate with f the function

$$\varphi(\xi, \eta, \zeta, \tau) = \lim_{X\to\infty} \frac{1}{2X}\int_{-X}^{+X} dx \cdots \lim_{T\to\infty} \int_{-T}^{+T} f(x+\xi, \cdots, t+\tau)\bar{f}(x, \cdots, t)dt$$

(assuming here and in the following that f is so constituted that the various processes involved are possible and have a meaning) by averaging over each of the four variables in turn. If f be the sum of complex exponential functions φ will also be such a sum, but its coefficients will be the squares of the absolute values of the corresponding coefficients of f. Taking the coherency of $\varphi(\xi, \eta, \zeta, \tau)$ with respect to τ we have

$$r(\xi, \eta, \zeta : \lambda') = \int_{-\infty}^{+\infty} \varphi(\xi, \eta, \zeta, \tau)\frac{(e^{-i\lambda\tau} - e^{-\tau})^2}{i\tau}d\tau.$$

But I shall deal with the difference

$$2\pi r(\xi, \eta, \zeta : \lambda'; \lambda'') = r(\xi, \eta, \zeta : \lambda'') - r(\xi, \eta, \zeta : \lambda')$$

of two such r's instead of with the original r itself, i.e., in Professor Wiener's terminology, with the energy between the two frequencies λ', λ'' instead of with the total energy up to frequency λ''. I then perform the analogous process for each of the variables ξ, η, ζ in turn by introducing variables α, β, γ in place of them in the same way that τ was replaced by λ; the result is a coherency function $r(\alpha', \beta', \gamma', \lambda'; \alpha'', \beta'', \gamma'', \lambda'')$ depending on eight parameters. A set of functions f_n would in the same way give rise to an Hermitean coherency matrix $\|r_{mn}\|$. On applying this to the function

$$f(x, y, z, t) = \sum a(\alpha, \beta, \gamma, \lambda) e^{i(\alpha x + \beta y + \gamma z + \lambda t)},$$

where the summation is taken over a range of values of the parameters α, β, γ, λ (integration if the range is continuous) we find

$$r(\alpha', \beta', \gamma', \lambda'; \alpha'', \beta'', \gamma'', \lambda'') = \sum |a(\alpha, \beta, \gamma, \lambda)|^2,$$

where the sum (or integral) is taken over all values of α, etc., between α', α'', etc.

Consider now the case in which such an f is a solution of Schrödinger's equation

$$\nabla^2 f + \frac{4\pi i \mu}{h} \frac{\partial f}{\partial t} = 0.$$

The parameters are then subject to the restriction

$$\lambda = -\frac{h}{4\pi\mu}(\alpha^2 + \beta^2 + \gamma^2) \qquad \left(= -\frac{2\pi}{h} E \right).$$

The term of f having these parameters represents the de Broglie wave for a free particle with total energy $-\lambda h/2\pi$ and direction determined by the ratios $\alpha : \beta : \gamma$. The (normalized) amplitude $a(\alpha, \beta, \gamma, \lambda)$ is interpreted as the probability that the particle will be in this state. Our coherency function $r(\alpha', \beta', \gamma', \lambda'; \alpha'', \beta'', \gamma'', \lambda'')$ is consequently the probability that the particle have energy E lying between $-\lambda'' h/2\pi$ and $-\lambda' h/2\pi$ and direction $\alpha : \beta : \gamma$

between $\alpha' : \beta' : \gamma'$ and $\alpha'' : \beta'' : \gamma''$. It seems likely that the extension of these ideas to other sets of orthogonal functions, as suggested by Professor Wiener in the latter part of his paper, would serve an analogous purpose for general quantum-mechanical systems.

I should also like to call attention to the connection between Professor Condon's discussion of momentum space and the theory of harmonic analysis. Expressed in the terminology employed by Professor Weyl in his paper, Professor Condon introduces in the unitary system-space $\mathfrak{R}$ a coördinate system in which the operator $\mathbf{p}$ representing the momentum is referred to principal axes, in which case the operator $\mathbf{q}$ may often be represented by $-\frac{h}{2\pi i}\frac{\partial}{\partial p}$. This coördinate system must be related to that used by Schrödinger (in which the principal axes of the operator $\mathbf{q}$ are taken as coördinate axes) by a unitary transformation of $\mathfrak{R}$. It is in fact readily shown—by finding the principal axes of the operator $\frac{h}{2\pi i}\frac{\partial}{\partial q}$ in Schrödinger's coördinate system—that the components $\psi(p)$ of the vector Ψ defining the state in the former coördinate system are related to its components $\psi(q)$ in the latter by the transformation

$$\psi(p) \sim \int e^{-(2\pi i/h)pq}\psi(q)dq$$

and that the operator $\mathbf{q}$ becomes $-\frac{h}{2\pi i}\frac{\partial}{\partial p}$. In other words —and this is the connection to which I wish to call attention —the function $\psi(p)$ is the Fourier transform of $\psi(q)$. In such cases the equation obtained by Professor Condon for $\psi(p)$ is the equation of the Fourier transform of $\psi(q)$. Darwin has in fact employed the inverse of this transformation in the quantum-mechanical discussion of certain simple types of motion.

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The Effects of War Gases. The Chemical Warfare School of the Edgewood Arsenal, Maryland, has prepared a pamphlet comprising a comparative study of gas and other weapons in the recent war. The pamphlet was prepared by Colonel H. L. Gilchrist, U. S. Army, Chief of the Medical Division, Chemical Warfare Service, issued from the Government Printing Office.

The question of the permissibility of poisonous gases in war was discussed many years ago without any definite decision. In the Hague Convention of 1899 action against the use of such gases was proposed. It was pointed out, however, that all explosives used in shells produce more or less dangerous products and it was suggested that action be taken prohibiting the use of any apparatus for the sole purpose of distributing asphyxiating gases. This agreement was signed by all the nations represented except the United States and Great Britain. At the convention of 1907 Great Britain signed, so that when the World War broke out the United States was the only nation of any importance that had refused to prohibit this method of warfare. The Hague Convention provides that if a belligerent takes an ally not signatory to any agreement the opposing parties are released.

When the United States went into the war Germany would have been permitted to use poisonous gases but Germany anticipated this with some very serious results to the allied forces. It can be shown, that it may have been fortunate that the Germans did anticipate in this matter, but this phase need not be discussed here. The pamphlet above noted gives a large amount of valuable information concerning the effects of poisonous gases as compared with other methods of attack. It seems clear that, notwithstanding all that has been said about these gases they represent a less disastrous ultimate effect than the other methods of warfare. Unfortunately the effects have been made responsible for a large amount of illness in subsequent years with which, in reality, the gassing had no concern. In the very large majority of cases the diseases which have appeared in men gassed during the war are due either to the natural decline of health in advancing years, improper habits of life, or causes commonly operative in the transmission of disease.

H. L.

THE KINETIC METHOD IN STATISTICAL MECHANICS.

BY

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So much has been said to-day about the statistical interpretation of quantum-mechanics, that perhaps one should like to see now quite clearly, how the fundamental postulates of the ordinary kinetic theory and statistical mechanics,[1] especially their connection with thermodynamics, are "explained" by the quantum theory. In principle this is nowadays certainly possible,[2] I think, though I am not able to show you exactly how. Therefore only a few remarks, especially on the so-called kinetic method.

We can distinguish in the kinetic theory *two* fundamental methods:

a. The statistical method; the equilibrium state is described as the most probable or as the mean state. The range of application is about the same as of ordinary thermodynamics,[3] because it deals principally with equilibrium states, with the exception [4] that it can explain also the *fluctuation-phenomena* around an equilibrium state.

b. The kinetic method or *method of collisions.* Here we try to explain the macroscopic properties by considering more precisely *the interaction* between the particles (atoms, light quanta, etc.), out of which our system is composed. It is the only method, for systems *not* in an equilibrium state, for finding the *rate* of physical and chemical processes. Especially it gives, in principle, by means of the H-theorem of Boltzmann, something about the velocity with which the equilibrium state is reached. A disadvantage is, that it

[1] Compare for instance my dissertation, Over statistische methoden in de theorie der quanta (The Hague, Nyhoff, 1927).

[2] Compare J. v. Neumann, *Gött. Nachr.*, 1927, p. 273.

[3] Partly, it is simply another language.

[4] This is, perhaps, not exactly true, in the sense that by a slight reformulation of the second law of thermodynamics one can also account for the fluctuation-phenomena. Compare L. Szilard, *ZS. f. Phys.*, v. 32, p. 753, 1925.

cannot account for the fluctuation-phenomena, at least not in a general way.

I think that quantum-mechanics will combine these two, now almost distinct, methods by sharpening the kinetic method, using statistical considerations.

In the kinetic method we can deduce, at least formally, all the thermodynamic laws by the following assumptions: [5]

1. The "Stoszzahlansatz": the number of collisions per sec. of particles in the quantum state i with particles in the state j, where k and l are the states after collision, is:

$$A_{ij}{}^{kl} = a_{ij}{}^{kl} n_i n_j (1 + \theta n_k)(1 + \theta n_l).$$

$a_{ij}{}^{kl}$ is a sort of transition probability, and

$\theta = -1$ for a Fermi-Dirac-gas,

$\theta \doteq 0$ in the classical case,

$\theta = +1$ for a Bose-Einstein-gas.

2. The principle of microscopic reversibility: [6]

$$a_{ij}{}^{kl} = a_{kl}{}^{ij}.$$

Introducing a generalized H-function of Boltzmann by [7]

$$H = \sum_i \{n_i \log n_i - \theta(1 + \theta n_i) \log (1 + \theta n_i)\},$$

we can show that $(dH/dt) < 0$ until the equilibrium-state is reached, which is determined by the *functional equation*

$$\bar{n}_i n_j (1 + \theta n_k)(1 + \theta \bar{n}_l) = \bar{n}_k n_l (1 + \theta \bar{n}_i)(1 + \theta \bar{n}_j),$$

where the bar denotes the values in the equilibrium state. From this follows immediately, when we take into account only the conservation of energy during each collision (no translational motion of the whole gas):

$$\log \frac{n_i}{1 + \theta n_i} = -\beta \epsilon_i + \log A$$

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[5] This is for a single gas. Compare P. Jordan, *ZS. f. Phys.*, v. 41, p. 711, 1927; L. S. Ornstein and H. A. Kramers, *ZS. f. Phys.*, v. 42, p. 481, 1927; L. Nordheim, *Proc. Roy. Soc.*, series A., v. 119, p. 689, 1928.

[6] See the discussion in R. C. Tolman, Statistical Mechanics (New York, The Chemical Catalog Company, 1927), chapter 15.

[7] The entropy is always given by $S = -kH$.

or

$$n = \frac{1}{\frac{1}{A} e^{\beta \epsilon_i} + \vartheta}.$$

It is easy to generalize this for mixtures, to find the analogue of the transport equation of Boltzmann, and so on.[8]

I wanted further only to point out that, in the case of the interaction of light and fixed atoms, all these formal assumptions are beautifully explained by the theory of Dirac.[9] Especially the principle of microscopic reversibility is there "proved" by showing that $a_{ij}{}^{kl}$ is equal to the square of the absolute value of an Hermitean matrix.

With the help of this we can see also more precisely how fast the equilibrium state (Planck's radiation formula) is established. As is well known, Planck always mentioned that we need "a little piece of coal dust" to establish the equilibrium, when we have the radiation in a vessel, of which the walls are perfect mirrors. We may ask now, how big this piece of coal dust has to be?

First it is clear that we need some matter, because light waves or light quanta have no way to interact, will therefore not "collide," or, we may say, are perfectly penetrable for each other. Let us therefore take first one free electron. One sees easily that, when we consider only the Compton-scattering, this will not establish the equilibrium.[10] Because then, also, the *number* of light quanta is conserved (besides their energy), and from the statistical treatment it follows that we can only get

$$\bar{n}_i = \frac{1}{\frac{1}{A} e^{\beta \epsilon_i} - 1}$$

and not Planck's formula, for which $A = 1$. We have to

[8] See L. Nordheim, *Proc. Roy. Soc.*, series A., v. 119, p. 689, 1928.

[9] P. A. M. Dirac, *Proc. Roy. Soc.*, v. 114, p. 243, 1927; v. 114, p. 710, 1927. Compare also the treatment in H. Weyl: Gruppentheorie und Quantenmechanik (Leipzig, S. Hirzel, 1928), § 20 and 44. For the extension to an electron gas, see esp. P. Jordan and E. Wigner, *ZS. f. Phys.*, v. 47, p. 631, 1928.

[10] P. Jordan, *ZS. f. Phys.*, v. 33, p. 649, 1925. W. Bothe, *ZS. f. Phys.*, v. 23, p. 214, 1924.

determine A from the condition that

$$\sum_i \bar{n}_i = N.$$

The function $N(A)$ will be zero for $A = 0$, and infinite for $A = 1$, and is always curved toward the A axis. It is clear, that the electron will *very slowly* make N infinite, namely by triple and higher collisions. It is therefore not a very effective "piece of coal dust." Neither is one Hydrogen-atom, as we see from Dirac's treatment, because that can only establish Planck's law for its own discrete frequencies; at least its effectiveness for other frequencies must be very slight, since here again collisions of higher order are necessary. So that we may say, perhaps, that we really need a piece of crystal, or at least some material system with a practically continuous energy spectrum, to establish the equilibrium *rapidly*.

In conclusion, I may say, perhaps, that it seems to me better now to stress more the *difference* between light quanta and electrons, than their similarity. I have the impression that light is still more "wavy" than electrons, for the following reasons:

a. For light quanta holds the Bose-Einstein statistics which is equivalent to quantized eigenvibrations; for electrons we have the Fermi-Dirac statistics, which cannot be translated in a natural way into a three-dimensional wave language.

b. To find the coupling terms between light and matter we use always the classical expressions following from the electro-magnetic *wave*-theory of light and from the *corpuscular* theory of matter (esp. Coulomb's law).

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NOTES FROM THE U. S. BUREAU OF STANDARDS. *

CHROMIUM PLATED LABORATORY WEIGHTS.

THE hardness of chromium plating is a considerable advantage in the case of weights that are much used, but this advantage is somewhat offset by the susceptibility of such a coating to attack by fumes of hydrochloric acid. However, it seemed worth while for the bureau to try this plating on three sets of weights which have been put into use in the chemistry division. In preparing these sets it was found that great care is needed in washing the plating liquid out from under the knob. If this is not done the weights vary with changes in atmospheric humidity, as is the case with gold-plated weights. Some indication of the degree of care needed was given by the fact that one of the three sets was at first too variable to warrant regular certification, although a careful laboratory assistant, familiar with electroplating work, thought that he had washed this set thoroughly, and just as carefully as the other two sets.

As in the case of gold-plated sets, it was found that this chromium-plated set could readily be made constant by proper boiling and rinsing.

QUALITY OF PURIFIED WOOD FIBERS.

THE following is a summary of a report on purified wood fibers presented at the annual meeting of the Technical Association of the Pulp and Paper Industry, held in New York on February 19.

As a result of an investigation in progress at the bureau, it is indicated that certain types of highly-purified wood fibers, which are commercially obtainable at the present time, are suitable for conversion into high grade bond and permanent record papers, as well as any other types of papers where the qualities of durability and permanence are essential.

* Communicated by the Director.

Such papers have hitherto been made only from the best grades of rag fibers, which are considerably more expensive than the purified wood fibers.

Pure cellulose, commonly called alpha cellulose, has a high degree of permanency. The cotton fiber is the purest form of cellulose found in nature. For this reason papers carefully prepared from high grade cotton rags have hitherto been used exclusively where permanence extending over hundreds of years is desired. The ordinary wood fibers, even those of the best grades, have impurities and degraded forms of cellulose present which seriously affect their permanence. By a series of chemical treatments, which have been developed by a pulp and paper manufacturer, these objectionable impurities are removed, leaving a fiber similar in its chemical composition to the cotton fiber and having the desired paper-making characteristics.

In order to find out how the purified wood fibers would endure as compared with other commonly-used paper-making fibers, tests are being made of the various types of paper-making wood fibers and of several grades of rag fibers. These tests include chemical purity, whiteness, and microscopical structure. A thorough investigation of the strength and durability of a series of representative commercial papers prepared from fibers similar to the above is also being made.

The relative durability of the various paper-making fibers is studied by means of accelerated aging tests, whereby a few hours' treatment simulates the effect of natural aging over a long period of years. The samples are baked in dry form at 100° C., are cooked with steam, and are exposed to intense light rays from an artificial sun. After all these severe treatments, the samples are again subjected to searching tests to find how much they have deteriorated both chemically and physically. In all cases, while the ordinary wood fibers show the expected rapid deterioration, the purified wood fibers withstand the aging tests practically as well as the highest grade of rag fibers.

While further study is being made, it would appear that fibers approaching closely to cotton fiber in paper-making value can be made from materials having originally much less value than cotton. This development is similar to that

of rayon and, in fact, is an off-shoot from a study of the utilization of wood fiber for making rayon.

CORNSTALKS FOR PAPER MAKING.

THE manufacture of paper from cornstalks is a matter of great interest to the Bureau of Standards, as an important phase of the general problem of utilization of waste products from the land.

For the past two years, the bureau has been coöperating with Iowa State College in producing a cornstalk insulating board, on a semicommercial scale. While the pulp used in making this board is of a different quality from that required for paper, the work has yielded rather definite information on the cost of cornstalks and on the nature of the cornstalk as a source of fibrous material. In connection with this work, an economic survey was made, which indicates that cornstalk pulp may now be able to compete with wood pulp if it can be made of satisfactory quality.

Previous investigators have shown that paper made wholly of cornstalks is so hard and harsh as to be objectionable for many purposes. However, most papers now on the market are made of blended pulps, such as sulphite and ground wood, sulphite and rag, etc. It would seem that cornstalk pulp might be used as an ingredient in such a blend, and that its characteristic harshness may even be used to advantage in making a paper of different properties from any now on the market. To work out such a blend, when the experiments are made on a full commercial scale, is both laborious and expensive. The newsprint used in printing the Sunday Commercial News of Danville, Illinois, December 16, 1928, is the result of one such experiment. This paper contains 55 per cent. cornstalk pulp, which is evidently too much, for the paper is too harsh to be satisfactory as newsprint.

Another well-known characteristic of cornstalk pulp is its "dirtiness." The cornstalk contains several kinds of fibers, which offer different degrees of resistance to being reduced to pulp. If the stalks are cooked but slightly, the more difficultly reduced fibers remain in a raw condition and show up as brown flecks in the paper. If the stalks are cooked hard,

the more easily reduced fibers are gelatinized, and the drying of these spots of jelly produce "shiners" in the papers. The paper used in printing the book "Farm Wastes in Industry" by G. M. Rommel, contains 25 per cent. of cornstalk pulp, and shows these flecks quite plainly. While they are not particularly objectionable in book paper, pulp containing them could not be used at all in making writing paper.

There are thus two technical problems to be solved in making cornstalk paper; finding out how to cook cornstalks to make a clean pulp, and finding out how to blend cornstalk pulp with other pulps to make papers of the desired qualities.

Private capital is making an attempt to solve these problems. In view of the obvious importance of the project to both the farmers and the paper industry, it is believed that the Government is justified in coöperating with private industry in giving such assistance as it can in the solution of these two technical problems. Work on the project has already been started at the bureau, and will be continued intensively during the next fiscal year.

Since both of these technical problems will affect the cost and selling price of cornstalk paper, the economic problem must await the development of further information.

STRENGTH OF SAND-LIME BRICK.

THE bureau, in the course of its investigation of the properties of sand-lime brick, has accumulated considerable data on the strength of the brick. However, the majority of tests of the commercial product have been made on small lots, usually five or ten brick from each manufacturer, as required by the specifications. It was considered desirable to obtain additional information on the strength of the brick, and particularly on the variation in strength of the product produced by an individual manufacturer. Consequently, 25 manufacturers have supplied 50 brick each, representing their regular production, for use in these tests.

After drying to constant weight, the absorption by total immersion in cold water was obtained at the five minute, five hour, and 24 hour periods. The absorption on boiling for five hours was also determined. Ten of each group of 50

brick were then selected, using care to obtain a range of absorption, and these ten were reserved for freezing tests, to determine the weathering properties of the brick. The other 40 were dried, and broken transversely to determine the modulus of rupture. The compressive strength of each half of each brick was then determined, one-half having been tested on edge, the other half tested flat. The results obtained thus far show that the absorption ranged from 13.7 to 21.6 per cent. of the weight of the dry brick; the modulus of rupture from 330 to 730 lbs./in.^{2}; the compressive strength flat, from 2,150 to 4,470 lbs./in.^{2}; and the compressive strength on edge, from 1,500 to 3,850 lbs./in.^{2}

COMPRESSIVE AND TRANSVERSE STRENGTH OF BRICK.

THE April issue of the *Bureau of Standards Journal of Research* will contain a paper reporting the compressive strength flat and on edge, and the transverse strength of 27 makes of bricks covering a range of conditions in method of manufacture and degree of burning. The attempt is made to correlate the variation in ratios of these different measures of strength with the various structural features of the brick.

The following conclusions apply to the data thus presented:

1. The tendency of soft mud brick is to give higher unit strengths when tested on edge than when tested flat.

2. The "compacting effect" on the structure of the edge-set brick by the superimposed weight of the other bricks in the kiln is offered as a tentative explanation of the tendency toward higher strength on edge.

3. This tendency toward higher strength on edge is overcome by laminar and cracking structure in the case of certain bricks.

4. Soft mud brick tend to display less deviation in the ratio modulus of rupture to flat compressive strength than any of the other methods of manufacture, but even with these the ratio ranged between 0.13 and 0.26.

5. Auger lamination in end-cut brick appears to be associated with high ratios for modulus of rupture to flat compressive strength.

6. Die lamination in side-cut brick appears to be associated

with low ratios for modulus of rupture to flat compressive strength.

7. The ratio of the compressive strength of brick tested flat to the compressive strength of brick tested on edge ranges from 0.74 to 2.3.

8. In view of the variation in the ratio of the compressive strength of brick tested flat to the compressive strength when tested on edge, there exists no general rule for converting values from one kind of test to the other kind.

9. The ratio of the modulus of rupture to flat compressive strength ranged from 0.426 to 0.070.

10. In view of the variation in the ratio of the modulus of rupture to compressive strength when tested flat the possibility of inferring a compressive strength from a transverse test or vice versa is open to very large errors for any given make of brick.

CAST STONE.

THE bureau has determined the absorption of numerous samples of cast stone. In connection with this work it has been suggested that the absorption may be due to a low dry weight of the sample caused by partial dehydration of some of the colloidal material present in the stone when the latter is dried at temperatures as high as 110° C. It is also possible that high absorptions may result from boiling the sample for the total absorption, thus producing continued hydration of some of the cement which has not set.

With these possibilities in mind, the following method of absorption was tried. Cores were cut from each of twelve typical samples. These were maintained at 35° C. in dry air until a loss of not more than 0.05 per cent. in 4 hours was obtained. The specimens were then totally immersed in 300° mineral seal oil and the absorption noted over a period of 84 hours. Then the specimens and oil were placed in a closed chamber, the pressure reduced and maintained at 4 mm. mercury for 5 hours. The absorptions were then calculated to the equivalent of water as per cent. of dry weight of specimens. The specimens absorbed from 1.6 to 9.3 per cent. in 48 hours, one specimen being below 2 per cent. After reducing

the pressure to 4 mm. the absorptions ranged from 2.2 to 13.4 per cent.

Thus, even under conditions where the possibility of dehydration is reduced and the rehydration eliminated, cast stone does absorb a considerable quantity of water and the absorption as generally carried out, using water, is a fair measure of the pore space of the stone.

REFRACTORIES INVESTIGATION.

REFERENCES to data of interest which have developed as a result of the study of fundamental properties of fire-clays and fire-clay bricks have appeared from time to time in the Technical News Bulletin, the most recent reference having been made in the August, 1928, issue. A progress report, giving in detail the complete method of procedure followed in this study and the results of tests, have been prepared for publication at early date. A brief summary of the most important results is here given.

Thermal expansion data determined on the fire-bricks "as received" from the manufacturer and after reheating in the laboratory kilns at 1,400, 1,500, and 1600° C. show that both the rate of expansion and total expansion of the majority of bricks decrease as the temperature of firing is increased. These data also indicate that the thermal expansion behavior of the finished bricks may not necessarily be the average rates of expansion of the clays entering into their manufacture.

The modulus of elasticity of the bricks increased greatly at 550° C. when compared to values obtained in tests made at 20° C. At 1000° C. the modulus of elasticity decreased greatly, i.e., the bricks became less rigid and more subject to plastic deformation. The modulus of elasticity and transverse strength determined at room temperature increased as the temperature of firing of the bricks was raised. Resistance of fire-bricks to spalling decreased with increase of modulus of elasticity and increase of thermal expansion.

Data were obtained which show that sufficient differences exist in the physical properties of bricks of the same brand to cause large variations in the number of quenchings required to cause spalling.

An empirical relation of the modulus of rupture, transverse strength, coefficient of expansion, and total grog content of the brick is given, which results in a factor expressing the relative resistance of the various bricks to spalling in the water quenching test.

STANDARD SAMPLES.

A STANDARD sample of opal glass of the following composition is now ready for distribution: Silica 67.5 per cent.; calcium oxide 10.5 per cent.; sodium oxide 8.4 per cent.; potassium oxide 3.2 per cent.; alumina 6.0 per cent.; fluorine 5.7 per cent.; arsenic trioxide 0.2 per cent.; arsenic pentoxide 0.1 per cent.; and ferric oxide 0.08 per cent. The price of this glass is $2 per sample of 45 grams. If remittance does not accompany the order the samples will be sent by parcel post C. O. D. to addresses in the United States.

STANDARD VISCOSITY SAMPLES.

THE bureau has for sale several types of oil samples (6 petroleum and 1 castor oil) for use in calibrating viscometers, ranging in viscosity from 20 to 8,000 centipoises, between temperatures of 15 to 30° C. The charge is $5 per 1-quart sample, which includes a statement of the exact viscosity (within ± 0.5 per cent.) at any one temperature at the time of purchase. Additional viscosities will be charged for at the rate of $5 for temperature.

Remittance payable to the "Bureau of Standards" must accompany order. Samples will be forwarded via express collect to addresses in the United States. Orders from foreign countries should include an amount to cover parcel post for a total weight of 5 pounds including packing.

NOTES FROM THE RESEARCH LABORATORY, EASTMAN KODAK COMPANY.*

EFFECT OF ENVIRONMENT ON PHOTOGRAPHIC SENSITIVITY. I.[1]

S. E. Sheppard and E. P. Wightman.

THE reversible pH effect upon the speed of emulsions as observed by Rawling and Rawling and Glassett has been investigated. Aged plates, treated by bathing, give small and uncertain effects. Fresh emulsions appear to show the effect more definitely. Considerably more work is required before the nature of the effect can be definitely established. The possible causes appear to fall in four classes: (1) An action on silver bromide; (2) an action on silver nuclei; (3) an exaltation of a bromine acceptor function of the silver sulfide nuclei during exposure as suggested by K. C. D. Hickman; and (4) a reversible effect on gelatin.

PHOTOGRAPHIC CHARACTERISTICS OF MOTION PICTURE STUDIO LIGHT SOURCES.[2]

L. A. Jones and M. E. Russell.

THE fundamental relationships which determine the visual and photographic efficiencies of light sources are discussed briefly. A direct experimental method of measuring relative photographic efficiencies is then described. Numerous data are given relating to all of the light sources commonly used in the motion picture studio, including those introduced recently, such as the high efficiency Mazda lamps, white, orange, and yellow flame carbons, and the mercury vapor are in combination with a new gaseous conductor neon lamp. The

* Communicated by the director.

[1] Communication No. 349 from the Kodak Research Laboratories and published in *Phot. J.* **69**: 22 (1929).

[2] Communication No. 358 from the Kodak Research Laboratories and published in *Trans. Soc. Mot. Pict. Eng.* **12**: No. 34, 427; *Phot. Ind.* **26**: 1051–1077 (1928).

problem is considered both from the standpoint of relative photographic candle power of the light source unmodified by selective reflection or absorption, and also from the point of view of their relative merit when it is desired to obtain correct orthochromatic rendering of colored objects. Values relating to the photographic efficiency of these light sources are obtained by using three types of film representing the three typical classes from the standpoint of color sensitivity.

THE LATTICE ENERGIES AND PHOTOCHEMICAL DECOMPOSITION OF THE SILVER HALIDES.[3]

S. E. Sheppard and W. Vanselow.

CONSIDERATION of the energy steps in the Born cycle for calculating the lattice energies of the silver halides leads to the following conclusions: 1. The photolysis of solid silver halide might occur directly to give halogen and metallic silver. This requires only a quantum at 800 mμ or longer, equivalent to the heat of formation of the halides from the elements. 2. This photolysis might occur indirectly, by way of separation of electrons from halide ions, followed by acceptance of the electrons by silver ions. This course would require a quantum at $\lambda = 1300$ Å., or beyond, to disrupt the lattice; then a quantum at $\lambda = 3000$ Å., or beyond, to liberate electrons. In subsequent reactions of electron acceptance, *etc.*, energy might be freed, either as radiation quanta or by radiationless collisions, so that the final difference of energy equals the heat of formation. In this case primary absorption of two quanta appears necessary at thresholds much higher than those known to be operative in the photochemical decomposition of the silver halides. The contradiction might be dismissed by abandoning the view that the photolysis involves the intermediate liberation of photoelectrons from halide ions—a hypothesis suggested independently by K. Fajans and S. E. Sheppard. The phenomena of photoconductance definitely point to a relative freeing of photoelectrons. F. C. Toy has shown (*Nature*, Sept. 24,

[3] Communication No. 360 from the Kodak Research Laboratories and published in *J. Phys. Chem.* **33**: 250 (1929).

1927) that the discrepancy between the photoconductance absorption spectrum of the silver halides and the spectrum photographically active is removed by consideration of the thickness factor. The authors have recently obtained independent evidence of the liberation of photoelectrons, concomitant with the production of free halogen, in a study of the photopotentials of silver—silver halide electrodes in electrolytes. The difficulty of the energy quanta required cannot be dismissed. The experimental evidence obtained with these cells led to a modified theory of the inner photoelectric effect which is consistent with Sheppard's orientation theory of photographic sensitivity and latent image formation. By considering that both the lattice energy and electron affinity are lowered at interfaces, particularly at true interfaces with conductors, it seems possible for the inner photoelectric effect to occur at the shorter wave-lengths.

THE SUPERMOLECULAR STATE OF POLYMERIZED SUBSTANCES IN RELATION TO THIN FILMS AND INTERFACES.[4]

S. E. Sheppard, A. H. Nietz, and R. L. Keenan.

THE compounds, gelatin, rubber, and cellulose esters, may be represented by the formulas: rubber, $(C_5H_8)_n$; cellulose, $(C_6H_{10}O_5)_n$; protein, $(NHCHRCO)_n$; where n indicates both an unknown magnitude and quality or kind of aggregation.

The constitutions of such substances are well contrasted between the "association" theory of Hess and Pringsheim and the "macromolecular" theory of Staudinger.

A study of the action of these high molecular bodies in thin films and at interfaces is in agreement with Staudinger's conception of extended chains of atoms held by homopolar primary valencies. The remaining interactions, *i.e.*, orientations and condensations of these chains, are effected by development of alternating polarity along the chains and consequent mutual induction. The binding forces in this case (between the chains) may vary from Van der Waal's forces to electrostatic constraints. Under the influence of

[4] Communication No. 361 from the Kodak Research Laboratories and published in *Ind. Eng. Chem.* 21: 126 (1928).

dissolvents, the macromolecular state may be brought to incipient dissociation—a state comparable with that postulated by the associationists.

A STILL FOR LIQUIDS OF HIGH BOILING POINT.[5]

K. C. D. Hickman.

SCARCELY volatile organic liquids which decompose on ordinary vacuum distillation can be made to volatilize and condense without harm if the apparatus is suitably shaped. The hot liquid surface is maintained without the hindrance of intervening pipes within a few centimeters of the cooling surface and vacuum is secured by a Hyvac pump. Benzyl citrate, castor oil, and fatty acid esters have been distilled.

A SUBLIMATION MERCURY STILL.[6]

K. C. D. Hickman.

DISTILLATION of mercury does not remove noble metal impurities. A three-stage distillation gives sufficient separation, however, to postpone washing the mercury residues to infrequent occasions. The still is made from three interconnected alembics supported at barometric height above four reservoirs. Mercury is transferred by sublimation through a torricellian vacuum three times in its passage from the soiled to the clean reservoir. The still is electrically heated and automatic in action.

SOME EXPERIMENTS WITH BINOCULAR AND MONOCULAR VISION.[7]

E. M. LOWRY.

AN account is given of a series of investigations, in which both binocular and monocular vision were used, of the bright-

[5] Communication No. 368 from the Kodak Research Laboratories and published in *J. Opt. Soc. Amer.* 18: 69 (1929).

[6] Communication No. 369 from the Kodak Research Laboratories and published in *J. Opt. Soc. Amer.* 18: 62 (1929).

[7] Communication No. 380 from the Kodak Research Laboratories and published in *J. Opt. Soc. Amer.* 18: 29 (1929).

ness sensibility of the retina. A new type of visual sensitometer is described which enables the operator to determine the brightness functions of the eye in a variety of ways. The instrument consists of a hemispherical bowl, one meter in diameter, equipped with an illuminating system which will give brightnesses from 0.01 to 600 millilamberts. This bowl serves as the sensitizing field. A mechanism is provided by means of which the threshold for brightness may be determined at any adaptation level. Contrast sensibility may also be measured at any brightness as well as the effect on threshold and contrast sensibility of introducing various objects into the visual field. In addition, monocular and binocular acuity may be determined by using the Ives form of acuity meter. Data and curves are given to represent the results secured for several observers. Emphasis is laid on the necessity of determining the various functions of the eye binocularly if they are to be correctly interpreted in practice.

The Rational Utilization of Coal. This important industrial problem has been recently discussed at some length at a meeting of the Northeast Coast Institution of Engineers and Ship Builders, of Newcastle-on-Tyne. Papers were read by Mr. W. J. Drummond and Dr. W. T. K. Braunholtz.

A most interesting and important point is the account in Drummond's paper of the manner in which Germany has recovered its control of the fuel necessary for its industries. The peace treaty took the coal mines, assigning the Upper Silesian to Poland and the western to France. After careful investigation, the lignite deposits within German territory, which have long been considered as of little importance for power purposes, have been utilized efficiently and Germany is now using these deposits as one of the chief sources of power production. The uneconomic result of the peace treaty is shown by the fact that Poland has no large use of coal within its borders and, therefore, has to seek export methods if the mines are to be worked. As the transit to shipping ports covers about 400 miles of railway, it has been necessary to subsidize the railroad largely in order to enable the mine owners to compete with the outside sources of coal.

The papers also describe in much detail the many methods of what may be called the indirect use of coal, among which is the low-carbonization procedures, which seem to promise furnishing a smokeless fuel of high heating power. Pulverized coal is also discussed and the obtaining of fuel-oils and use of the gas produced by the low temperature methods. It is noted that soft coal is now considered as consisting of essentially four heat-producing proximate principles: fusain, durain, clarain, and vitrain. The approximate compositions and qualities of these are set forth.

H. L.

NOTES FROM THE U. S. BUREAU OF MINES.*

TWENTY YEARS OF EXPLOSIVES TESTING.

THE Pittsburgh Experiment Station of the United States Bureau of Mines recently marked the twentieth anniversary of the official opening of that station, and coincidentally of the initiation of the work of testing explosives for permissibility for use in coal mines.

As the testing of explosives for permissibility was an entirely new undertaking and one which involved passing on the merits of the products of established manufacturers, there were some apprehensions as to its outcome, states Dr. Charles E. Munroe, chief explosives engineer of the Bureau of Mines. These doubts were speedily removed through such prompt and active cooperation of the leading explosives manufacturers that, within a year from the date of the opening, it was announced, through the published lists, that 31 different explosives, made by 11 different manufacturers, had passed the tests and were approved for use in blasting in coal mines.

The interest shown by the coal mine operators was equally gratifying, for, almost immediately, the substitution of permissible explosives for those previously employed in mining coal began (cautiously at first since they must needs be used in a definitely prescribed manner to which the miner must become habituated) and they met with such acceptance that during the year 1912 the total sales of Permissible Explosives in the United States amounted to 24,630,270 pounds, of which 18,150,618 pounds were used in coal mines. From then on there has been a general annual increase in the quantities of permissibles sold (though naturally fluctuating with the fluctuations of the coal mining industry as affected by war or labor conditions) until, in 1926, the quantity of permissibles used in the United States was, during that year, 67,685,416 pounds, of which 65,143,319 pounds were used in coal mines. Owing to conditions in the coal mining industry, through which

* Published by permission of the Director, United States Bureau of Mines.

there was a stoppage of mining in many mines, the total amount of permissibles used in 1927 fell to 63,847,161 pounds and the indications now are that the figure for 1928 may not differ greatly from that last mentioned.

When the testing of explosives for permissibility was under consideration it was known that there were many factors involved about which little or no precise knowledge existed. It was true that for upwards of a quarter of a century prior to 1908 the governments of several countries, and notably those of England, France, Belgium and Germany, had been testing explosives for use in coal mines, but there was no agreement among them as to details of methods or criteria of safety, though the procedure of each tended to detect and eliminate the most unfit of the explosives which had been used or were offered for use.

Consequently in testing for permissibility it was determined to conform so far as possible, both in materials used and in methods, to the actual practices followed and conditions which then obtained in coal mining in the United States. A notable example of this was the decision to use *natural* gas rather than *manufactured* gas in all those tests in which gas was to appear as a factor, since natural gas was the particular gas encountered in American coal mines. And there were other differences from foreign testing practices which were quite as radical.

To make sure the procedure adopted was justified and to gain more precise knowledge as to the characteristics, behaviors and best use of explosives the Explosives Division of the Bureau of Mines has constantly been engaged during these twenty years in researches on these problems at the testing station, in the laboratory and in the field, the results of these researches having been embodied in many publications and, as applied, they have led to many changes in procedure in testing and use and in the compositions of permissible explosives.

Because of these, the general progress in the explosives art, and the necessity of finding substitutes for important components owing to shortages growing out of war-time and trade conditions, during these twenty years, 633 different explosives have been submitted for tests for permissibility, of

which 372 passed. For various reasons 252 of those which passed have been removed or transferred, so that on December 31, 1928, the number remaining on the Active List was 130.

The chief features that may be noted in the general development of Permissible Explosives during this period are (1) The introduction of components that depressed the freezing point, thereby producing what are styled "low-freezing," or "L.F." explosives, through which the menace from the use of frozen explosives, or in the thawing of frozen explosives, has been eliminated: (2) The greatly increased use of ammonium nitrate as the oxidizing component of explosives, this substance having been found to possess characteristics rendering it especially well suited to the production of permissibles: (3) The substitution of nitrated sugars or of nitrated glycols for nitroglycerin: (4) The reproportioning of the components so as to produce explosives giving on explosion the minimum of poisonous and inflammable gases, together with the recognition that the paper in which the explosive is wrapped must be considered as one of these components: and (5) the development of the bulky or "high-count" cartridges.

Besides modifications in the form of apparatus or method of procedure or of preparation of materials with which to conduct the tests, there have been invented and perfected the pendulum friction device with which to determine the sensitiveness of explosives to glancing blows; the sand test bomb with which to determine the relative efficiencies of detonators; and the Crashaw and Jones apparatus by which information is obtained as to the kind and quantities of the substances occurring in the products of the explosion of an explosive.

Although the investigation of explosives for use in mining coal was the primary object sought in the organization of this work, explosives for use in other mining operations, in quarrying and in engineering operations have been the subject of many suitability tests. All classes have been studied in field use, as well as in the laboratory and at the testing station, and the safest and most efficient methods of employing them pointed out. The results of some of these investigations are presented in Bureau of Mines reports entitled "The effect of stemming on the efficiency of explosives," "The effect of

cartridge diameter on the strength and sensitiveness of certain high explosives," "The selection of explosives used in engineering and mining operations" and many other publications from the Division.

In the Chemical Section new methods of analysis are the constant subject of study while many new explosive substances especially those which, as initiators, may be used in substitution for mercury fulminate have been the subject of critical investigations. Since liquid oxygen explosives have certain obvious characteristics of value, they also have been the object of extensive investigation and their limiting characteristics developed.

The Explosives Division of the Bureau of Mines has rendered expert services to various divisions of the Federal and State governments and to municipalities. From the outset it has served as the explosives inspection service of the Panama Canal Commission. Through its efforts in demonstrating the methods for their safe and efficient use in industrial operations more than 126,000,000 pounds of the military explosives left as a surplus on the conclusion of the Great War have been used in governmental engineering operations and agricultural projects. As an inspection corps the officials of this Division have investigated a large proportion of the more serious accidents and outrages involving explosives or explosions which have occurred during the past twenty years.

In 1928 a distinct novelty was introduced when the blasting device, styled "Cardox," was approved for permissibility and placed upon the Active Permissible List. This device consists of a cylindrical steel tube having an aperture in its base which may be closed by a steel disk of desired thickness and shearing strength. On the interior of the shell, but enclosed in a tube, is a quantity of a combustible material which may be ignited electrically from without. With this "heater element" inclosed and the aperture at the bottom of the shell covered by its steel disk properly secured, the shell is charged with the determined quantity of liquid carbon dioxide. Leading wires through which to transmit the electric current being attached, the closed shell is placed in the bore hole in the coal, stemmed and tamped and the current turned on, thus igniting the composition in the heating element, which pro-

duces in burning sufficient heat to raise the pressure of the carbon dioxide to such an extent that it shears a hole through the disc, and escaping through the aperture thus made pushes down the coal. For over fifty years there have been repeated attempts made to utilize the expansion of carbon dioxide in mining coal. This seems to be the first practical solution of the problem.

USE OF CAUSTIC SODA, CAUSTIC POTASH AND SODALIME IN OXYGEN BREATHING APPARATUS.

AN investigation conducted by the laboratories of the Drägerwerk at Lübeck, Germany, has shown that the absorption of carbon dioxide by solid caustic soda or caustic potash is paralyzed, so to speak, when traces of certain organic vapors as gasoline, benzol, and alcohol are present in the gas mixture. This finding is of considerable importance in the use of caustic soda and potash in self-contained oxygen breathing apparatus to remove carbon dioxide from the exhaled air. If the carbon dioxide is not removed it will accumulate in what is known as the breathing bag and aside from causing undesirable physiological response as increased respiration and headaches, it will exclude the oxygen and rapidly cause collapse from oxygen deficiency—often without warning symptoms. The seriousness of such a situation may be appreciated when it is considered that the wearer of this type of apparatus is usually encountering an atmosphere that is irrespirable due to oxygen deficiency, poisonous gases, or both.

When ordinary precaution is taken the chances for contamination of the oxygen in the breathing bag or the caustic are not great when the apparatus is used in mines. Briefly, the main sources of trouble would be, contamination of the caustic during manufacture, storage and use of the material; cleaning of parts of the apparatus with paralyzing agents and the sterilizing of mouth pieces with alcohol. Also, as a small portion of alcohol taken in beverages is excreted through the lungs and more may be added to the breath by that remaining in the mouth, contamination may occur from that source. When the self-contained breathing apparatus of the type using

caustic is employed for exploring tanks, tankers, and other places where benzol, gasoline, or other similar hydro-carbons may occur, the danger from these paralyzing agents is greater. In addition to the above-mentioned sources they may gain access by permeating the rubber parts of the apparatus after long exposure, by faulty or leaky mouth pieces, by the wearer inhaling around the mouth piece or through the nose, or through carelessness in upkeep and place of storage.

Caustic soda or potash is not, however, widely used in this country for the purpose previously described. The reagent commonly used in breathing apparatus to absorb carbon dioxide is a specially prepared granular compound composed of a mixture of sodium hydroxide and calcium hydroxide. In view of the results reported by the Drägerwerk Laboratory and the chemical similarity of this compound to the ordinary caustic soda or potash, the Pittsburgh Experiment Station of the United States Bureau of Mines is making experiments in order to ascertain whether the same paralyzing phenomena is true for sodalime. Results of experiments made with the most widely used brand have so far shown that the vapors of ethyl alcohol, gasoline, or benzol have no effect on the efficacy of sodalime for removing carbon dioxide.

UTILIZATION OF WASTE NATURAL GAS.

The utilization of the enormous quantities of natural gas which are going to waste in localities where the gas cannot be disposed of as fuel, is a problem of great importance. The Pittsburgh Experiment Station of the United States Bureau of Mines has undertaken the preparation of a series of reports dealing with the economics of possible processes for converting this natural gas into valuable products which can be economically used and transported to their respective markets. Natural gas, of course, cannot be economically transported great distances except in pipe lines. However, liquid or easily liquefiable products such as formaldehyde, ammonia, methanol, and motor fuel can be made from natural gas. The proposed reports will indicate which of these products can be most profiitably prouced at the locations where natural gas is available.

DETERMINING THE CAPACITY OF NATURAL GAS WELLS.

The method in common use for determining the capacity of a natural gas well is responsible for a large waste of gas and, in many cases, serious damage to the producing capacity of the well, as it is allowed to "blow" unrestricted for fifteen minutes in order that the maximum flow of gas can be determined. It is evident that a method of determining the potential capacity of a well without allowing it to blow wide open would be of great economic value.

The Petroleum Experiment Station, maintained jointly by the United States Bureau of Mines and the State of Oklahoma at Bartlesville, Oklahoma, has therefore started an investigation of the fundamental relations involved in the flow of gas through gas sands and the relationships between gas flow and well capacity. Studies of over 400 gas wells located in different parts of the country have been made and the data correlated. The results of the investigation up to date indicate that a fairly simple and practical field method for determining the capacity of a gas well can be evolved.

An Investigation of the Absorption Spectra of Water and Ice, with Reference to the Spectra and the Major Planets. J. C. MCLENNAN, R. RUEDY and A. C. BURTON. (*Proc. Royal Soc.*, A 785.) Slipher in 1909 photographed the spectra of sunlight reflected from Jupiter, Saturn, Uranus and Neptune. All the spectra thus obtained showed absorption bands superimposed on the usual solar spectrum. Some bands may be due to atomic hydrogen in the atmospheres of the planets. The intensity and width of the absorption bands increase in general from Jupiter to Neptune. There have been three notable bands awaiting identification, (1) At 5,430 A.U., weak in the spectra of Jupiter and Saturn but very strong in Uranus and Neptune. This is in the green part of the spectrum. (2) At 6,190 A. U. Conspicuous for all four planets, broadening from a width of 50 A.U. in Jupiter to 200 A.U. in Neptune. (3) A strong double band 7,200 to 7,260 A.U., recorded only for Jupiter and Saturn. Slipher held that this last band was in the right position to be the strongest absorption band of water vapor but the other bands due to water vapor were not present on the plates.

Since a continuous spectra had never been photographed through thicknesses of water the three authors proceeded to do this, especially as there was some reason to believe that the band at 6,190 A.U. might be due to this substance. A continuous spectrum was photographed through 1, through 4 and finally 21.5 meters of water. Plates were sensitized with the same dye as that used by Slipher. While the comparison of planetary and artificial absorption spectra is far from showing identity in all details it seems probable that the second and third bands are due to water. The work of Coblentz has shown the surface temperature of the major planets to be as low as $-140°$ C. Since ice could exist at this temperature it seemed worth while to photograph the absorption spectrum of this solid. Blocks of ice about a meter long, with clear ends, were placed in a row and light was sent through as much as 14 meters of ice. The photographs of the transmitted light show marked differences from those obtained through water." There is evidently a considerable shift of the absorption bands toward the longer wave-lengths, there being in ice very little absorption where, with water, the absorption is very strong."

"It is concluded that the absorptions in the red common to the four major planets can be attributed to water, not as vapor or as ice, but in the liquid state. The absorption in the green at 5,430 Å. cannot be attributed to water and its identity is unknown. Search is being made for it in the absorption of certain liquefied gases."

G. F. S.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, MARCH 20, 1929.

The regular monthly meeting was called to order by the President, Mr. Nathan Hayward. He announced that the minutes of the preceding monthly meeting had been published in full in the March number of the Journal of the Institute and that unless objection were uttered, the minutes would be approved as printed. No objection was offered, so the President ruled the minutes approved.

The Secretary reported the following changes in membership since the last regular meeting:

New members: one life member; six resident members; four non-resident members; one transfer from resident to resident life membership.

He reported, with sincere regret, the death of three members—Messrs. C. B. Cochran of West Chester, Pa.; S. P. Hutchinson of Philadelphia; and Edward Lupton of Philadelphia.

There was no further business except the reading of the paper of the evening, so the President introduced Dr. S. L. Hoyt of the General Electric Company, who spoke on "The New Alloy, Carboloy; Its Development and Uses." The lecturer first of all showed a moving picture film which displayed the use of the new alloy 'carboloy' in the industries. He then outlined the development of this alloy and dwelt particularly upon the upbuilding of the new art involved in its production. Dr. Hoyt's interesting and important paper was received with much enthusiasm and was followed by an animated discussion. The meeting adjourned at ten-eight p.m., with a rising vote of hearty thanks to the lecturer.

HOWARD MCCLENAHAN,
Secretary

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, March 6, 1929.)

HALL OF THE INSTITUTE,
PHILADELPHIA, March 6, 1929.

DOCTOR THOMAS D. COPE *in the Chair.*

A second reading of the report of the Special Committee on the Revision of the Regulations of the Committee on Science and the Arts was presented by Mr. Franklin, Chairman, and after discussion was adopted.

The following reports were presented for final action:

No. 2896: The Work of Mr. Elmer A. Sperry.

The Sub-Committee of Investigation decided to limit its field to those inventions that are based as was Mr. Sperry's former invention of the Gyro Compass on the gyroscope. These include a Gyro-Pilot which automatically steers a vessel upon a prescribed course; a Gyro-Stabilizer which largely eliminates the roll of a vessel; a Gyro Roll and Pitch Recorder which permits a comparison to be made

between the motion of a vessel with and without the stabilizer; and a Gyro Track Recorder which enables the irregularities in a railroad track to be located and measured.

All of these inventions have been put into extensive practical use and have proved most helpful in their respective fields.

The award of the Elliott Cresson Medal was recommended to Mr. Sperry "In consideration of his application of the principle of the gyroscope to the development of means whereby vessels may be navigated with greater safety, comfort, and economy, both in time and fuel, and whereby the irregularity of movement of vessels and trains may be accurately recorded."

No. 2901: Self-Aligning Coupling.

The Fast Flexible or Self-Aligning Coupling is a device for coupling together the ends of a driving and driven shaft in such a way as to permit the transmission of power with a minimum of loss.

The device consists of a coupling that does not make use of any flexible material but fastens to the end of each shaft a gear, the teeth of which mesh into the internal teeth of a pair of collars that are bolted together over the ends of the coupled shafts.

The form of the teeth is such as to permit a considerable misalignment of the shafts. The casing containing the collars retains a quantity of oil that is distributed to the gear teeth by the centrifugal force of rotation.

The award of the John Price Wetherill Medal to Mr. Gustave Fast, of Annapolis, Maryland, was recommended, "In consideration of the novelty of design of this coupling, the stability of its construction and of its successful practical application."

The following reports were presented for first reading:

No. 2856: Vapor Phase Process.

No. 2906: Literature.

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, March 13, 1929.)

LIFE.

MR. RANDAL MORGAN, III, Student, St. Paul's School, Concord, N. H.

RESIDENT.

MR. ALFRED THOMAS BRATTON, Patent Attorney, Fraley and Paul, 1815 Land Title Building, Philadelphia, Pa. For mailing: 1008B West Erie Avenue, Philadelphia, Pa.

MR. JOHN PIM CARTER, Research Chemist in Manufacturing, 3144 Passayunk Avenue, Philadelphia, Pa. For mailing: 314 Maple Avenue, Drexel Hill, Pa.

DR. EDWARD HILL COX, Professor of Chemistry, Swarthmore College. For mailing: 8 Whittier Place, Swarthmore, Pa.

DR. DUNCAN GRAHAM FOSTER, Teacher, Department of Chemistry, Swarthmore College. For mailing: 504 Walnut Lane, Swarthmore, Pa.

MR. FRANK C. ROBERTS, JR., Taylor-Wharton Iron and Steel Company, Widener Building, Philadelphia, Pa.

MR. ANDREW SIMPSON, Engineer, Swarthmore College, Swarthmore, Pa.

NON-RESIDENT.

MR. DEWITT D. BARLOW, Civil Engineer, President, Atlantic Gulf and Pacific Company, 15 Park Row, New York City.

MR. JOHN J. CATON, JR., Textile Engineer, Wallace Silk Company, Phillipsburg, N. J. For mailing: 306 Mercer Street, Phillipsburg, N. J.

MR. ISAAC HARTER, Engineer and Manufacturer, Vice-President, Babcock and Wilcox Company, 85 Liberty Street, New York City.

MR. JAMES P. WALLACE, President, Wallace Silk Company, Phillipsburg, N. J. For mailing: 306 Mercer Street, Phillipsburg, N. J.

TRANSFERS.

FROM RESIDENT TO RESIDENT LIFE.

MR. MORRIS CLOTHIER, Merchant, Philadelphia.

CHANGES OF ADDRESS.

MR. WILLIAM DEKRAFFT, Fidelity-Philadelphia Trust Building, Broad and Walnut Streets, Philadelphia, Pa.

MR. CHARLES F. FORSTALL, Reading Company, Spring Garden Street Station, Philadelphia, Pa.

MR. OTTO T. MALLERY, 225 South Fifteenth Street, Philadelphia, Pa.

MR. EDWARD W. PARKER, Room 607, Morris Building, 1421 Chestnut Street, Philadelphia, Pa.

MR. G. R. REBMANN, 502 North Nineteenth Street, Philadelphia, Pa.

MR. A. T. SMITH, 565 Park Avenue, New York City.

MAJOR JOHN A. VOGELSON, 2031 Locust Street, Philadelphia, Pa.

MR. W. CHATTIN WETHERILL, Chestnut Hill Post Office, Chestnut Hill, Philadelphia, Pa.

NECROLOGY.

J. Snowden Bell

J. Snowden Bell was born in Philadelphia on July 11, 1843, and died in Brooklyn, N. Y., on November 27, 1928. He was educated in the public schools of Philadelphia, the Polytechnic College, and the University of Pennsylvania, receiving from the latter institution the degree of Bachelor of Laws in 1879.

In 1862 he entered the service of the Baltimore and Ohio Railroad as draftsman but soon after received the appointment of assistant engineer on the U. S. Frigate Minnesota, serving in this capacity, with distinction, to the end of the Civil War.

In 1865 he returned to the Baltimore and Ohio shops as chief draftsman where he supervised the designing of several of the passenger and freight engines. For a brief period he was engineer and superintendent of the Pittsburgh Locomotive and Car Works and about 1870 opened an office in Philadelphia as consulting mechanical engineer, and solicitor of patents.

In 1879 Mr. Bell was first admitted to the bar—being admitted to practice before the Philadelphia Bar on June 14, 1879; in all he was admitted to practice before some 23 courts, including the United States Supreme Court.

From 1883 to 1909 his office for the practice of patent law was in Pittsburgh, Pa. In the latter year he removed to New York where he continued his patent practice. Mr. Bell was an authority on locomotive boilers, front ends, superheaters, feed-water heaters, etc., frequently contributing papers on these subjects to the several associations with which he was connected. He also wrote numerous papers on the history and development of locomotives and is also noted for his work on patent law known as "Bell on Expert Testimony." He was an active member of the Master Mechanics Association, the mechanical section of the American Railway Association, Engineers' Club of New York, and numerous other engineering organizations.

He became a member of The Franklin Institute on March 10, 1875.

Mr. C. B. Cochran, West Chester, Pa.
Mr. S. Pemberton Hutchinson, Philadelphia, Pa.
Mr. Edward Lupton, Philadelphia, Pa.

LIBRARY NOTES.

RECENT ADDITIONS

Archivio di Storia della Scienza. Volumes 1–8. 1919–1927.

BAKER, CLYDE. Modern Gunsmithing. 1928.

BRADY, GEORGE S. Materials Handbook. First edition. 1929.

CLOUD, JOHN WILLS. Castles in the Ether. Revised edition. 1928.

DAHL, O. G. C. Electric Circuits: Theory and Applications. Volume one: Short-Circuit Calculations and Steady-State Theory. First edition. 1928.

DUNCAN, RICHARD. Air Navigation and Meteorology. Second edition. 1928.

ELLIS, CARLETON, AND JOSEPH V. MEIGS. Gasoline and Other Motor Fuels. 1921.

FAIRBANKS, ERNEST E., editor. The Laboratory Investigation of Ores: a Symposium. First edition. 1928.

FLETCHER, HARVEY. Speech and Hearing. 1929.

FLETTNER, ANTON. The Story of the Rotor. 1926.

FREEMAN, JOHN R., Editor. Hydraulic Laboratory Practice. 1929.

HOLLAND, MAURICE, AND HENRY F. PRINGLE. Industrial Explorers. 1929.

HUMPHREYS, W. J. Physics of the Air. Second edition, revised and enlarged. 1929.

International Critical Tables of Numerical Data, Physics, Chemistry and Technology. Volume 5. 1929.

Jahrbuch für Photographie und Reproduktionsverfahren für die Jahre 1921–1927, Herausgegeben von J. M. Eder. Dreizigster Band, 1 und 2 Teil. 1928.

Klimschs Jahrbuch. Band XXII. 1929.

KUNS, RAY F. Automotive Essentials. 1928.

LEE, WILLIS T. Stories in Stone. 1926.

MACKENZIE, CATHERINE. Alexander Graham Bell: the Man Who Contracted Space. 1928.

MARTIN, GEOFFREY. A Treatise on Chemical Engineering Applied to the Flow of Industrial Gases, Steam, Water, and Liquid Chemicals. 1928.

MELLOR, J. W. A Comprehensive Treatise on Inorganic and Theoretical Chemistry. Volume IX. 1929.

OLSEN, J. K. Production Design. 1928.

ORNSTEIN, MARTHA. The Rôle of Scientific Societies in the Seventeenth Century. 1928.

Photograms of the Year 1928. Thirty-fourth year.

REINTHALER, FRANZ. Artificial Silk. Enlarged and revised edition, translated from the German by F. M. Rowe. 1928.

SEVERNS, WILLIAM H., AND HOWARD E. DEGLER. Steam, Air, and Gas Power. 1929.

STOCK, ALFRED. The Structure of Atoms. Translated from the second German edition by S. Sugden. No date.

STOTT, VERNEY. Volumetric Glassware. 1928.

TINKLER, C. K., AND HELEN MASTERS. Applied Chemistry. Volume I. 1929.

TOMMASINA, THOMAS. La Physique de la Gravitation et la Dynamique del' Univers. 1928.

TRAVERS, MORRIS W. The Discovery of the Rare Gases. 1928.

VALIER, MAX. Raketenfahrt. 1928.

WIEN, W. Das Wiensche Verschiebungsgesetz. Ostwald's Klassiker der exakten Wissenschaften. Nr. 228. 1929.

World Almanac and Book of Facts for 1929.

BOOK REVIEWS.

APPLIED CHEMISTRY. By C. Kenneth Tinkler, D.Sc., F.I.C., and Helen Masters. B.Sc. Volume I, second edition revised. xii–296 pages, illustrations, 12mo, London, Crosby Lockwood & Son, 1929. Price, 15 shillings.

Applied chemistry has a wide significance. It may comprehend the vast field of modern industry, or the manifold phases of medicine. The book in hand is a text-book for students in Household Science and Public Health. It contains many tests and quantitative processes, and is, of course, a compilation from standard literature with occasional additions from the personal experiences of the authors. It is a second edition and the first volume of a series. It relates to: Water, Detergents, Textiles, and Fuels with some minor topics. Volume II is announced as covering Foods, being written by the same authors.

The information offered is of standard character, and does not call for detailed criticism. Emanating from British sources, one notes not unexpectedly, for instance, that in the procedure for the determination of nitrates in water, the extensive and important work done some years ago in the United States concerning the phenol-sulphonic method is entirely ignored. Similarly, the

delicate method for nitrites with alpha-aminonaphthalene is unnoticed. The routine given for water analysis is, however, full enough to secure judgment of its potability and industrial applications according to generally accepted standards.

In the section on textiles, the familiar photomicrographs of the more important raw fibres are given, but no indication of the degree of magnification. Photographs are also given to show the effects of treating mixed textiles for determination of the several fibres. They are of no use. The only proper method for instruction in such work is to let the student carry out the processes and make the examination under the microscope. Textiles of known composition should be submitted and the student obligated to make the analysis.

A reference to "rayon" gives the information that efforts to have the British call this by some name other than "artificial silk" have so far failed. Within a comparatively small space, making a handy volume, a large amount of information is given, and the careful use of the book will serve to introduce the student to much material of practical application. It is not stated how many volumes will constitute the series, but it is evident that a very comprehensive treatment of some phases of applied chemistry is in view.

HENRY LEFFMANN.

LA GOMME DE BALATA. By A. D. Luttringer. Pamphlet, 50 pages, 12mo. Paris, A. D. Cillard, 1929.

Balata is a material, closely resembling gutta-percha, derived from the latex of a tree *Mimusops globosa* Gaert. of the Sapotaceae, the family to which our common persimmon belongs. The tree grows widely in northern South America especially in the Guianas. The pamphlet gives a full account of the methods of extraction of the material, its purification, composition and adaptability as a substitute for ordinary gutta-percha for which it seems to be highly suitable.

H. L.

PHYSICS OF THE AIR. By W. J. Humphreys, C.E., Ph.D. Second edition, revised and enlarged. 654 pages, illustrations, 8vo. New York, McGraw-Hill Book Company, Inc., 1929. Price, $6.

Doctor Humphreys' comprehensive volume covered at its first appearance a wide and interesting field. Its success has led to a second edition, in the preparation of which the author has made extensive revisions and considerable additions, among the latter especially noted a chapter on meteorological acoustics, grouping many of the sounds due to the relation of the air in motion to obstructing conditions, as observed for instance in the soft and soothing murmuring of the forest or the terrifying noise of the tempest. Much information is also given as to the laws of transmission of sound through air in different conditions. A very recent development that might be classified under the above title is the relation of weather conditions to radio transmission. The weather bureau has begun to advise the "fans" concerning probabilities in this matter.

The book is liberally illustrated with maps and graphs, also apparatus. It would have been worth while to insert one of the several available excellent pictures of the funnel-shaped cloud constituting the tornado, and, perhaps a couple of simple maps showing the approximate course and width of recent tornadoes, of which, indeed the middle west of this country has had sad experi-

ence. Pictures of the destruction wrought by such storms are of no particular interest.

Doctor Humphreys has considerable of the saving grace of humor and writes in a more or less colloquial style. His large experience and intimate acquaintance with subjects of the work have been fully utilized in its text. It is to be hoped that someone equally qualified will give us a book on the *chemistry* of the air. Many years ago a most interesting and valuable series of lectures appeared in the Chemical News on "The Supposed Nature of Air, Prior to the Discovery of Oxygen."

The reviewer feels that the canons of criticism would not be fully observed, if the general character of the text was passed over without some animadversion. There is certainly a good deal of carelessness in the literary form. Many sentences would be likely to secure a flogging for Macauley's learned school-boy of the fifth form. There is a good deal of indifference to proper phraseology now noticed in American books. It is possibly another expression of determination to have "personal freedom." We consider Lindley Murray as much a trespasser on our privileges as Volstead.

It is gratifying to see a work of such scientific character find a hearty response from those to which it is addressed. In a future edition some specific notice might be given of the peculiar "ricochet" effect observed in the operation of sirens and fog-horns along the coast. HENRY LEFFMANN.

LA PHYSIQUE DE LA GRAVITATION ET LA DYNAMIQUE DE L'UNIVERS, BASÉES SUR LA DÉCOUVERTE DU MÉCANISME RÉEL DES RADIATIONS DÉMONTRANT LEUR FONCTION GRAVITATIONNELLE RÉCIPROQUE. Par Thomas Tommasina, Docteur ès Sciences. vi–302 pages, 8vo, paper. Paris, Gauthier-Villars et Cie, 1928. Price, 50 francs.

Most books that analyze the fundamental concepts of physical science employ every generally known mathematical device (and often some not generally known) in their demonstrations, the argument being interwoven with analytical deduction that imposes upon the reader a concentration upon mathematical transformations more or less at the expense of a desirable reflective attitude in viewing the subject. Doctor Tommasina employs the exceptional course reminiscent of Cornot's classic, Reflections on the Motive Power of Heat; it is wholly philosophical and reflective. The aim of the book is to pass in critical review the concepts of modern physics and to furnish a guide, sure, precise and understandable of the purely mechanical and material nature of all the phenomena in the field of astronomy and physics.

In an introduction, the author makes a summary review of the evolution and present state of modern physics. He then proceeds with a lengthy consideration of the fundamental notions upon which scientific deduction may be based. An equally lengthy account upon when and how the physics of universal gravitation was initiated and developed includes quotations of the works of many of the most celebrated physicists of modern times which are in agreement with his own views. Next to pass in review is the work of that famous leader in modern mathematical physics, H. Poincare and of a celebrity of a century earlier J. H. Lambert as forerunners in the theory of relativity. A discussion of the work of Einstein under the title "A Einstein, his two relativist theories and his erroneous

interpretation of physical phenomena" occupies a chapter of satisfying length, and as the title indicates, of a distinctly critical character. A note on the recent work of Jean Chazy on relativity, also critical, and a list of the author's 119 scientific memoirs from 1899 to 1914 conclude the work.

The work is a timely contribution toward bringing modern views of the fundamental laws of physical science into concordance with observed phenomena which should prove of inestimable value as a reference for investigators in the fundamental principles of physics.

L. E. P.

RAKETEN-FAHRT (ROCKET-TRAVEL). By Max Valier, fifth edition. 252 pages, illustrations, 8vo. Munich and Berlin, R. Oldenbourg, 1928. Price, in paper 8.50 marks, bound 9.50 marks.

"Glücklicher Säugling! dir ist ein unendlicher Raum noch die Wiege; Werde Mann und dir wird eng die unendliche Welt."

Schiller's epigram might well be the motto of this book, for it expresses the modern Alexandrine desire to pass from the conquests that have been made in this world to the worlds beyond. Wide notice has already been given of Valier's exploits with the rocket-driven auto, and a short time ago, newspapers presented discussions of Goddard's suggestion for a rocket shot at the moon. That such proposals are met with doubts and even with open and severe condemnation need not astonish us. Such distinguished scientists as Lord Kelvin and Simon Newcomb deprecated Langley's efforts and expressed doubts as to whether a machine heavier than air could be raised by its own power.

Valier was the first and fearless institutor of a peculiar method of automobile propulsion, and is enthusiastic as to its wide application, considering it not only available for surface travel, but entirely feasible for aviation and even for sailing through the interplanetary space. He would probably modify our well-known English phrase and say "No pent-up Utica contracts our powers, but the whole boundless universe is ours."

The book is a comprehensive and extended discussion of the problem, throughout most optimistic. Theoretical as well as practical questions are to be solved. The preface opens with a romantic account of a telescope study of Mars in 1924 when its approach to the earth was unusually close. The preliminary discussion on the relations of gravity and interplanetary conditions must be left to Einstein and his disciples. The reviewer is quite out of his depth.

Much information is given concerning the make-up of rockets for the several purposes for which they are used. A historical note exhibits rather over-trustful acceptance of early allusions, among other statements, the suggestions that Moses and his contemporaries were familiar with high explosives and incendiary methods of warfare. It is, of course, well known that a work purporting to be written by "Marcus, the Greek," entitled "The Book of Fires to burn Enemies" dates from at least as early as the Eighth century and probably derived from much earlier oriental sources. Many instances of fiery projectiles employed in war are given, the definite invention of Congreve in 1799 being discussed.

The possibility of travelling through interplanetary space is discussed more seriously than would be expected, but the author is naturally deeply interested in the question. About sixteen pages are given to the "Theory of Travelling in

the Cosmos" and considerably more space to the difficulties and interferences connected with these ventures. A doubter might be inclined to ask what would the traveller to the moon do when he got there? Surely there is not to be expected any possibility of "re-fueling." In our common experience a rocket goes up brilliantly and rapidly in defiance of gravitation, but it comes down as a stick under the ordinary law of acceleration, except as influenced by the resistance of the air.

Undoubtedly this book will awaken much thought on the part of those who are striving after new and startling performances. There is no limit apparently to human ambition. The book is liberally illustrated and excellently printed. The literary style is typically German, many of the sentences being long and involved, but owing to the rule as to the locating of adjective phrases, German lends itself to such construction more than any of the familiar languages. The fact that in four years five editions have been published shows the wide-spread interest in the subject. HENRY LEFFMANN.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 296. Pressure Distribution Tests on PW-9 Wing Models From − 18° Through 90° Angle of Attack, by Oscar E. Loeser, Jr. 21 pages, illustrations, quarto. Washington, Government Printing Office, 1929, price, fifteen cents.

At the request of the Army Air Corps, an investigation of the pressure distribution over PW-9 wing models was conducted in the atmospheric wind tunnel of the Committee. The primary purpose of these tests was to obtain wind-tunnel data on the load distribution on this cellule to be correlated with similar information obtained in flight tests, both to be used for design purposes. Because of the importance of the conditions beyond the stall as affecting control and stability, this investigation was extended through 90° angle of attack. The results for the range of normal flight have been given in N. A. C. A. Technical Report No. 271. The present paper presents the same results in a different form and includes, in addition, those over the greater range of angle of attack, − 18° through 90°.

The results show that

At angles of attack above maximum lift, the biplane upper wing pressures are decreased by the shielding action of the lower wing.

The burble of the biplane lower wing, with respect to the angle of attack, is delayed, due to the influence of the upper wing.

The center of pressure of the biplane upper wing (semispan) is, in general, displaced forward and outward with reference to that of the wing as a monoplane, while for the lower wing there is but slight difference for both conditions.

The overhanging portion of the upper wing is little affected by the presence of the lower wing.

Report No. 297. The Reduction of Observed Airplane Performance to Standard Conditions, by Walter S. Diehl. 27 pages, illustrations, quarto. Washington, Government Printing Office, 1928, price, fifteen cents.

This report shows how the actual performance of an airplane varies with air temperature when the pressure is held constant. This leads to comparatively simple methods of reducing observed data to standard conditions. The new

methods which may be considered exact for all practical purposes, have been used by the Navy Department for about a year, with very satisfactory results.

The report also contains a brief historical review of the important papers which have been published on the subject of performance reduction, and traces the development of the standard atmosphere.

Report No. 314. Drag and Cooling with Various Forms of Cowling for a "Whirlwind" Radial Air-Cooled Engine—II, by Fred E. Weick. 22 pages, illustrations, quarto. Washington, Government Printing Office, 1929, price, twenty cents.

This report gives the results of the second portion of an investigation in the Twenty Foot Propeller Research Tunnel of the Committee, on the cowling and cooling of a "Whirlwind" J-5 radial air-cooled engine. The first portion, which is reported in N. A. C. A. Technical Report No. 313, pertains to tests with a cabin fuselage. This report covers tests with several forms of cowling, including conventional types, individual fairings behind the cylinders, individual hoods over the cylinders, and the new N. A. C. A. complete cowling, all on an open cockpit fuselage. Drag tests were also made with a conventional engine nacelle, and with a nacelle having the new complete cowling.

In the second part of the investigation the results found in the first part were substantiated. It was also found that the reduction in drag with the complete cowling over that with conventional cowling is greater with the smaller bodies than with the cabin fuselage; in fact, the gain in the case of the completely cowled nacelle is over twice that with the cabin fuselage. The individual fairings and hoods did not prove effective in reducing the drag The results of flight tests on an AT-5A airplane (reported in the Appendix to N. A. C. A. Technical Report No. 313) have been analyzed and found to agree very well with the results of the wind tunnel tests.

R.

BIBLIOGRAPHY OF AERONAUTICS, 1926. Published by the National Advisory Committee for Aeronautics. 154 pages, 8vo. Washington, Government Printing Office, 1928, price, twenty cents.

BIBLIOGRAPHY OF AERONAUTICS, 1927. Published by the National Advisory Committee for Aeronautics. 184 pages, 8vo. Washington, Government Printing Office, 1928, price, thirty-five cents.

These volumes cover the aeronautical literature published during 1926 and 1927. The first Bibliography of Aeronautics was published by the Smithsonian Institution as Volume 55 of the Smithsonian Miscellaneous Collections and covered the material published prior to June 30, 1909. Supplementary volumes of the Bibliography of Aeronautics for the subsequent years have been published by the National Advisory Committee for Aeronautics. They are the Bibliography of Aeronautics for the years 1909–1916, 1917–1919, 1920–1921, 1922, 1923, 1924, 1925.

As in previous volumes, citations of the publications of all nations are included in the languages in which these publications originally appeared. The arrangement is in dictionary form with author and subject entry, and one alphabetical arrangement. Detail in the matter of subject reference has been omitted on account of the cost of presentation but an attempt has been made to give sufficient cross reference for research in special lines. R.

YEAR BOOK OF THE AMERICAN PHARMACEUTICAL ASSOCIATION, 1926, volume 15, containing the Sixty-Ninth Annual Report on the progress of pharmacy, and the Constitution, By-laws and Roll of Members. lxxi–613 pages, illustrations, 8vo. Baltimore, American Pharmaceutical Association, 1928.

The series to which this volume belongs is well-known in Pharmacy, and is framed upon the principles now general in such reports. The value of a publication of this type will depend upon the extent to which the literature has been examined and the judgment by which the abstracts have been selected. Collaborators will always differ as to what is suitable and what not, but taken all in all the collection here presented is a broad and comprehensive summary of the progress of pharmacy during the year 1926. It seems regrettable that the volume is so late, for more than two years have elapsed since the literature abstracted was closed.

A detailed examination is impossible, not only for lack of time and space (an Einsteinian would probably say "space-time") but comparison of the abstracts with the originals would be necessary. The reviewer turns naturally to a topic with which he has some familiarity practically as well as from the literature. The detection of methanol in the presence of ethanol has become a very important analytic problem in the United States for obvious reasons. The abstracts here given on this subject have not been collated. American chemists and pharmacists have been for several years urging that the term "methyl alcohol" should be abandoned to avoid any popular notion that the substance has use as a substitute for common alcohol. It is the duty of abstractors to support this effort or if they fail the editor should take care of the matter. Yet "methyl alcohol" is prominently indexed with two references to page 196 and "methanol" with reference to page 346. There is some confusion in the statements in the several paragraphs, and a lack of definiteness in some parts. Thus, referring to Kolthoff's method of detecting methanol in presence of ethanol the phrase is "(oxidation of the methyl alcohol by phosphoric acid solution and detection of the resulting formaldehyde by means of Schiff's reagent) is more sensitive than the method of Denigès (oxidation by means of acid permanganate)". Phosphoric acid is not an oxidizing agent and the standard procedure of the U. S. P. X. is by permanganate in solution made acid by phosphoric acid. Later in the text the fact that the phosphoric and sulphuric acid methods are mere variants is indicated, but the group of three abstracts on the detection of this important adulterant of common alcohol leaves the matter in some confusion. No mention is made of the fact ascertained several years ago by LaWall that glycerol will simulate methanol in the test and the liquid to be tested should therefore always be distilled.

HENRY LEFFMANN.

STEAM, AIR, AND GAS POWER. By William H. Severns, M.S., Member A.S.M.E., Associate Professor of Mechanical Engineering, University of Illinois, and Howard E. Degler, M.E., M.S., Member A.S.M.E., Assistant Professor of Mechanical Engineering, University of Illinois. vii–425 pages, illustrations, 8vo, cloth. New York, John Wiley & Sons, Inc., 1929. Price $4.

College courses which deal with apparatus for the production of power from fuel cover an exposition of the principles and a description of the construction and operation of an innumerable variety of appliances. A descriptive treatise in

which representative types of such apparatus are discussed with an account of the principles of their operation is necessarily an essential feature of such a course of study. It is not so long ago that texts on that subject, were not altogether satisfactory in representing current practice. Besides progress goes on apace and when new appliances and methods come into use text-books must follow the procession. In that connection, it may be noted that 150 lbs. was a high boiler pressure in 1883 while 1800 lbs. is the present day record, and in large units. Since the early years of the century books of that sort have been getting better and better, and this one in point of scope, arrangement and quality of execution is well within the "limits of error of observation" in that progressive improvement. The field covered is extensive, and to successfully condense into one volume of moderate dimensions the "necessary and sufficient" proclaims rare discrimination on the part of the authors.

The work covers a progressive study of the power-plant, the boiler and auxiliaries and the engine. Water-tube boilers predominate as is consistent in this era of high pressures. The auxiliaries include economizers, feed-water heaters and purifiers, water softening, air preheaters; it is all very complete and in excellent form. Among the engines, we have the high-speed, corliss, poppet-valve unaflow and a separate chapter on the principles and structure of the steam turbine. Pumps, compressed air and compressors, and internal-combustion engines each have an analytical and descriptive chapter. The theory of each type of apparatus is fully considered with ample descriptions and their 262 superior cuts and diagrams. Besides, there are chapters in the beginning of the book on thermodynamic principles, steam calorimetry, fuels and combustion; the first named containing a surprising amount of applicable theoretical information in a comparatively small space.

The work will doubtless satisfy the requirements of many engineering courses as well as meet the needs of numerous other readers who seek accurate information on the apparatus employed in power generation and their principles of operation.

LUCIEN E. PICOLET.

INFRA-RED ANALYSIS OF MOLECULAR STRUCTURE. By F. I. G. Rawlins, M.Sc., Trinity College, and A. M. Taylor, M.A., late scholar of Trinity College, Ramsay Memorial Research Fellow. 176 pages, illustrations, 8vo. New York, The Macmillan Company, 1929.

This is the first volume of a new series of monographs on Physical Chemistry to be prepared under the auspices of the Cambridge Press. This volume is devoted to data obtained by means of infra-red spectroscopy. This part of the spectrum seems to have attracted, so far, little detailed attention compared to that which has been given to the ultra-violet, the main reason being probably that the latter rays are more brilliant in their visible optical effects, more widely applicable in some departments of research and more useful in medical science. The existence of rays beyond the red of the visible spectrum was discovered by Herschel, but for many years not much attention was given to them. Wood made some interesting observations, especially the fact that ordinary leaves throw off considerable infra-red, as is shown in his well-known picture of a tree in full leaf, appearing as if covered with snow. The discovery made a few years ago that some flowers reflect ultra-violet light introduces us to a remarkable color contrast which

we cannot formulate, as both rays are invisible to us. If they are visible to any of the insect tribe, great aid to distinction of leaf and flower will be given.

The general scientist, that is, one who has not specialized in this field turns with some curiosity to the methods of producing infra-red rays. It is generally known that they are at the "heat-end" of the spectrum, and we find that heat-producing apparatus such as a Nernst glower or a silica pencil is used. It is stated that in the early work the Welsbach mantle was used, but this is discarded. Any one who has worked with a Nernst lamp will be familiar with the heat it emits, and it appears that much of this is of the type useful in these researches. The special topic of the book is presenting in much detail, the application of the rays to study of molecular structure, presumably somewhat in the manner in which the ultra-violet is used, especially that group called conventionally X-rays. Much space is given to construction and operation of the apparatus for producing the dispersion of the rays. A remarkable effect on an ordinary photograph plate is described by which a plate slightly inpressed by visible rays is exposed to the infra-red spectrum when reversal of image occurs on development, positive images, that is, bright lines in the spectrum, are obtained. Details are given as to the method to be used to obtain this result. It has also been noted that some phosphorescent paints, especially the calcium sulphide preparations, will be affected by infra-red, in such a manner that the phosphorescence will be materially diminished if subjected to such rays and even by visible red rays.

The data presented in the book are of a highly specialized character and correspondingly recondite, but a very large amount of information is presented. It is the first book in English devoted to the special subject and will be a welcome addition to the literature of chemical physics.

HENRY LEFFMANN.

ENCYCLOPÉDIE LEAUTÉ. LES COMBUSTIBLES LIQUIDES ARTIFICIELS. By A. Mailhe, Professeur a la Faculté des Sciences de Paris. 280 pages, illustrations, 8vo. Paris, Gauthier-Villars & Cie, Masson & Cie., 1929. Price 30 francs.

This is a summary of chemical and physical data concerning the many forms of combustible liquids available for heat and power, the main motive in the making of the volume being the wide-spread interest in obtaining substitutes for natural petroleum. The Paris Faculty of Sciences has established recently a chair for the study of combustibles and the author of the book, who has devoted himself to extensive researches in this field, has been appointed to the position. The subject-matter covers a wide field, including, of course, both aliphatic and cyclic compounds. The methods and effects of "cracking" are discussed at length. The English word has been admitted without change of spelling into the French language. It would be interesting to know how it is pronounced by our Gallic friends. However, this is merely a little persiflage. Apart from the combustible materials themselves, accessory compounds are often described. The utilization of lignite, oil-bearing shales, and products of redistillation of tar receive considerable notice.

We, therefore, have in this work, an excellent summary of the many sources from which the world may recruit its supply of heating and power fuel if the natural sources are exhausted, a condition that is beginning to be feared by some authorities.

H. L.

STEAM TURBINES, A PRACTICAL AND THEORETICAL TREATISE FOR ENGINEERS AND STUDENTS, INCLUDING A DISCUSSION OF MERCURY AND GAS TURBINES. By James Ambrose Moyer, S.B., A.M. Sixth edition, revised and enlarged. ix–557 pages, illustrations, 8vo, cloth, with an entropy chart folder. New York, John Wiley & Sons, Inc., 1929. Price $4.50.

"Light for a million homes"—all at one time, is the striking illustrated announcement in the pictorial section of a recent issue of a New York daily by the Westinghouse interests of the capacity of one of their steam turbines installed at the Hell Gate electric power station in New York City. That a generator of such dimensions is possible is eloquent testimony of the progress which has been made in large-scale power production. A considerable number of years have passed since the supremacy of the turbine over the piston engine in economy, capacity and cost became established for large outputs. During that period the engineering profession has been as generous in supplying constructive literature to that development as it was in prior years in dealing with piston engines. Professor Moyer has an honored place among these contributors as engineer, teacher and author. His "Steam Turbines" first appeared in 1908, and its present issue in a sixth edition is ample evidence of its value in effecting its intended purpose.

The work, in general, covers principles, design and descriptive matter of the types of turbines which have survived the test of performance. The discussion of the subject is preceded by a short historical survey, the elementary theory of heat, and a detailed explanation of the application of the temperature-entropy diagram. The present edition has been amplified to keep pace with progressive refinements in design and construction. Particular reference may be made to new material on "bleeder" turbines, variation of steam consumption with age, shaft design and casings for high pressures. Mechanical difficulties with the early designs of reduction gears brought about the Melville-Macalpine floating-frame reduction gear. This appears to be given more exclusive prominence than is justified by its omission in recent practice, careful workmanship having apparently overcome the need of that refinement of design. With revisions and additions in keeping with recent progress, the book will doubtless continue its successful career.

L. E. P.

PUBLICATIONS RECEIVED.

Max Valier. *Raketenfahrt.* 5 Auflage von Vorstoss in den Weltenraum. Eine technische Möglichkeit. 252 pages, illustrations, 8vo. München und Berlin, R. Oldenbourg, 1928, price, in paper, 8.50 marks.

Physics of the Air, by W. J. Humphreys, C.E., Ph.D. Second edition, revised and enlarged. 654 pages, illustrations, folded plate, 8vo. New York, McGraw-Hill Book Company, Inc., 1929, price $6.00.

The Elements of Practical Mechanics, by Charles Ranald MacInnes. 131 pages, illustrations, 8vo. New York, D. Van Nostrand Company, 1929, price $2.25.

Applied Chemistry. A practical handbook for students of household science and public health by C. Kenneth Tinkler, D.Sc., and Helen Masters, B.Sc.

Volume I—Water, detergents, textiles, fuels, etc. Second edition, revised. 296 pages, illustrations, 8vo. London, Crosby, Lockwood and Company, 1929, price 15 shillings.

Encyclopédie du Caoutchouc et des Matières Plastiques. La gomme de Balata par A. D. Luttringer. 50 pages, 12mo. Paris, A. D. Cillard, 1929.

Statistical Mechanics. The theory of the properties of matter in equilibrium. Based on an essay awarded the Adams prize in the University of Cambridge, 1923–24, by R. H. Fowler, M.A. 570 pages, illustrations, 8vo. Cambridge, University Press, New York, The Macmillan Company, 1929, price 35 shillings, net.

Year Book of the American Pharmaceutical Association, 1926, vol. 15, containing the sixty-ninth annual report on the progress of pharmacy and the constitution, by-laws and roll of members. 613 pages, illustrations, portrait, 8vo. Baltimore, The Association, 1928.

Bibliography of Aeronautics, 1926. National Advisory Committee for Aeronautics. 154 pages, 8vo. Washington, Government Printing Office, 1928, price twenty cents.

Bibliography of Aeronautics, 1927. National Advisory Committee for Aeronautics. 184 pages, 8vo. Washington, Government Printing Office, 1928, price thirty-five cents.

CURRENT TOPICS.

The Terminal Velocity of Drops. W. D. FLOWER. (*Proc. Physical Soc.*, London, Vol. 40, Pt. 4.) Liquid drops falling through quiet air after they have descended some distance get a constant velocity. This paper has to do with the measurement of the distance fallen and of the constant terminal velocity. Incredible as it may seem the author was unable to discover any previous determinations of the distance fallen before this constant velocity is established. A drop was liberated and fell upon the pan of a ballistic torsion balance turning about a horizontal axis. The distance from the point of release to the pan was progressively increased until no greater deflection was obtained by further increase. The drop was caught upon a horizontal sheet of filter paper. A fresh sheet was used for each drop, care being taken that all sheets occupied exactly the same position. The drops fell from a burette. When the drops form slowly and break away under the force of gravity they vary but little in volume. The range of volume for drops of water was from .2 to .7 c.c. As one would expect the larger drops fell a greater distance before they attained their constant velocity. When this state has been reached the acceleration due to gravity is just equal to the retardation caused by the air. For drops having volumes of .0194, .0415 and .0688 c.c. respectively the corresponding distances fallen were 225,316 and 340 cm. Drops of methyl salicylate also were tried. This liquid has 19 per cent. greater density than water, about half its surface tension and three times its viscosity. Drops of this liquid and of water having volumes of about .04 c.c. need to fall the same distance to attain constant velocity, but for larger sizes the distance for water is greater and for smaller volumes it is greater for the other.

To determine the constant terminal velocity a photographic method was used. For drops of water having volumes of .006, .01, .0414, and .083 c.c. the velocities attained were respectively 556.7, 611.5, 774.9 and 810.7 cm./sec. All the results obtained for the salicylate are larger than those for water drops of comparable size. The author sets up formulæ involving the coefficient of resistance and the radius of the drop and from these deduces that water drops with a radius of .25 cm. and salicylate drops with a radius of .21 cm. will fall faster than those of any other dimensions. Unfortunately in the case of both liquids the radius for maximum final speed lies

near the end of the series of radii investigated and hence it is impossible to see whether there actually is a maximum speed or not. Lenard of Heidelberg has reached the same conclusion for water and his explanation "is that the friction of the air causes deformation of the drops, so that instead of retaining the shape of spheres they become flattened out, thus presenting an increased surface to the air through which they are falling. This deformation is slight for the smaller drops, but increases rapidly in the case of the larger drops. After a certain size of drop has been attained, any further increase in volume produces a greater flattening, and instead of the velocity being increased it is slightly decreased." G. F. S.

The Influence of Electric and Magnetic Fields on the Crystallization of Supercooled Liquids. W. KONDOGURI. (*Z. f. Physik*, Vol. 47, page 589.) About twenty years ago it was discovered that the crystallization of molten sulphur was hastened by exposing it to the rays coming from radium. In following a line of experiments suggested by this result Kondoguri found that the electric field acts as radium does but has a stronger effect. In the present paper he goes on with his investigations using the magnetic as well as the electric field. Two liquids, piperin and salol were examined. Piperin melts at 128° and salol at 42.5°. Both of them are quite permanent in the liquid state when cooled below the melting temperature. Films of the liquids, .05 or .06 mm. in thickness, contained between coverglasses were put under the microscope so that the centers of crystallization could be seen and counted. The direction of the electric field was parallel to the surface of the film, while that of the magnetic field was perpendicular to it. The highest voltage applied was about 420 and the magnetic field lay between 9,000 and 10,000 gauss. An electric battery supplied the difference of potential and a Dubois electromagnet the magnetic field. Salol is much more productive of results than piperin. With no external influence in operation it is true that piperin develops centers of crystallization more rapidly than salol under comparable conditions relative numbers being 15 : 2, 33 : 5, 55 : 7, 7 : 1. When, however, an electric or magnetic field is applied, piperin manifests a much smaller change than salol, the ratio being roughly 1 : 10. Thus because salol is more sensitive and in addition more stable in the supercooled state it was chiefly used.

Counts were made at intervals extending over as much as sixteen days of the numbers of crystallization centers seen through the microscope in definite portions of the supercooled liquid, *a.* exposed to radium rays, *b.* acted on by the electric field and, *c.* left to itself without external influence. The numbers of centers for these three conditions were as follows:

After interval of	2	4	6	9	11	16 days
a. Radium	3	17	51	76	79	80
b. Electric Field	23	67	118	160	176	200
c. Control Liquid	1	2	6	10	14	19

Under the action of radium or of the electric field the number of centers reaches a limit after the lapse of sufficient time. After this has been attained the application of a higher voltage causes a further increase which continues until a new limit has been reached. The number of centers is certainly greater for strong electric fields than for weak ones. The author claims proportionality between field strength and the rate of initial increase of centers.

For the study of the influence of a magnetic field piperin was used because the heat from the electromagnet melted the centers that formed in salol. Only a part of the liquid could be used for counting the centers since in other parts they formed compact groups. However it was clear that with the passage of time the number of centers increased. In the case of piperin in the electric field a variation of the normal procedure was noticed. After several days, the number of centers having almost reached the limit, there was a sudden drop. Thereupon the number rose to its former value or even exceeded it. Once there was no drop after the limit had been almost attained instead a sudden rise toward a new limit took place. G. F. S.

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NATHAN HAYWARD

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MAGNETIC ALLOYS OF IRON, NICKEL, AND COBALT.*

BY
G. W. ELMEN, M.A.,
Research Department, Bell Telephone Laboratories, Inc.

IF the three ferromagnetic metals, iron, nickel and cobalt, are melted together in various proportions, all of the resultant alloys are magnetic when the materials used are reasonably pure. The magnetic properties of the alloys, however, vary greatly with composition and heat treatment. Some alloys have been found to be nearly non-magnetic; others have higher permeability than any hitherto known magnetic material and may be magnetically saturated in the earth's field. Still others are superior at high field strengths or excel in constancy of permeability at low fields.

In the Bell Telephone Laboratories we have been especially interested in these alloys and have made a fairly complete survey of the magnetic properties of the whole field of compositions of these metals. In this address I wish to tell you how our interest in these alloys was aroused, what our procedure was in carrying out an investigation, and what some of the principal results were. I shall also refer to some of the particular applications which we have made as a result of our discoveries.

I first became interested in these alloys in 1913. At that time I was looking for a magnetic material which would be more suitable for certain uses in the electrical communication

* Presented at a meeting held Thursday, November 8, 1928.

(Note.—The Franklin Institute is not responsible for the statements and opinions advanced by contributors to the JOURNAL.)

field than the iron then used. Of the large number of alloys investigated, several contained principally iron and nickel. One of these was particularly interesting. The composition of this alloy was approximately 70 per cent. nickel and 30 per cent. iron; it was a commercial alloy used as a special resistance material. In the hard worked condition in which it was supplied, its magnetic properties were not even as good as iron in a similar magnetic condition, but when heat-treated it was superior to the iron at low field strengths, the region in which I was especially interested.

This alloy differed from iron in another important characteristic. Experience with iron had shown that the best magnetic quality was obtained when the material was heated to a high temperature, and then cooled slowly to room temperature. It was considered particularly important to cool slowly in order to give the iron time to pass through its transformations and to allow it to build up a large grain structure. When the nickel-iron alloy was heat-treated in this manner, it was found to have lower permeability than it had when cooled fairly rapidly.

The discovery that rapid cooling was required for this alloy to give the best magnetic quality was one of the major contributions from our early work. It showed us that one of the important factors in developing the magnetic properties of new alloys was the determination of the rate of cooling for the best magnetic quality for each alloy.

Another difference between this alloy and iron relates to the energy loss caused by the hysteresis at low flux densities. In this range iron has lower hysteresis loss when it is in a mechanically hard condition than when it is well annealed. As the flux density increases, however, the hysteresis loss of the hard material increases more rapidly than does that of the annealed and at medium and high flux density, the mechanically hard iron is much inferior. Tests on the nickel-iron alloy show that both for high and low flux densities the hysteresis loss was a great deal higher for the mechanically hard material than for the one heat-treated to give the best magnetic quality.

The discovery of the unusual magnetic properties of this 70–30 per cent. nickel-iron alloy gave us the lead that we were

looking for, and started our investigation of the magnetic properties of the whole series of nickel-iron alloys. We found, as we had reason to expect, that the 70–30 per cent. alloy was one of a large group of alloys in the same series which had similar magnetic properties. In fact substantially all of the alloys containing more than 30 per cent. nickel had similar characteristics except that some of them were not as sensitive to heat-treatment as the first alloy we had tested.

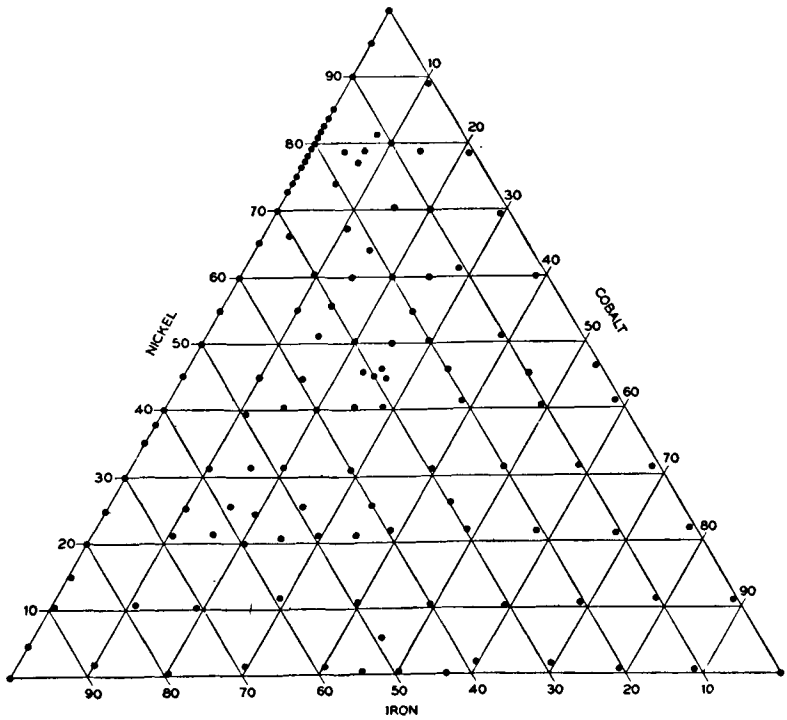
Because of the technical possibilities of the nickel-iron alloys, we spent several years in their investigation and their commercial application. We were especially concerned with increasing the resistivity of a number of these alloys and with this in view we added other elements to the iron and nickel in order to determine their effect both on the resistivity and the magnetic properties. Copper, chromium, molybdenum, tungsten, and tantalum are a few of those we added; also the magnetic element cobalt. The striking results obtained with the addition of cobalt to the iron-nickel alloys led us to make a complete survey of the whole field of alloys containing the three magnetic metals.

A convenient way of showing graphically the number of alloys we investigated and the distribution of compositions of these alloys is afforded by the equilateral composition triangle in Fig. 1. In this triangle the metals, iron, nickel, and cobalt, are represented by the corners; the binary alloys by points on the three sides, and the ternary alloys by points within the area of the triangle. Each alloy we investigated is indicated by a dot and the location of each dot indicates the proportion in per cent. of the three metals in the alloy. A glance at the diagram will give you an appreciation of the completeness with which the field was covered.

In preparing these alloys we used the best commercial materials available. These contained small amounts of impurities some of which affect unfavorably the magnetic properties. It was considered beyond the scope of this investigation, in which such a large number of alloys were required, to attempt to remove these impurities completely or even partially. Moreover we were especially interested in alloys which could be reproduced on a commercial scale, without too great cost due to refinements of raw materials or to methods of preparing the alloys.

Throughout the investigation we have followed a standard procedure for preparing our samples. We have also followed a standard procedure for heat-treating and in magnetic measurements. The result is that we have accumulated a large mass of data over a number of years all of which may be significantly compared.

FIG. 1.

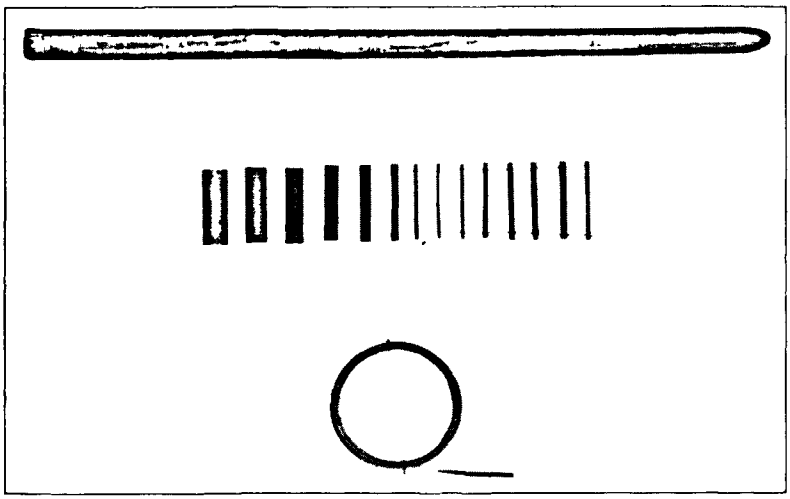


Composition diagram—Ni-Fe-Co series. Dots show compositions tested.

The alloys were cast from Armco iron, electrolytic nickel, and a very high grade of commercial cobalt containing only small amounts of impurities. They were melted together in the desired proportions in a silica crucible in a high frequency induction furnace. The metal was cast into bars 18″ long and ¾″ in diameter. The bars were rolled or swaged into ¼″ rods; then they were drawn into wire and flattened and trimmed into ⅛″ by .006″ tape. The material was annealed several times in the reduction process, for the cold working hardened the alloys rapidly and made them difficult to work.

To prepare the tape for heat-treatment and subsequent magnetic measurements, about 30 ft. of it was wound spirally into a ring of 3″ inside diameter, the ends being spot welded to the adjacent turns. Care was taken to wind the rings loosely to prevent the turns from sticking to each other during annealing.

FIG. 2.



Cast alloy bars and intermediate stages of samples in reduction to tape. Also ring from tape ready for heat treatment.

For convenient comparison of the magnetic properties of the alloys with those of the metals from which the alloys were cast, sample lots of the iron, nickel, and cobalt which we used in making the alloys, were melted, cast, and prepared in the same manner as the alloy test samples.

In Fig. 2 the various steps through which the alloys had to pass in the mechanical reduction to tape are shown. Between each step in the reduction, as indicated by a sample, the alloy had to be annealed to remove the mechanical hardness. A sample ring wound from the finished tape, ready for heat-treatment, is also shown in the figure.

The next step in the process of preparing these alloys for magnetic measurements is the heat treatment. Early experience with the nickel-iron alloys indicated the need for a

variety of such treatments, but of course it was impossible to apply to every alloy such a complete variety of heat-treatments, that all of the combinations of heat-treatment and composition could be known. Our procedure was to apply three types of heat-treatment.

A number of rings of a given composition were packed in a nichrome pot. The pot was placed in an electrical resistance furnace, the temperature of the furnace raised to between 900° and 1000° C. and held at that temperature for one hour. The current was then turned off and the pot cooled with the furnace. Ten hours were required for the furnace to cool to the temperature of the room. Between 700° C. and 400° C. the rate of cooling was approximately 1.5° per minute.

At least two rings of each composition were always annealed together. One of these rings received no further heat-treatment. The second ring was heated for fifteen minutes in a furnace held at 600° C., then removed and cooled rapidly on a copper plate in the air. With this cooling the rate was approximately 20° C. per second. In some cases a third annealed ring was heated 24 hours at 425° C.

In the discussions and in the figures, the rings which received the first heat-treatment only are referred to as "annealed," those reheated to 600° C. and rapidly cooled as "air-quenched," and those held for a long time at 425° C. as "baked." The magnetic measurements were made on these rings.

The magnetic induction, or flux density, was determined for a large number of magnetizing forces, beginning at a few hundredths of a gauss and increasing in uniform steps up to 100 gauss. Magnetization curves were plotted from these measurements. The induction was also determined for each composition at a magnetizing force of 1,500 gauss.

The permeabilities were computed from the induction measurements, and were plotted either against the flux density or the magnetizing force, depending on which graph illustrated best the characteristics of the material. At the lower ends these curves were extended to zero field strengths. Their intercepts on the permeability axis are the initial permeabilities. The maximum permeabilities were also obtained from these curves.

For determining hysteresis loss, two methods were used. In some cases hysteresis loops were plotted from ballistic measurements, and in other cases a direct determination of hysteresis loss was made from the apparent alternating current resistance of a winding wrapped around the sample. Ordinarily the hysteresis loop was obtained for one condition only in which the flux density was varied between plus and minus 5,000 gauss. For some of the alloys of special interest, a large number of loops were obtained for different magnetizations, the maximum flux densities varying from 100 or less to 5,000 gauss.

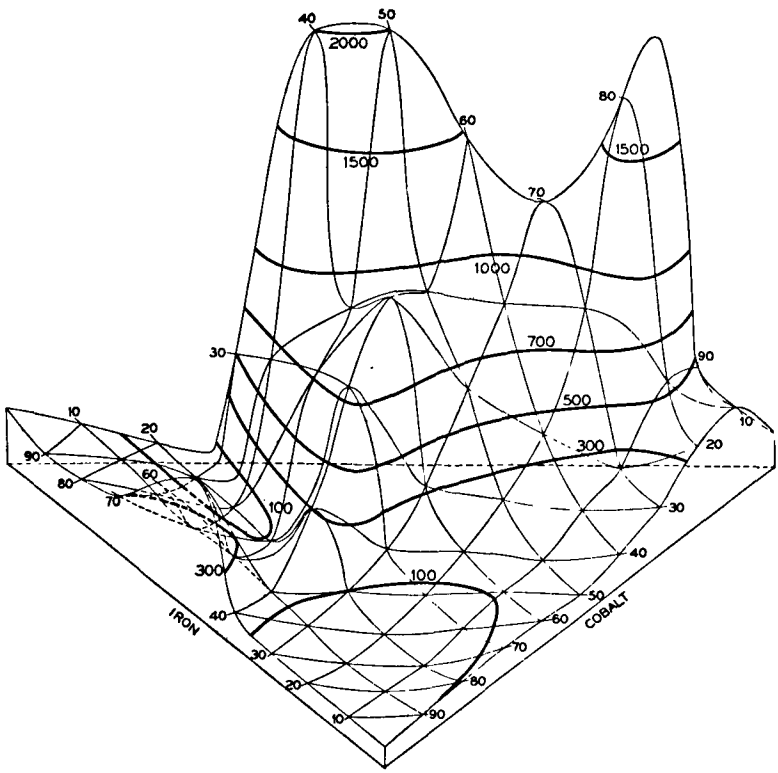
In illustrating the magnetic properties I will be forced to limit the discussion to a few outstanding values for each alloy, and by comparing these, obtain a general view of the relation of the magnetic properties to composition. The values I have selected are the intrinsic inductions for two magnetizing forces, 50 and 1,500 gauss respectively, the initial and the maximum permeabilities and the hysteresis loss for a maximum flux density of 5,000 gauss. For a number of alloys which represent regions of composition with magnetic properties of special interest, curves for magnetization and permeability will be shown. A number of hysteresis loops for different maximum flux densities will also be given.

In illustrating graphically the relations between the magnetic properties and compositions of ternary alloys, it is convenient to plot these quantities in the form of solid diagrams. Such a diagram is shown in Fig. 3 constructed for initial permeabilities of the alloys in the annealed condition. In this figure the composition triangle, Fig. 1, is used as the base. On this triangle, verticals are erected proportional to the numerical values of the initial permeability. The ends of these verticals give a contour of the upper face of the figure. With a sufficient number of alloys this surface represents fairly accurately the values of the initial permeabilities for all compositions. The edges of the surface give the initial permeabilities of the binary alloys and the rest of the surface those of the ternaries.

The coördinates of the composition triangle, the intersections of which give the compositions of the alloys in 10

per cent. variations, are projected to the surface and are represented by the narrow black lines. The heavy black lines on this surface are contours such as you see on topographical maps and represent the intersections with the surface of planes parallel to the base at definite elevations. Each line

FIG. 3.



Initial permeabilities for annealed alloys.

locates the compositions of those alloys which have equal values of initial permeability.

In this solid diagram of the initial permeabilities for the alloys in annealed condition, there are three regions of composition which are of special interest. The first of these is in the binary iron-nickel series represented by the back face of the diagram, which may be referred to as the nickel-iron

plane. The nickel percentage is indicated by the numbers at the intersections of the permeability coördinates with the surface. The left-hand corner represents 100 per cent. iron and the right-hand corner 100 per cent. nickel, with initial permeabilities of 250 and 200, respectively. Between these limits of composition the initial permeability varies from approximately 100 to more than 2,000. With small additions of nickel to iron the permeability drops gradually until the added amount is approximately 28 per cent. of the composition. From that point there is a rapid rise in the permeability with the increase in the nickel content reaching a maximum of over 2,000 for the alloy containing approximately 45 per cent. nickel. It then decreases as more nickel is added, reaching a second minimum of about 1,200 for 70 per cent. nickel and increases again reaching a second maximum of approximately 1,900 at 83 per cent. nickel.

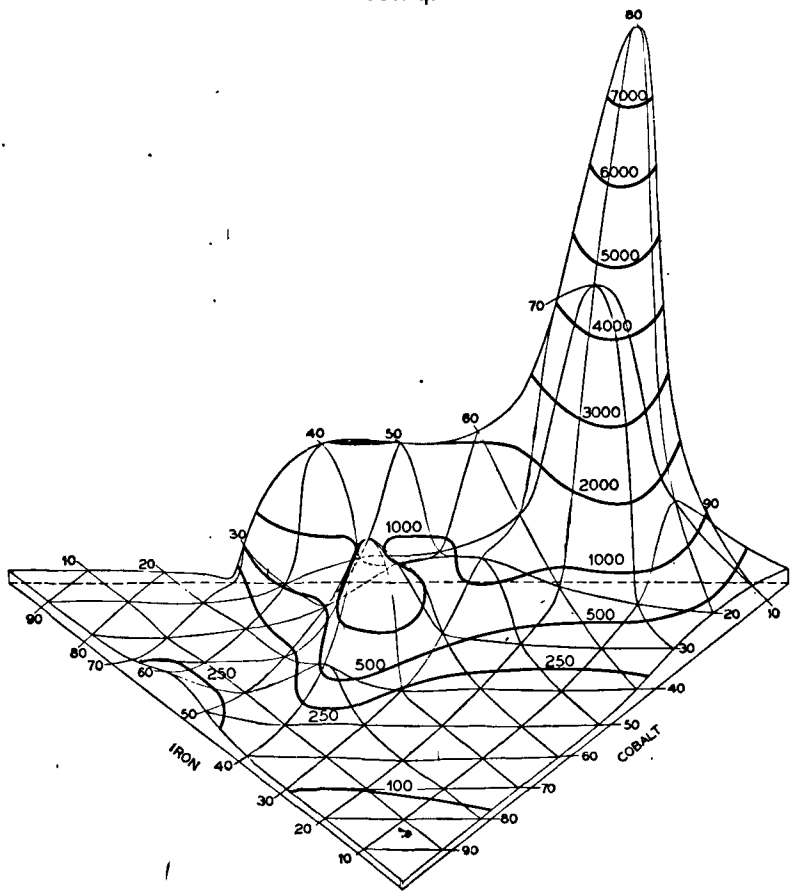
Another region of special interest is found in the ternary series in front of the first maximum in the nickel-iron series. If you follow the coördinate from 40 per cent. Ni, you will note an elevation between the 700 and 1,000 permeability contour lines. The ternary alloys of this region are remarkable not because of their high permeabilities although they are higher than for the surrounding region, but because of the constancy of permeability and low hysteresis loss at low magnetizing forces.

Another region of interest is located in the iron-cobalt plane between 40 per cent. and 70 per cent. iron. This is the plane in the figure on which the iron percentages are marked. The initial permeability for the highest point is over 600; more than twice the initial permeability of Armco iron.

When the alloys are air-quenched the initial permeabilities change in some regions very materially and give us an entirely different looking solid diagram as shown in Fig. 4. Because of the very high values of the permeabilities a different scale was used in this figure from that used in Fig. 3. It is interesting to note how the rapid cooling has affected some of the binaries in the iron-nickel plane. The rise in permeability begins at about 28 per cent., the same as for the annealed alloys, and the initial permeabilities are substantially the same, up to about 45 per cent. nickel. Between 45 per cent.

and about 90 per cent. nickel the permeabilities have increased to a remarkable degree. The valley we saw in the region 45 per cent. to 75 per cent. nickel for the annealed alloys has disappeared, and from 55 per cent. upward there is a rapid

FIG. 4.



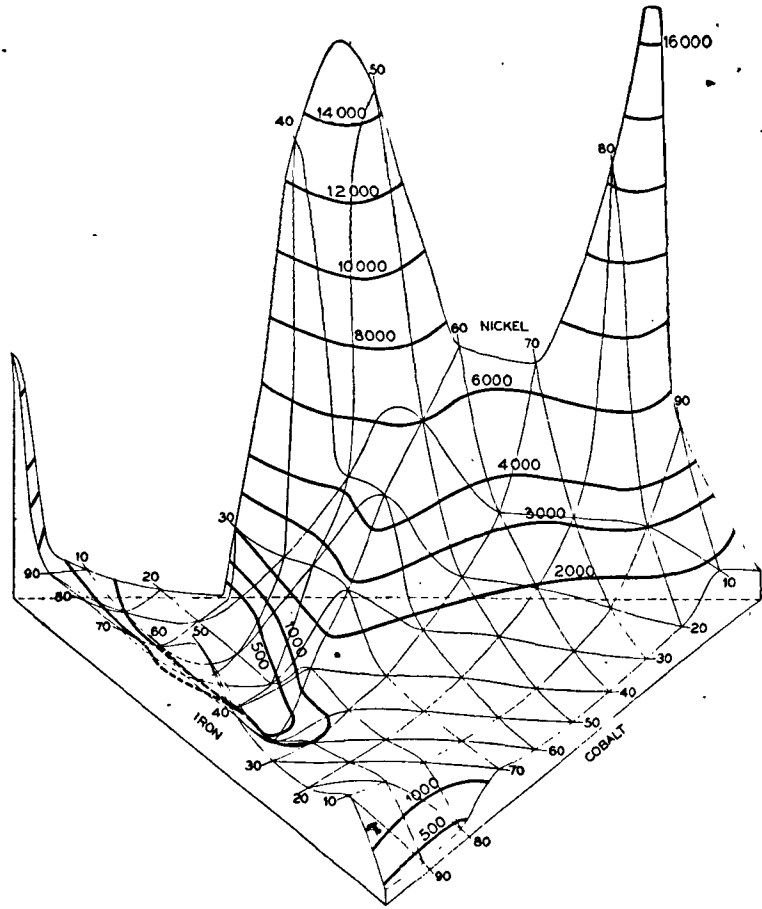
Initial permeabilities for air-quenched alloys.

increase reaching a peak value of approximately 8,000 for the composition containing 78½ per cent. nickel, an increase of over four times the permeability of the annealed alloy of the same composition.

Air-quenching also increases the initial permeabilities of

the group of ternary alloys which in the annealed condition showed a maximum characterized by a low hysteresis loss and a substantially constant permeability. This increase in permeability, however, is accompanied by a material increase

FIG. 5.



Maximum permeabilities for annealed alloys.

in the hysteresis loss at low flux densities and a decrease in the constancy of permeability.

In contrast to the alloys just discussed, the iron-cobalt series in the neighborhood of 50 per cent. iron gives a lower

permeability upon rapid cooling. In this respect they behave the same as iron and cobalt.

The maximum permeabilities for the annealed alloys shown in Fig. 5 give a surface very similar to that for the initial permeabilities. The most interesting difference is the rapid decrease in permeability of iron as indicated at the left-hand corner of the diagram by the addition of small amounts of cobalt or nickel, 10 per cent. of either being sufficient to reduce the maximum permeability 85 per cent. All of the iron-nickel alloys between 35 per cent. and 85 per cent. Ni have as high or higher permeability than Armco iron.

I have not prepared a diagram for the maximum permeabilities for the air-quenched alloy. However, I can say that the surface obtained resembles the one shown for the initial permeabilities for the rapidly cooled alloys, although, of course, the maxium permeabilities are of different magnitudes. As high as 120,000 has been obtained for the alloy containing 78½ per cent. nickel.

The group of alloys of high permeability for low magnetizing forces in the iron-nickel series we have named permalloy.[1] The permalloys, therefore, include nickel-iron alloys containing more than approximately 30 per cent. nickel. In referring to specific compositions in this group we have found it convenient to use as a distinguishing prefix for each composition its nickel content. For example, 78.5 permalloy is the alloy containing 78.5 per cent. nickel and 21.5 per cent. iron. When other elements are added to the permalloys the chemical symbol and the percentage of the added element are also added to the prefix. For example, 3.5–78.5 Mo-permalloy is an alloy containing 3.5 per cent. molybdenum, 78.5 per cent. nickel, and 18 per cent. iron.

The ternary alloys of constant permeability and extremely low hysteresis loss at low flux densities we have also grouped together under a common name, perminvar.[2] The limits of composition for the perminvars are less easily defined than for the permalloys, because the transition in magnetic proper-

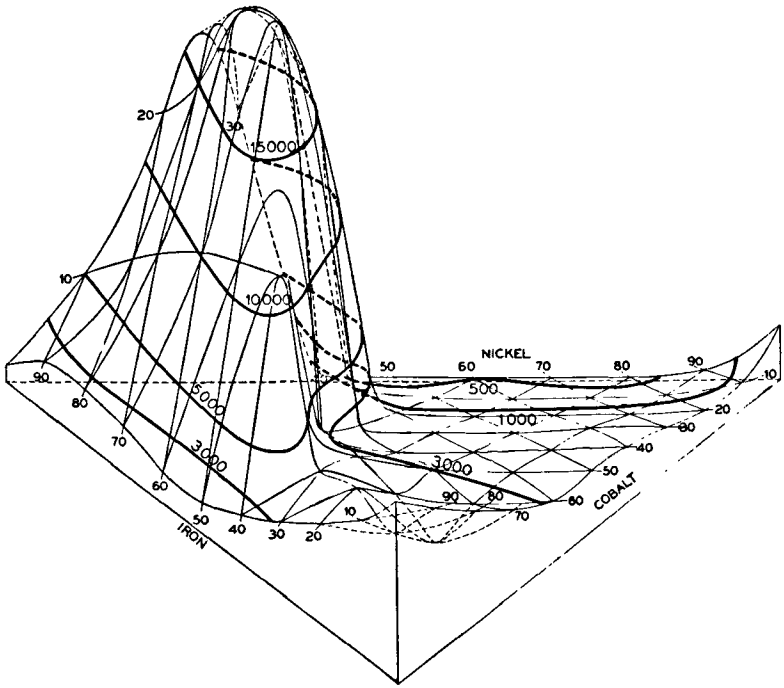
[1] Arnold and Elmen, JOURN. OF FRANKLIN INST., May, 1923, p. 621.

[2] G. W. Elmen, JOURN. OF FRANKLIN INST., Sept., 1928, p. 317.

ties is not as marked with small changes in composition. We have found that compositions between 10 per cent. and 40 per cent. iron, 10 per cent. and 80 per cent. nickel, and 10 per cent. and 80 per cent. cobalt have marked perminvar characteristics.

The hysteresis losses of the alloys in the annealed condition are plotted in Fig. 6 for a maximum flux density of

FIG. 6.

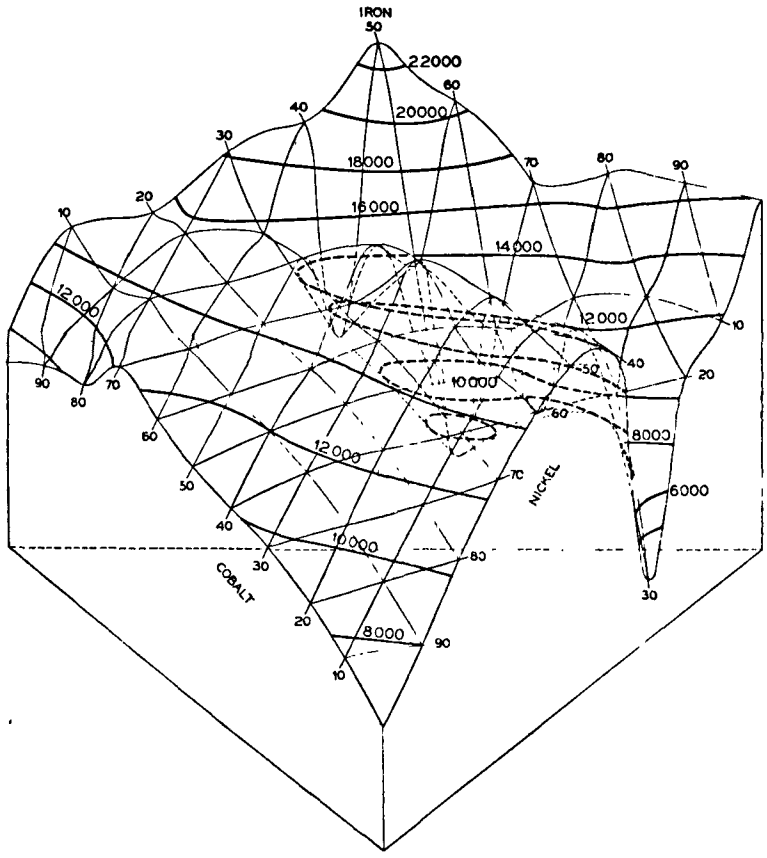


Hysteresis losses, ergs per cm.³ per cycle, for maximum flux density of 5,000 gauss for annealed alloys.

5,000 gauss. This diagram illustrates again the abrupt change in the magnetic properties as we pass from alloys with large iron content to those in which nickel predominates. In this figure the lowest energy losses are those for alloys in the neighborhood of the 78.5 permalloy composition. In the iron-cobalt series the 50 per cent. cobalt alloy has the lowest hysteresis loss.

The intrinsic inductions, which are those parts of the inductions contributed by the magnetic material, are shown in Fig. 7 for a magnetizing force of 50 gauss. In the figure

FIG. 7.



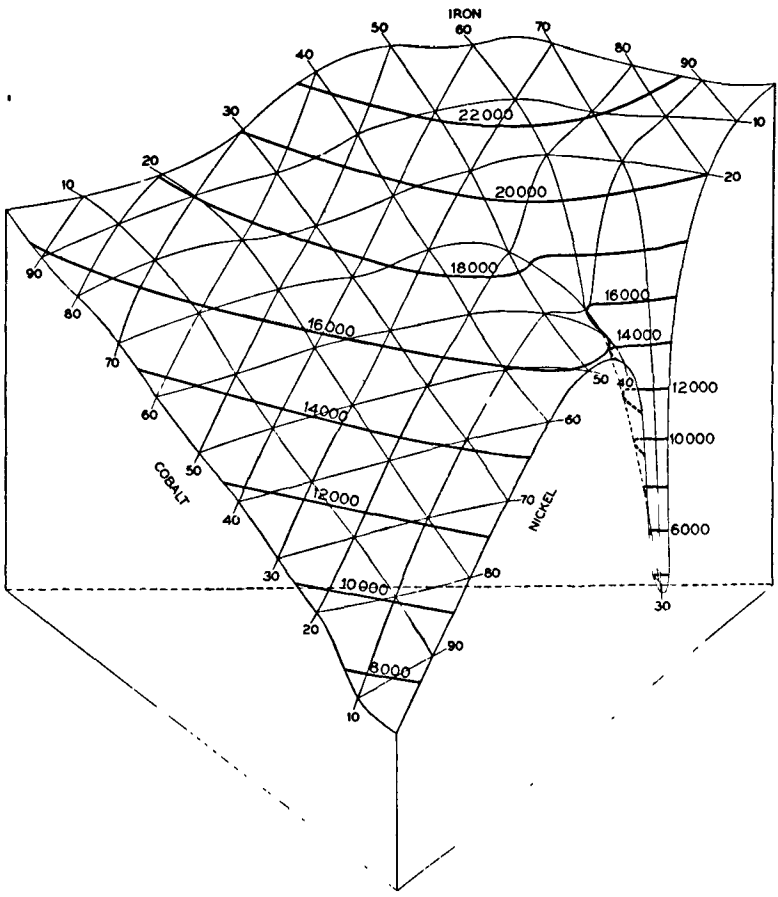
Intrinsic inductions for annealed alloys at $H = 50$.

the triangle has been turned through 120° clockwise from its position in the previous figures, placing the iron-cobalt alloys in the back of the diagram. For this magnetization the 50 per cent. iron-cobalt alloy is superior to any of the others.[3] Another interesting part of this diagram is the deep depression

[3] This was also found by Ellis, Engineering and Science Series No. 16, June 1927, Rensselaer Polytechnic Institute.

at about 30 per cent. nickel, in the iron-nickel plane—now the right-hand front face of the diagram. This depression extends back for some distance into the ternary alloys.

FIG. 8.

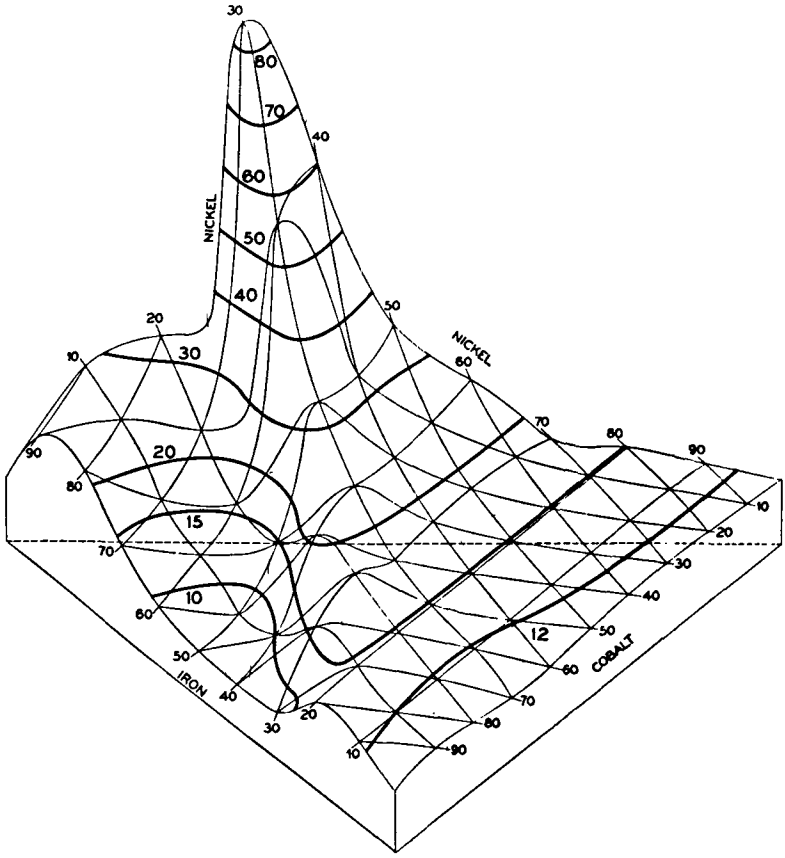


Intrinsic inductions for annealed alloys at H = 1,500

The intrinsic inductions are also shown for a magnetizing force of 1,500 gauss in Fig. 8. Here we find that the irregularities of the surface in the previous figure have largely disappeared and the surface has become fairly flat. The inductions for the alloys in the neighborhood of 34 per cent. cobalt and 66 per cent. iron have now increased so that they

are practically the same as for the 50 per cent. composition. The valley in the neighborhood of 30 per cent. nickel has largely disappeared except for the alloys without cobalt which are much harder to saturate.

FIG. 9.

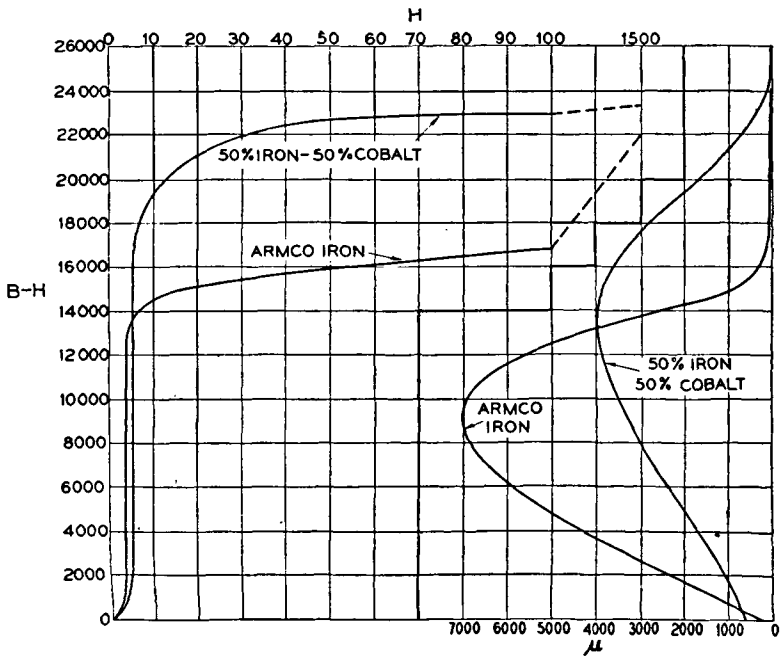


Resistivities, microhm-cm., for annealed alloys.

In Fig. 9 a solid diagram has also been constructed for the resistivity of these alloys in the annealed condition. It is interesting to note the high resistivities of some of the nickel-iron alloys—again in the back of the diagram—and the low values for some of the cobalt-iron alloys in the 45 per cent. to 70 per cent. cobalt range of compositions.

In the discussion of the magnetic properties of these alloys, I have pointed out that there are three regions of composition which are of special interest from the standpoint of applications because of their unusual magnetic properties. One of these is in the iron-cobalt series. A representative alloy of this group is the 50 per cent. cobalt composition. In Fig. 10 the magnetization curves for this alloy and for a sample

FIG. 10.



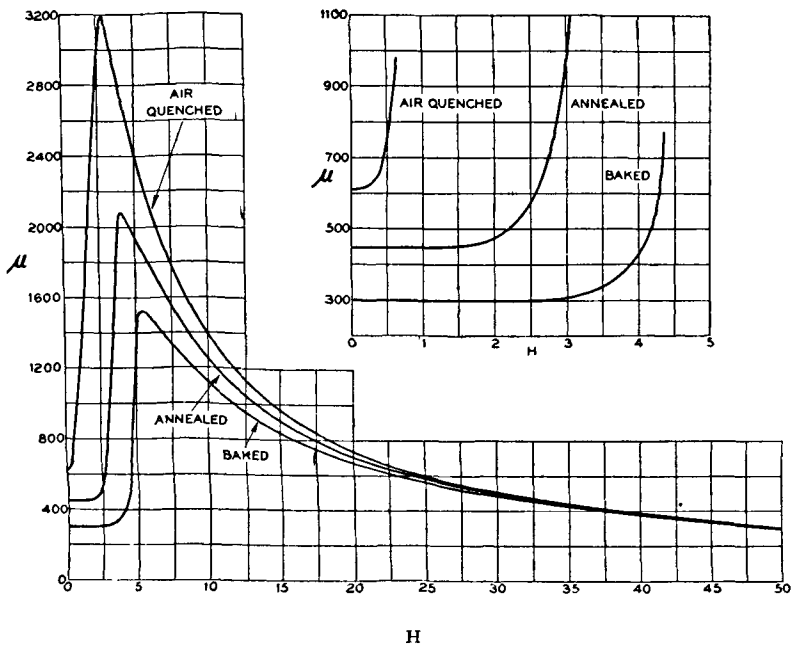
Magnetization and permeability curves for the 50 per cent. iron, 50 per cent. cobalt composition and for Armco iron.

of iron are shown on the left and the permeability curves plotted against the same scale of intrinsic inductions on the right. The scale for the magnetizing force of the magnetization curve is at the top of the figure. Below an induction of 1,000 and above 13,000 the permeability of the alloy is higher than for iron. Its initial permeability is 600 and the maximum permeability is 4,000. For a magnetizing force of 100 gauss the intrinsic induction is nearly 23,000 gauss. For Armco iron the initial and maximum permeabilities are 250

and 7,000, respectively, and the intrinsic induction for a magnetizing force of 100 gauss is 17,000.

The perminvars are the next group with interesting characteristics. Their remarkable constancy of permeability at low magnetizing forces and their extremely low hysteresis loss at low flux densities make them of unusual interest. The composition 45 per cent. nickel, 25 per cent. cobalt and 30 per

FIG. 11.



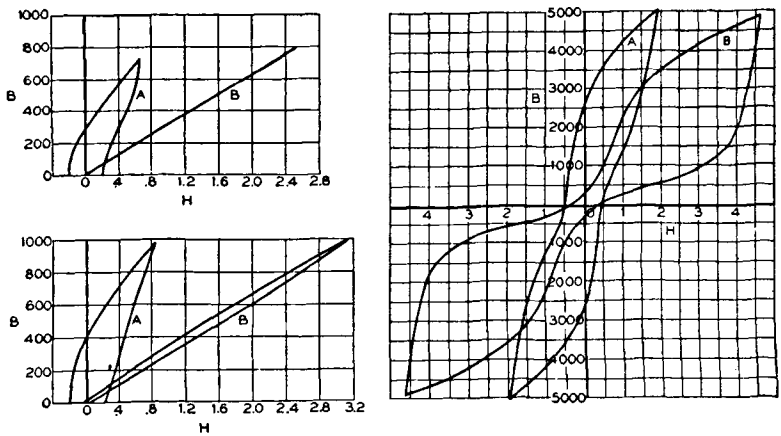
Permeability curves for a perminvar alloy of 45 per cent. Ni, 25 per cent. Co, 30 per cent. Fe.

cent. iron is a typical alloy for this group. Permeability curves for this alloy for three types of heat-treatment are plotted against magnetizing force in Fig. 11. The insert in the upper right-hand corner shows the lower parts of these curves plotted to a larger scale. For the baked alloy the permeability is constant at 300 with an increase of the magnetizing force from zero to $2\frac{3}{4}$ gauss. The initial permeability for the air-quenched alloy is more than twice that of the baked, but the constancy of permeability is a great deal less. This illustrates

the close relation of these two properties. As the magnetizing force approaches 40 gauss the differences in the permeabilities, resulting from different rates of cooling, disappear.

The hysteresis loops for this composition are also of unusual interest. Fig. 12 illustrates the hysteresis charac-

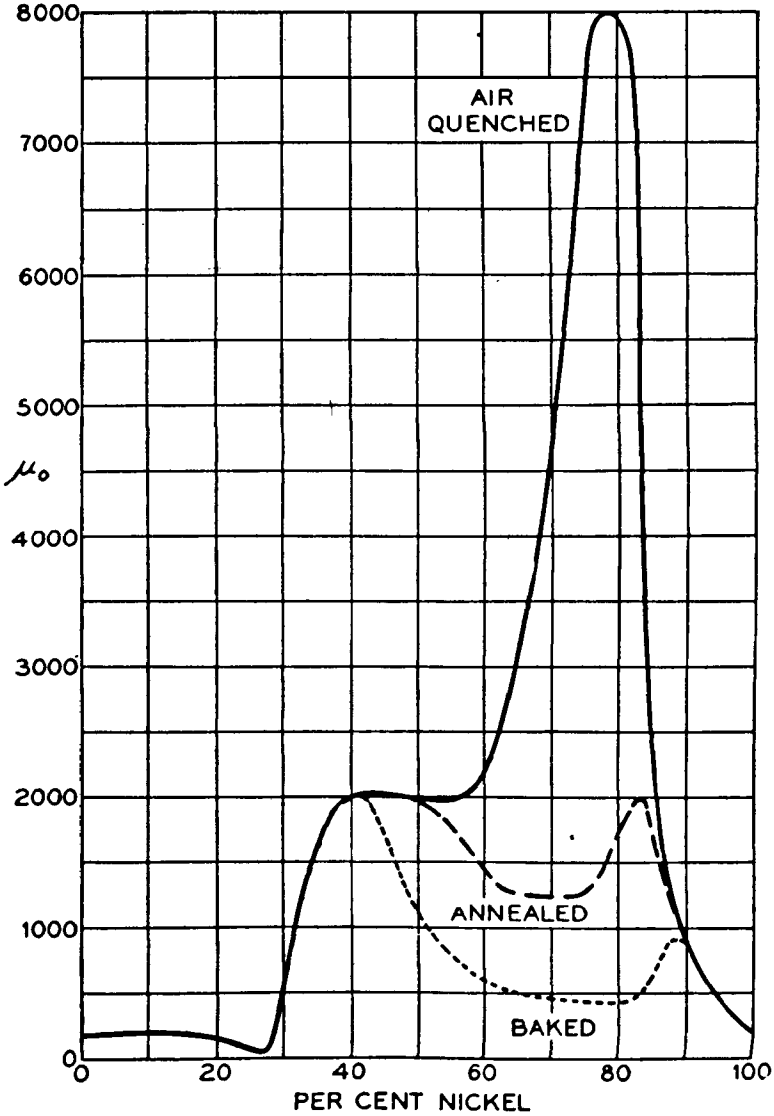
FIG. 12.



Hysteresis characteristics for a perminvar alloy containing 45 per cent. Ni, 25 per cent. Co, 30 per cent. Fe. A = air-quenched; B = baked.

teristics for this alloy, in air-quenched and baked condition, for three maximum flux densities, 750, 1,000 and 5,000 gauss, respectively. For the lowest flux density of the baked sample the ascending and the descending branches of the loop coincide and the loop is represented by a straight line. For the next higher flux density the loop, for the same heat-treatment, begins to have a measurable area. At low values of induction, however, the two branches of the loop for the baked alloy approach each other and often come together completely at the origin. The complete loop for a maximum flux density of 5,000 gauss also shows this peculiar shape for the baked alloy. The loop is constricted in the middle, the two branches almost passing through the origin. The air-quenched alloy also shows tendency of constriction but much less than the baked. The areas of the two loops show that for this value of maximum flux density the hysteresis loss for the baked sample is greater than for the air-quenched. This condition is the reverse of that for lower flux densities.

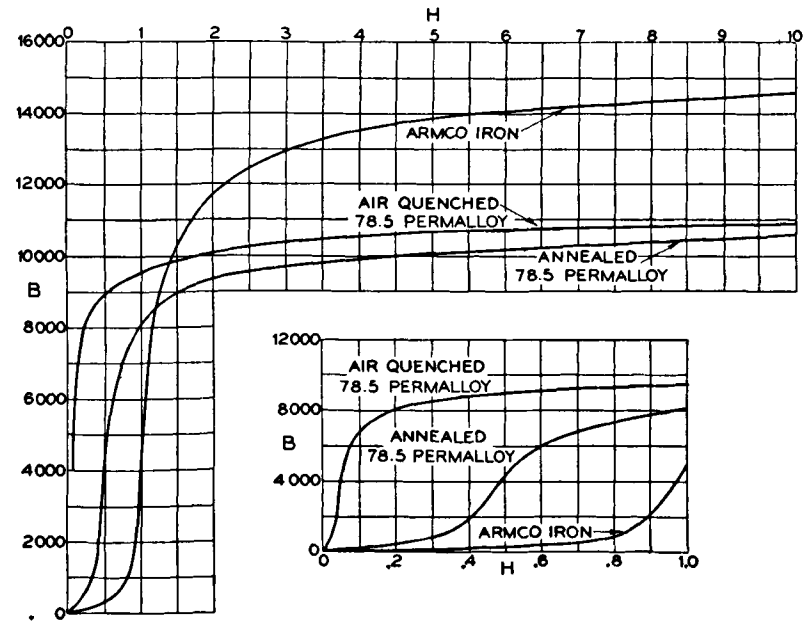
FIG. 13.



Initial permeabilities for the Fe-Ni series.

The last group of alloys of special interest are the perm-alloys. In Fig. 13 I have taken the curves for initial permeabilities of the iron-nickel series from the solid diagrams for the air-quenched and the annealed conditions and plotted them on the same scale of coördinates. A curve of initial permeabilities for a series of baked alloys is also plotted in this figure. The baking process for these alloys differed from the one we usually employed in that each alloy was baked

FIG. 14.

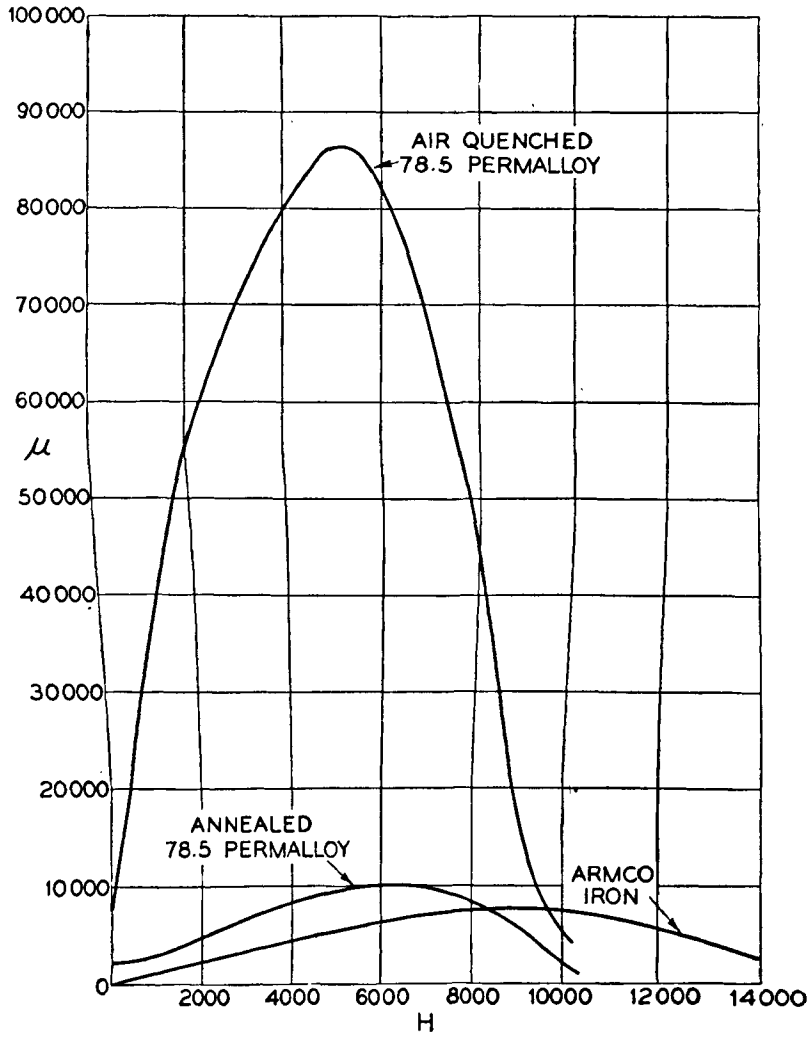


Magnetization curves for 78.5 permalloy and for Armco iron.

until no decrease in the permeability resulted from further baking. For some alloys several weeks were required before this condition obtained. The time was shortest for compositions between 60 per cent. and 80 per cent. nickel. On both sides of this range the time necessary for stabilization increased both with increasing and decreasing nickel content. With nickel content of less than 42 per cent. and more than 88 per cent., approximately, no difference sufficiently large to be attributed to the baking could be observed, even after the alloys had been baked for several weeks.

The 78.5 permalloy composition in this series is of special interest because this alloy when air-quenched gives the highest

FIG. 15.

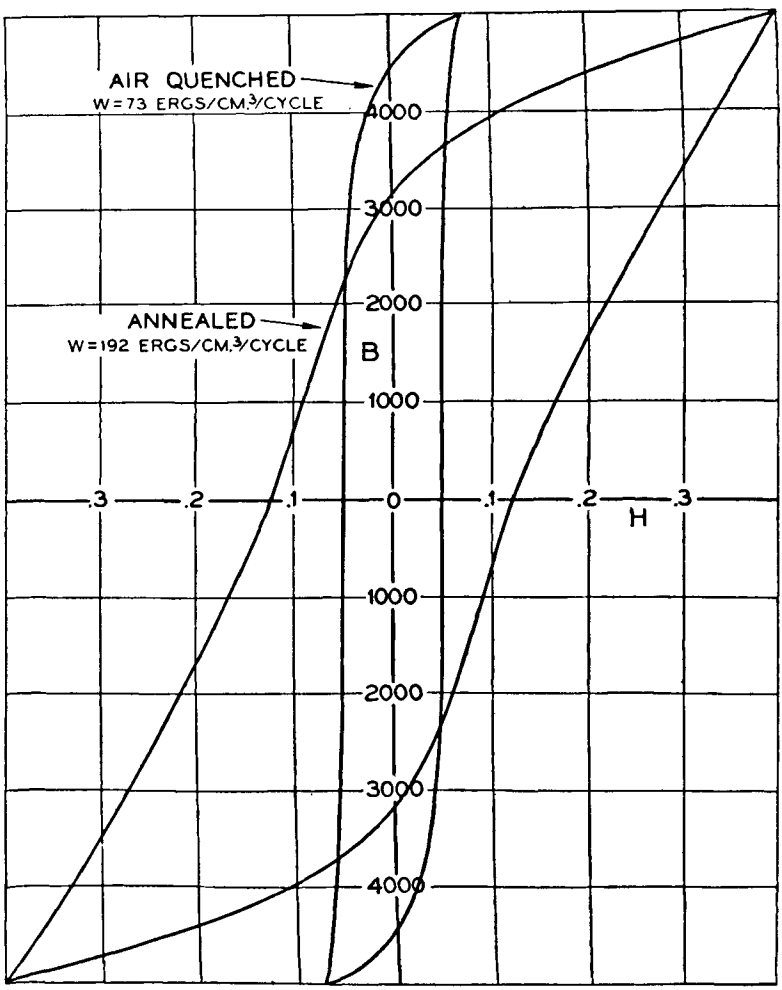


Permeability curves for 78.5 permalloy and for Armco iron.

initial and maximum permeabilities. The magnetization curves for this composition are plotted in Fig. 14 for annealed and air-quenched samples, and for a sample of

annealed iron. The lower part of this graph is shown in the insert on a larger scale. The curves are plotted for magnetizing forces between 0 and 10 gauss only. In these curves the

FIG. 16.



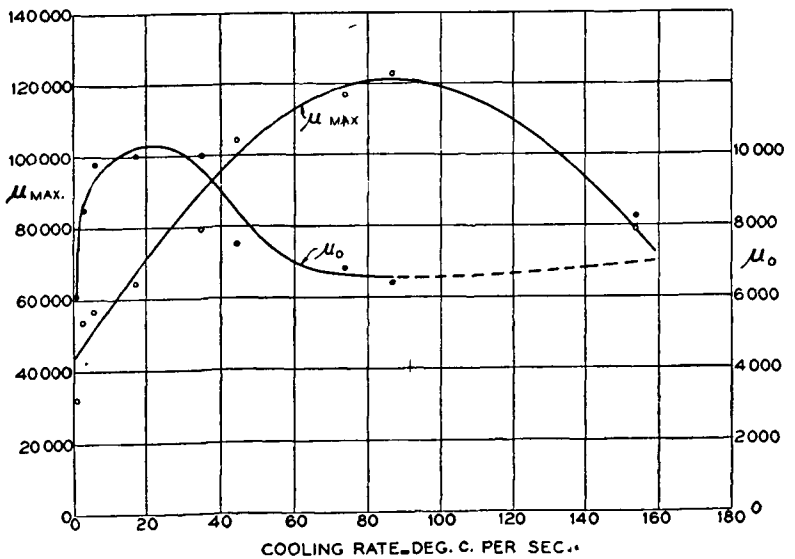
Hysteresis loops for 78.5 permalloy.

induction for the annealed sample remains lower than for the air-quenched sample, but as the force increases there is a tendency for the two curves to approach each other. This

continues with still further increase in the magnetizing force and beyond 50 gauss the two curves coincide.

The permeability curves for these samples are plotted in Fig. 15, illustrating the great difference in their maximum permeabilities. For the annealed and the air-quenched samples, the initial permeabilities are 2,000 and 9,000, and the maximum permeabilities 10,000 and 87,000, respectively. The initial and the maximum permeabilities for the iron sample are 250 and 7000, respectively.

FIG. 17.



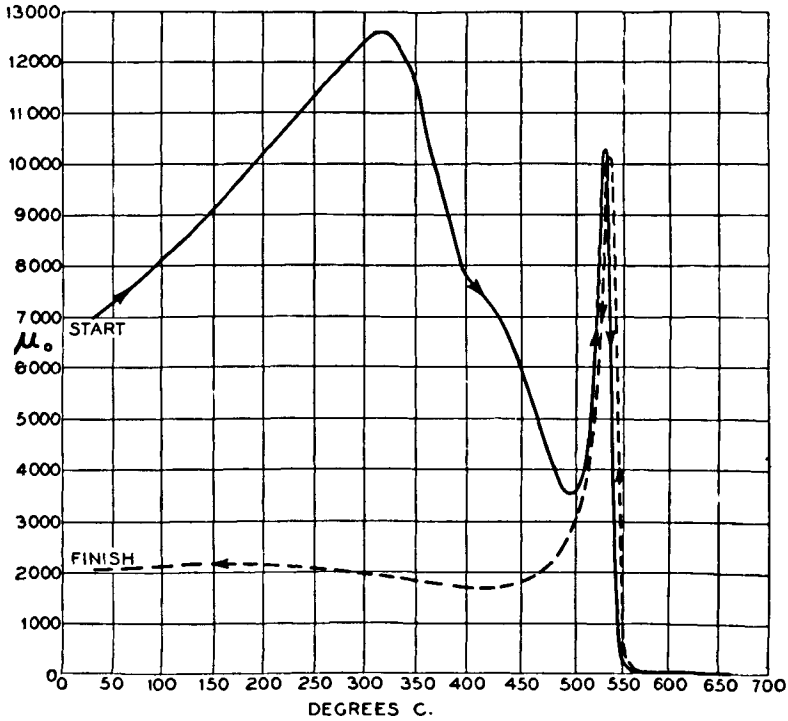
Initial (μ_0) and maximum (μ_{max}) permeabilities for 78.5 permalloy for different rates of air-quenching from 600° C.

The effect of air-quenching on the energy loss is illustrated in Fig. 16 in which hysteresis loops for a maximum flux density of 5,000 gauss are plotted for the air-quenched and the annealed samples. The energy loss for the rapidly cooled sample is only 35 per cent. of that for the annealed.

In deciding on a rate for air-quenching we selected the rate which gave the highest initial permeability for the 78.5 permalloy. In Fig. 17 I wish to illustrate how small changes in this rate affect the initial and the maximum permeabilities

for this composition. The scale for maximum permeability is on the left and for initial permeability on the right. The cooling rate is in degrees centigrade per second. It is interesting to note that the highest initial permeability was obtained when the cooling rate was approximately 20° C. per second, while a rate four times as rapid gave the highest

FIG. 18.



Temperature-permeability curve for 78.5 permalloy: magnetizing force .003 gauss.

maximum permeability. 10,000 and 120,000 were the highest initial and maximum permeabilities, respectively, for these particular samples. These values are not the highest we have obtained for this composition. Test samples from other castings have given as high as 13,000 for initial permeability and upwards of 400,000 for maximum.

Fig. 18 illustrates the manner in which the permeability at a low magnetizing force for a rapidly cooled 78.5 permalloy

composition is affected by passing it through a temperature cycle between room temperature and 650° C. The permeability was measured for a constant magnetizing force of .003 gauss as the temperature was passed through this cycle. The rate at which the sample was heated and cooled was rather slow, requiring upwards of two hours to complete the heat run.

The permeability of the air-quenched sample was 7,000 at the start, increasing rapidly up to about 315° C. With further increase in temperature there is a rapid drop in permeability until 500° C. is reached. With further increase in temperature the permeability rises very rapidly to a sharp peak at about 530° C. and then decreases, the alloy becoming non-magnetic at 590° C. With decreasing temperature the alloy again becomes magnetic at about the same temperature, at which the magnetism disappeared and the curve for decreasing temperature is the same as for increasing temperature over a short range. At 500° C. the curve for decreasing temperature does not follow its original path but continues to drop until the temperature reaches about 425° C. From that point on until room temperature is reached the return curve is nearly horizontal.

At room temperature the permeability is only 2,000, a drop of 5,000 from its value at the beginning of the run. With the alloy in this condition, if a second cycle is run, the permeability both for increasing and decreasing temperature will be substantially the same as the one in the figure for decreasing temperature. By heating of the sample to 600° C. and air-quenching, the magnetic properties at the beginning of the test are restored.

The connection between the permeability of this composition and the heat treatment in the temperature range below 600° C. is also illustrated by a series of tests in which annealed rings were air-quenched from temperatures below 600° C. The rings were placed in a furnace and heated at 600° for fifteen minutes. The temperature was then decreased slowly to 550° C. and held until the alloys had reached a constant condition. One of the samples was then taken out of the furnace and air-quenched. The temperature of the furnace was then dropped another step and the process of

stabilization and air-quenching repeated for the next ring. This was repeated for a number of temperatures until there was substantially no change between two rings quenched from successive temperatures. Another series of annealed rings was slowly heated in successive steps to the same temperatures, stabilized, and then air-quenched.

A comparison of the initial permeabilities of these rings showed that each ring attained a definite permeability, characteristic of the temperature from which it was air-quenched whether it reached that point from a lower or a higher temperature, provided it was stabilized before air-quenching. The air-quenched rings stabilized between 500° and 400° C. showed the greatest change in permeability. The time required for stabilization at the higher temperatures was very short, only a few minutes being required in the neighborhood of 500° C., but it increased progressively as the temperature was lowered, and in the lower end of the range several days were required for the alloy to reach a constant condition.

Tests on other compositions showed that those alloys in which the magnetic properties depended on the rate of cooling below 600° C. gave results similar to those for the 78.5 permalloy. The temperature range was also found to be substantially the same as for the 78.5 permalloy, although in some cases there were indications that small changes occurred above 600° and below 400° C.

This connection between heat treatment and magnetic properties led me to conclude that the differences in these properties in the alloys which had been heat-treated differently below 600° C. were caused by constitutional changes in the alloys. Such changes are common in alloys in the solid state, often at low temperatures. The progressive change in the permeability as the temperature of the alloy decreases slowly below 600° C., the gradual increase in the time required for a change to complete itself as the temperature drops, and the prevention of the change by rapid cooling through this temperature range support this conclusion.

It is well known that some alloys are in the state of homogeneous solid solutions at high temperatures, but segregate into two or more phases as the temperature drops.

Such segregation ordinarily takes place in a definite temperature range, and is progressive in nature. It is a change of this type which I picture as taking place in these alloys during slow cooling. At the upper end of the critical temperature range for each alloy, the homogeneous solid solution begins to segregate into constituents of different composition. This segregation continues until the temperature has dropped to a point where no further changes take place. Rapid cooling prevents this segregation and the alloys remain after cooling in a metastable condition.

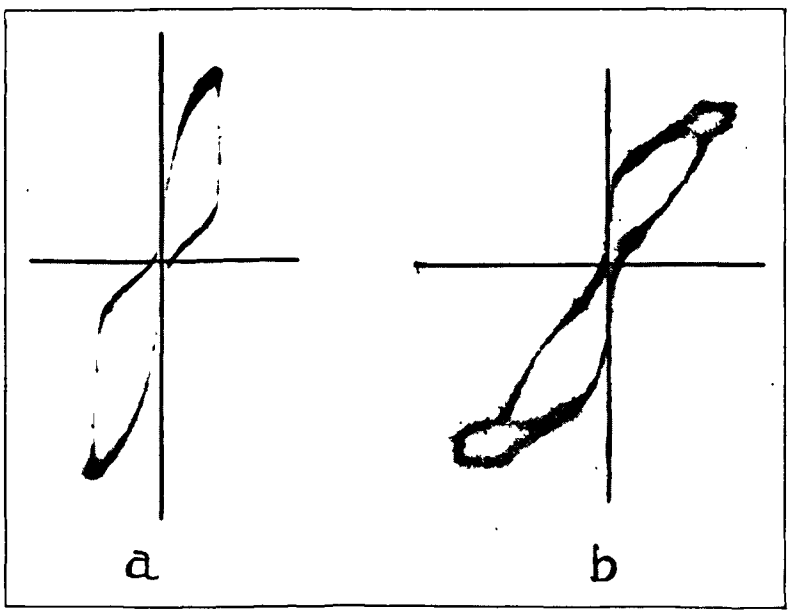
We would suppose that if such a change takes place, confirmatory evidences might be found. I shall refer, briefly, to some of our attempts to obtain evidence to confirm our speculations as to the nature of these magnetic changes.

The resistivity was measured for rapidly cooled and for baked alloys both of permalloy and perminvar compositions. These measurements showed that for both types of alloys, baking reduced the resistivity. For example, the resistivity of a 78.5 permalloy sample was measured after annealing and after air-quenching. The resistivities for the two conditions were 14.2 and 15.8 microhm-cm., respectively. Upon reannealing the resistivity again dropped to 14.5, about the same as before it was air-quenched. This change is in accordance with what would be expected if a segregation took place with annealing. A homogeneous solid solution has the highest resistivity and segregation tends to lower it.

Other interesting evidence is obtained from the study of the hysteresis loops. For the perminvar alloys the constricted loops are very marked and easily produced. In the permalloys containing between 60 and 80 per cent. nickel also there is a tendency to constriction in the slowly cooled alloy, not so marked but sufficiently prominent to lead me to believe that the same general changes occur in both groups of alloys, differing only in the nature of the segregates and the ease with which segregation occurs. Now we know that homogeneous magnetic materials have a characteristic type of hysteresis loop. For such materials there is no constriction in the middle, but the widest part of the loop is generally at that point. We also can construct hysteresis loops which have constrictions by making up cores of several materials in a

parallel or parallel-series arrangement.[4] This is illustrated by the hysteresis loop *b* in Fig. 19. This loop is traced by a cathode ray oscillograph for a bi-metallic rod, 15″ long consisting of a core of .04″ diameter unannealed piano wire and a .006″ wall permalloy tube, heat-treated to give high permeability and fitting closely to the wire. Curve *a* is a loop similarly traced for a perminvar core. Though the magnetic circuit conditions for the two cores are not the

FIG. 19.



Hysteresis loops: *a*, perminvar (45 per cent. Ni, 25 per cent. Co, 30 per cent. Fe); *b*, bi-metallic rod. Loop traced with cathode ray oscillograph.

same, the marked similarity of the two loops favors the view that the constricted loop of the perminvar results from segregation.

It is interesting to note in this connection that the examination by X-ray crystal analysis methods of these alloys has not given evidence of a segregation such as other evidence leads me to think must occur. Perhaps the reason for this is

[4] E. Gumlich, *Arch. f. Elektrotechnik*, Vol. 9, p. 153, 1920.

that the different constituent metals are so closely related that small changes in the structure cannot be detected by this means, or perhaps the reason is that the size of the groups of atoms making up the constituents is too small to be detected by X-ray methods.

While the variation in the permeability and in the other magnetic characteristics which could be affected by heat treatment may be explained by the theory of segregation, this theory does not explain why these alloys have such high permeabilities at low magnetizing forces.

Nor does this theory explain the unexpected magnetic characteristics, such as high saturation values of induction, or the low electrical resistance which characterize a large proportion of the alloys in the iron-cobalt series. It has been suggested by Weiss [5] that when the saturation values of an alloy are higher than they are for the constituent metals it is an indication of the existence of an intermetallic compound. On this ground he has accounted for the high saturation values he found for an iron-cobalt alloy containing 34 per cent. cobalt. In our investigation, which was not carried up to the high magnetizing forces used by Weiss, the 50 per cent. cobalt alloy gave us as high flux densities as any in the series for magnetizing forces upwards of 1,500 gauss.

It has been found in the study of intermetallic compounds that one indication of their existence is a low resistivity. It is generally believed that if an alloy has lower resistivity than any of its constituent metals, an intermetallic compound exists. In our measurements of the resistivity of the iron-cobalt series we found that the alloys with lowest resistance were those containing between 25 per cent. and 60 per cent. iron. There is a rather abrupt decrease in the resistivity as the percentage of iron increases beyond 25 per cent. Beyond 50 per cent. iron there is a gradual increase with a maximum at about 85 per cent. iron. From these measurements we would conclude that if an intermetallic compound exists, it is of a higher cobalt percentage than that suggested by Weiss. From our measurements of the resistivity of the alloys in this series the most probable intermetallic compound would be

[5] *Transactions of the Faraday Society*, Vol. 8, p. 149.

one containing approximately 66 per cent. cobalt of the chemical formula $FeCo_2$.

The data which I have presented are those for the compositions of iron, nickel and cobalt. In addition to these alloys, we have studied the effects of adding non-magnetic elements to numerous alloys of particular compositions. I cannot at this time go into a detailed discussion of the results with these, but I shall mention briefly some of them. We found that the addition of some of the non-magnetic metals to both the permalloys and ther perminvars made those alloys less sensitive to heat treatment. The resistivity was also generally increased. For some compositions the addition in small percentages of a non-magnetic metal, particularly chromium and molybdenum, increases the permeability at low magnetizing forces. This is illustrated in Fig. 20 where I have plotted the permeability curves for Armco iron, 78.5 permalloy, and a 3.5–78.5 Mo-permalloy. You will note that the initial permeability of the molybdenum-permalloy alloy is 21,000 as compared with 9,000 for the permalloy without molybdenum. As a rule the maximum permeability was generally decreased when a non-magnetic element was added and so were the saturation values of induction.

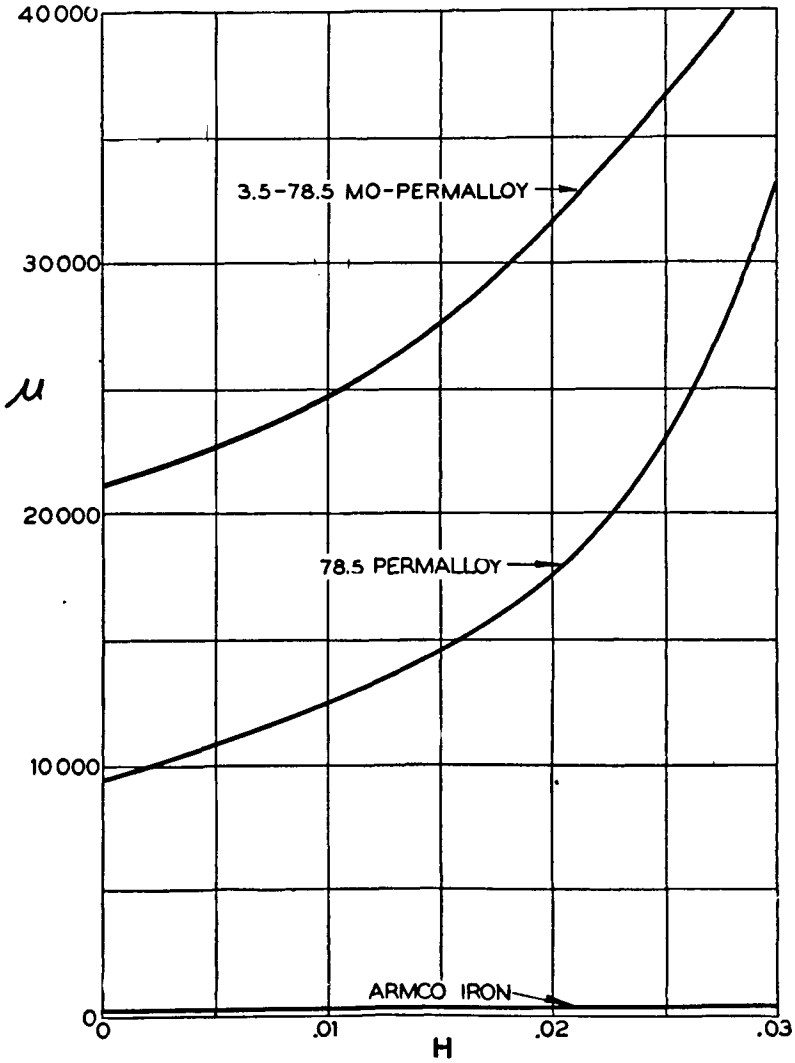
As our investigation was undertaken primarily for the purpose of searching for magnetic materials which could be used to advantage in the electrical communication field, it may be of interest to describe a few of the principal uses to which some of these alloys have been put.

Of the three groups of alloys which are of special technical interest, the permalloys are now used extensively in electrical communication circuits. Perhaps their most spectacular use is for continuous loading of submarine telegraph cable.

The term loading is used in the electrical communication art to designate a system of adding inductance to a transmission circuit for the purpose of overcoming the unfavorable transmission characteristics resulting from the electrical capacity of the circuit. This system has been used in telephone transmission circuits for over a quarter of a century. The standard method used for telephone circuits, in which inductance coils are placed at equally spaced intervals along the transmission line, was not considered practical for deep sea

cables. The only suitable method from a mechanical standpoint was a continuous loading in which a magnetic material

FIG. 20.



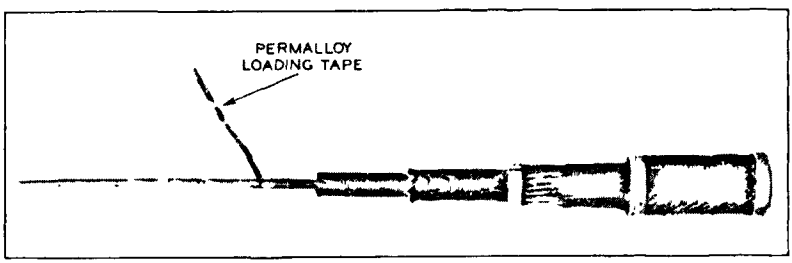
Permeability curves for permalloys.

is distributed uniformly along the whole length of the cable. Before permalloy was developed the best magnetic material

available was iron. It could not be used economically for long submarine cables because of its low permeability. With permalloy having between 40 and 50 times the permeability of iron in the range of magnetic field strength encountered in such cables, it was found that beneficial results could be attained and cables of more than five times the carrying capacity of the old type could be built.

The first permalloy loaded submarine telegraph cable was laid in 1924 between New York and the Azores, a distance of approximately 2300 nautical miles. A sample of the deep sea section of this cable is shown in Fig. 21. Numerous other

FIG. 21.

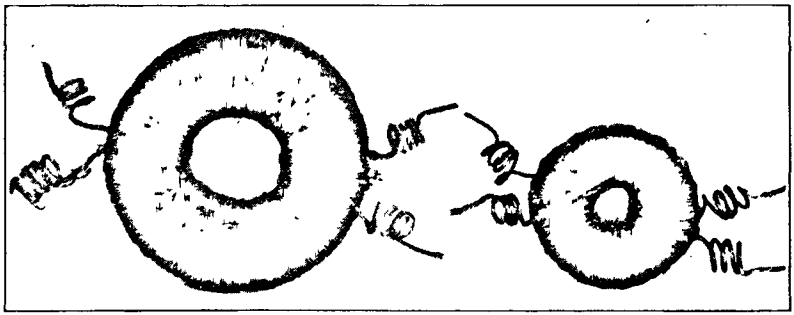


Sample of submarine loaded cable, showing the loaded conductor.

loaded cables have been laid since 1924; the total mileage of loaded submarine telegraph cables is now upwards of 16,000 nautical miles.

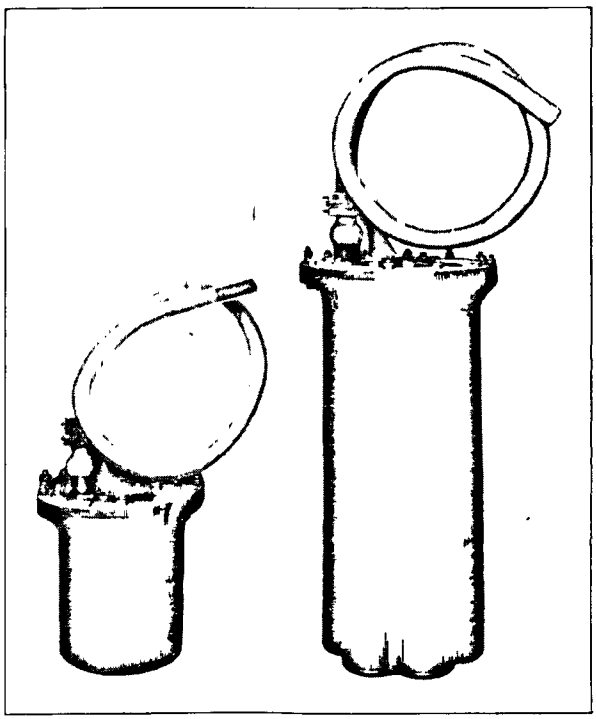
Another purpose for which permalloy is now used extensively is for loading coils for telephone transmission circuits. Before the introduction of permalloy, finely powdered, insulated and compressed iron dust was used for the cores of these coils. Permalloy has now replaced iron for loading coil cores, and upwards of a million cores per year are used by the Bell System. For these cores the permalloy is used in the form of compressed insulated powder. Some of the advantages in using permalloy result from its lower hysteresis losses and higher permeability. Taking advantage of these qualities in the design of the coils, it has been possible to reduce materially their sizes. This is illustrated in Fig. 22 where two standard loading coils for use in the same circuits are shown. One of these has a permalloy core and the other

FIG. 22.



Compressed powdered core loading coils: left, electrolytic iron core; right, permalloy core.

FIG. 23.



Cast-iron cases each containing 200 loading coils: left, permalloy core coils; right, iron core coils.

a core of iron. The former is approximately $\frac{1}{3}$ the size of the latter.

When these coils are used in service usually a large number are placed in containers which are placed at different points in manholes or on poles along the telephone circuits. In Fig. 23 two such iron cases with their cable connecting stubs are shown; each contains 200 coils. The small case contains the permalloy coils, and the large case the iron core coils. The use of permalloy has reduced the combined weight of the coils and the cases from approximately 1,700 lb. to about 700 lb.

Another use of permalloy in the telephone plant has been found in the case of relay cores. Relays are used to connect and disconnect mechanically the telephone circuits at the central office and at other points. These relays are magnetically operated and their efficiency depends largely on the magnetic quality of the core material. Certain groups of these relays are required to operate under very severe circuit conditions where the operating currents are small and where the relays are required to distinguish between very small changes in current values. For such circuits, relays with permalloy cores are now used extensively.

The permalloy alloys are also used extensively for cores in high quality audio transformers, in telephone receivers, and earphones, and in electrical measuring instruments.

The permalloy series was first developed, and is also the one which was first used, for commercial purposes on a large scale. The other alloys are still in the commercial development stage, but interesting results have been obtained which make us feel confident that both the perminvars and the iron-cobalt alloys will take their places beside permalloy as important magnetic materials in electrical communication.

The Rate of Emission of Alpha Particles from Radium. (*Proc. Roy. Soc.*, A 787.) H. J. J. BRADDICK and H. M. CAVE. The number of alpha particles emitted by a unit mass of radium in unit time is quite an important constant in the interpretation of radioactive changes, especially where energy relations are involved. The heat evolved by radium is in the main accounted for by the energy of the α particles and of the recoil atoms. When the quantity of heat is calculated it is found that all of it is not accounted for, provided use is made of the experimentally determined rate of emission of α particles. This investigation is undertaken with the intent of securing an entirely reliable determination of this rate. It is found that one gram of radium in one second emits 3.68×10^{10} alpha particles. Geiger and Werner recently publish as their result 3.40×10^{10} while Rutherford and Geiger obtained 3.57×10^{10}. When the new result is used for the calculation of the heat developed excellent agreement is seen between the calculated and the experimentally obtained quantities of heat. G. F. S.

Visible Radiation Characteristics of Incandescent Oxides. MARCELLA L. PHILLIPS. (*Physical Review*, Nov., 1928.) A study was made of the visible radiation characteristics of urania, ceria, lanthana, neodymia, erbia, yttria, zirconia, thoria, alumina, beryllia, magnesia and of certain mixtures. These were heated to red brightness temperatures ranging from 1,400 to 2,000° K. and the total emission at each temperature in candles per sq. cm. and the blue brightness in terms of the blue brightness of the black body at the temperature of melting palladium are tabulated. At the lowest observed temperature of 1,400° K. a black body has a blue brightness of .0061. The following substances have values of this quantity lying between .006 and .01, viz., tungsten, urania, ceria neodymia, lanthana, zirconia, thoria, and beryllia. At the temperature considered magnesia has the greatest blue brightness, .071. Magnesia has also the greatest emission per sq. cm., viz., about one candle per sq. cm. This becomes 79 at 2,000° K., but at that temperature it is exceeded by alumina and beryllia. Elevating the temperature from 1,400° to 2,000° K. produces the largest relative increase in radiation per sq.cm. in the case of yellow ceria where the emissions at the respective temperatures are .36 and 88.0. For a black body the values for the two temperatures are .26 and 44.0. G. F. S.

COSMIC RADIATION AND RADIOACTIVITY.

BY

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Communication No. 37.

In 1898 S. Currie [1] suggested that there may exist in all space some unknown highly penetrating radiation which is capable of being absorbed by the elements of high atomic weight. This radiation would manifest itself by its secondary radiations in the form of Becquerel rays. Later Perrin [2] proposed a theory of radioactivity in which he assumed that the atoms of the radioactive elements are capable of absorbing ultra penetrating radiation which passes unabsorbed through non-radioactive elements. Experiments which have afforded a test of this theory are those in which an investigation is made in order to determine if the rate of decay of a radioactive substance is altered by placing it underground where it will be shielded from cosmic radiations. Elster and Geitel [3] have measured the activity of Joachimisthal (a uranium ore) in a mine at a depth of 500 meters and found that its activity remained constant. Piccard and Stahel [4] have found no change in the activity of UX when left in the Simplon tunnel for two months at a depth of 2,200 meters. Also A. H. Compton and Hagenow lowered a tube of radium emanation to the bottom of the Calumet Heckla Mine in northern Michigan. After allowing it to remain there for a period of two weeks it was compared with a similar tube which had been left in the laboratory for the same length of time. Their results were not published, but Prof. Compton has informed the writer that their measurements showed no greater change of the activity of the two specimens than about two per cent in the remaining activity of the emanation. This difference was of the same order as the experimental error.

In the present work a radioactive substance was taken to the bottom of a mine where its activity was measured. These results were then compared with previous measure-

ments which were taken while the substance was at the top of the mine. The specimen which was chosen for the investigation was polonium because of its relatively short half-value period and its simple transformation. The most accurate of the experiments which have been mentioned above are those of Piccard and Stahel. They prepared four specimens of UX and for a period of two months they were left in the following places: the first at Zurich (500 m. alt.), the second at Brigue (680 m. alt.), the third at Junfraujoch (3,500 m. alt.) and the fourth in the Simplon tunnel under 2,200 m. of rock. The measurement of the relative intensity of the specimens were made at Zurich before and after the exposures and were found to differ by not more than one-tenth of one per cent. If we assume the fundamental postulate of radioactivity the method used in the present work is equivalent to that of Piccard and Stahel. The great difference between the two experiments is the entirely different measure of transformation which it concerns. The UX used by Piccard and Stahel emitted β and γ rays while in the present work α particles are the primary agent involved.

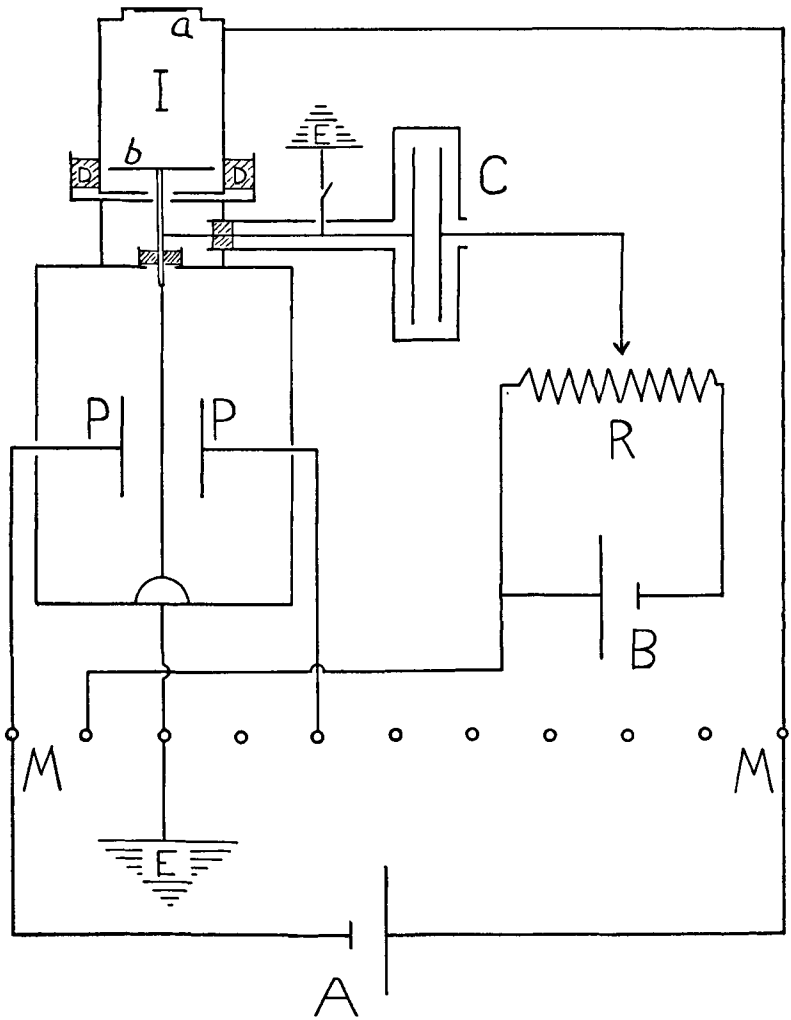
The polonium was taken 1,150 feet below the surface to the bottom of the zinc mine (owned by the New Jersey Zinc Company) at Franklin, New Jersey. The mine contained a large amount of Willemite (Zn_2SiO_4) and in order to calculate the absorption of cosmic radiation for this depth of Willemite in terms of water or lead we can use the following formula of Klein and Nishmia:

$$\mu = \frac{2\pi Ne^4}{m^2c^4}\left\{\frac{1+\alpha}{\alpha^2}\left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{1}{\alpha}\log(1+2\alpha)\right]\right.$$
$$\left. + \frac{1}{2\alpha}\log(1+2\alpha) - \frac{1+3\alpha}{(1+2\alpha)^2}\right\},$$

where μ is the absorption coefficient. N the number of electrons per cc. which produce scattering, i.e. $N = ZL\rho/A$ where Z is the atomic number, ρ the density, A the atomic weight and L Loschmidts number (the number of atoms per gram atom). m is the mass of the electron and e its charge. $\alpha = \frac{h\nu}{mc^2}$ where ν is the frequency of the cosmic radiation.

Thus for a given frequency the absorption depends directly upon the number of electrons of the atom which produce

FIG 1.



Experimental Arrangement.

scattering. From this equation we find that the ratio of μ for lead to that for Willemite is 2.8 for any given frequency. Thus 1,150 feet of Willemite should absorb as much of the

cosmic radiation as would about 400 feet of lead or 3,200 feet of water. Since Millikan has found that it only requires approximately 68 meters of water to absorb practically all of the cosmic radiation we are lead to believe that at a depth of 1,150 feet in the mine there should be no appreciable cosmic radiation.

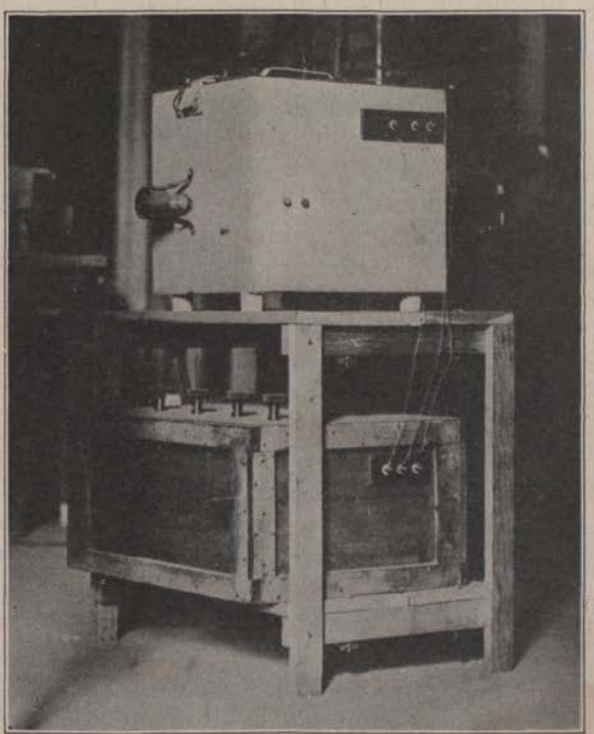
The method for measuring the activity of the polonium is illustrated in Fig. 1. The polonium plate *a* was mounted in the ionization chamber *I* with a difference of potential of 300 volts between *a* and *b* supplied by the battery *A*, while *b*, the inner part of the ionization chamber, was connected to the fiber of the single-fiber electrometer. The potential for the plates of the electrometer *PP* was obtained by using a portion of the potential maintained upon the megohm resistance *MM*. The ionization chamber was supported by the case of the electrometer and insulated from it by a hard rubber ring the cross section of which is shown at *D*. The ionization chamber was 6.5 cm. high and 5.5 cm. in diameter, and the distance between the polonium plate and plate *b* was greater than the range in air of the alpha particles. The charge accumulating on *b* was neutralized by altering the potential of the condenser *C* by known amounts by means of the battery *B* and resistance *R*, as shown in the figure. Thus throughout the readings, the fiber and *b* were always kept at the same potential as the case of the electrometer, so that the possibility of leakage was greatly reduced.

Figure 2 shows a photograph of the portable apparatus used. The box on top of the stand contained the apparatus shown in the preceding figure. It was made nearly airtight and contained drying agents. The readings of the electrometer and all of its adjustments could be made without opening the box. The longer box in the lower part of the stand contained the batteries and resistances which were also kept dry. The wires connecting the two boxes were mounted on plugs of sulphur set in the sides of the boxes to insure good insulation.

The polonium plate was prepared from old radium emanation tubes in which the emanation and products had gone over to RaD. The tubes were broken up and placed in dilute HCl. The HCl was then evaporated on a copper plate,

giving an active deposit. By measuring the absorption of the radiation emitted by the plate it was found to consist of α rays, hence the activity must be due to polonium. In fact, over a period of 140 days the radiation from pure polonium

FIG. 2.



View of the apparatus which was taken into the mine.

consisting entirely of alpha rays, will diminish to half value. It was found that while the activity of the plate prepared was due almost entirely to alpha rays, its strength remained constant to two per cent for a period of ninety days. The inference is, that the amount of Radium E which remained on the plate after evaporation of the hydrochloric acid was just sufficient to generate polonium at a rate which balanced the loss by decay. While the condition obtained was accidental

and unexpected it was not without its advantages in the present work.

RESULTS AND DISCUSSION.

The activity of the plate was measured at the top of the mine and then 1,150 feet below the surface. The ionization current produced in I varied only between the values of 9.6×10^{-11} amperes and 9.7×10^{-11} amperes during the entire course of the measurements. We thus conclude that there is no effect of cosmic radiation on the disintegration of the polonium to within one per cent. The natural ionization of the ionization chamber when at the bottom of the mine was 6×10^{-13} amperes. This current, which must be due to the activities of the rocks is small compared to the ionization produced by the polonium.

Swann [5] has shown that a single cosmic ray, by using all of its energy in ionization, is capable of producing 5.4×10^6 ions and has pointed out that since Millikan has found that cosmic rays produce 1.4 ions per cc. per sec. in air at sea level, we would have $1.4/5.4 \times 10^6$ cosmic rays absorbed per cc. per second in air or 1.3×10^{-4} cosmic rays absorbed per second per gram of absorbing material so that the 10^{-11} grams of polonium contained on the plate would absorb 1.3×10^{-15} cosmic rays per second. This would mean that a cosmic ray would be absorbed by one of the polonium atoms once in 10^{15} seconds or once in about 2×10^7 years, a situation which entirely precludes any attempt to measure the change in activity if we assume that the absorption of one cosmic ray is to be associated with the disintegration of one atom.

While the considerations which have already been involved leave very little hope for a plausible explanation of radioactivity as a result of the action of cosmic rays, it is worthwhile to consider the possibilities of a different view point with regard to the action of these rays than the particular mechanism which we have already considered. Let us then trace the results which would follow from the assumption that the ray is to be considered as of the nature of a particle, which in its passage by an atom must approach within a certain distance in order that it should produce the necessary disintegration. A simple calculation of the number of polonium atoms on the plate used will show that the total number per

sq. cm. is of the order of 1×10^{10}, and that therefore the distance between the atoms is of the order of 1×10^{-5} cm, that is about 1,000 atomic diameters. We should naturally expect that a cosmic ray operating as a corpuscular entity would have to approach nearer than this distance in order to produce a disruption of the atom. The condition is in fact one of single impact of the cosmic rays in passing through the plate and the maximum effect to be expected is one in which each cosmic ray disrupts one atom while penetrating through the plate. The problem now remains to calculate the number of cosmic rays per second which at sea level will pass through the polonium plate.

We shall assume that cosmic rays are generated in interstellar space and let $N(R)$ be the number of cosmic rays generated per unit volume per second between R and $R + dR$. Then $N(R)R^2 \sin\theta\, d\theta d\phi dR$ (spherical coördinates) will be the number of cosmic rays created per second in the polar element of volume dv in interstellar space. The distance R is measured from an element dS of a surface which incloses a volume V of air at sea level. The angle θ is measured with respect to the vertical. Then the total amount of energy falling on dS per second due to those rays originating in dv is given by

$$\frac{kdS\cos\psi}{4\pi R^2} N(R) e^{-(\mu H/\cos\theta)} R^2 \sin\theta d\theta d\phi dR,$$

where k is the energy of one cosmic ray and ψ is the angle which the normal to dS makes with R. H is the height of the earth's atmosphere if it were of uniform density throughout. We have here neglected any absorption which the rays might experience before coming into the earth's atmosphere. The amount of energy dE which will be absorbed in V will be

$$dE = \sum \frac{\mu l k dS\cos\psi}{4\pi} N(R) \sin\theta e^{-(\mu H/\cos\theta)} d\theta d\phi dR,$$

where the summation is taken with respect to all elements of surface dS over one-half of the area of the surface, i.e. over that part of the surface on which the energy is incident.

l is the distance traversed by these rays while going through V. Since the angle θ will not change appreciably during the summation we shall assume that it is constant and hence we have

$$dE = \frac{\mu k V}{4\pi} N(R) \sin \theta e^{-(\mu H/\cos\theta)} d\theta d\phi dR.$$

If we assume on the average that all of the energy of the cosmic ray goes into energy of ionization, then the number dQ of ions produced by the absorption of cosmic ray energy to amount dE is given by

$$dQ = KdE,$$

where $K = 1/Ue$, Ue being the energy necessary to ionize one ion. Thus we can obtain the following for q the total number of ions formed per second per cc. in V

$$q = \frac{\mu k K}{4\pi} \int\int\int N(R) \sin \theta e^{-(\mu H/\cos\theta)} d\theta d\phi dR. \qquad (1)$$

The integration will be taken over a portion of the region outside of the earth's atmosphere where cosmic rays are being generated, i.e. over the space for which θ varies from 0 to $\pi/2$, ϕ from 0 to 2π, and R between the values R_1 and ∞. The function $N(R)$ is assumed to become zero at infinity in such a manner that $\int_{R_1}^{\infty} N(R)dR$ will be finite.

The number of cosmic rays dn passing through ds, an element of a horizontal surface, is given by

$$dn = \frac{ds}{4\pi} \int\int\int N(R) \cos\theta \sin \theta e^{-(\mu H/\cos\theta)} d\theta d\phi dR.$$

In this equation $\cos\psi$ has been replaced by $\cos\theta$ since now ψ and θ are equal. Thus we have the following for S the number of cosmic rays per second per sq. cm penetrating through the horizontal surface.

$$S = \frac{dn}{ds} = \frac{1}{4\pi} \int\int\int N(R) \cos\theta \sin \theta e^{-(\mu H/\cos\theta)} d\theta d\phi dR.$$

From equations 1 and 2 we now have on integrating

$$S = \frac{q}{\mu k K} \frac{\int_1^\infty \frac{e^{-\mu H x}}{x^3} dx}{\int_1^\infty \frac{e^{-\mu H x}}{x^2} dx},$$ [1]

where $x = \sec \theta$. kK gives the number of ions a single cosmic ray is capable of producing and is equal to 5.4×10^6 for a cosmic ray having an energy corresponding to an absorption coefficient of approximately 1×10^{-6} per cm. of air. For this value of μ we now have

$$\int_1^\infty \frac{e^{-\mu H x}}{x^2} dx = .136 \quad \text{and} \quad \int_1^\infty \frac{e^{-\mu H x}}{x^3} dx = .103,$$

where $H = 10.5$ meters for the water equivalent of the earth's atmosphere measured at sea level. q for air has been found to be 1.4. By using these values, S comes out equal to about .2 cosmic ray per sq. cm. per sec.

The area of the polonium plate used in this experiment was 10 sq. cm. and therefore two cosmic rays would pass through it per second. There were about 3,000 atoms of polonium disintegrating per second and if each cosmic ray striking the plate were capable of causing one atom to disintegrate, irrespective of the fact that it may be necessary for the cosmic ray to approach within a certain distance of the atom, then the total cosmic radiation would contribute to less than one-tenth of one per cent. of the activity of the polonium. According to these considerations we should not expect to find an effect of cosmic radiation on the rate of disintegration for the present work.

The Writer wishes to thank the New Jersey Zinc Company for the permission to carry on the experiment on their mine at Franklin, New Jersey: and desires to express his gratitude to Dr. F. C. Benson of the Hahnemann Medical College and Hospital of Philadelphia for supplying the radium emanation

[1] This expression is of the same form as that deduced by Swann for the ratio of the influx of atmospheric corpuscles per sq. cm. per sec. into the earth, to the corpuscular current absorbed per sec. per cc. by a mass of metal.

tubes used in the preparation of the polonium, and to Dr. A. Bramley for his assistance in taking the measurements at Franklin, New Jersey. The Writer also wishes gratefully to acknowledge the helpful suggestions of Professor W. F. G. Swann.

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THE REFLECTION OF HYDROGEN ATOMS FROM CRYSTALS: INTENSITY MEASUREMENTS OF THE SPECULARLY REFLECTED BEAM.

BY

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BARTOL RESEARCH FOUNDATION.
Communication No. 35.

IN a previous paper in this journal [1] some qualitative results were reported on some intensity characteristics of the specularly reflected beam of hydrogen atoms from a crystal of rock salt. These results were based chiefly upon visual comparisons of the intensities of the traces produced by the specular beam on the detecting plate which was employed for recording the reflected atoms. Since then an attempt has been made to make quantitative measurements of the intensity of the specular beam as a function of the angle of incidence and of the temperature of the crystal. Although the precision of these measurements is still rather poor and they have not been extended as far as seems desirable, yet certain definite and interesting results have been obtained and as the work is temporarily interrupted it seems advisable at this time to publish what has been done.

The apparatus and general method of the investigation were the same as reported in the earlier paper. A beam of hydrogen atoms, originating in a discharge tube and collimated by two slits, was allowed to fall upon a crystal of rock salt which could be heated to expel the adsorbed gas layer. The reflected atoms were recorded on a glass plate coated with molybdenum oxide which became darkened by reduction where the hydrogen atoms struck it. The intensity of the reflection was then quantitatively measured by making a densitometer analysis of this plate.

In order that these measurements might be relied upon as representing the true density of the reflected atom beam,

[1] JOUR. OF THE FRANK. INST., Vol. 206, p. 301.

several precautions were necessary. As in the earlier work the detecting plates were made by smoking a piece of glass over an oxygen-gas flame in which metallic molybdenum was being burned. To improve the uniformity of the oxide deposit, so that measurements of the densities over different parts of the plate might be compared, it was found of great advantage to use a chimney for the smoking operation. This consisted of a brass tube, one inch in diameter and six inches long, in the center of which were one or two wire gauze diaphragms. At the lower end, along with the smoke from the flame, a fine jet of air was introduced which produced a thorough mixing, so that with the proper adjustment of smoke and air, a dense and very uniform cloud emerged from the upper end of the chimney. A glass plate held in this cloud became coated with a very uniform deposit of about the right density in a minute or two.

The intensities of the specularly reflected atom beams at two different angles were compared by making two exposures on the same plate. More than two such exposures on the same plate were found impractical because of the resulting great intensity of the diffuse background which was always present. In order to annul any inequalities in the specular beams at the two angles arising from changes in the intensity of the primary beam, the crystal was rotated first to one position and then to the other at frequent and regular intervals. Although this precaution was taken when possible, it was not considered necessary, for with the arrangement which was used for regulating the flow of gas, the pressure in the discharge tube, after coming to equilibrium, remained constant to within what could be read on the McCleod gauge over an indefinite period.

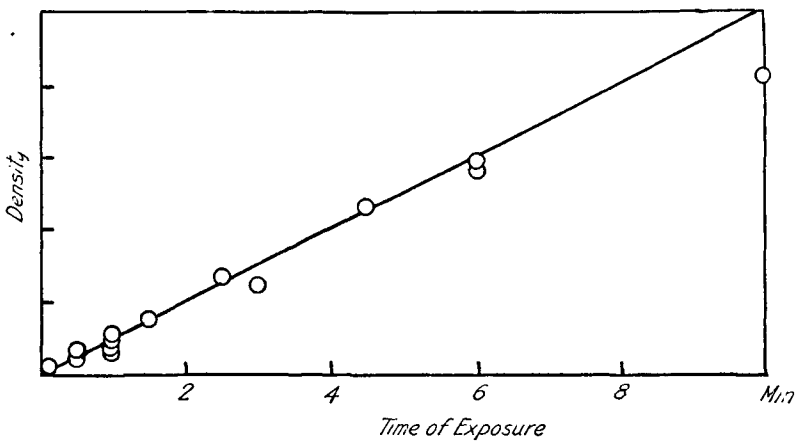
A calibration plate was taken to determine the relation between the measured density of the detecting plate and the number of atoms which had struck the plate per unit area. This plate was mounted for exposure in the apparatus in such a way that it could be moved parallel to itself and perpendicularly to the beam. It was exposed in different positions for varying times to the primary atom beam, which was of constant intensity, so that after the exposure, it was crossed with a series of dark lines of varying densities.

The plate was then analysed with a densitometer and the densities D of the exposed regions were calculated by means of the relation

$$D = \log I_0/I,$$

in which I is the intensity of the light transmitted by the densitometer through the exposed part and I_0 is the intensity of the light which would have been transmitted had the plate remained unexposed. I_0 was found by interpolating between the measurements which were made on either side of the exposed line. The results of the measurements of the

FIG. 1.

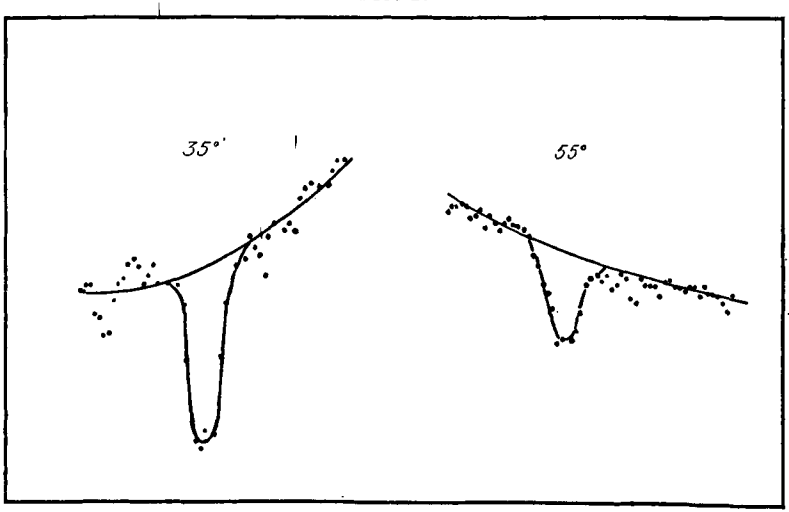


calibration plate are shown in Fig. 1 in which the densities of the centers of the exposed lines are plotted against the time of exposure. It is evident that the density is proportional to the time of the exposure, or what is the same thing, to the number of atoms per unit area, unless the plate has been over-exposed. We may therefore use the densitometer measurements for determining the relative intensities of beams if they are exposed on the same plate for equal times.

If the plate is left exposed to moist air for any length of time after exposure it again turns white by oxidation. A convenient way of avoiding this was to seal a second glass plate over the oxide coated surface. In this way some records have been kept for a year without any very noticeable fading.

To illustrate the procedure by which the actual measurements were carried out, some results of the densitometer analyses of the regions around the specularly reflected beams are shown in Figs. 2 and 3. The two beams shown in Fig. 2 were reflected for equal times onto the same plate with the crystal maintained at a constant temperature but at the angles of 35° and 55° respectively. The calculation of the

FIG. 2.

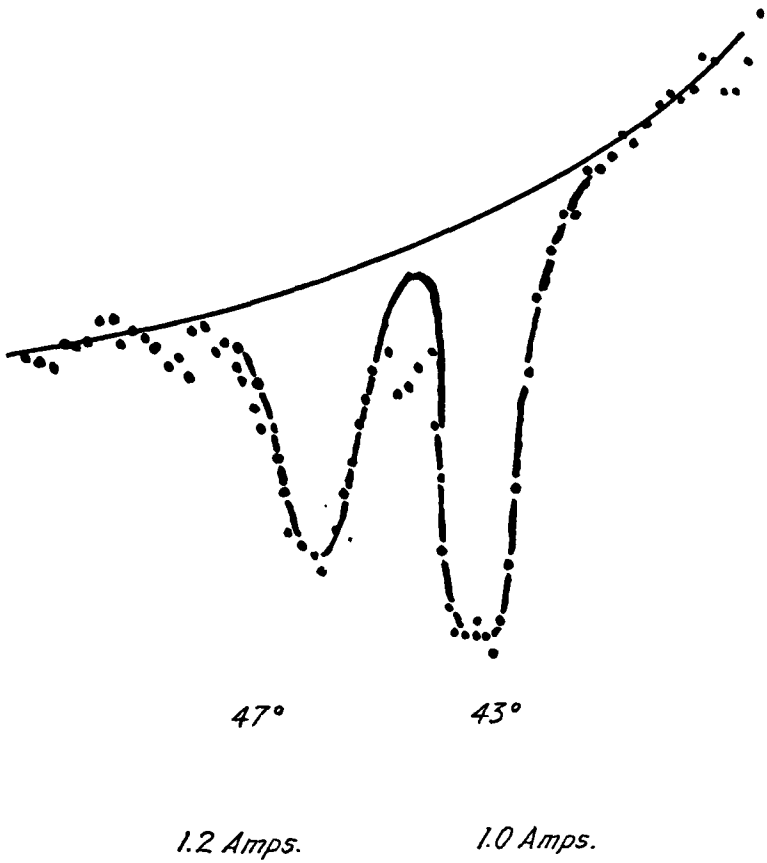


Densitometer analyses of specular beams.

beam intensities was carried out by either of two methods. With the first method the density, as defined above, was calculated for each position in the beam, using the values of I_0 found by interpolation between the measurements in the adjacent regions which were exposed to only the diffuse reflection. The calculated density was then plotted as a function of the position, and the area under the resulting curve was determined with the use of a planimeter. If the detecting plate had been circularly cylindrical with the crystal at its axis, the area of these curves would at once represent the beam intensities. The plate, however, was flat and was located with respect to the beam and crystal as shown in Fig. 4, P_1 or P_2. In the position P_1 comparisons could be made directly from the integrated density curve if

the two beams were symmetrically located with respect to the 45° position. Otherwise a correction was necessary because of the difference in the beam paths r between the detecting point and the source slit S_1. This correction is proportional to r. The intensities could also be calculated

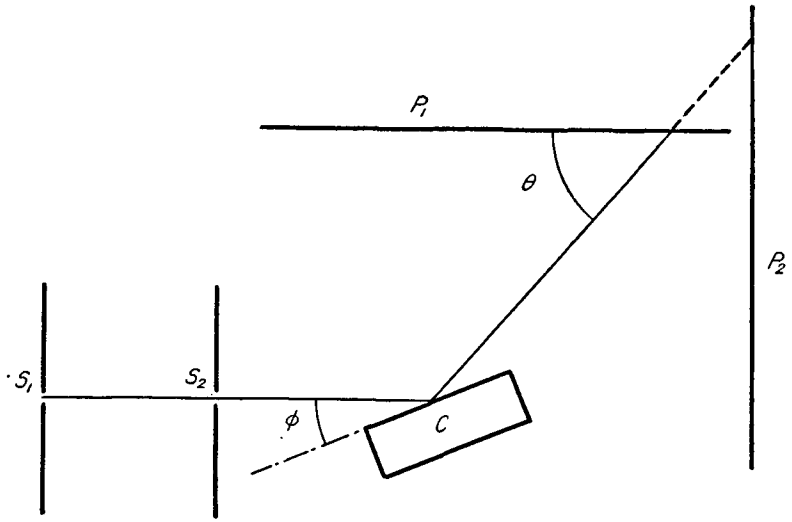
FIG. 3.



from the values of I and I_0 at the position of greatest density. To obtain the beam intensity in this case corrections had to be applied to the measured densities which were proportional to $r^2 \sec \theta$ where θ is the angle between the direction of the reflected beam and the detecting plate. Figure 3 shows the results of the densitometer analysis of a plate made for the

purpose of studying the variation of the beam intensity with temperature. In this case the crystal was rotated through only a small angle in order that the change of intensity because of the different angles might also be small. The error which was incurred from this change in angle is discussed below. A better method for determining the effect of temperature on the intensity of the beam would have been to move the plate keeping the crystal stationary, but this method happened to be less convenient in the arrangement which was used.

FIG. 4.



Schematic diagram of the arrangement of the crystal C, and the detecting plate P_1 or P_2.

So far in this work no direct determinations have been made of the crystal temperatures. So much uncertainty would attach to direct thermocouple measurements because of the difficulty of placing a thermocouple where it would measure the surface temperature that the entire problem of determining the temperature has been left for a future time. Hence the temperatures are only known in terms of the currents which were used to heat the crystal holder.

Since the contrast between the exposed and the unexposed parts of the plate is much greater by reflected than by transmitted light it was possible to notice effects visually

which were often unnoticeable from the densitometer measurements. For this reason many plates could be studied visually although they were either too weakly exposed or the oxide coating was not sufficiently uniform for accurate measurements. From a comparatively large amount of qualitative data, it is therefore possible to state that the effects shown by the quantitative data, meager though they are, are definite and reproducible. It is impossible, however, to make an accurate estimate of the reliability of the quantitative work. One of the principle errors doubtless arises from the scattering of the points in the densitometer analysis graphs as illustrated by Figs. 2 and 3. This can only be accounted for by irregularities in the glass or the oxide deposit, and not until a further study is made will it be possible to state to what degree the reflecting power of the crystal is reproducible. A summary of the more reliable measurements is given in Table I. Data 6 and 7 which were taken under as nearly as possible identical conditions give the ratio of the intensities at 55° and 35°. They differ by nine per cent., but it seems that most of this can be accounted for by the uncertainties in the densitometer measurements.

TABLE I.

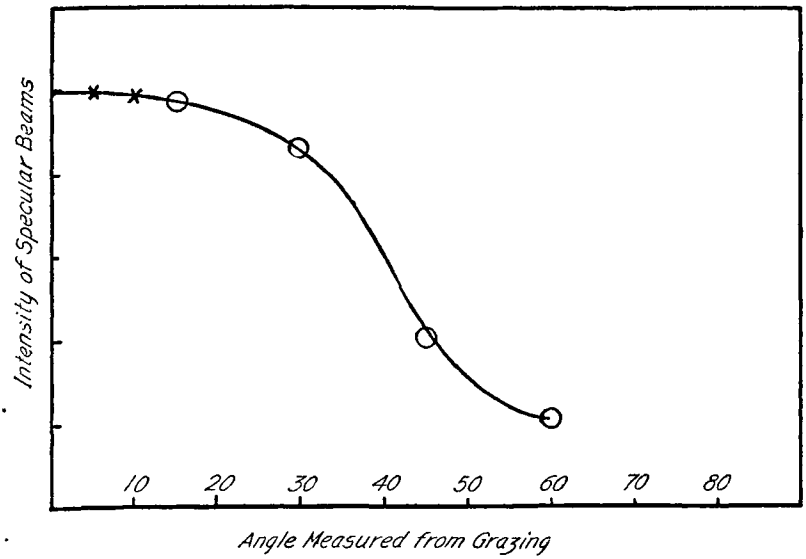
The Relative Intensities of Specular Beams.

	Angles from Grazing.		Intensity *a* : Intensity *b*.	Crystal Heating Current.
	a.	*b*.		
1	60°	30°	.26	1.4 amps
2	45°	30°	.48	" "
3	15°	30°	1.10	" "
4	5°	10°	1.00	" "
5	45°	35°	.56	1.0 "
6	55°	35°	.56	" "
7	55°	35°	.61	" "
8	55°	35°	.39	1.1 "
9	55°	35°	.38	1.3 "
10	43°	47°	1.66	1.0 " 1.2 "

The first four data are shown graphically in Fig. 5. These were all obtained from one crystal at a temperature corresponding to a heating current of 1.4 amperes. This temper-

ature was just under that at which the sublimation of the crystal began to be noticeable. Unquestionably this curve represents the general features of the intensity variation with angle, although the precision is not sufficient to make the numerical values reliable.

FIG. 5.



The variation of the intensity of the specular beams with angle.

The other data were obtained from a second crystal and bring out two other interesting points. (1) *The ratio of the intensity at an angle near normal incidence to the intensity at an angle near grazing, becomes greater as the temperature of the crystal is reduced.* (2) *The intensity of the specular beam at any particular angle is greater at low temperatures than at high temperatures.* Although the measurements upon which these statements have been based are confined to but a few angles nothing has been observed to suggest that the statements are not true generally, and in fact they are supported by several plates exposed at other angles which were examined only visually. Datum 10 upon which (2) is based must of course be corrected for the fact that the two beams were not reflected at exactly the same angle. If the variation were as great as that shown in Fig. 5 for which the temperature was

high, an intensity ratio of 1.2 could be accounted for by a change in angle of 4°. It has been shown, however, that this ratio is considerably less for temperatures corresponding to the heating current of 1.0 amperes, and we may safely conclude that the temperature effect is the important cause of the intensity difference of the two beams studied in 10.

Unfortunately the temperatures at which intensity measurements are possible, are limited to a narrow range between the point at which sublimation begins to be important, corresponding to a heating current of 1.5 amperes, and a point where a change in the crystal surface takes place. This change may probably be attributed to the formation of NaOH under the combined action of atomic hydrogen and adsorbed water vapor and it takes place at a temperature corresponding to about .6 amperes. It seems particularly desirable to establish the law of the intensity variation and the law of the variation of the angular distribution of the intensities of the specular beams as a function of the temperature, even if only over this short range, in order to determine the part played by the thermal agitation of the surface molecules. Qualitatively the variations are in the right direction to be explained by a roughening of the surface by thermal agitation, assuming the hydrogen atoms may be regarded as plane de Broglie waves. Since the average wavelength of the atoms which were used was about 1 Å and therefore of the same order as the expected amplitude of the oscillations of the surface atoms, it may be expected that a large variation of intensity such as is observed might well take place from this cause.

Determination of the Velocity of Light by Means of the Kerr Electro-optical Effect. A. KAROLUS and O. MITTELSTAEDT. (*Phys. Zeit.*, No. 19, 1928.) In Fizeau's determination of the velocity of light a ray of light passed between two teeth of a rotating wheel, proceeded over a distance of some eight kilometers to a mirror by which it was reflected so as to traverse again its previous path. Upon its arrival at the wheel it would be caught upon a tooth, provided a tooth had moved ahead and occupied exactly the position formerly held by the space between two teeth. The time taken by the light to go from the wheel to the mirror and back would then equal the time required for the tooth to occupy the position of the space. From the speed of rotation of the wheel and the distance from wheel to mirror the velocity of light can be calculated. The accuracy of the result depends upon the exactness with which these two quantities can be measured.

The method described in this paper goes back to a suggestion made by Des Coudrès in 1893. The rotating wheel is replaced by a Kerr cell placed between crossed Nicol prisms. A ray of light is directed upon the first prism from which it goes to the cell and thence to the second prism. Under ordinary conditions no light can get through. If however, arrangements of direction being propitious, an alternate-potential be applied to the cell, then twice every cycle light will get through for an instant. There is thus an optical interrupter whose frequency is dependent on that of the applied potential. A second Kerr cell was used in the arrangement for receiving the light after its passage through the interrupter and across the distance separating this from the receiver. The applied frequency was measured with an error of about .02 per cent. The distance traversed by the ray of light amounted to about 332 meters all within the compass of a laboratory since the light was reflected several times by mirrors before finally reaching the receiving end. This distance was measured with an accuracy of .04 per cent. The total error thus did not exceed the sum of .02 per cent. and .04 per cent. or .06 per cent. From about 750 determinations the mean value of the velocity of light is given as 299,778 km. per sec. Michelson's result is 299,796 km./sec. By an error of arithmetic the authors give the mean error of their result as 20 km./sec. whereas it should be 180 km./sec. G. F. S.

THE REFLECTION OF HYDROGEN ATOMS FROM CRYSTALS: VELOCITY SELECTION IN THE SPECULAR BEAM.

BY

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BARTOL RESEARCH FOUNDATION.
Communication No. 36.

IN an earlier publication [1] some experiments were described in which it was found that hydrogen atoms were specularly reflected from a natural crystal surface of rock salt if the latter was maintained at a temperature just under that at which sublimation takes place. More recently the same result has been found to hold for KCl crystals and is presumably true for certain other crystals as well. At that time it was pointed out that if one is to understand this phenomenon in a way which is consistent with the roughness of the interatomic forces as assumed in the well-developed theory of crystal structure as worked out by Born and Lennard-Jones, it is necessary to think in terms of wave mechanics rather than classical mechanics. It is, of course, to be expected that a difference between the classical and the wave mechanical predictions should exist for this phenomenon since the wave-lengths ($\lambda = h/mv$) associated with hydrogen atoms of the velocities used are of the order of the distances between the atoms of the crystal. It was further pointed out in the earlier paper that, although diffraction beams from the surface of the crystal lattice of higher order than the zeroth (specular beam) would be expected to appear if atoms of homogeneous velocity could be used, yet in the beam which is available the Maxwellian velocity distribution covers such a wide range that even the first order beam would be a band distributed through at least 50°.

In addition to the elastically reflected atoms which should be found in these diffraction beams there is also a very large number of inelastically reflected atoms which are distributed

[1] JOUR. OF THE FRANK. INST., Vol. 206, p. 301.

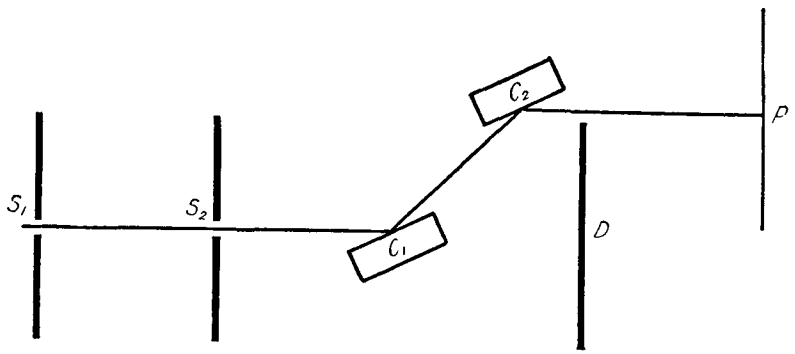
in all directions in about the ratios of the cosines of the angles from the normal to the crystal surface. For this reason only a relatively small number of the atoms reflected at any angle outside the specular beam may be expected to have undergone diffraction so that until a beam of homogeneous velocities is used it will doubtless be impossible to demonstrate the existence of diffraction from the surface lattice. In the direction of specular reflection, however, the number of elastically reflected atoms is a very considerable part of the total. Therefore it seems that whatever evidence is to be gained with the use of inhomogeneous beams, pertaining to the problem of elastic reflection, must be obtained from measurements made upon the specular beam itself.

In the investigation about to be described some evidence has been obtained concerning the velocity selection in the specularly reflected beam. A particular interest attaches to this work in the light of a recent publication by Ellett and Olson. These investigators have found that a beam of cadmium atoms, specularly reflected from a crystal of rock salt, is of homogeneous velocity, selected according to the angle in a manner analogous with Bragg interference of X-rays. Their results were quite unexpected for two reasons. In the first place the resolving power of the crystal in the experiments of Davisson and Germer has shown that even electrons penetrate the crystal to a depth of only a few atoms, and it would be expected that cadmium atoms could penetrate much less deeply, if at all, and still undergo elastic reflection. In the second place, the wave-length which they have measured is that given by $\lambda = h/m_H v$, where m_H is the mass of one proton and not that of the cadmium atom. As these authors have themselves pointed out, this result is quite inconsistent with the wave mechanical expectations.

In order to test whether or not a beam of hydrogen atoms which has been specularly reflected from a crystal of rock salt is of homogeneous velocity, this beam was again reflected from a second crystal at the same angle. If the first specularly reflected beam contains only atoms with certain selected velocities it should be reflected from the second crystal with greater intensity than a beam containing the same number of atoms distributed according to the Maxwellian law.

A schematic diagram of the experimental arrangement is shown in Fig. 1. Atomic hydrogen, formed in a discharge tube, by the dissociation of molecular hydrogen was collimated into a beam by means of two slits, S_1 and S_2, between which the excess gas was eliminated by a high speed diffusion pump. The beam thus formed was reflected successively from the two crystals C_1 and C_2 and thence it fell upon the molybdenum oxide coated detecting plate P. The technical details were in most respects the same as were described

FIG. 1.



more fully in the earlier paper. Both crystals were mounted on compound eccentric ground glass cones which permitted adjustment both as regards position and inclination. The actual adjustments were made optically and the reflecting angle could, by this method, be made accurately the same for both crystals to well within the angular width of the beam (about 1°). The diaphragm D protected the region on the detecting plate, where the specular beam was expected, from diffuse atoms from the higher pressure region between the two collimating slits as well as from the diffusely reflected atoms from the first crystal. This precaution made it possible to run an exposure for ten hours with no more general darkening than was usually obtained in one hour without the diaphragm. The general darkening which eventually covered the plate and limited the exposure to about ten hours was due to the incomplete effectiveness of the surrounding walls in catalysing the recombination of the stray hydrogen atoms, in spite of the fact that these walls were coated with

platinum black and protected from grease and wax vapor by the guard rings described in the earlier paper.

Before making a long exposure to detect the reflection from the two crystals in succession, each crystal was first tried alone. Since no appreciable variation has ever been found to take place in the reflecting power of a crystal because of exposure to the air, these tests could be made separately and before the final plate was inserted. A visible trace of the specular beam from a single crystal appeared after an exposure of about one-half hour. Since the primary beam under the same conditions appeared with an exposure of twenty seconds it is estimated that the specular reflection from the first crystal is about one per cent. of the primary beam. If there were no velocity selection and specular reflection from the second crystal were no more probable than from the first, an exposure of fifty hours would be required to record the second specular beam. Because of the general darkening of the detecting plate by stray atoms an exposure of fifty hours was impractical but on two different attempts, when conditions were good, exposures of ten hours failed to record a visible trace of the specularly reflected beam from the second crystal. These exposures were made at angles close to 20° from grazing.

It is concluded, as a result of this experiment, that the velocity selection in the specular beam is less than enough to increase the probability of specular reflection from the second crystal by more than a factor of five. This result may be stated more precisely as follows. If $N(V)dV$ is the number of atoms in the primary beam with velocity between V and $V + dV$, and $P(V)$ is the probability that one of these be specularly reflected, then the experiment shows that

$$\frac{\int_0^\infty N(V)P^2(V)dV}{\int_0^\infty N(V)P(V)dV} < \frac{1}{20}, \tag{1}$$

whereas it is known that

$$\frac{\int_0^\infty N(V)P(V)dV}{\int_0^\infty N(V)dV} = \frac{1}{100}. \tag{2}$$

If P were independent of the velocity equation (2) would yield the value 1/100 for P and the value of the left side of (1) would then be equal to 1/100. On the other hand, if complete velocity selection occurred so that P were unity for certain values of V and zero for all others, the left side of (1) would be equal to unity. The fact that it is less than 1/20 and therefore quite small, expresses the fact that P is nearly independent of V and there is little, if any, velocity selection.

This result agrees with an experiment which was reported in the earlier paper in which it was found that an interference phenomenon of the Bragg type depending upon the lattice spacing of the rock salt crystal and wave-length $\lambda = h/mv$ for the hydrogen atom, was ineffective in determining the probability that an atom be specularly reflected. The result is, however, at variance with that found by Ellett and Olson for cadmium atoms, but, since the mass of a cadmium atom is 112 times that of the hydrogen atom and its deBroglie wave-length is one-tenth that of the latter, it is perhaps not surprising that differences should exist even though the reason for these differences is not known.

Demonstration of the Doppler Effect in Sound. J. ZENNECK. (*Phys. Zeit.*, No. 11, 1928.) A loud speaker is attached to a flat piece of iron and the whole is swung from the ceiling of the lecture room by a bifilar suspension. An alternating current of frequency 1,000 per sec. is sent through the loud speaker while it swings back and forth in a plane perpendicular to a wall nearby. A person in the audience receives two sound waves, one direct from the loud speaker, the other by reflection from the wall. When the apparatus swings toward the auditor he gets from it a note whose pitch is higher than when it is at rest, while at the same time the note he gets by reflection from the wall is lowered in pitch because the acoustical image of the loud speaker is receding from him. He therefore hears beats. These are fastest when the pendulum is at its lowest point for there its velocity is greatest. The frequency of the beats can be changed through a wide range by changing the amplitude of the swing. G. F. S.

Up to the year 1925 helium was liquefied only in the cryogenic laboratories situated in Leyden and in Toronto. In that year a third apparatus for the same purpose was installed in the Physikalisch-Technische Reichsanstalt. In 1927 a new building for the production of low temperatures was erected in connection with the institution. 20 liters of liquid nitrogen per hour and 10 liters of liquid hydrogen per hour can be produced.

The helium gas for liquefaction is derived either from a mixture of neon and helium furnished by the Linde Company or from crude helium coming from the Auer Co. and obtained from monazite sand. From the first of these sources only about 70 liters of pure helium are produced per day. To get this 7 l. of liquid hydrogen must be used. The yield per day of helium gas from the second source of supply is 500 liters and again 7 l. of liquid hydrogen are employed. The stock of helium gas kept on hand amounts to about 6 cu.m. In 9 minutes from the appearance of liquid helium 400 cu.cm. are collected. G. F. S.

THE DEVELOPMENT OF A FIVE-STRINGED CELLO.*

BY

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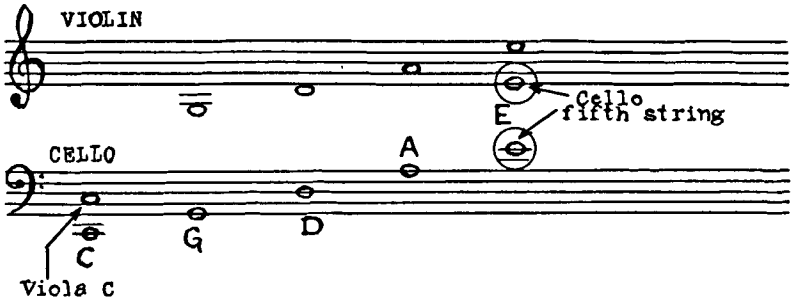
The usual cello has four strings tuned as shown in Fig. 1. For comparison, the four open strings of the violin are also indicated. By "stopping" or pressing a string down against the fingerboard any intermediate note may be obtained. On the ordinary cello, the range one octave above the open A is obtained with comparative ease, after a few years of

FIG. 1.



The open strings on the violin, viola, and cello, showing the added fifth string.

practice. Above that note, there still is a useful range of at least two octaves, but these notes are not only more difficult to produce and to play in tune, but the tone quality becomes nasal and the volume decreases as the string is shortened. Generally speaking, the same limitation applies to the upper range of the violin, with the following differences: (*a*) The whole range of the violin is the useful "solo" range, whereas in the cello the pitch of the lower two strings is too low for a melody and these strings are mostly used in ensemble playing, for the bass part and the accompaniment. Even the D-string of the cello is somewhat low for solo playing, because the

* The substance of this article was presented by the author in a talk before the Franklin Institute on December 6, 1928, at which time the instrument itself was demonstrated.

piano accompaniment has to lie above it. Thus, the A-string on the cello is the only one preëminently used in solo playing, and the range of one octave is not sufficient for anything but a very simple melody. (*b*) The fraction of the length of a string above which playing is practically impossible is a function of the string diameter. The thinner the string the farther it may be stopped without producing unpleasant partials. Thus, on the violin higher notes are more pleasant and are easier to produce than the notes corresponding to the same fraction of the length on the cello.

In other words, the range of the ordinary cello as a solo instrument is insufficient without going to high positions which are difficult to play and on which the tone quality is not nearly on par with that of the lower positions. Yet composers continue to write works for the cello using unreasonably high notes, for the same reasons for which they write songs just above the comfortable range of the singers. The reasons given are, first, that the composer could not possibly express his ideas keeping within the best range, and secondly that his music has been written primarily for the best performers who know how to overcome tone difficulties. Even in orchestral playing, and in chamber music, where the violas and the violins are available, one often finds cello parts written on quite a high range, partly because the composer wanted a cello tone quality and partly because the other instruments have other parts to carry. Whatever the reasons for writing unreasonably high music for the cello, the situation which actually confronts a performing cellist is that the best cello music is to a considerable extent written on high positions. No wonder that since the days of Johann Sebastian Bach (1685–1750) cellists and cello makers have been interested in adding a fifth string, to extend the range of the instrument upward without resorting to high positions.

Bach's biographer, Spitta, gives rather meager information about Bach's five-stringed cello. We know that it had an E-string (Fig. 1) one-fifth above the open A, and that it was called "viola pomposa" and "cello piccolo." Some of these instruments apparently were intended to be played resting on the floor with a pin, like the present day cello (Fig. 2), whereas others were supposed to be held against the chin,

FIG. 2.

Prof. Vladimir Karapetoff with his five-stringed cello.

like the violin. Bach also wrote a Suite (No. 6) for a five-stringed cello. It may be of interest to note that one of these early instruments, supposed to be made for Bach, is now in Philadelphia, a cherished possession of Mr. Ben Stad. Of course, the fifth string is gone, and we have no idea of the material of which it was made.

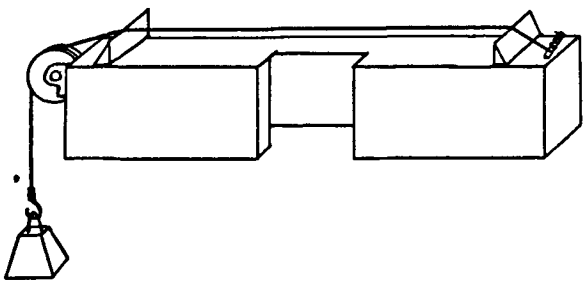
While Bach's instrument was not a success, there are somewhat indefinite records of later cellists having tried to attach a fifth string to their cellos, but without success. Yet, most any advanced performer would welcome such an instrument if he were presented with one, fully developed and ready to perform upon. The difficulties encountered in the construction of the earlier instruments will become clear from the following account of the development work done by the writer of this article.

I became interested in the five-stringed cello about 1913, for the same reason as my predecessors, that is, to be able to play advanced and more interesting cello pieces without using high positions. Not knowing anything about the physical difficulties, I simply removed the A-string, procured a triple length of the violin E-string and tried to tighten it up to the pitch of E (Fig. 1). It broke long before it came up to the pitch. I tried the best violin strings I could obtain, of different thickness, I had special gut and silk strings made for me by one of the leading manufacturers of strings, to possess the greatest tensile strength they knew how to obtain. All these strings broke either before being fully tightened or shortly afterward. I also tried a fine steel wire, but it also broke before coming up to the pitch.

Then I decided to investigate the subject scientifically, and either to solve the problem or to find the reason for which it could not be solved. I took a monochord (Fig. 3) and placed its two bridges at the same distance as the vibrating length of a cello string (about 27.5 inches). Then I fastened an ordinary cello D-string in the manner shown and loaded the free end with weights until the string came up to its required pitch of D. The necessary tension was about 20 lbs. Then I fastened an ordinary gut A-string and found that it

took about 28 lbs. to keep it at the required tension corresponding to the pitch of A. After that, I turned my attention to steel wire as the only one that looked promising for the E-string. The kind of wire with which I experimented and which I now use for the E-string and for the A-string is steel

FIG. 3.



A. monochord.

wire which one can obtain in music stores, wound on small wooden spools. It is used in some plucked stringed instruments and is drawn to the Webster and Horsefall gage, which is as follows:

Gage number.....	0	1	2	3	4	5	6	7	8
Diameter in mils.....	9	10	11	12	13	14	16	18	20

I began with a wire of No. 1 gage and found that it could be easily stretched to respond to the pitch of E, but the tone was poor. Then I tried wires of larger diameter and determined that a No. 5 wire required about 35 lbs. I did not dare to subject the cello to a greater tension and did not try wires of larger diameter. The then incomprehensible part of the experiment was that on the monochord all these wires could be tightened considerably beyond the pitch of E without breaking, whereas in the cello a No. 1 wire broke before reaching the pitch of E.

Before going any further, I checked the observed value of tension for No. 5 wire with the familiar formula for the number of double vibrations for a string per second. Namely

$$N = [1/(2L)]\sqrt{Pg/w} \qquad (1)$$

In this formula, L is the length of the string between the

bridges, in inches; P is the longitudinal tension in lbs.; $g = 386$ in./sec.^{2} is the acceleration due to gravity, and w is the weight of the wire, in lbs. per inch of its length. Assuming the middle C to correspond to 256 vibrations per sec., the E, a major third above it, is of the pitch $N = (5/4)256 = 320$ vib. per sec. The weight of the wire was computed on the basis of 484 lbs. per cubic foot and it was found that $w = 0.431 \times 10^{-4}$ lbs. per inch; $2L = 55$ inches. Substituting these values in eq. (1) and solving for P, the theoretical tension was found to be equal to 34.5 lbs.

Before a steel wire could be tried on the cello, it was necessary to provide a machine peg, that is, one which is operated through a worm gear. The tension of a steel wire is too great to tune it conveniently by an ordinary wooden peg, and, moreover, a very small displacement changes the tension and the pitch quite considerably. These machine pegs are similar to those used on a doublebass, only smaller. The machine peg for the E-string worked so well that ultimately all five pegs were changed to the worm-gear type (Fig. 2). When a steel wire was first tried on the cello, the wire broke again and again before coming up to the pitch, no matter how carefully it was tightened. Since it did not break on the monochord, and since it usually broke at the peg, the evident conclusion was that there were large secondary stresses at the peg which were added to the main tension. A close observation showed that a steel wire under tension twisted considerably when wound on a peg.

To remedy this, the wooden body of the peg was replaced by a brass cylinder, and the end of the string was fastened as carefully as possible, to minimize the secondary stresses. At the present time, the E-string breaks perhaps once in two years, and even then the break is not at the peg and is usually due to rusting. Within the last few years a heavier wire, No. 6, has been used, because it gives a greater volume of tone. The tension is probably about 45 lbs.

Before all the five strings could be used, it was necessary to raise the bridge, in order to provide a sufficient clearance for the bow past the waist of the instrument (the C-holes). Anyone familiar with a bowed stringed instrument will realize that the bridge must have a certain curvature on top in

order that each string may be bowed independently, without touching the adjacent ones. With five strings and the same curvature, the bridge has to be wider and somewhat higher. The fingerboard had to be raised accordingly, by gluing a maple wedge between it and the neck of the instrument. The steel wire is liable to deepen the groove in the bridge, and it is advisable to place a small piece of leather under the wire, so as to distribute the pressure.

It was not feasible to attach the lower end of the E-string to the tail-piece, because of a too great one-sided tension, and for this reason the string was fastened directly to the tail pin. Some skill is necessary to make a proper fastening which will not pull out as the string is tightened. It is necessary to wind the end of the string several times around itself, back and forth, and finally to pinch the end against the ebony edge.

With the foregoing changes, it became possible to try the instrument, using the ordinary four strings and a steel fifth string. The fingerboard was not widened, but the outer strings were placed nearer the edges, giving a little more room. After several years of playing on the five-stringed instrument, the writer does not consider it essential to have a wider fingerboard, although a wider one and a wider neck could be easily provided, if desired. The first trials showed that the instrument was mechanically sound, but from the musical point of view it left much to be desired. Not only the tone of the new string was too metallic, but the redistribution of the stresses in the body of the instrument took away much of the sonority of the other strings. Besides, the break in the tone quality between the metal E and the gut A was too great for satisfactory connected passages on the two, and it was decided to try a steel A. There was no question of ultimate strength, as with the E-string, the stresses being much lower. It was simply the question of using a wire that was heavy enough to give the desired volume of tone. So far, wire No. 7 has been found satisfactory for the purpose.

Having thus done away with the break in the tone quality between the two upper strings, the next step was to remedy the break between the steel A and the gut D-string. Since

the steel A is just barely tight enough for proper bowing, it was out of the question to use a steel D. It would be too flabby, no matter what size wire might be chosen. To see this point clearly from formula (1), divide the numerator and the denominator under the radical, by the cross-section of the wire. The result is

$$N = [1/(2L)]\sqrt{P'g/w'}, \qquad (2)$$

where P' is the unit stress and w' is the weight per unit length and per unit cross-section, in other words, the specific weight of the material. With the pitch N, the length L, and the material constant w' given, P' becomes a definite quantity, no matter what the cross-section is. Increasing the cross-section merely increases the total tension P and also the volume of the tone, but "flabbiness" will remain. An inspection of the formula indicates that in order to increase P' and thus make the string stiffer and more convenient for bowing, it is necessary to increase w' accordingly.

Thus, the problem became one of finding a material heavier than steel and capable of standing the required unit stress P'. The only two materials which suggested themselves were hard-drawn brass and hard-drawn tungsten. The latter proved to be unsuitable partly because its w' is too great for the purpose and partly because it cannot be wound and unwound on a peg many times without breaking. In other words, while its tensile strength is quite high, it becomes readily fatigued in bending and twisting. This shortcoming was later remedied by providing steel jaws connected to steel wires, so that the tungsten wire was used only on the straight portion, a little above the upper nut and a little below the bridge. However, the tone quality proved to be somewhat harsh, and the tungsten wire was laid aside.

Hard-drawn brass proved to be a fairly satisfactory material for the D-string, especially for sustained notes where some flabbiness was not noticeable after the first attack. However, the difference between the specific weights of brass and steel is not sufficient to give the string the required tension. This was particularly noticeable in spiccato passages (springing bow) in which the string gave a noticeable twang.

The first remedy sought was to wind a brass string with some metal, such as silver, to increase its *w*. Such metal strings wound with another metal are used in the guitar and in the piano. A manufacturer of strings wound my brass string with silver strip according to my specifications, but the very first trial showed that such a string was suitable only for being plucked or struck with a hammer, but not bowed. It gave a harsh uneven sound. It was then decided to gold-plate the brass string so as to increase its *w* by about 20 per cent. and thereby to increase the tension in the same ratio. This solution proved to be entirely satisfactory, and at the present time a hard-drawn brass wire is used, of about 23 mil diameter, coated with gold about one and a half mil thick. Such a wire can be obtained from the jewelry trade. By using more gold, the string can be made tighter and probably more sonorous, although even the present string is more sonorous and mellow than the usual gut D-string.

Some difficulty was experienced in bowing metal strings, on account of their being smooth. The bow was therefore re-haired with coarser horse-hair, such as is used in double-bass bows. A more viscous rosin than the kind ordinarily used for cello bows also helped to remedy the trouble.

The tone quality proved to be quite uniform, without any break, from the lowest C to the highest notes on the E-string, over a range of nearly five octaves, except for the open E and the open A, where the tone was somewhat metallic. True, the tone quality of the upper open strings of any violin or cello differs from that of the same strings when stopped, and a skilled player carefully avoids open strings, except for special effects. Nevertheless, the metallic quality was too pronounced, and a piece of leather on the bridge did not damp it out sufficiently. A simple remedy was finally found in tying an ordinary rubber band across these two strings, about 3/4 inch below the upper nut. This rubber band may be seen in Fig. 2, just above the player's left hand. It is not tight enough to affect the pitch and yet it causes an open string to give about the same mellow tone as when stopped with a finger.

It would hardly be of sufficient interest to enumerate here various other small changes tried and abandoned, or adopted,

in order to improve the tone quality. Moreover, it was not always possible to tell whether an improvement was due to a change or to a gradual adjustment of the instrument itself to the new stresses, to a shift in the position of the sound post, etc. In some respects, even an old cello, when provided with a fifth string, behaves like a "green" instrument, and it takes time and patience to adjust it and to practice on it to limber it up. Many a time, during the last ten years, the achievement of the goal seemed almost hopeless, in so far as the tone quality was concerned, and then a noticeable improvement would unexpectedly come within a few days. The instrument is not finished yet in the sense that its tone quality is still improving, and various suggestions occur from time to time which help to release more tone.

Almost simultaneously with the writer and quite independently of him, a violin maker by the name of Perin, in Rushville, Indiana, constructed a few five-stringed cellos. A description of the present writer's instrument was published in the *Violinist* for February 1922, and the Perin cello was described by him in the April issue. We both were surprised to find that we followed similar paths unknown to each other. Perin built special bodies, larger than the usual cello, although he kept the length of the strings about the same, that is, 27 inches. In his letter to the writer, dated December 15, 1928, he makes the following statements:

"Lack of time for experimenting has prevented us from further work in developing this instrument. Also the attitude of cello teachers toward the invention has been such that we have concluded there is no call for an easier method of playing the instrument.

"We placed a small number of these instruments in different parts of the country, only a few of the users mentioning the use for which they desired them, school, concert, business (dancing), novelty (the latest thing), covering all statements. We answered inquiries from China, Africa, Australia, Canada, Japan, Austria, Brazil, in fact, almost the whole world.

"(*a*) We use a specially processed silk string and also harp wire in various sizes, smaller and larger than 26 B and S as required by the particular instrument. (*b*) Our calculations

showed more latitude for adjustment with a slightly larger body. (*c*) Noticing the apparent tension required to bring different instruments to pitch we worked to secure an apparent low tension and with our calculations for the size of body we got tension that will allow us to use gut (occasionally), silk (in the majority of cases) and, of course, steel in the remainder.

"We cannot scientifically explain some of the conditions we have experienced in our work, and can only take advantage of them as experience has taught us."

Not having seen or heard any of the Perin instruments, the writer merely records these statements as coming from a fellow-worker in the same field.

Coming now to the purely musical significance of the five-stringed cello, its importance for an individual musician depends entirely upon his aims and point of view. That which may be of primary importance to one cellist, may be of no interest to another. The following situations are typical:

(*a*) A professional or amateur cellist of limited skill and with limited time for practicing usually has a comparatively small repertoire of pieces, mostly those written on the first four positions. At least half of these are transcriptions of works originally written for other instruments or for the voice. We shall assume that he can play them adequately, even at a small concert, before not too critical an audience, which enjoys the singing tone of the cello and simple melodious pieces. There are a number of more interesting and musically more significant pieces, many of which were written especially for the cello, and which in a few places require medium-high positions with the use of the thumb. Our player may play these pieces adequately in all but these few places, but even one place played inadequately bars him from including these pieces in his repertoire. With the fifth string, all these pieces will come within the range of the first four or five positions and can be readily played by our musician. This type of player is quite common throughout the world, and to him the fifth string will mean all the difference between being and not being a solo performer.

(*b*) Many a professional or an amateur cellist can play a few thumb positions creditably, say, up to the open violin E (Fig. 1) but feels the limitations of the nasal tone quality on the high positions, and also aspires to more difficult cello pieces of the concert caliber. To him, after a few months of practice, the fifth string will prove to be a source of great enjoyment in enabling him to play his former pieces with much more facility and sonority, and also opening up a new repertoire of advanced pieces. Of course, the fifth string primarily helps the left hand, but because of easier tone production and lighter bowing, the right hand is also materially assisted when playing at the point of the bow or executing staccato and spiccato.

(*c*) A professional player of the highest caliber, who travels and gives solo recitals, can add materially to his prominence and message by using the fifth string and avoiding much playing on the highest positions, which is often of acrobatic nature and not in keeping with the natural dignity of the instrument. Anyone who will take the trouble of looking over the concert repertoire of half a dozen prominent cellists, say for the last ten years, cannot help but be impressed by the paucity of the material used, not because the cello literature does not offer sufficient variety, but because comparatively few pieces are "effective" and sound well on high positions. This applies particularly to cello concertos with the orchestra accompaniment: the volume of cello sound on high positions is totally inadequate, and the orchestra usually drowns much of the soloist's work. A steel E can be made to "cut through" the accompaniment so that the melody will come out clear and rich. This has been demonstrated by the writer on such well known compositions as the Saint-Saens Concerto, op. 33, Boellmann's Symphonic Variations, etc.

(*d*) The possibility of playing much of violin music on the five-stringed cello is a great advantage which can be enjoyed by all classes of cellists, from a beginner to the greatest virtuoso. In fact, since having added the fifth string, the writer finds that about ninety per cent. of his repertoire consists of regular violin pieces. The reason for this may be clearly seen from Fig. 1. The usual cello is more than an

octave and a half below the violin. Therefore, it is not practical to transpose violin pieces an octave lower. They would still lie on much higher positions than on the violin. On the other hand, the four upper strings on the new cello are exactly an octave below those of the violin, so that when playing them an octave lower most of the original positions are preserved, except where cello distances are beyond the reach of the human hand. This does not mean that it is necessary to re-write every violin piece which it is desired to play on the five-stringed cello. The performer simply reads from the original violin score and imagines himself playing the violin. It is only necessary to remember the corresponding positions. This can be readily accomplished after a few weeks of practice, and the results are certainly worth the trouble. The violin literature is many times more extensive and interesting than the cello literature, for the reason previously mentioned, namely that the solo range of the ordinary cello is too narrow for many of composers' ideas. Compare, for example, Bach's Suites or Beethoven's Sonatas for the violin with those for the cello, and the difference will be fully apparent.

When a violin piece has a part which lies mainly on its two lower strings, it may be played as written, without transposing, and then the next part played an octave lower. Actual experience has shown that this procedure can be effectively used in many cases. Thus, the cellist of the future can make his recital programs partly of cello pieces and partly of violin pieces, in this manner making his concerts more interesting and varied. A few changes here and there may be necessary, for example, in large chords, but as a rule the writer has found violin pieces even of concert character quite playable on the five-stringed cello.

(*e*) In chamber music, especially by modern composers, the fifth string has proved to be very valuable in making difficult parts comparatively easy. In an ensemble each note must be exactly in tune, and this makes higher positions on the cello doubly difficult and undesirable for this particular purpose. Even in some of Beethoven's chamber music unreasonably high positions are used in the cello part, and the fifth string not only permits of a better execution but also

brings out those solo passages more effectively. Of course, the player has to reduce the sonority of the strings in some places by bowing lightly.

(*f*) Composers will find new inspiration in the five-stringed instrument, in that it will do away with some important limitations of the regular cello with respect to higher positions and narrow range. There are pieces, as for example Bach's celebrated Chaconne, which should be played on the cello rather than on the violin, so as to give more depth to the lofty and somber moods. With the range limitations removed, composers should feel inclined to express their inspiration of this sort on the cello. The same applies to sonatas and concertos of which there are not nearly enough for the cello. Sonatas by Grieg, Locatelli, and others, with high positions, sound immeasurably better with the fifth string, and the performer does not have to worry about being drowned by an over-ambitious accompanist.

(*g*) No string ensemble is complete without one or two violas. The range of the viola is shown in Fig. 1, the lowest string being a C, an octave above the cello C and one-fifth below the violin G. The remaining three upper strings of the viola are the same as the three lower strings on the violin. In other words, the viola is an octave above the ordinary cello and one-fifth below the violin. Outside large musical centers it is comparatively difficult to find satisfactory viola players, because most violinists do not care to learn the alto clef and some new positions. Besides, they claim that using the viola affects their violin playing unfavorably. Many viola parts in trios, quartets, quintets, and orchestral pieces can be played on the five-stringed cello without much difficulty. The player simply has to learn to read the alto clef. No separate instrument is necessary and there can be no interference with his regular cello technique because the performer is playing his regular instrument and is using the same positions. A cellist has to know the treble clef, the tenor clef, and the bass clef anyway. It should not be much trouble for him to learn to read a fourth clef, especially since it is not much different from the tenor clef. The writer has played viola parts on his cello on many occasions, and while he forgets the alto clef in between, it comes back to him

within a day or two of practicing. Thus, the new cello will permit a further development of orchestral and chamber music playing in smaller musical centers.

Note: During the talk, the following pieces were performed by the lecturer:
Bach: Suite for the five-stringed cello (unaccompanied).
Massenet: Meditation from the opera "Thais."
Romberg: Andante from the Second Concerto.
Mendelssohn: Andante from the E-minor Concerto for the violin.
deBeriot: Scene de Ballet, for the violin.

Evidence Obtained by X-Ray Analysis of Films of Iron in Magnetic Fields as to the Ultimate Nature of Magnetism. T. D. YENSEN. (*Phys. Rev.*, July, 1928.) K. T. Compton and Trousdale examined the X-ray pattern obtained with single crystals of hematite and magnetite and found that the application of a magnetic field to the crystals made no difference in the patterns. Later A. H. Compton and Rognley studied the intensity of an X-ray reflected from the surface of a magnetite crystal and found that magnetizing the reflector made practically no difference in the intensity of the reflected ray. The joint conclusion from these experiments is that the ultimate magnetic particle is not the atom because an orientation of the atoms by the field would have been revealed by some change in the X-ray pattern. It must be something smaller than the atom. "These conclusions, however, do not preclude the possibility that in metallic substances like iron, there may be minute crystal aggregates whose orientation is affected by the magnetic field and which consequently may cause dissipation of energy and otherwise influence the magnetic properties." This paper reports an investigation made to detect the presence of such crystals that change their position when a magnetic field is produced. "Based on the results obtained, the answer must be a negative one. Even with the minutest crystals of submicroscopic dimensions, such as exist between the main crystals of iron, the evidence indicates no change in the random orientation, even when a magnetic force sufficient to produce magnetic saturation is applied. The results therefore incidentally confirm the hypothesis that the ultimate magnetic particle is an intra-atomic unit."

G. F. S.

THE HORIZONTAL RAINBOW.

BY

W. J. HUMPHREYS,

Professor of Meteorological Physics,
U. S. Weather Bureau.

VARIOUS observers have reported seeing a rainbow on a calm surface of water, often following a fog, and two, at least, have even seen rainbow clusters on calm lake surfaces. C. Juday,[1] who also cites earlier observations by others, saw the primary bow with two of its supernumeraries, and also the secondary, on Lake Mendota, Wisconsin, October 23, 1914. The other, Dr. E. D. Ball, observed on Lake Monroe, Florida, in the spring of 1928, and following a fog, three bows well separated from each other. These appear to have been the primary bow, the secondary bow, and the reflected primary, that is, the light or colors of a primary bow that left the droplets in such direction as to be reflected by the lake surface to the eye of the observer. Dr. Ball did not publish his observations, but gave them to me with the kind permission to do with them what I liked. They describe an unusual rainbow complex, and are the occasion of this note.

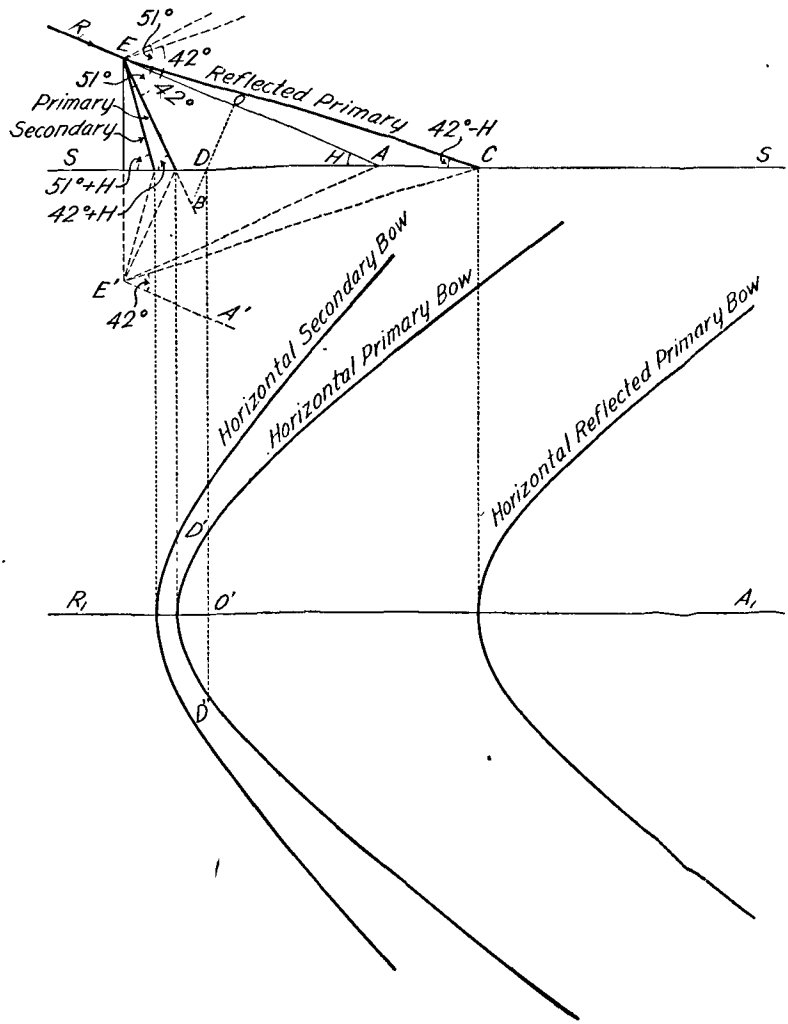
Apparently, all horizontal rainbows are caused by either a layer of droplets floating in the air just above the surface, or a sheet of droplets, probably of drizzle size, usually resting on an oil film which keeps them from merging with the water beneath. The general explanation of how these bows are formed, provided they are due, as here assumed, to a horizontal sheet of droplets, is simple and direct.

Let *RA*, Fig. 1, be the incident ray from the sun through the point *E* (position of eye when making observations) and making the angle *H* with the water surface *SS*. Evidently, then, from the theory of the rainbow, all drops, and only those drops, that are on the surface of a right cone about *RA* as axis, and having an angular spread (axis to element) at the vertex *E* of 42°, or thereabouts, as determined by color, will

[1] *Monthly Weather Review*, 44, p. 65, 1916.

contribute to the light of the primary rainbow seen from E. In the present case these drops evidently are at the intersection of the given cone with the plane SS, and therefore

FIG. 1.



along a conic section—a circle when the sun is in the zenith; an ellipse when the solar altitude is greater than 42° and less than 90°; a parabola when the altitude of the sun is exactly 42°

(of course for the appropriate color); and an hyperbola when this altitude is less than 42°.

Exactly the same things hold for the secondary rainbow seen from E on merely substituting its angular spread for that of the primary, that is, 51° in place of 42°. Similarly, the supernumeraries are just inside the primary, and outside the secondary, respectively, each with its appropriate angular distance from the axis RA, and forming on the water surface its particular curve, in the same manner exactly as does the primary bow, explained above.

Obviously an eye at E may receive not only that rainbow (primary, secondary and supernumerary) light which comes directly from the drops, but also such light after reflection from the water surface. The condition that this reflected light shall reach the point E is that it must leave the drops in the direction of E', the image below the water surface of E above it. The reflected primary bow, therefore, lies along the intersection of the water surface with a right cone of 42° angular spread, with vertex at E', and axis $E'A'$ parallel to REA. Similarly, the reflected secondary is along the intersection of the water surface with a right cone having the same vertex and axis, that is, E' and $E'A'$, respectively, with a spread of 51°; and the reflected supernumeraries just inside the reflected primary and outside the reflected secondary, respectively. It is not certain, however, that the reflected secondary has ever been observed, hence its theoretical position is not shown in the Figure. The reflected supernumeraries surely are too faint to be seen.

Furthermore, there also may be reflection bows, that is, bows produced by light reflected from the water surface into the drops, and passed from them to the observer, either directly or after reflection. Evidently the light that produces these reflection bows enters the drops in the direction of, or parallel to, $E'A$, and forms primary and secondary bows exactly like those described above due to the original, or non-reflected, incident light. Clearly, too, these bows, since they might have been seen from E', are, after reflection, seen from E as superimposed upon the direct primary and secondary bows observed from that point, E. Similarly, that light of a reflection bow which reaches E directly is superimposed upon the corresponding reflected bow.

There remains the problem of finding the actual traces of the horizontal bows. For this purpose let R_1A_1 be the projection of the incident ray onto the horizontal surface, here viewed from above. To find a point of intersection of the cone of the primary bow, for instance, vertex E, axis EA, spread 42°, with the horizontal surface SS, cut this cone with a plane normal to its axis, thus obtaining a circular intersection of radius OB, to take a special case; then, on a circle of this radius, draw a chord through a point at the distance OD from the center and normal to the radius through that point: half this chord is the distance $O'D'$, $O'D''$ from (and normal to) the axis R_1A_1 to the corresponding points $D'D''$ on the horizontal primary bow. Similarly, for as many points as desired on each of the horizontal bows.

This is not, of course, the only way, nor, perhaps, the most expeditious, of constructing these curves, but, in addition to being the way those here given were constructed, it is direct and easily understood, and therefore adapted to the present purpose—to impart a clear understanding, rather than to supply a means for speedy procedure.

NITROGEN AND THE FERTILIZER INDUSTRY.*

BY

CHARLES J. BRAND,

Executive Secretary and Treasurer,
The National Fertilizer Association.

Mr. President, Members of The Franklin Institute and Fellow Guests:

May I first express my appreciation of the honor the Institute has conferred upon me by asking me to be its guest speaker this evening. I am not unmindful of the high character, wide note and distinguished ability of those who have preceded me at this forum. May I also express the hope that the facts that I shall relate may command your attention and consideration, even if my presentation of them may not fitly or fully express their interest and importance.

There are many ways in which we might consider the use of nitrogen in modern industry and agriculture. It is not easy to decide which facts out of myriads may prove most interesting and challenging to an audience interested in all endeavors to promote the general economic welfare. I will begin by setting forth some facts relating to nitrogen itself, and will then broaden the view by considering the use of other essential plant foods, and finally will make a detailed statement regarding the modern development of nitrogen fixation and its value in peace and war.

THE NATURE AND IMPORTANCE OF NITROGEN.

Nitrogen, a colorless, tasteless and odorless gas, was discovered by Rutherford, an English chemist, in 1772. Because an atmosphere composed of nitrogen will not support life, the French named it "azote" (a, without; zoe, life); the Germans called it "Stickstoff" (a substance that chokes). Nitrogen is one of the most widely diffused of the elements. It constitutes four-fifths of our atmosphere, in which it occurs not in combination but in a balanced mixture with other

* Presented at a meeting held Thursday, March 14, 1929.

gases, chiefly oxygen. It is an essential constituent of our most destructive explosives and of some of our most deadly poisons. In its many forms, simple and complex, it supplies food for plants, animals and man. Indeed the proteins, which are nitrogenous compounds, are indispensable constituents in the human diet. In chemical technology, where it is used largely in the form of nitric acid, it is employed in the manufacture of dyes, of substitutes for silk and leather, of photographic films and of many other products. It is also used in the manufacture of artificial ice and in cold storage and household refrigeration.

Nitrogen is indispensable in both peace and war. In war it is a destructive, death-dealing explosive; in peace it is used constructively in mining, quarrying, clearing land for cultivation and road building. As a plant food it is of paramount value for application to soils that have had their natural fertility exhausted.

Taking the world as a whole, agriculture uses approximately 80 per cent. of all the inorganic nitrogen produced. The proportion used in our own country is smaller, because our soils have not yet been so greatly exploited as those in the older parts of the civilized world. With us only 60 per cent. of the inorganic nitrogen produced is consumed by agriculture, 40 per cent. being used in chemical industries.

DEVELOPMENT OF THE ART OF FERTILIZATION.

I shall not attempt to trace the history of the world's agriculture—of the development of crop plants, domesticated animals and the art of fertilization. 'At some time early in the history of the human race it was discovered, probably by accident around graves, that organic matter introduced into the soil produces luxuriant growth and free fruiting of the seeds of the wild plants that provide a part of the food of land animals. The Greek, Roman, Mesopotamian, Chinese, Hebrew and other early writings show a gradual development of the application of fertilizing materials to soil to promote the production of food for man and domestic animals. Cato, Virgil, Columella and Pliny make many references to the use of fertilizing substances. The ancient Chinese encyclopedias, dating back to periods no less than 1200 years before Christ,

when China already had a fully developed agriculture, likewise leave no doubt as to the relatively common knowledge that crop wastes, animal and human manures and other substances containing organic materials are useful to fertilize land.

It was not until many centuries later, after the blight of the Middle Ages, during the Renaissance, that a more conscious and thorough knowledge of the art of fertilization began to develop. By the beginning of the seventeenth century, many fertilizing materials had come into use, including not only dung but marl, green manure, lime, ashes, rags, wool, bones, horns, feathers, hoofs and fish. Then, for a period while the art of tillage was being learned, apparently less progress was made in the discovery of useful fertilizing substances. Indeed, it was not until Liebig, the great German agricultural chemist, made his classic investigations, in the period around 1840, that a new era of such progress was begun. Liebig not only made noteworthy researches but collated and summarized the knowledge contained in the available literature of past centuries.

THE AGRICULTURAL TRIAD.

It was not an accident that the triad which we are about to consider was found to be of great service in plant nutrition. Liebig's analyses first disclosed the prime necessity of nitrogen, phosphorus and potassium in the metabolism of plants. Of these three elements undoubtedly the most essential is nitrogen, although the other two are almost invariably associated with it, particularly in American agronomic literature, in any consideration of plant nutrition. This triad is of intrinsic interest not only because of its economic value but because of the individual peculiarities of each of its members.

Nitrogen is one of the most inert gases and is widely diffused, but despite its universal occurrence it was not isolated until 1772, and the fact that it is not a "permanent" gas was not discovered until 1878. Phosphorus ignites spontaneously in the presence of air and forms poisons so deadly that the laws of most nations forbid the use of one of its forms in the manufacture of matches. When the metal potassium is dropped into water, it produces a flame.

Neither one of the three members of this triad occurs in nature in a form that can be directly taken up by crops, and

consequently it has become almost a fixed habit to speak of nitrogen in the form of ammonia or nitrates; of phosphorus as phosphoric acid, the pentoxide, or as phosphates, and of potassium as potash, the oxide.

More recently other substances are coming to play important parts in agriculture. Among these are copper, manganese, boron, zinc and magnesium.

Commercial chemical fertilizers contain one, two or all three of the essential plant foods, and some that are used for fertilizing certain crops and soils contain also other substances that may be deficient in quantity or unavailable in form in the soil to be fertilized.

Nitrogen, usually considered as ammonia, increases the growth of stem and leaf, promotes the tillering or stooling of grasses and grains, augments the growth of new wood on apple, peach and other fruit trees, and in general produces luxuriance of foliage, vigorous growth, and the healthy green color seen in crops that produce good yields of high quality.

Phosphorus, available for the growing plant in the form of phosphoric acid, produces vigorous growth of roots early in the season and insures early ripening. Its value may be seen in its effects on the American corn crop, of which we produce about three billion bushels annually, for phosphoric acid, applied in the form of superphosphate, shortens by a week or more the period required to get mature grain. The opportunity thus afforded for maturing a crop before the autumn frosts may mean the difference between a crop of which 80 to 90 per cent. is merchantable and one of which only 50 to 75 per cent. is merchantable.

Potash increases greatly the formation of starch and helps to make heavy, flinty grains, such as are most desirable in milling and baking. It also stiffens the stems of grasses and grains, increasing their ability to bear heavier crops without the "lodging" that occurs when the ear of the grain is too heavy for the culm that bears it. In many crops potash also increases the power of the plants to resist disease.

PROPORTIONS OF THE ELEMENTS OF THE TRIAD IN FERTILIZER.

The elements of the triad must be used in fertilizer in suitable ratios one to another in order to insure increased

production and optimum quality. A given fertilizer is usually described by a series of three numerals, such as 3–9–3, 4–8–4, 2–12–6, 5–15–5, which express the proportions in which the elements of the triad have been combined in it. Probably no less than a thousand of such combinations are in present use, a number that is grossly excessive and that the industry is trying to reduce by a program of simplification. The series of figures thus used states the percentage of each plant food in an analysis, giving them in the order of (1) nitrogen, (2) phosphoric acid and (3) potash. The figure for nitrogen usually represents ammonia, of which only 82.25 per cent. is nitrogen as such. Thus, a 3–9–3 fertilizer contains 3 per cent. of ammonia, 9 per cent. of phosphoric acid and 3 per cent. of potash. In such a fertilizer the ratio of the plant foods to one another, as distinguished from the percentages, stated in the same order, would be 1–3–1, 1–2–1, 1–6–3. Upon the series of ratios can be built a group of analyses that would probably extend far beyond the needs imposed by the diversity of soils and crops. Practical experience has taught that under given conditions the best results are obtained by using specific ratios.

A STRIKING ILLUSTRATION OF THE EFFECT OF FERTILIZATION.

The efficacy of chemical plant food in increasing the yield and quality of a crop is shown by the results of experiments made in growing onions on peat soils by the State Agricultural College and Experiment Station of Michigan. Nitrogen was

Results of Experiments in Growing Onions

Pounds of Fertilizer Applied per Acre (Broadcast).	Marketable Onions		Percentage of Immature Onions.	Percentage of Bulbs Sprouted by May 15.
	Bushels per Acre.	Average Weight per Onion, Ounces.		
No fertilizer	65	1.2	82	36.4
Muriate of potash, 500 pounds	141	1.7	72	15.2
Superphosphate, 750 pounds	217	1.5	32	51.0
Phosphate, 750 pounds; potash 500	571	2.6	12	11.5
Nitrogen, 300 pounds; superphosphate, 750, and potash, 500	730	3.3	7	13.4

not used alone in the experiment, but its effect would have been small unless it had been used with one or both of the other members of the triad. The average results of the experiments, which were made for six years, from 1922 to 1927, inclusive, are shown in the preceding table.

Note not only the astounding progressive increase in quantity from 65 bushels per acre with no fertilizer to 730 bushels per acre when nitrogen, superphosphate and potash were used in complete fertilizers, but the astonishing fact that the number of immature onions decreased from 82 per cent. where no plant food was used to 7 per cent. where a complete chemical fertilizer was used.

We really should not be astonished at this result, however, for we have ample other illustrations of the power of chemicals to modify the metabolism of organisms. Perhaps the most striking illustrations of this power may be found in the internal secretions of human beings. For example, Dr. Graham Hoskins reported that the hormone known as epinephrine, obtained from the suprarenal glands and also made synthetically, is readily detectable by biological assay in a dilution of 1 part in 300,000,000. This means that one ounce of epinephrine crystals dissolved in 5,000,000 gallons of water can be detected. Hoskins put it more picturesquely by saying that to produce this degree of dilution "the contents of 40 miles of water carts, each holding 625 gallons and deployed 200 carts to the mile, would be required."

OCCURRENCE OF NITROGEN.

Dr. J. G. Lipman, in an address delivered in 1927 before the First International Congress of Soil Science in Washington, estimated the quantity of nitrogen in the soils, forests, and animal and human population of the world as follows:

Soils	40 billion tons
Forests	592 million tons
Animals	9 million tons
Human population	2 million tons

As most of the world's nitrogen lies near its surface, these figures might seem to indicate an adequate supply, but nitrogen is more quickly exhausted from soils, perhaps, than any other element of plant food. The peat deposits of the

world contain about 3,620,000,000 tons of nitrogen, and the coal deposits contain 98,000,000,000 tons. Our truly inexhaustible supply of nitrogen is in the atmosphere. There are about 39,000 tons of nitrogen over each square acre or about 25,000,000 tons in the air over each square mile. There are four main sources of nitrogen for agriculture and industry:

1. Organic substances, such as cottonseed meal, packing and rendering plant tankages, dried blood, fish scrap and other organic waste materials. These have grown more and more valuable as animal food, and hence less and less available for use in producing crops. Consequently only those of the lower grade are now available to the fertilizer industry.

Different compounds of nitrogen supply that element to plants at different rates, the rate depending on the rapidity with which the microorganisms in the soil attack the compounds and make the nitrogen available for the use of the plants. The makers of fertilizer use different carriers of nitrogen in plant food prepared for different soils and crops. By the use of proper carriers the plant is fed not only when the seedling first germinates and the cotyledons are thrust up through the soil, but during the later stages of the growth of the plant up to the time of its full fruition. Nitrate of soda provides nitrogen quickly. The nitrogen in sulphate of ammonia is somewhat less quickly available, because the ammonia must first be changed to nitrate by the microorganisms in the colloids around the particles of soil and in the soil solution. Organic ammoniates provide nitrogen for the plant still more slowly.

On certain soils and for certain crops it is essential that some of the nitrogen be supplied by organic materials. This is particularly true of the tobacco crop. At present only about 15 per cent. of the nitrogen in commercial plant foods is obtained from organic material. The remainder is obtained from chemical by-products in the form of sulphate of ammonia, and from the fixation of atmospheric nitrogen. In 1913, 42 per cent. of the ammonia supplied to plants in the United States was obtained from organic material; in 1926 the quantity so supplied amounted to 22 per cent.

2. Nitrate of Soda from Chile. For several decades before 1900 the largest source of nitrogen for use in agriculture and

the chemical industries was the natural deposit of nitrate minerals in Chile. Nitrate was first shipped from Chile in 1830, but it was not at once used in agriculture. As a matter of fact the art of producing crops by the use of artificial fertilizers is less than a century old. It was not until 1840, 89 years ago, that von Liebig published his pioneer work on "Organic Chemistry in its Application to Agriculture and Physiology," and the thought of previous centuries on the use of artificial promotors of plant growth seems to have come to a sort of focus around 1840. It was just after this time that the production of ammonia salts, including sulphate of ammonia, in great quantities as a by-product of the manufacture of gas was begun. The production of these salts has continued to grow until the present time, and they are now among the most commonly used chemical plant foods.

It was also in 1840 that the first South American guano was shipped from Peru to Europe. Its use grew rapidly, because it produced marvellous results. Its great value was due to its content of quickly available nitrogen in the form of ammonia, ranging from 5 to 14 per cent. according to the deposit from which it was obtained, and to its content of phosphoric acid, ranging from 10 to 25 per cent.

At this time also Sir John Lawes was developing his process of producing available phosphoric acid from natural rock phosphates by treating them with sulphuric acid. He patented his process in 1843. The first potash shaft was sunk in the Stassfurt district, in Germany, in 1851. The Germans were not seeking potash but good commercial supplies of common salt, the discovery of potash salts being purely an incidental result of their work. The systematic exploitation of the German deposits, however, was not begun until 1861.

The development of the Chilean nitrate deposits is of more than passing interest, for it shows what may happen to an industry based upon a natural resource that constitutes a monopoly when the chemist applies himself to the problem of finding independent supplies. By 1885 75 per cent. of the total nitrate nitrogen used in the world was shipped from Chile. By 1900 this percentage had dropped to 50, and by 1925 to less than 25, which is nearly the percentage now used.

The total consumption of nitrogen, however, has been increasing by leaps and bounds.

In 1898 Sir William Crookes expressed a fear that in a few decades—indeed, by 1931—the crops of the world would be suffering so much from nitrogen starvation as to threaten world hunger unless some new means of supplying nitrogen to agriculture could be found. His statement of a fear might almost have been construed as a prophecy, for already the nations of Europe, particularly Germany, England, France and Italy, are practically free from the necessity of importing Chilean nitrate, and by 1932 the United States will be similarly free from the need of importing nitrogen for making explosives in a time of war and for use in agriculture in a time of peace.

Since Sir William made his statement new deposits have been discovered in Chile, so that even after nearly 30 years it is now estimated that the Chilean deposits will supply the world's need at the present rate of consumption for 200 years longer.

3. By-Product Nitrogen from the Coking Process. By converting all the forms of inorganic nitrogen used in 1883 into terms of pure nitrogen, we find that the world then used about 95,000 tons, which is the equivalent of 613,000 tons of Chilean nitrate or 457,000 tons of sulphate of ammonia, the nitrogen content of the Chilean product being 15.6 per cent., and that of sulphate of ammonia being nearly 20.8 per cent. By 1900 the world used 330,000 tons of nitrogen; by 1913, 750,000 tons; by 1923, 925,000 tons, and at present about 1,500,000 tons. By 1913 the production of sulphate of ammonia had become so great that it represented 37 per cent. of the world's consumption; by 1926, it had risen to 50 per cent., a proportion which, in consequence of the growing production of synthetic compounds, will probably not be much exceeded in the future.

Coke and gas plants are practically the sole source of ammonium sulphate. New by-product coke ovens are rapidly replacing the wasteful old beehive ovens. About 90 per cent. of our production now comes from ovens of the modern type. The process of recovery is being improved, and we are now obtaining an average of a little over 22 pounds of sulphate of ammonia from each ton of coal that is coked in by-product

ovens. Inasmuch as less than 15.5 per cent. of the coal now mined is coked before burning (the production of bituminous coal in 1928 was nearly 500,000,000 tons, of which only 76,000,000 tons were coked), an increase in the production of ammonium sulphate is probable, although the percentage of its production to that of synthetic nitrogen may remain unchanged or may even decrease, because of the more rapid increase in synthetic production.

The increase in the production of ammonium sulphate in the United States can be shown at one stroke by saying that in 1915 its total production was about 250,000 tons; whereas in 1928 it exceeded 780,000 tons. During the next ten years its production will probably be increased to more than 1,250,000 tons. This increase would be equal to 125,000 tons of pure nitrogen.

Ten years ago, when the much-talked-of cyanamid nitrate plant was built at Muscle Shoals, its capacity was to be 40,000 tons of nitrogen. Since then the annual production of sulphate of ammonia alone has grown from 403,000 tons to 788,000 tons, the output in 1928. This means an increase from about 84,000 tons to 164,000 tons of pure nitrogen—almost double. As private enterprise is so adequately meeting the growing need of the country for nitrogen, it is proper here to raise the question of both the necessity and the desirability of governmental intervention, either directly or by subsidy, in the production of nitrogen by the Muscle Shoals plant.

4. Synthetic Fixation of Air Nitrogen. Two decades ago a wholly negligible quantity of nitrogen was obtained from the inexhaustible resources of the atmosphere. In the year 1929 about a million tons will be obtained from the atmosphere. This figure represents pure nitrogen in terms of world production and not substances carrying nitrogen. Expressed in terms of nitrate of soda, it would be equivalent to more than six million tons.

Self-preservation, speaking somewhat euphemistically, or, to be slightly more brutal, military preparedness, has been the motive or purpose behind the enormous increase in the fixation of air nitrogen. It is freely stated, and not without considerable truth, that Germany postponed a warlike attitude until she could, by reason of her freedom from the Chilean

nitrate beds, wage independent and effective war, even though blockaded. Every nation naturally has an ambition to be sufficient unto itself with respect to the commodities that are absolutely essential for its defense. The ambition for military preparedness has greatly benefited the farmers of the world, for in times of peace agriculture provides the only relatively sufficient outlet for the immense quantity of nitrogen that has been produced during the last twenty years.

PROCESSES OF MANUFACTURE.

Air-derived nitrogen is produced by three distinct processes, usually termed (1) the arc process, (2) the cyanamid process and (3) the synthetic process. The synthetic process really involves a number of different processes or variations of the same fundamental process. The essential features of these processes can be described very briefly.

The arc process, as its name indicates, involves the use of the electric arc in a manner not essentially different from that in which it is employed for street lighting. It is an imitation of the natural fixation of nitrogen by lightning.

The cyanamid process involves the fusing of coal and limestone to form calcium carbide, a compound of calcium and carbon. This compound is heated to a high temperature (approximately 1,000 degrees Centigrade) and nitrogen is then passed over it in great autoclaves in the presence of steam and is fixed in the mass in the form of calcium cyanamid.

In *the direct synthetic process*, the one now most widely used, a mixture of nitrogen and hydrogen, which are first obtained separately, is passed under high pressure (1,500 to 15,000 pounds per square inch) over a catalyst at a high temperature (500° to 600° Centigrade) to produce ammonia gas. This gas is condensed or absorbed, as may be desired, and is marketed either in the form of ammonia (NH_3) or aqua-ammonia (NH_4), or is combined with other elements to form sulphate of ammonia, synthetic nitrate of soda or other materials.

Some of the synthetic processes now in use are the Haber, the Haber-Bosch, the Casale and the Claude. In these various processes different methods are employed to produce the pure nitrogen and the hydrogen required for the catalytic

process, but the ultimate result is substantially the same. The Fixed Nitrogen Research Laboratory of the United States Department of Agriculture has devised one of the methods of fixation now used, and in particular has improved catalysts and increased our knowledge of them.

POWER REQUIREMENTS OF SYNTHETIC PROCESSES.

Recently Sir Alexander Gibbs, President of the Institute of Chemical Engineers of England, estimated that the arc process, which recently has been used only in Norway, where hydroelectric power is very cheap, and even there its use is diminishing, demands about 80,000 kilowatt hours per ton of nitrogen fixed. Other authorities have stated that this process requires only 60,000 kilowatt hours. Gibbs states that the cyanamid process requires about one-fourth of the electrical energy of the arc process, or from 16,000 to 20,000 kilowatt hours. The direct synthetic process requires about one-sixteenth of the power required by the arc process and one-fourth of that required by the cyanamid process. It uses from 4,000 to 5,000 kilowatt hours per ton of nitrogen fixed.

These facts are worthy of careful thought in connection with the present effort to reconstruct and modernize at government expense the cyanamid plant at Muscle Shoals. Why should we undertake to put into use a plant that requires 16,000 to 20,000 kilowatt hours per ton of nitrogen if the modern process requires only 4,000 to 5,000 kilowatt hours, particularly in view of the fact that it will cost from $10,000,000 to $20,000,000 to modernize the plant?

TREND OF THE INDUSTRY AS TO PROCESSES.

In 1913 seven plants in the world were using the arc process and producing 19,800 tons of nitrogen. At that time this quantity represented nearly 32 per cent. of the total output of synthetic nitrogen. In 1926 the same seven plants were still producing nitrogen, but with increased capacity and greater efficiency. They were then making 40,500 tons of nitrogen, yet that quantity was only 5.5 per cent. of the total synthetic nitrogen made in the world.

In 1913 fifteen plants were producing nitrogen by the cyanamid process and their total product was 36,000 tons, or 57.4 per cent. of the world's total output of synthetic nitrogen.

By 1926 the number of plants had increased to 28 (nearly double) and the production had increased to 174,000 tons, but the percentage of the total nitrogen produced by this process had dropped from 57.4 to 23.7.

In 1913 (note the contrast) only one plant was producing nitrogen by the direct synthetic process. It was making 6,000 tons of nitrogen, only about 11 per cent. of the total synthetic output, but by 1926 the number of plants producing nitrogen by this process had risen to 49 and the total production had increased from 6,000 to 519,000 tons, or 70.8 per cent. of the total synthetic nitrogen produced in the world.

The figures for 1926 are given because they are the most readily available and perhaps more authoritative than those for later years. The same trend continues, even with some accentuation, for if the market will absorb it Germany will produce this year about 600,000 tons of nitrogen by direct synthetic processes.

IMMENSE DEPLETION OF OUR SOIL RESOURCES.

Almost no layman and few agronomists realize clearly the extent to which crop production and erosion are making drafts on the nation's soils. Mr. H. H. Bennett, of the United States Department of Agriculture, recently made a statement from which we may infer that it takes 10,000 years for Nature to form a foot of fertile soil from the rocks of the earth's crust. Every year we carelessly and thoughtlessly permit erosion to wash away a vast amount of the soil that has been formed during a period covering more than twice our known historical era. According to Bennett erosion takes 126 billion pounds of fertilizing elements annually from our soils, which is 20 times as much as that removed by all of our crops. He estimates that the plant nutrients thus removed could not be replaced by an expenditure of two billion dollars a year.

The National Industrial Conference Board recently estimated that crops remove annually 9,000,000 tons of nitrogen from our soils. Only about 40 per cent. of the nitrogen thus removed is restored by the use of manures. An enormous quantity of both phosphoric acid and potash is also removed by crops, which cause a loss of plant food above that restored of about 5,900,000,000 pounds, valued at $400,000,000 annually.

It is not difficult to see what an annual overdraft of $2,400,000,000 on our bank of soil resources ultimately means to the country if continued for many years. Declining returns from agriculture have already served notice to farmers and economists that even the richest soils are not inexhaustible. Our agricultural plant is depreciating and no sinking fund is being provided to cover its depreciation. Many blame the farmer for the increasing loss. Liebig once said, "Agriculture is of all industrial pursuits the richest in facts and the poorest in their comprehension." He was no doubt hurling a shaft at the German farmer of the middle of the nineteenth century. His statement has less force now than it had then, and even then it was not altogether fair, nor would it be fair if applied to the American farmer.

THE FARMER'S HANDICAPS.

For a long time farmers have operated at a disadvantage in exchanging their goods and services for the goods and services of others. The purchasing power of farm products in terms of other commodities has been so low as to create a handicap that they could not overcome and that lies at the root of the present demand for agricultural restoration, or farm relief, as it is commonly called. Many a farmer would have a better home, would educate his children better, would buy better machinery and undertake a program of soil improvement, including the use of chemical plant food, if he could only gain a fair net income. Of the commodities that he must use only feeds and fertilizers are now cheaper than the products of his farm, taken as a whole. The prices of the commodities and services he must buy from others are from a few per cent. to more than 100 per cent. higher than in 1913. His labor costs him about 70 per cent. more; some of his taxes are more than 100 per cent. higher; his railroad transportation charges are from 75 to 100 per cent. greater, and the prices of many other things are much higher. We therefore need not wonder at the astounding continued depletion of our soil resources, despite the fact that the farmer is aware of the loss. He cannot solve the problem; it is too big for him. We need a national agricultural policy in which all the elements of our population, from producer to consumer, must share if it is to be effectively carried out.

Not all the plant food and particularly not all the nitrogen that is removed by crops and erosion need be returned through agricultural practices involving the growing of crops with associated nitrogen fixation, or through restoration by the use of natural manures or commercial fertilizers. Every year the electric discharges shown by lightning cause the fixation of no less than 100,000,000 tons of nitrogen, which through rain and snow ultimately finds its way into the water and soil. Unfortunately, the arable acres receive only as much nitrogen as equal surfaces of the ocean and of the barren deserts.

NITROGEN FIXATION IN SOILS BY BACTERIA.

Competent soil bacteriologists estimate that as much as 200 pounds of nitrogen per acre may be fixed in the soil by the bacteria that live on the roots of many legumes—members of the pea family. The nitrogen in organic matter is complex and is therefore not directly available for the use of plants. It must first be broken down into simple water-soluble compounds. This action involves the conversion of combined nitrogen into ammonia, although, perhaps, some ammonia is taken up as such. A fraction of the vast bacterial population of the soil changes the ammonia to nitrites, and these are then changed to nitrates, which are directly available to plants as food.

Certain plants are more efficient than others in obtaining nitrogen from the air and soil. Especially efficient are the members of the legume family, to which belong not only our familiar garden peas and beans, but also the clovers, alfalfa, the vetches, soy beans, cow peas, and many other garden plants, as well as hundreds of species of trees, like the locusts, the cassias and the acacias. The apparent power of these plants to fix atmospheric nitrogen is due to their symbiotic relations with a group of nodule-forming and nitrogen-fixing soil bacteria. If you pull up a bean or pea or clover plant and examine it, you can find hundreds of little tubercle-like nodules on its roots. These nodules are the homes of billions of colonies of bacteria. We little realize the immensity of the number of inhabitants of our soils, even of those of the desert. Dr. S. A. Waksman of Rutgers University reported in 1927 that the surface portion of a brown loam soil contains 1,518,000,000 microorganisms per gram. Dr. H. J. Conn

found as high as 800,000,000 bacteria alone in a gram of soil. Numerous fluctuations take place in their numbers in a short time. A sample of soil containing 7,500,000 bacteria at one time as determined by the culture method had decreased in 16 days to but a little more than 2,000,000. Great variations occur in different parts of the same plot of ground. Waksman reported that a bacterial count of a small plot of earth showed a range from 8,000,000 to 25,000,000 bacteria per gram of soil. The number varies also in soil of different types, at different depths, at different seasons of the year and in soils that have received different agricultural treatment. Great differences also occur with variations in the acidity or alkalinity of the soil.

OUTSTANDING ACHIEVEMENTS OF THE LAST TWO DECADES IN THE FIXATION OF NITROGEN FROM THE ATMOSPHERE.

In the fixation of nitrogen the World War stimulated much greater progress in Europe than in the United States, and the European experience may show fairly well what may be done in the United States.

In 1913 the world production of nitrogen was as follows:

	Tons N.
Chilean nitrate	429,897
By-product ammonia	319,667
Synthetic ammonia	90,491
Total	840,055

Mr. H. R. Bates, in a paper read before the Nitrogen Symposium of the American Chemical Society in 1928, estimated the production capacity of the world in 1927 as follows:

		Tons N.
Synthetic ammonia	700,000	
Norwegian salpeter	30,000	
Cyanamid	200,000	
		930,000
By-product ammonia		370,000
Chilean nitrate		320,000
Total		1,620,000

These figures represent an increase of almost 100 per cent. within 14 years. Only the stimulation of national self-defense and military ambition could have achieved such a result. Preparedness has been foremost in the minds of

European statesmen or politicians, who have given every possible assistance to their domestic nitrogen industries.

In Germany, where the most remarkable development has occurred, this encouragement has been given by the construction of government plants to be leased to private corporations at nominal rentals or transferred to them outright at very low prices. There are two plants of special interest in Germany, both owned by the great German Dye Trust, the Interessen Gemeinschaft Farbenindustrie, commonly called the I. G., one of which has a fixation capacity of 100,000 tons of nitrogen a year, and the other a capacity of about 500,000 tons. One plant is at Oppau and the other at Merseburg. In 1910 the plant at Oppau was a small experimental unit. In 1911 it was turning out only about 25 pounds of ammonia a day. In 1912 it was making about one long ton (2,240 pounds) a day. In 1914 it was making 7,000 tons of ammonia a year. In 1917 it was producing 100,000 tons of fixed nitrogen a year. This single illustration shows the possibilities of growth in the fixation of nitrogen.

In 1927 the nations participating in the manufacture of fixed nitrogen had estimated production capacities shown by the following table:

World Capacity of Nitrogen Production in 1927, in Tons.

	Synthetic.	Cyanamid.	By-Product.	Arc Process.	Total.
Germany	450,000	114,000	100,000	.....	664,000
France	116,700	53,500	5,000	.....	175,200
England	55,000	.....	100,000	.....	155,000
United States	25,000	40,000	147,000	.....	212,000
Italy	63,700	20,000	3,500	.....	87,200
Japan	43,500	20,000	5,000	.....	68,500
Belgium	22,000	.....	10,000	.....	32,000
Spain	7,200	.....	.....	.....	7,200
Poland	3,000	30,000	.....	.....	33,000
Czecho-Slovakia	4,500	6,000	.....	.....	10,500
Norway	.....	15,000	.....	46,500	61,500
Russia	7,300	.....	.....	.....	7,300
Sweden	2,500	6,000	.....	.....	8,500
Roumania	.....	5,000	.....	.....	5,000
Canada	.....	60,000	5,000	.....	65,000
Switzerland	2,200	.....	.....	.....	2,200
Jugoslavia	.....	14,000	.....	.....	14,000
	802,600	383,500	375,500	46,500	1,608,100
Actual Production	700,000	200,000	370,000	30,000	1,300,000

DEVELOPMENT OF THE FIXATION OF NITROGEN IN THE UNITED STATES.

Manufacturers and others who want to get possession of the power plants at Muscle Shoals and politicians who would be benefited or think they would be benefited by the government's operation of these plants are accustomed to belittle the progress made in the United States in the fixation of atmospheric nitrogen. In 1927 we used approximately 250,000 tons of inorganic nitrogen as a source of commercial plant food. In a time of peace our industries other than agriculture use about 100,000 tons of inorganic nitrogen a year. It would therefore appear that the total nitrogen required annually in a time of peace would be about 350,000 tons. Experts on munitions estimate that the nitrogen annually needed for carrying on a first-class war would amount to about 144,000 tons.

By the end of 1929 we shall have the capacity to produce at least 100,000 tons of synthetic nitrogen. The production of sulphate of ammonia in 1928 increased nearly 10 per cent. over that in 1927. The production in 1928 was 788,000 tons. Assuming a slower rate of increase in 1929, we may expect a production of at least 850,000 tons. With a nitrogen content of 20.8, this would amount to more than 175,000 tons of pure nitrogen. Our total capacity to produce inorganic nitrogen would therefore be approximately 275,000 tons against an approximate peace-time need of 350,000. We therefore appear to be about 75,000 tons short of covering our need of inorganic nitrogen, and the demand is still increasing. Highly satisfactory progress is being made, however, and by the end of 1932 we shall probably be measurably self-sufficient. In 1913 we produced no synthetic nitrogen whatever; in 1921 we produced 200 tons; by the end of 1929 we shall have the capacity to produce about 100,000 tons. The recent and prospective increase has been due chiefly to the breaking in of the units of the great chemical plant at Hopewell, Virginia. Six or seven other corporations are increasing their production, so that by the end of 1932 we may have a synthetic production capacity of possibly 200,000 tons. In the event of war we could easily raise this total to 250,000 by modernizing the obsolete cyanamid plant at Muscle Shoals. When we make war we are not likely to consider economy of production.

To show our progress toward freedom from foreign nitrogen, I need only say that in 1900 the United States produced 13 per cent. of its requirements; in 1910, 18 per cent; in 1913, 25 per cent; in 1928, 50 per cent. In 1929 it is estimated that it will produce 65 per cent. of its requirements, and in 1930, 80 per cent.

One of the interesting features in the progress of the past twelve months has been the direct use of liquid ammonia in the manufacture of fertilizer, a use that is a means of economy. It is shipped in tank cars to seventy-one plants, operated by some twenty companies. Both synthetic nitrate of soda and synthetic sulphate of ammonia of remarkably fine quality are also being produced.

EUROPEAN EFFORTS TO MARKET NITROGEN IN THE UNITED STATES.

Germany, England, France, Italy, Norway, Czecho-Slovakia, and other countries are producing nitrogen in excess of their own domestic consumption. The military urge, of course, is behind this great production, but in the quiet of peace some outlet must be found for the product. The United States, China, India and other countries are looked to as lands of promise. Germany, for instance, has a consumptive

Typical German Concentrated Fertilizers Containing Two or More Plant Foods.

	Nitrogen (N).	Phosphoric Acid (P_2O_5).	Potash (K_2O).	Total.
Mono-ammonium phosphate...	11.6	60.6	0	72.2
Di-ammonium phosphate.....	20.7	52.3	0	73.0
Potassium ammonium nitrate..	16	0	28	44
Urea ammonium phosphate....	20	49.4	0	69.4
Potassium ammonium phosphate.....	5.25	54	18.5	77.75
Potassium nitrate.....	13	0	44	57
Leunaphos.....	20	20	0	40
Nitraphoska No. 1.....	17.5	13	22	52.5
No. 2.....	15	11	26.5	52.5
No. 3.....	16.5	16.5	20	53
No. 4.....	15.5	15.5	19	50
No. 5.....	15	30	15	60

capacity of only about 400,000 tons of fixed nitrogen and a producing capacity of more than 600,000 tons. To obtain entry to the American market, the German chemical industry, always progressive and vigorous in its policy of trade develop-

ment, has prepared a number of valuable chemical plant foods containing one, two or all three of the agricultural triad—nitrogen, phosphoric acid and potash. Without taking time to describe these products in the course of my oral statement, I insert at this point a short table giving some typical materials containing two or more plant foods.

EFFORTS TO INCREASE PLANT FOOD CONTENT OF FERTILIZERS.

We can realize the value of these German fertilizers and their high content of plant food (50 to 77 per cent.) if we remember that the combined average percentage of nitrogen, phosphoric acid and potash contained in the average American fertilizer is between 16 and 17 per cent. The fertilizer industry and the official agricultural forces of the United States have been working for many years to raise the plant-food content of fertilizers to the level now reached. As a consequence, the German fertilizers, when used in the United States, must be diluted in order to meet the present demand of the American farmer. Incidentally, their importation duty free constitutes one of the problems of the American industry.

The National Fertilizer Association has devoted ten years of earnest effort to increase the concentration in American fertilizers, but its progress has been limited by the economic condition of the farmer, the relative slowness of agricultural education, and the fact that our fertilizer-distributing machinery is not yet sufficiently perfected to place in its proper relation to the seed or the growing plant the small quantities of these concentrated fertilizers that must be applied.

MUSCLE SHOALS AND THE NITROGEN PROBLEM.

No discussion of the production of nitrogen in the United States would be complete without mention of the problem of disposing of the government properties at Muscle Shoals, Alabama. It would be like discussing Macbeth and failing to mention Banquo's ghost. During the last ten years Muscle Shoals has ranked with prohibition and farm relief in its demand for newspaper space. I have already commented on the fact that the ambition to attain military independence with respect to essential munitions of war now marks the psychology of practically all countries. The United States

has not escaped this ambition. In addition agricultural distress has stimulated the desire of farmers in all countries to obtain cheaper nitrogen, despite the fact that nitrogen is now cheaper relative to farm products than it was before the World War. Politics and propaganda of interested groups have been great breeders of distorted pictures and misleading half-truths. Great expectations have been created in the farmer's mind with a view to using his political power to get possession, in his supposed interest, of the great property at Muscle Shoals. False hopes have been raised in the farmer's mind of the benefits that may come to him from this source to relieve him of his present serious economic burdens.

The United States has almost unlimited supplies of phosphate rock for the production of phosphoric acid. Our phosphate reserve is seven to ten billion tons.

We are heavily dependent upon Germany and France for our potash salts, but we are now producing about 75,000 tons of potash (K_2O) annually, and the output can be increased, though perhaps not immediately, until it is sufficient to supply our needs. Potash deposits are being explored in Texas and New Mexico, and the greensands of New Jersey may some day, by the aid of our chemists, make us self-sufficient in potash, as we are in phosphorus.

Our situation as to nitrogen has been fully disclosed in the course of this address. We come, then, to a consideration of a sensible economic disposal of the plants at Muscle Shoals, which shall take into account the rights of private enterprise that must carry on without subsidy from the government. We now have three valuable assets at Muscle Shoals:

1. The Wilson Dam, which cost approximately $47,000,000, with its fine hydroelectric power plant, with which should be considered Steam Power Plant No. 2, built for the use of the big cyanamid plant at a cost of approximately $12,000,000.

2. Nitrate Plant No. 1, which was built at a cost of $13,000,000 to utilize a modification of the synthetic process known as the Haber-Bosch but which was never run successfully because of our lack of precise knowledge of the details of operation.

3. Nitrate Plant No. 2, the cyanamid plant, with which should be included the Waco limestone quarry, the cost of the

two together, with their incidental properties, being about $68,000,000.

PLAN PROPOSED IN MADDEN BILL.

The Madden bill, which was pending in Congress when it adjourned on March 4, proposes to turn over the properties described above to a private corporation without charge for interest except on $17,000,000 of the cost of building the Wilson Dam, the sum expended after May 31, 1922. In other words, assets that cost the United States above $140,000,000 are to be leased for a period of fifty years without rental interest except on the $17,000,000 mentioned. About eighty million dollars of new money is to be expended, and the products of the expenditure are to be turned over to a private corporation at a rental of 4 per cent. The cost of construction immediately after the war was no doubt excessive, but property representing an eventual expenditure of more than $216,000,000 is to be turned over at an average rental that will pay only about 2½ per cent. Private enterprise is supposed to compete with the company that is to be given such largess by the government.

The unreasonableness or the impossibility of such a plan is made evident by the fact that the ordinary manufacturer of fertilizer pays from 6 to 8 per cent. for all the money he borrows, both for fixed and operating capital. Furthermore, he must pay state, county and municipal taxes, from all of which the government's lessee is to be relieved. The private operator must also carry fire and other forms of insurance; the government will carry all the insurance on the plant leased. Furthermore, the fertilizer industry at present has a capacity to manufacture 10,000,000 tons of complete fertilizer, with a demand for only about 7,000,000.

Muscle Shoals is intrinsically a power enterprise. This fact is readily disclosed by the present plans. It is planned to provide dams and generating facilities in addition to those already installed, for the production of approximately 1,250,000 horsepower years of power. The lessee promises, if a group of meticulous conditions are fulfilled, to produce, after a period of several years, a maximum of 50,000 tons of nitrogen. The maximum amount of power required for such a production would be 150,000 horsepower years. As more than

a million horsepower above this requirement are to be produced, it is clearly evident that the purpose of those to be benefited is to obtain the possession and use of enormously valuable water-power plants rather than to manufacture cheap fertilizer. It should be said, however, in fairness to the company named in the Madden bill, that it does propose in good faith to manufacture a large quantity of fertilizer. The objection of the existing fertilizer industry, of which the company leasing the plant is a part, is that the plan will create unfair competition that is in every aspect un-American.

PLAN FAVORED BY THE FERTILIZER INDUSTRY.

The fertilizer industry, through The National Fertilizer Association, has suggested what it believes to be a fair and practical plan for the disposal of the plants at Muscle Shoals. Its suggestion briefly is:

1. That the Wilson Dam and Steam Plant No. 2 shall be sold or leased for the generation of power for any purpose for which there may be an economic demand. If no satisfactory bidder is found, the Engineer Corps of the War Department shall continue to produce power as it is now doing and sell it at the switchboard to the highest responsible bidders. The average income for the last two fiscal years from the sale of power at Muscle Shoals has been approximately $1,300,000 yearly.

2. That Nitrate Plant No. 1, which is a pilot-size plant, shall be turned over to the United States Department of Agriculture for technological and chemical research in nitrogen fixation, including the production of nitrates for making fertilizers on a scale sufficient for farm demonstrations under the direction of federal and state experiment station workers, but not in quantity sufficient for commercial purposes.

3. That the obsolete cyanamid Plant No. 2 shall be kept in stand-by condition by the War Department for from one to three more years, and that it shall then be sold for scrap or otherwise, just as other surplus war materials have been sold, but that the nitric acid or oxidation part of the plant shall be retained by the War Department for such value as it may have for national defense, particularly in the manufacture of powder.

The cyanamid process, as has been shown, is wasteful of power, requiring no less than 16,000 kilowatt hours per ton of nitrogen, whereas the synthetic process requires only 4,000 to 5,000. The plant at Muscle Shoals has been built ten years, and during that time the art of nitrogen fixation has progressed with what might fairly be termed meteoric rapidity. There is a constantly growing need of power for all kinds of technological and industrial purposes in the Muscle Shoals area. It would be an economic crime to waste this valuable resource in an obsolete plant using an obsolescent process. It would cost between $10,000,000 and $20,000,000 to modernize the plant, and for that sum of money a new synthetic plant of greater capacity could be built near to coal, which is more important for nitrogen fixation than water power because it is a valuable source of the hydrogen that must be used in the more efficient catalytic synthesis.

The United States government should encourage, not throttle, private enterprise in the remarkable progress it has already made in nitrogen fixation. It should protect a service-giving industry that is indispensable to American agriculture and not threaten it with competition from a governmentally subsidized corporation drawn from its own ranks. It should encourage private endeavor in every possible manner, and should not engage in competition with its citizens except, as Lincoln said, when it is necessary to do something for the citizen which he cannot do for himself.

ECONOMIC BENEFITS OF FERTILIZATION.

The economic returns from the use of commercial plant nutrients can be illustrated in many ways. I have already given a single illustration of the effects of their use on onions grown on peat lands in Michigan. In order to leave a more accurate and generally applicable impression, I will briefly show how much the use of fertilizers may increase yields.

There are about 6,250,000 farms in the United States. In 1928 about 2,250,000 of these farms used artificial plant food in some form.

The results of official experiments, many of them carried on in large commercial field tests, and of the experience of hundreds of thousands of farmers show that, as a general

average, one dollar invested in fertilizer returns three dollars in increased crop values. The following examples show the increases that may be gained by the use of fertilizer.

One ton of fertilizer, costing approximately $35, produces an increase of 75 to 125 bushels of potatoes, worth $85 to $140 (5-year average, farm price on December 1).

One ton of fertilizer, costing approximately $30, produces an increase of 1,500 to 2,000 pounds of seed cotton (approximately one-third lint and two-thirds seed), worth $115 to $155 (5-year average, farm price on December 1).

One ton of fertilizer, costing approximately $30, produces an increase of 70 to 100 bushels of wheat, worth $86 to $119.

One ton of fertilizer, costing approximately $30 to $40, produces an increase of 1,000 to 1,400 pounds of tobacco, worth $200 to $275 (5-year average, farm price on December 1).

Another way of expressing the economic benefits to be obtained by the use of fertilizer is afforded by the official records of the Ohio Agricultural Experiment Station, which found that an hour's labor on unfertilized land produced a little over a half bushel of corn; whereas an hour's labor on fertilized land produced over a bushel and a half. An hour's labor on unfertilized oats produced two bushels; on fertilized oats, four bushels. On unfertilized land an hour's labor produced $1\frac{1}{8}$ bushels of wheat; on fertilized land it produced more than 3 bushels. Unfertilized cotton in South Carolina yielded about 500 pounds of seed cotton per acre; immediately adjacent land treated with 800 pounds of chemical plant food yielded nearly 1,200 pounds of seed cotton per acre.

PERIOD OF CROP NUTRITION IN AMERICAN AGRICULTURE.

Modern agriculture in western Europe and the United States began with Jethro Tull, who was born in 1700 and died in 1760. The period of progress that was ushered in by Tull's discoveries and practices lasted until about the end of the Napoleonic wars, in 1815. It was characterized by new crops, improved methods of cultivation, pronounced advancement in animal husbandry in all of its phases, and the substitution of individual farming for the ancient system of commons.

The period of greatest advancement in American agriculture, which was reflected in European agriculture, began with the invention of the harvester and the many other modern farm implements that are now commonplaces to all of us. This period of mechanization still continues, and in its modern phases is marked by the development of the tractor, the combine harvester and many types of improved cultivating and other machines. It is also marked by notable improvements in the control of plant and animal diseases, in the breeding of superior varieties of plants and animals, and in the increased use of power, as well as by other similar evidences of progress.

The agricultural depression of the last decade has forced the American farmer to recognize the necessity of intensive cultivation. His future ideals will be to reduce the cost of production and to obtain larger yields. Our present agricultural condition is not new, for despite the fact that our progress in the use of commercial plant food has been slow, we are now using between 7,000,000 and 8,000,000 tons of fertilizer a year. We made our first practical application of fertilizer in the decade between 1840 and 1850, almost immediately after the discoveries of Liebig in Germany and Lawes in England. So we may properly say that we are now decidedly over the threshold of a period of scientific plant feeding, a period not unlike the period of scientific animal feeding that has been staged during the last fifty years. In this unfolding scene nitrogen will play a most important part

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

PAPER TESTING STANDARDS.

At the annual meeting of the Technical Association of the Pulp and Paper Industry held in February, B. W. Scribner of the Bureau of Standards, chairman of the paper testing committee, presented a report of the activities of the committee for the past year. The following is a summary of the report:

The committee is still concentrating on the development of official association testing methods. During the past year three new methods were completed—gloss, opacity, and bulk—and the first two have been adopted by the association as standards. The adoption of these two optical methods gave considerable impetus to their use as a part of the testing routine for control of paper-making processes.

Considerable development work was done on specific projects allotted to four different subcommittees. Under the leadership of E. O. Reed, much progress was made in further standardizing a method for measuring the resistance of papers to printing inks, and a satisfactory method for this important determination will no doubt be available in the near future. The subcommittee on microscopical methods, headed by Prof. C. E. Libby, made an intensive study of the "dot" method for determination of fiber composition. They proved that this modification gives increased accuracy, particularly in the case of the more inexperienced analyst. Miss H. U. Kiely's subcommittee on chemical methods obtained test data which will lead to considerable improvement of the present official methods for quantitative determination of starch and paraffin. Progress was made in developing a method for quantitative determination of acidity and an improved method for detection of formaldehyde. For acidity, it is believed that a modified Kohler-Hall method, whereby a water extract of the paper is tested in the presence of the

* Communicated by the Director.

fibers in an atmosphere freed of carbon dioxide, will permit much greater accuracy. The subcommittee on permeability to liquids, Allen Abrams, chairman, obtained considerable valuable data through coöperative tests by various laboratories, of the degree of concordance to be expected in determination of degree of sizing and grease resistance. For the former, the methods proposed by the Bureau of Standards gave very promising results. Various modifications of H. A. Smith's method for grease resistance gave rather conflicting results and considerable further development of it is necessary.

During the current year, in addition to the continuing work indicated, development work will probably be initiated on determinations related to permanence, such as alpha cellulose, copper number, effect of light, and effect of heat.

COÖPERATIVE DEVELOPMENT OF PERMANENT LEDGER PAPER.

A STRIKING example of the value of correlating laboratory tests and mill practice was recently experienced. At the request of a paper manufacturing concern, the bureau is coöperating with them in improving the quality of a rag fiber permanent ledger paper found to be deficient in some respects. Exhaustive permanence tests showed that the paper was exceedingly strong, having a folding endurance of around 25,000 folds. When subjected to the heat treatment, however, the paper darkened, increased excessively in copper number, and decreased excessively in alpha cellulose, all of which indicated that the paper contained degraded celluloses and therefore that it would be of doubtful permanence. As the highest grade of raw materials was used in its manufacture, it was surmised that some detail of the manufacturing processes was faulty. The original copper number was low, therefore the cellulose degradation was attributed to either excessive hydration in beating or to the high rosin content. The manufacturer made a subsequent paper with exactly the same raw materials but with much milder beating, as is indicated by the lower folding endurance of 6,000 folds. This paper in all respects appeared to have the maximum quality for permanence as it was practically unaffected upon heating for 72 hours at 100° C. Apparently excessive hydration was the prime cause of the previous poor quality.

TRANSPARENCY OF TRACING CLOTH TO ULTRA-VIOLET LIGHT.

IN view of the inquiries received regarding the transparency of tracing cloth, about which there has been considerable publicity in the press, transmission tests have been made on this material.

Tracing cloth consists of thin, loosely woven fabric of linen or cotton, impregnated with a highly soluble, transparent starch. The drawback in the use of such a material for a window is the high solubility of the sizing which dissolves when touched with water, leaving the cloth in a rough condition which is then no better than other thin, bleached, loosely woven material, such as, for example, nainsook or balloon cloth.

The bureau has determined the transmission of tracing cloth and finds that the total ultraviolet of wave-lengths 250 to 310 millimicrons diffusely transmitted through a new sample of tracing cloth amounted to about 26 per cent. before wetting, and 14 per cent. after removing the sizing and drying the cloth.

In Table 1 of Bureau of Standards Research Paper No. 6, which may be obtained from the Superintendent of Documents, Government Printing Office, Washington, D. C., at 5 cents per copy, are recorded the ultraviolet transmissions of a sample of balloon fabric (transmission, 21 per cent.), of batiste (transmission, 16 per cent.), and of nainsook (transmission, 31 per cent.), showing that tracing cloth is not superior to other fabrics for use as a window for transmitting ultraviolet radiation.

EFFECT OF SERVICE ON THE ENDURANCE PROPERTIES OF RAIL STEELS.

A SERIES of tests has been carried out to determine the effect of service on the endurance properties of rail steels, and the complete report will appear in a forthcoming number of the *Bureau of Standards Journal of Research.*

In a previous report (Technologic Paper No. 363, obtainable from the Superintendent of Documents, Government Printing Office, Washington, D. C., at 35 cents per copy) data were given on the endurance properties of steels from new rails from 11 distinct heats. An hypothesis based on the

phenomena of "overstressing" and "understressing" was presented whereby it was considered possible to determine whether the steel of the rails in service was subjected to stresses greater or less than its endurance limit by a comparison of the endurance limits before and after service.

Through the coöperation of the Baltimore and Ohio Railroad and the Canadian Pacific Railway, a total of 18 rails were removed from track after being subjected to over 20,000,000 and 12,000,000 tons of traffic, respectively. These were principally *A* and *B* rails from 6 heats tested before service, and 1 heat not previously tested.

The endurance properties were determined in the same manner as for the rail steels before service.

In the rails returned from service on the Baltimore and Ohio Railroad the endurance properties of the steel were found to be the same as before service.

Tests were made to determine the effect of appreciable surface flow and consequent hardening of the top of the head of the rail in service on the endurance properties of the steel in that rail. No evidence was found of any "overstressing" or "understressing" having occurred.

It was concluded that under the service conditions to which the rails were subjected on the Baltimore and Ohio Railroad the steel in the head of the rail was not subjected to fatigue stresses greater than its endurance limit.

The results of similar tests of rails after service on the Canadian Pacific Railway were not as definite in their indications. In most cases the test specimens showed a very marked "scatter" in endurance properties indicating appreciable inhomogeneity in the steel. This "scatter" was found to be due to the presence of groups of minute transverse cracks or so-called "shattered zones" in the rail as removed from service.

Microscopic examination showed the cracks to be principally transcrystalline but in a few instances evidence of intercrystalline failure was apparent indicating failure while the metal was hot, that is, before service.

Examination of a new rail from one of the same heats showed the presence of shatter cracks; premature failure of a fatigue test specimen from a new rail of the same heat was also found to be due to the presence of an internal crack.

It was therefore, believed that the cracks found in the service rails, and consequent wide scatter of results obtained in the endurance tests, may have been due to cracks present in the new rail and not to conditions which originated in service.

In cases where the endurance tests were not affected by the presence of cracks, the endurance properties were found to be comparable with those given by tests before service, indicating that service had not imposed repeated stresses greater than the endurance limit of the steel. It was concluded that fatigue failure in service, transverse fissures, spread from a pre-existed nucleus.

Comparative tests were also made on steel from a rail which failed in service due to a transverse fissure, and an adjacent rail in track which had been subjected to the same traffic conditions. The endurance limit of the steel from the fissured rail was found to be appreciably greater than that from the unfissured rail from which it was concluded that failure due to transverse fissure is not due to fatigue stresses alone but develops from a preëxistent nucleus.

A MODIFIED METHOD FOR THE DECOMPOSITION OF CERAMIC SILICATES FOR CHEMICAL ANALYSIS.

THE method ordinarily used for decomposing silicates preparatory to their chemical analysis consists of fusing them with several grams of sodium carbonate. In connection with the routine analysis of some china clays, fireclays and other aluminous silicates, the bureau has recently used a method for decomposing them which is not well known, but which apparently can be used quite advantageously for certain silicates.

The method consists essentially of intimately grinding together a weighed amount of the silicate with not more than an equal weight of sodium carbonate, transferring the mixture to a platinum crucible and heating it for about an hour over a Tirrill or Meker burner. Ten milliliters of water are added to the cooled crucible, which is then placed on a steam bath. After a few minutes ten milliliters of hydrochloric acid are added and the mixture is evaporated to dryness. Ten milliliters of concentrated hydrochloric acid are added, after

which the crucible is filled practically full of water, digested for a few minutes and the dehydrated silica filtered and washed. The combined R_2O_3 is precipitated and reprecipitated in the usual manner and the analysis completed in the ordinary way.

The amount of sodium carbonate required and the temperature of burning depend on the nature of the silicate. The most desirable condition is obtained when neither sufficient sodium carbonate nor heat is used to yield a glassy or vitrified mass in the crucible. For different materials the amount of soda ash may range from 0.2 to 0.5 grams and the temperature from that of a dull red to the highest temperature obtainable with a Meker burner. No recommendation can be made at present as to the amount of soda ash that should be used or the temperature required, but a few trials by chemists dealing with materials of a similar nature will indicate the most desirable combination.

The evidence on which the value of this procedure is based is that the non-volatile residue after treating the dehydrated silica with hydrofluoric and sulphuric acids generally does not exceed 1 mg. A decided advantage in this procedure is that the decomposition of the silicate and the dehydration of the silica can be carried out in the crucible in which the "sintering" was done. The greatest advantage, however, lies in the fact that the amount of sodium oxide to be washed out of the various precipitates may in some cases be reduced from 4.5 to 0.2 grams.

COEFFICIENT OF EXPANSION OF ENGLISH CHINA CLAYS.

DETERMINATIONS of the coefficient of linear expansion of 17 English china clays which had been fired to seven different temperatures, were made by the Fizeau-Pulfrich method, which has been in use at the bureau for the measurement of small dilatations for a number of years. Because of the fine grained character of the clays and the fact that there was no admixture present, this method was considered to be the best for the purpose. The clays had been fired to cones 3, 5, 7, 11, 14, 18 and 23.

The thermal expansion curves obtained are practically linear in character over the range investigated, from room

temperature to 600° C. An interesting fact brought out in the investigation is the peculiar behavior noted in the linear expansion curves for all the clays that had been fired to cones 3, 5, and 7. In each case a "kink" was shown in the curve in the approximate range of 120 to 280° C., where the slope of the curve decreased considerably. This was probably caused by the presence of some water in the sample, but in just what form it was present it is impossible to say without further investigation of the matter. In each case a second determination resulted in a linear thermal expansion curve over the range from room temperature to 600° C., from which the mean coefficient of linear expansion was calculated. The calculated mean coefficients showed a minimum value of $.287 \times 10^{-5}$ and a maximum of $.501 \times 10^{-5}$, with an average value of $.387 \times 10^{-5}$.

From an examination of the results obtained, it would appear that the average tendency was for a gradual increase in the value of the mean coefficient of linear expansion from cone 3 to cone 11, followed by a gradual decrease in the values obtained for the clays fired at the higher temperatures.

MEASUREMENT OF pH IN NICKEL PLATING SOLUTIONS.

A CONFERENCE was held recently at the bureau of those persons who have coöperated in a research on the methods of measuring pH in nickel plating solutions. It was agreed that the hydrogen electrode measurements should be made the basis of all pH reports. Various methods of coördinating the results of colorimetric and electrometric measurements were discussed. It was recommended that the full report of the investigation be incorporated in a paper to be prepared at the bureau and to be presented at the fall meeting of the American Electrochemical Society.

Electricity Due to Air-blown Particles. P. E. SHAW. (*Proc. Roy. Soc.*, A 789.) "Electric charges due to mutual impact of dust or other particles arise in various ways: (*a*) electric dust- or sand-storms in the Tropics; (*b*) electric snow storms in the Antarctic; (*c*) electric flashes seen in the ejectamenta from volcanoes; and (*d*) electric charges, and possibly sparking, brought about by the raising of organic powders in certain industrial processes." It is the impact of like particles that produces such electrical charges, though it has been customary to hold unlike substances as necessary for the production of electricity by shock. A few years ago the author showed "that *identical* solid surfaces can charge one another by friction or impact, and, further, that this property of the surface changes as rubbing continues. It was there shown that such organic insulators as ebonite and celluloid yield considerable charges, whereas hard inorganic materials such as quartz, calcite and glass, produce smaller effects. When the 'like' solids meet in violent impact, not rubbing, the combined net charge is not nil, as might be expected according to Faraday's law of equal and opposite frictional charges, but finite, generally negative; so that the air surrounding the surfaces must attain an equal positive charge."

In the series of experiments here described measurements were made with an electroscope on the potential developed by blowing with a speed of 54 miles per hour successive charges of particles, each .15 c.c. in volume, through a tube into a vessel, both tube and vessel being lined with the same substance. When both were lined with copper the charge developed on the vessel decreased progressively from + .04 to − .27 electrostatic units as the driven particles were successively Cu, Fe, Brass, Zn. When both were lined with zinc a similar decrease from + .98 to + .14 E.S.U. was noted as the particles passed through the same change from Cu to Zn. The electrical charges on the solid particles are usually considerably larger than those on the air, but this is not so when copper meets copper or zinc meets zinc. Sand on sand causes very large charges on the grains, as well as on the sand-lined container.

The development of electricity is enhanced by having all surfaces clean, by raising the temperature of the blast and, by increasing the velocity of the blast. Even when the two surfaces brought into sharp contact are chemically alike electrical charges are produced by impact. G. F. S.

NOTES FROM THE RESEARCH LABORATORY, EASTMAN KODAK COMPANY.*

A CONTRIBUTION TO THE PREPARATION OF STANDARD GELATIN.[1]

J. H. Hudson and S. E. Sheppard.

IT is recommended that the first standard gelatin prepared should be for scientific purposes; that is, it should be as near as possible a definite chemical compound, with physical properties equal to, or better than, the highest grade gelatin now obtainable. The question of its suitability for photographic or other industrial uses should not enter the question of its merits, as standards for a special purpose material can be prepared later.

Preferably the standard gelatin should be prepared from well-limed calfskin, and only the first extract taken. The finished product should contain not more than 0.05 per cent. ash and be at its isoelectric point of pH 4.7. More definite specifications for a standard gelatin should be decided upon only after several laboratories have completely tested a purified gelatin.

DYE TONING WITH SINGLE SOLUTIONS.[2]

J. I. Crabtree and C. E. Ives.

PREVIOUS experimenters employed single solution dye toning baths consisting essentially of a mixture of the mordanting bath and a basic dye, together with a protective agent; but since the basic dye invariably precipitated on standing, none of these formulas has been satisfactory.

A bath has been devised which contains no heavy metal

* Communicated by the Director.

[1] Communication No. 364 from the Kodak Research Laboratories and published in *Ind. Eng. Chem.* **21**: 263. 1929.

[2] Communication No. 372 from the Kodak Research Laboratories and published in *Amer. Phot.* **22**: 656. 1928; *Trans. Soc. Mot. Pict. Eng.* **12**: No. 36; *Brit. J. Phot.* **75**: 768. 1928.

but only potassium ferricyanide, acetic acid, a basic dye, and acetone. Acetone is a necessary solvent to maintain the dye in solution.

Nearly two hundred basic dyes were examined and ten were chosen as suitable for use in this bath. The optimum dye concentration varies with the dye used. A typical formula is the following:

Victoria green	0.4 gram
Acetone	100.0 c.c.
Potassium ferricyanide	1.0 gram
Acetic acid (glacial)	5.0 cc.
Water to make	1.0 liter.

The time of toning varies from five to fifteen minutes at 70° F., according to the depth of tone desired. The image does not intensify appreciably during toning so that it is possible to commence toning with images of normal photographic quality. This process of toning seems to have a less propensity to produce rack marks when toning by the rack and tank systems than the usual two-solution dye toning method.

Intermediate tones can be secured by using suitable mixtures of the dyes.

NOTES FROM THE U. S. BUREAU OF MINES.*

MICROSCOPIC STUDIES OF ORES RENDER REAL AID TO MINING INDUSTRY.

WHEN the microscopic study of ores was first undertaken at the Intermountain Experiment Station of the United States Bureau of Mines, Salt Lake City, Utah, the mining industry was, for the most part, totally unaware of the assistance that the microscopist could render the mill man in connection with the concentration of ores, the prevention of losses of values in mill tailings, etc. Gradually, however, as a result of the work that was done in the microscopic laboratory of the Intermountain Station, the value of such work was eventually realized by the industry. Last year one of the most progressive of the western mining companies installed a microscopic laboratory of its own. In doing so they first sent their ore-dressing engineer to the Intermountain Station so as to permit of his familiarizing himself with the Bureau's method of conducting its microscopic work. The Bureau was asked to make up a list of supplies and equipment that would be needed and after the laboratory had been installed the microscopist of the Intermountain Station assisted in putting it into operation. Now the mining companies of the district adjacent to Salt Lake City have arranged to install and maintain a service microscopic laboratory in the Department of Mining and Metallurgical Research of the University of Utah coöperating with the United States Bureau of Mines.

This laboratory will give its attention to those problems which are faced daily in plant operations and which involve intensive microscopic study of the various mill products including the ores which are treated. This work will be in the nature of fact-finding research and will serve as a basis for the application of theoretical consideration and study of problems whose importance is justified by the facts obtained. The scope of the work conducted in the laboratory will not be

* Published by permission of the Director, United States Bureau of Mines.

confined entirely to a study of ores and the problems encountered in milling, but will be extended to the field of geology. Microscopic study of rock formations and their relation to ore occurrence will form an important part of the laboratory work. There is also a field for microscopic study of smelter products, not only the slags which are produced under certain conditions, but of the materials entering the charge in the form of fluxes and ores.

Microscopic equipment of the latest and most refined character has been purchased and will offer facilities for using the most recent methods for conducting the various types of studies that will be made. Under this heading may be enumerated facilities for the study of extremely fine material, using dark field illumination. The various milling products examined will be subdivided into several minus 200-mesh products by means of elutriation, which has proven of great value in a study of ball mill practice now being conducted at the Intermountain Station. The study of sands or coarser material will be conducted through the agency of briquets prepared from acid-resisting substances and made under heat and hydraulic pressure.

LOW-FREEZING DYNAMITES.

THE thawing of frozen explosives was at one time a hazardous duty which fell to the lot of every user of explosives during the winter time. Many accidents occurred as a result of attempts to warm up the explosive before an open fire or on steam pipes or radiators. This hazard has almost been eliminated during the past 10 years by the development of low-freezing dynamites.

The physical characteristics of low-freezing dynamites have been the subject of a recent investigation by the Pittsburgh Experiment Station of the United States Bureau of Mines, Department of Commerce. Nitroglycerin, the explosive ingredient of dynamites, freezes easily, but the addition of nitropolyglycerin, ethylene glycol dinitrate, or certain nitro-substitution compounds to the nitroglycerin lowers the freezing point to such an extent that dynamites made with the mixture resist freezing under severe winter

conditions. The commercial use of ethylene glycol dinitrate was made possible recently by the development of manufacturing processes for the production of ethylene glycol on a large scale. The Bureau's investigation has shown that propulsive strength, rate of detonation, and sensitiveness are not adversely affected by the substitution. Investigation of the effect of the substitution on the safety of explosives of the permissible type has shown that the low-freezing explosives are equally safe for use in coal mines. The majority of permissible explosives are now of the low-freezing type.

TERMINOLOGY IN COAL RESEARCH.

AN effort to clarify the meaning of various terms used in studying the constitution of coal has been made by representatives of the United States Bureau of Mines, Department of Commerce, and the British Safety in Mines Research Board in connection with the coöperative work being done by the two organizations with the object of finding methods of preventing or lessening accidents in coal mines. Because some of the mine-safety researches relate to spontaneous fires in mines and to the explosibility of coal dust in air, it has been necessary to study how coal is constituted and the characteristics of different coals of the same and different periods of formation, a study which in turn leads to the question of how coal is formed and the origin of the different portions of the coal. Since many investigators in different lands have described the constitution of coal and have used different terms in doing so, it is necessary in order to properly discuss the matter when comparing the coals of the different countries to have a full understanding of what the different terms mean in the different countries.

In Technical Paper 446, recently published by the Bureau of Mines, the authors, Reinhardt Thiessen and Wilfred Francis, have given particular attention to the terms "vitrain," "clarain," "durain," and "fusain," recently coined in England and to the terms "anthraxylon," and "attritus," evolved contemporaneously in America.

The conclusion is drawn that the English term "vitrain" has the same significance as the American term "anthraxylon."

These terms apply to those portions of coals of bituminous rank which are made up of brilliant bands with conchoidal fractures. These bright bands consist of woody portions of trees which were turned into coal during the accumulation of vegetable matter in the watery swamps of earlier coal-forming ages of the earth. The word "anthraxylon" is from the Greek anthrax (coal) and xylon (wood).

The English term "clarain" is considered to have the same meaning as the American term "attritus." These terms are used to describe those portions of coal consisting of dull bands which had their origin in a miscellaneous assortment of vegetable debris such as small splinters of wood, cellular structures, spores and pollen grains, cuticles and bark. The thin bright streaks in clarain are bright streaks of vitrain (anthraxylon) which cannot be readily separated from the duller portions. Inherently laminated, it breaks sharply with a small surface at the break.

The English term "durain" refers to a dull-band granular coal which has no counterpart in American coals. It is difficult to break, and surfaces are always granular or matt. It is largely composed of opaque matter and spores in varying proportions. The English term "fusain" is applied to what has been called "mineral charcoal" or "Mother of coal" in America. This dark, fibrous and powdery component is present universally in coal in small quantities.

The terms "anthraxylon," "attritus," and "fusain" may be used irrespective of the rank or geologic age of the coal. The terms "vitrain," "clarain," and "durain" should only be used for coals of bituminous rank.

The older terms of the Germans—namely, Glanzkohle and Mattkohle—have been used in various ways by different authors. Glanzkohle may have been used for what is now termed "vitrain," for clarain, or for both. Mattkohle is probably more nearly related to durain, although it may also have been used for the darker portions of clarain.

It is considered that this report should enable those who study coals, whether from a mine-accident standpoint or a fuel standpoint, to compare and understand the literature of the several countries on the subject of the constitution of coals.

NEW METHOD FOR CALCULATING TERRANE CORRECTION FOR GRAVITY GRADIENT IN GEOPHYSICAL PROSPECTING.

A new graphical method for calculating the terrane correction for gravity gradient in geophysical prospecting is outlined in Technical Paper 444, by Donald C. Barton, just issued by the Department of Commerce, through the United States Bureau of Mines. The paper gives a graphical method that, states the author, is as accurate as possible under the conditions of ordinary field surveying with the torsion balance, that is rapid, simple, and elastic, and that allows the observer to visualize readily the extent to which rugged topography will affect the gradient at his station. This graphical method is not meant to supersede the Eötvös or older Schweydar methods of calculating the terrane correction for the gradient but to supplement them where the terrane around the station has a relief of more than one-half meter above the level of the base of the instrument or more than several meters below it. The method given does not take into account the distinction made by Eötvös, Heiland, and others between the terrane correction for the zone within 100 meters and the cartographic correction for the topography further than 100 meters from the instrument; it can be used with ease to obtain the gradient terrane correction for all the topography further than 5 meters (5 feet if used with the Lancaster-Jones Shaw terrane formulæ).

TREATMENT OF ORES CONTAINING LEAD VANADATE OR LEAD MOLYBDATE.

ORES containing vanadium or molybdenum combined with lead are a minor source of these metals. A large number of complex minerals have been identified in such ores, but for metallurgical purposes the rare metals may be considered to occur as vanadinite and wulfenite, the vanadate and molybdate of lead respectively.

During periods of unusual demand domestic vanadinite and wulfenite have been produced and a small part of the present production of vanadium continues to come from foreign vanadinite.

The known vanadinite and wulfenite deposits in this country have not been of sufficient grade or size to interest the

large rare metal producers, and it is difficult for a small producer to find a market for such ores because they require a metallurgical treatment which is different from that used on the prevailing types of ore.

Vanadinite, wulfenite and related minerals are usually associated with other oxidized lead minerals, chiefly the carbonate. In some cases such deposits are worked for lead and the rare metal content of the ore is invariably lost because the smelter does not recover or pay for such metals.

Producers of such mixed ores should investigate the possibility of separating and refining the vanadium or molybdenum into marketable form. Due to their higher specific gravity and more compact crystal form, vanadinite and wulfenite travel higher on a concentrating table than the other lead minerals. A combination of selective mining and concentration may give a product of sufficient grade to justify refining operations. The Rare and Precious Metals Experiment Station of the United States Bureau of Mines in coöperation with the University of Nevada, Reno, Nevada, has prepared several papers dealing with the treatment of such concentrates.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, APRIL 17, 1929.

The regular stated monthly meeting of The Franklin Institute was called to order by the President, Mr. Nathan Hayward, at eight-twenty-three P.M.

The Secretary announced that the minutes of the last monthly meeting had been published in full in the April number of the Journal and moved that the minutes be approved as printed. This motion was unanimously adopted and the minutes were declared approved.

The Secretary stated the only item of business to come before the meeting was a report upon the membership acquisitions since the last meeting. He stated that two Resident and two Non-Resident members had been elected.

He announced with extreme regret the death of Dr. Samuel Rea, the former President of the Pennsylvania Railroad System and an Honorary Member and Franklin Medalist of the Institute.

There was no further business and the President asked Dr. Hobart A. Hare, as the representative of the Board of Directors of City Trusts, to assume charge of the meeting. Dr. Hare, as the representative of that Board, presented to Dr. Lee deForest of New York City, a member of The Franklin Institute, a John Scott Medal and premium of one thousand dollars. Dr Hare spoke as follows:

I am here to-night as a member of the Board of Directors of City Trusts, a Board which has the administration, among other funds, of that fund established in 1816 by John Scott of Edinburgh, Scotland. Evidently from such information as we have, John Scott was a great admirer of Benjamin Franklin. He devised to the City of Philadelphia as Trustee, the sum of $4000, and directed that the interest thereon should be used to reward ingenious men and women that make useful inventions, "the premium to be not more than $20," and a medal to be known as the "John Scott Medal," to be made of bronze and to bear the inscription "To the most deserving."

In the course of time the John Scott Medal Fund increased so largely that under an order from the court, the Board of Directors of City Trusts is now permitted to increase each award to a much greater sum.

The Board of Directors of City Trusts feeling that it would be wise to have expert evidence as to those who might be worthy for such an award, appointed a Commission consisting of Mr. Samuel Vauclain, President of the Baldwin Locomotive Works, Mr. Solomon M. Swaab, City Engineer of Philadelphia, Mr. Francis Shunk Brown, the Vice-President of the Board of City Trusts, and the speaker, the second Vice-President, to make recommendations as to those to whom the John Scott Medal and Premium should be given.

On their recommendation, the Board of Directors of City Trusts have voted that an award should be made to Dr. Lee DeForest, Electrical Engineer of New York City, for his invention of the audion which is used in radio and long distance communication, the more so as he is one of the pioneers in the development of wireless telegraphy and telephony in America. Indeed it may be said that his development of the audion made possible long distance telephoning and the

audion is, furthermore, the detector amplifier of the present radio apparatus. The device which Dr. DeForest developed acts as a valve which controls the flow of electrons from the hot cathode or filament to the anode or plate. It is, furthermore, interesting to note that his invention proved the key to the synchronization which has rendered possible the talking motion picture.

In order that the Board of Directors of City Trusts might be enabled to honor Dr. DeForest in the presence of those who also hold him in high regard, The Franklin Institute has been graciously pleased to permit this award to be made at this meeting.

Representing the Board of Directors of City Trusts, the administrator of the John Scott Medal Fund, I, therefore, take pleasure in presenting the medal and premium to Dr. DeForest.

Dr. deForest responded and made graceful expression of his recognition of the honor which had been done him and of his gratitude for the same. He spoke briefly in outline of the past development of the radio as based upon the audion, and ventured certain predictions as to the likely development of the future.

The President of the Institute then reassumed the Chair and presented as the speaker of the evening Dr. W. F. G. Swann, Director of the Bartol Research Foundation of The Franklin Institute. Dr. Swann made a "Report of the Year's Activities of the Bartol Research Foundation." He spoke as fully as time permitted of the many and important investigations which are being carried on in the Bartol Laboratories, and gave a convincing summary of the researches which have been completed or are in process at this time.

The meeting adjourned at ten-twenty P.M.

HOWARD MCCLENAHAN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, April 3, 1929.)

HALL OF THE INSTITUTE,
PHILADELPHIA, April 3, 1929.

DOCTOR THOMAS D. COPE *in the Chair.*

The following report was presented for final action:

No. 2906: Literature.

This report, recommending the award of the Louis Edward Levy Medal to Mr. L. J. R. Holst, Consulting Engineer and Vice President, of Brock and Weymouth, Inc., Philadelphia, for his paper on "Topography from the Air," published in the October, 1929 issue of the Journal of The Franklin Institute, was adopted.

The following report was presented for first reading:

No. 2905: Work of Doctor Chevalier Jackson.

This report recommended the award of the Elliott Cresson Medal to Doctor Chevalier Jackson, of Philadelphia, Pennsylvania, "In consideration of his researches on the mechanical principles involved in the removal of foreign bodies from the respiratory and food passages and of his designing of many instruments

for the purpose, and for his development of a technique for the use of these instruments."

On motion duly seconded it was voted to pass to the final reading, on the completion of which the recommendation was approved.

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(*Stated Meeting, Board of Managers, April 10, 1929.*)

RESIDENT.

MR. HOWARD E. CORNELL, Naval Architect and Engineer, Public Ledger Building. For mailing: 1035 Public Ledger Building, Philadelphia, Pa.

MR EDWARD B. HALSEY, Radnor, Pa.

NON-RESIDENT.

DR. BERTRAM FEUER, Consulting Bacteriologist, 612 North Michigan Avenue, Chicago, Illinois.

MR. GEORGE N. SIEGER, Executive, Carboloy Company, Inc., 350 Madison Avenue, New York City.

CHANGES OF ADDRESS.

MR. WALTER H. DILKS, 8201 St. Martins Lane, Chestnut Hill, Philadelphia, Pa.

MR. MAURICE FELS, Garden Court, Forty-seventh and Pine Streets, C 510, Philadelphia, Pa.

MR. H. ARTHUR GROSSCUP, 315 Pearl Street, Hartford, Connecticut.

DR. JOSEPH S. HEPBURN, Hahnemann Medical College, 235 North Fifteenth Street, Philadelphia, Pa.

MR. NATHAN M. LOWER, Lower Stoker Company, 932 North Lincoln Avenue, N. S., Pittsburgh, Pa.

DR. WALTER A. MACNAIR, Bell Telephone Laboratories, Inc., 463 West Street, New York City.

MR. H. W. MAJOR, 602 Heartwell Building, Long Beach, California.

MR. WILLIAM CLARKE MASON, 2107 Fidelity–Philadelphia Trust Building, Philadelphia, Pa.

MR. CHARLES H. NORDHAUS, 3920 Cullom Avenue, Irving Park Station, Chicago, Illinois.

MR. FRANK H. SAUER, 129 Manor Avenue, Oaklyn, New Jersey.

MR. ARTHUR W. THOMPSON, Dunwoddie, West Chestnut Avenue, Chestnut Hill, Philadelphia, Pa.

MR. ROBERT L. WOOD, Bateman Brothers, Inc., Malvern, Pa.

NECROLOGY

Dr. Samuel Rea, Philadelphia, Pa.

LIBRARY NOTES.

RECENT ADDITIONS.

American Medical Association. New and Non-Official Remedies. 1928.

American Pharmaceutical Association. Year Book 1926. Volume 15. 1928.

BACON, ROGER. The Opus Majus of Roger Bacon: a Translation by Robert Belle Burke. Two volumes. 1928.

BRODETSKY, S. Sir Isaac Newton: a brief Account of His Life and Work. 1928.

CREIGHTON, H. JERMAIN. Principles and Applications of Electrochemistry. Volume I, Principles. Second edition, revised and enlarged. 1928.

MAILHE, A. Les Combustibles Liquides Artificiels. 1929.

MOYER, JAMES AMBROSE. Steam Turbines. Sixth edition, revised and enlarged. 1929.

National Advisory Committee for Aeronautics. Bibliography of Aeronautics, 1926 and 1927. Two volumes. 1928.

NEWBOLD, WILLIAM ROMAINE. The Cipher of Roger Bacon. Edited with foreword and notes by Roland G. Kent. 1928.

PAGE, LEIGH. Introduction to Theoretical Physics. 1928.

Philadelphia College of Pharmacy and Science. Popular Science Talks. Volumes 3–6. No date.

RAWLINS, F. I. G., AND A. M. TAYLOR. Infra-Red Analysis of Molecular Structure. 1929.

U. S. Patent Office. Index of Patents, 1928. 1929.

WIEN, W., AND F. HARMS, editors. Handbuch der Experimentalphysik, Band 20, 1 Teil: Physiologische Optik von A. König. 1929.

BOOK REVIEWS.

THE BIOCHEMISTRY OF THE AMINO ACIDS. By H. H. Mitchell, Professor of Animal Nutrition, College of Agriculture, University of Illinois, and T. S. Hamilton, Associate in Animal Nutrition, College of Agriculture, University of Illinois. 619 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1929. Price $9.50.

This is one of the many monographs prepared under the auspices of the American Chemical Society. This series has been of great value to chemists because while each volume is devoted to a special field and often to a somewhat narrow field, yet the series has been extended to cover a very large part of chemistry both pure and applied. "Biochemistry" is a comparatively new term, but an expansive one, and relating to a very important and interesting department of applied science. The particular field of this book shows the high specialization of the modern science for it relates to only one line of compounds. These compounds have an intimate relation with protein chemistry and of course for that reason with the higher problems of animal nutrition.

The field of animal biochemistry and the physiology of nutrition have, as the authors remark, experienced a very rapid expansion in the last score of

years and like most other expanding fields of science the rate of progress is itself accelerating. In such rapid development mistakes are apt to occur, results may be reported without due checking of interfering conditions, and indeed two careful investigators may by reason of slightly different technic get different results, and impurities of reagents may be overlooked. In a progressive science of this type much material becomes out of date often in a few years, and the author of a volume presenting the existing state of a science has to sift carefully the material that is offered. It is not possible for the author to verify all the statements that he records; he must use his judgment in many cases, and even the best judgment may go astray. The authors point out that the science of nutrition so rapidly expanding is suffering from the complacency of investigators, arising largely from the fact that definite information seems to be acquired comparatively easy.

This review of the present state of knowledge of the amino-acids and their function in animal life has been presented by the authors after careful consideration. They are not unaware of the danger of being too positive as to the data over a wide field, only a limited portion of which they have themselves traversed.

The book deals with principles and not to any appreciable extent with methods. It is therefore, a treatise on the subject rather than a laboratory guide. The bulk is limited to recent work, no attempt being made to deal in detail with the earlier story of the subject. This is quite correct, for the purpose of these monographs is to present present day knowledge in the field. The book exhibits the usual excellence of make-up that characterizes the publications of the Chemical Catalog Company and is a welcome addition to the important field of physiological chemistry, and gratifying as a product of work from the Middle West.

HENRY LEFFMANN.

GAS CHEMISTS' HANDBOOK. January 1, 1929. Revised by the Committee on Analysis, Tests and Editing Gas Chemists' Handbook. A. F. Kunberger, Chairman, W. H. Fulweiler, C. A. Lunn, N. F. Prince and C. C. Tutweiler. Third Edition, 795 pages, 131 figures, 8vo. Published by the American Gas Association, 420 Lexington Avenue, New York. Price $7.00.

The degree to which advancement and refinement can take place in testing and analytical technique is exemplified in the new edition of the Gas Chemists' handbook. This third edition which came from the press just recently has been thoroughly revised in many of its sections while others have been rewritten in their entirety.

The subject matter has been divided into four main parts or chapters under the headings Raw Materials, Products of Gas Manufacture, Impurities in Gas and Miscellaneous. An excellent feature, for which the members of the committee are to be commended highly, is the adoption for many materials the methods of analysis already accepted as standards by the American Society for Testing Materials. These methods in addition to those developed by the sub-committees all have met the approval of the Chemical Committee of the American Gas Association.

Chapter one on raw materials is divided into three parts—coal and coke, gas oil, and purification material. Methods of sampling the material are de-

scribed in detail. Similarly great care has been taken to include all the tests of practical value and to describe them with a clarity and directness which will leave no uncertainty in the mind of the worker. The greater portion of Chapter two, Products of Gas Manufacture, is given to the various methods of analyzing gases. The remainder of the chapter is taken up with Light Oils, Ammonia, Tar and Cyanogen. The principal methods of gas analysis are described with great care and the whole subject has been brought up to date by the writers, Yant and Fieldner of the Bureau of Mines. Special attention has been paid to the methods for determining small amounts of carbon monoxide, oxygen and olefines. This includes a description of the application of the thermal-conductivity method of gas analysis to the testing of gas appliances. Various types of apparatus for the determination of density, heating value and dew point are described and include descriptions of the operations of the Ranerex Density Recorder, the Thomas calorimeter and the Tutweiler hygrometer. Chapter three on Impurities in Gas describes hydrogen sulfide tests which afford quantitative works control and also has included the latest methods used in the determination of naphthalene in gas. A description of the Conductivity Bridge Method for naphthalene has been added. Under the heading, Miscellaneous, Chapter four, will be found methods for the analysis of water, paint and paint materials, lubricants; solder, bearing metals, brass and bronze; pipe deposits, refractories, cement, lime and ferrous metals. The methods of water analysis are those adopted by the American Public Health Association. The appendix consists of a series of tables which the gas chemist will find to be of immense value as an aid to calculations and a convenient source of accurate information. The table of constants for certain gases and vapors has been carefully revised and brought up to date by the chairman, A. F. Kunberger.

This recent issue of the Handbook is a marked improvement over the two previous editions. Apparently the committee has spared neither time nor trouble in order to give this book a high standard of quality evidenced by its accuracy and completeness of detail.

T. K. CLEVELAND.

INTRODUCTION TO THEORETICAL PHYSICS. By Leigh Page, Ph.D., Professor of Mathematical Physics in Yale University. x–587 pages, 201 figures. New York. D. Van Nostrand Company, 1928. Price $6.50.

During the decade just passed a relatively large number of books have been published on the subject of physics. Many of these books deal with the study of physics in a general way and possess a high standard of excellence in regard to thoroughness, clarity and a logical presentation of its subject matter. This "Introduction to Theoretical Physics" is no exception to the rule. The author, a professor of mathematical physics, has written accordingly. The various physical laws describing conditions of the universe are here expressed in the form of mathematical equations which have been developed step by step from differentials representing the experimentally observed conditions and facts.

The first part of the book contains an introduction on Vector Analysis. Sufficient attention and detail has been accorded the subject so that the reader may follow intelligently the application of vectors in subsequent chapters.

Part One of the book deals with the dynamics of particles, of rigid bodies, and of deformable bodies as well as containing a chapter covering the subject in its more advanced phases. In this section may be found a very comprehensive treatment of the motions of tops, gyroscopes, pendulums and falling bodies. Part Two on hydrodynamics is a treatment of the laws governing the motions of and in perfect and viscous fluids. Part Three on thermodynamics should be of particular interest to physical chemists and chemical engineers. The section contains a comprehensive exposition of the laws of classical thermodynamics and of the kinetic theory of gases. Part Four on electromagnetism is concerned chiefly with relations developed from the experimental laws of Coulomb, Ampère and Faraday. In the chapter on electromagnetic theory may be found sections upon radiation pressure, principle of relativity, and various effects related to spectra. Part Five on optics and spectroscopy deals with that section of physical science attracting most attention at the present time. A chapter on geometrical optics is followed by one on physical optics which deals with the properties of gratings and interferometers. The final chapter on the origin of spectra lays particular emphasis upon the mathematical treatment of the subject and is not given to the elaboration of the various possible corollaries pertaining to the more recent theories.

The work of compilation of such a book must have been indeed tedious. The unstinted use of mathematical equations should make the book immensely valuable to students of research and instructors in physics. The instructive value of the book has been greatly enriched by the introduction of practical problems involving the application of the principles elaborated.

T. K. CLEVELAND.

THE SPECTROSCOPY OF THE EXTREME ULTRA-VIOLET. By Theodore Lyman, Ph.D., Hollis Professor of Natural Philosophy, Emeritus, in Harvard University. Second edition, vii–160 pages; illustrations, 8vo, cloth. New York, Longmans, Green and Company, 1928. Price, $3.75.

Always a subject of prime importance in modern physical science, radiation has come to play a capital rôle in its relation to matter, and especially in recent times physicists have gathered an enormous amount of precise data on the visible, infra-red and ultra-violet parts of the spectrum. Research in the ultra-violet has been particularly prolific in results of value but only by overcoming unusual difficulties entailed in dealing with these short waves. Progress was slow in the advancement of knowledge of the ultra-violet region in the early days of spectroscopy with occasional rapid strides, notably by Becquerel and by Stokes and later by Mascart, followed by the classic research of Ångström and the brilliant invention by Rowland of the concave grating. During the period terminating with the work of Cornu the causes of limitation of the spectrum by absorption were made clear, and a little later Schumann succeeded in overcoming these difficulties and extended the known spectrum wave-length to 1200. It is in this latter region that Doctor Lyman's researches have been recorded, described and analyzed in this volume.

Says the author "the title of the book might well have been, 'Spectroscopy of the Schumann region' for it deals with that end of the ultra-violet spectrum where

the absorption of air plays the determining rôle. The present title is chosen however because a more extended field is necessary in order to trace the beginnings of these phenomena which becomes pronounced as the region beyond wave-length 2000 is explored. Thus the absorption of the air begins to manifest itself in the ordinary ultra-violet, though it only becomes critical near wave-length 2000, and the absorption of many solids, usually considered transparent in the ultra-violet begins to suffer a modification even before the region of shortest wave-lengths is reached. The book is, therefore, divided into two parts. The first and shorter portion deals with that part of the spectrum lying between wave-length 4000 and 2000; this is the region usually referred to as the ultra-violet. The second and longer portion treats the spectrum on the more refrangible side of wave-length 2000; this is the extreme ultra-violet or Schumann region."

Preceded by a brief historical survey, Part I includes data on photometry in the ultra-violet, on the absorption of optical systems, and on the absorption of gases. Part II deals with apparatus and methods of investigation in the extreme ultra-violet, specific data on absorption and reflection in the extreme ultra-violet, emission spectra of various substances, radiations from gases and solids in the extreme ultra-violet, photo-electric phenomena and the extension of the spectrum. At the end of the volume are the author's early measurements of the spectra of hydrogen and carbon monoxide and other measurements collected from various sources of the spectra of carbon, nitrogen, oxygen and aluminium. A copious bibliography alphabetically arranged by authors, a list of the papers of V. Schumann and a description of his methods of preparing dry plates conclude the volume. Subject and author indexes are provided. A second edition of this important work which now records the approach of the extreme ultra-violet region into the domain of soft X-rays with its many details of recent research and technique is a welcome addition to the literature of leading and original work on spectroscopy.

L. E. P.

THE INDUSTRIAL DEVELOPMENT OF SEARLES LAKE BRINES WITH EQUILIBRIUM DATA. By John E. Teeple, Ph.D., American Chemical Society Monograph Series. 182 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1929. Price, $3.

The outbreak of the World War, with its establishment of the strict British blockade, gave the United States a severe lesson as to the danger of implicit reliance on other lands for materials essential to its manifold activities. The reaction to the failure to secure the coal-tar dyes may have been partly sentimental, but the interruption of the potash supply struck at the heart of one of our great industries. There was an immediate search for sources of this important fertilizer. The price rose enormously, a condition that was in itself a strong incentive to research. Among the sources that were made available was the content of the brines of Searles Lake in California, the principal solid ingredients being salt, sodium carbonate, sodium sulphate, borax and potassium sulphate, the latter being closely associated with sodium sulphate. The lake, much larger in ancient times, now covers about twelve square miles, the several salts having been deposited in considerable quantity in its shallow basin and existing in strong solution in the water.

The present volume, one of the long series of monographs by the American Chemical Society, is devoted to the history of the utilization of the Searles Lake products, especially borax and potash. As might be expected, this industry, developed with rapidity and under severe stress, has some energetic features. Many difficulties were encountered, but in the early period of the operation, the high price of the product enabled satisfactory results to be obtained, and Searles Lake potash, with potash from several other hitherto overlooked sources, became important supplies to American agriculturists. With the signing of the Armistice, the foreign supplies were again available. Interested parties, especially representatives of the foreign firms, importers and their accessory agents were loud in their advocacy of the view that no effort should be made in America to establish a potash industry.

Dr. Teeple's book is not a descriptive technology, but a summary of a large amount of scientific data accumulated in the laboratory of the plants of the American Potash and Chemical Corporation, at Trona, utilizing the Searles Lake brines. Commendation is due to the Company for permitting the publication of these data. The physical chemistry of the problem is very thoroughly set forth. The work of supplying from American sources two important chemicals, potash salts and borax, in competition with world supply has been undertaken and after much struggle is successful although only recently established. The book also gives a succinct account of the application of research, technology and common sense to the problem. The motive has been not only to set forth scientific data of moment but to show the peculiar attitude that has been manifested towards the effort to establish an American industry as noted above. Few things are more aggravating than loud and dogmatic assertions that something cannot be done. We are reminded by this thought that Lord Kelvin and Simon Newcomb strongly deprecated Langley's efforts towards securing a heavier-than-air flying machine. The man who is explaining that a thing cannot be done, is often interrupted by somebody doing it. Dr. Teeple writes in a forceful style and his thorough familiarty with all phases of the work to which the book is devoted will place it in the front rank of contributions in a very important field of American industry.

HENRY LEFFMANN.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 298. Effect of Variation of Chord and Span of Ailerons on Rolling and Yawing Moments in Level Flight, by R. H. Heald and D. H. Strother. 19 pages, illustrations, quarto. Washington, Government Printing Office, 1929. Price ten cents.

This report presents the results of an investigation of the rolling and yawing moments due to ailerons of various chords and spans on two airfoils having the Clark Y and U. S. A. 27 wing sections. Some attention is devoted to a study of the effect of scale on rolling and yawing moments and to the effect of slightly rounding the wing tips.

The results apply to level flight with the wing chord set at an angle of attack of $+4°$ and to conditions of zero pitch, zero yaw, and zero roll of the airplane. It is planned later to extend the investigation to other attitudes for monoplane and biplane combinations.

The work was conducted in the 10-foot wind tunnel of the Bureau of Standards on models of 60-inch span and 10-inch chord.

Report No. 302. Full Scale Tests on a Thin Metal Propeller at Various Tip Speeds, by Fred W. Weick. 14 pages, illustrations, quarto. Washington, Government Printing Office, 1928. Price ten cents.

The investigation described in this report was made in order to determine the effect of tip speed on the characteristics of a thin-bladed metal propeller. The propeller was mounted on a VE-7 airplane with a 180-HP. E-2 engine, and tested in the 20-foot propeller research tunnel of the National Advisory Committee for Aeronautics. It was found that the effect of tip speed on the propulsive efficiency was negligible within the range of the tests, which was from 600 to 1000 ft. per sec. (about 0.5 to 0.9 the velocity of sound in air).

Report No. 305. The Gaseous Explosive Reaction. A Study of the Kinetics of Composite Fuels, by F. W. Stevens. 18 pages, illustrations, quarto. Washington, Government Printing Office, 1929. Price fifteen cents.

This report deals with the results of a series of studies of the kinetics of gaseous explosive reactions where the fuel under observation, instead of being a simple gas, is a known mixture of simple gases. In the practical application of the gaseous explosive reaction as a source of power in the gas engine, the fuels employed are composite, with characteristics that are apt to be due to the characteristics of their components and hence may be somewhat complex. The simplest problem that could be proposed in an investigation either of the thermodynamics or kinetics of the gaseous explosive reaction of a composite fuel would seem to be a separate study of the reaction characteristics of each component of the fuel and then a study of the reaction characteristics of the various known mixtures of those components forming composite fuels more and more complex. This is the order followed in the simple studies herein described. R.

PUBLICATIONS RECEIVED.

Gas Chemists' Handbook, January 1, 1929. Revised by the Committee on Analysis; Tests and Editing Gas Chemists' Handbook. Third edition, 795 pages, illustrations, 8vo. New York, American Gas Association, no date, price $7.00.

Theoretical Mechanics. An introduction to mathematical physics by Joseph Sweetman Ames and Francis D. Murnaghan. 462 pages, illustrations, 8vo. Boston, Ginn and Company, 1929, price $5.00.

Encyclopédie Léaute. Les automobiles sans Pétrole. Les Combustibles liquid artificiels par A. Mailhe. 281 pages, 12mo. Paris, Gauthier-Villars et Cie, 1929, price 30 francs.

Infra-red Analysis of Molecular Structure, by F. I. G. Rawlins, M.Sc., and A. M. Taylor, M.A. 176 pages, illustrations, 8vo. Cambridge, University Press, New York, The Macmillan Company, 1929.

Ontario Department of Mines. Thirty-seventh annual report, being volume 37, part II, 1928. Kirkland Lake Gold Area (a detailed study of the Central Zone and vicinity) by E. W. Todd. Text, maps, plates, 2 vols., 8vo. Toronto, King's printer, 1928.

Operational Circuit Analysis, by Vannevar Bush, Eng.D., with an appendix by Norbert Wiener, Ph.D. 392 pages, illustrations, 12mo. New York, John Wiley & Sons, Inc., 1929, price $4.50.

The Biochemistry of the Amino Acids, by H. H. Mitchell and T. S. Hamilton. American Chemical Society Monograph Series. 619 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1929, price $9.50.

National Advisory Committee for Aeronautics. Technical Notes No. 304. Corrosion embrittlement of duralumin. V. Results of weather exposure tests by Henry S. Rawdon, Bureau of Standards. 26 pages, illustrations, quarto. Washington, Committee, 1929.

U. S. Bureau of Standards. Research Paper No. 45. Apparatus and methods for the separation, identification, and determination of the chemical constituents of petroleum by Edward W. Washburn, Johannes H. Bruun and Mildred M. Hicks. 22 pages, illustrations, 8vo. Washington, Government Printing Office, 1929, price ten cents.

U. S. Bureau of Standards. Recombination spectra of ions and electrons in cæsium and helium by F. L. Mohler and C. Boeckner. 12 pages, illustrations, 8vo. Washington, Government Printing Office, 1929, price five cents.

CURRENT TOPICS

Erosion by Water-hammer. S. S. Cook. (*Proc. Roy. Soc.*, A 783.) The matters discussed in this paper are connected with the pitting of condenser tubes and the erosion of propeller blades and of steam turbine blades. It is not always easy to distinguish erosion from corrosion. Erosion may furnish conditions favorable to corrosion by removing a surface layer or protecting scale or by carrying away the products of corrosion. "In many cases the reason for attributing the action to corrosion rather than to erosion is merely that the mechanical nature of the cause is not sufficiently apparent, through a failure to recognise the remarkable ability of water and other nearly incompressible fluids under certain circumstances, to produce momentary pressures far in excess of the yield-point strength of ordinary materials."

When a spherical cavity in an incompressible liquid suddenly collapses very high pressures are caused. Such a case may arise when a stream of water flowing into a tube is suddenly shut off. The momentum of the liquid would tend to keep it moving but with the formation of cavities that would collapse a moment later. The author calculates the pressure developed in a suppositious case and finds it to be 65 tons per sq. in. What takes place in the water-hammer is this. "If a moving column of water is suddenly arrested by a fixed surface, without any interposing gas to cushion the blow, there is a sudden arrest of the front layer of the impinging column. The pressure generated by the blow in such a case is only limited by the fact that both solids and fluids possess a slight degree of compressibility, so that the velocity energy of the front layer of the column is converted on impact into potential energy. It should be noted, too, that the water-hammer pressure induced by a suddenly arrested column of water is independent of the length of the column. It follows that drops of water, however minute, will at the first moment of impact produce the same water-hammer effect as large volumes of water, the only difference being in the area of attack and the duration. The duration will depend upon the length of the column, being the time required for the shock wave to travel to the end of the column and back again to the surface of impact. For example, if the column is 1 meter in length the pressure endures for about 1/700 part of a sec." In general the pressure due to the sudden stopping of water is much greater than that caused by the impact of a jet of the same liquid moving with the same velocity. It is only when the speed of the jet attains the velocity of sound in the liquid that the pressures become equal.

The author calculates the pressure due to impact in a steam turbine between the blades and drops of water in a high vacuum. On blades moving with a speed of 120 meters per sec. it amounts to 12 tons per sq. in., and for 300 m./sec. it is 30 tons per sq. in. "so that it will be seen to be of the same order as the yield-point of materials usually employed." Turbine blades 4 inches long were mounted on a shaft and rotated within a cylindrical casing. At two diametrically opposite places jets of water were sprayed across the moving blades. The blades were rotated for 18½ hours at 8,800 revolutions per min. The speed of the impact of the water was 140 meters per sec. at the part of the blades nearest to the axis and 233 m./sec. at the outermost part. The respective pressures were 12.4 and 20.7 tons per sq. in. The results were as follows:

Material of Blade.	Elastic Limit Tons/Sq. in.	Condition After Test.
"Hecla A.T.V." steel	51	Unattacked
Tungsten steel	40	Rusted and slightly eroded
"Staybrite" steel	16	Considerably eroded
Monel metal	18	Considerably eroded
Mild steel	16	Etched as by sand blast

G. F. S.

Experiments on the Diffraction of Cathode Rays. G. P. THOMSON. **The Diffraction of Cathode Rays by Thin Celluloid Films.** A. REID. **The Diffraction of Cathode Rays by Thin Films of Copper, Silver and Tin.** R. IRONSIDE. (*Proc. Roy. Soc.*, A 783.) According to de Broglie's theory of mechanics a particle in motion acts as a group of waves whose velocity and wave-length depend upon the mass and velocity of the particle. If the particles are electrons that have the velocity acquired by falling through a difference of potential ranging from 10,000 to 40,000 volts the corresponding wave-lengths would vary from 1.22×10^{-9} to $.66 \times 10^{-9}$ cms. Hard X-rays have such wave-lengths. Hence it was a fair supposition that crystals would accord the same treatment to both X-rays and to electrons. On page 417 of the September 1928 issue of this JOURNAL is published an account of experiments made by G. P. Thomson, son of Sir J. J. Thomson and Professor of Natural Philosophy in the University of Aberdeen, showing that the crystals in thin films of metal and of celluloid actually follow the predicted course. This group of three papers report further work done upon the same subject in the Aberdonian laboratory.

Prof. Thomson states that improved methods have produced better agreement between the quantitative results derived from

X-ray measurements and those based on the diffraction of electrons. After considerable difficulty he succeeded in extending his method to platinum films with which he obtained diffraction effects in agreement with theory. A beam of electrons fell at right angles upon the film and after passing through was received on a photographic plate. Upon development this showed a central image surrounded by rings due to diffraction.

An approximately uniform magnetic field was applied to the electrons issuing from the film. Both the central image and the diffraction pattern were deflected through 3 cm., this distance being dependant upon the electron velocity. Measurements show that there is so little difference between the distances through which the central and the diffracted rays are displaced that "the speed of the diffracted rays is certainly the same as that of the main beam to less than 1 per cent." Since a magnetic field would not deflect light waves the fact that deflection takes place is evidence, were evidence needed, that the diffraction pattern is not produced by light. "The question of the number of waves accompanying an electron is of great theoretical importance. The large resolving power seems definitely to show that the waves extend with appreciable amplitude over a distance at least of the order 50×10^{-8}, which makes the wave system of the free electron larger than an atom. The only way of reconciling the experiments with an electron, the amplitude of whose waves is concentrated in a small region, seems to be to suppose that the different electrons in the beam influence each other, so as to produce a coherent wave."

From the fact that the speed of electron waves increases with the wave-length an interesting state of affairs results. Even in a vacuum the waves do not stay together but the longest waves outrun the rest. Unless only waves of one wave-length are present the train will grow longer as the distance separating the long from the short waves becomes progressively greater.

Reid working with films of celluloid obtained results that confirm previous investigations of this material. To get thin films he dissolved celluloid in amyl acetate and placed a little of the solution upon the surface of water. When the solvent has disappeared a film of celluloid about 500 Å. U. thick remains. Ironside had little difficulty in getting silver films sufficiently thin. Copper presented more of a problem. The thinnest possible leaf was dissolved in part by floating it on nitric acid. It was impossible to get tin foil as thin as was desired for the experiment, so that the results were rather poor. The length of the lattice constants of these three metals determined by electron diffraction diverged only one or two per cent. from that derived from X-ray measurements. G. F. S.

Geophysical Methods of Prospecting, with Special Reference to Magnetic, Radioactive, and Electric Methods. C. A. HEILAND. (*Ter. Mag.*, Sept., 1928.) This is an abstract of a paper presented to the American Geophysical Union, April, 1928. It contains a valuable tabulation of the various geophysical methods of prospecting. Two general groups are recognized. "The first group comprises methods (for example, gravitational) by which direct effects of subterranean inhomogeneities are observed. The second group consists of procedures (for example, seismic, magnetic radioactive, and electric) whereby artificial fields of force are produced and their distortion by subterranean bodies of different physical properties is measured." In the first group we find gravitational methods, where the pendulum is used to detect large structural features and where the torsion balance, giving gravity gradients and curvature values, is applied to locating ore deposits, salt domes and the relief of buried terrain; natural earth-currents used especially for sulphide ores; thermic action, where the thermal gradient is measured. This method is used for mineral-oil structures and ores in wells and shafts; magnetic and radioactive methods, both of which will be discussed later in this abstract. In the second group are collected methods depending on reaction to elastic waves whether of acoustic or seismic nature, applied to the location of ores; on reaction to wireless waves, used for sulphide ores and brine accumulations; on reaction to electric currents discussed below.

The Magnetic Method.—This, perhaps the oldest geophysical method, depends on different minerals and formations possessing different magnetic permeabilities. It serves to locate deposits of iron ore and is valuable in oil geology, if the oilbearing structures go with igneous rocks, for these always contain magnetite. Gold and platinum can be located in placer deposits since they are accompanied by magnetite. Even non-magnetic deposits can be detected provided they are associated with magnetic formations or are as strongly diamagnetic as salt-domes. For coarse work for ore bodies the dip needle and the magnetometer are sufficient. For the search for oil vertical and horizontal field balances are chiefly relied upon. The theory of the effect of subterranean bodies on magnetic quantities has been investigated by at least fourteen writers.

Radioactive Methods.—Samples of earth are taken at intervals and the radioactivity is determined or the radioactivity of air accumulating in holes made in the earth some time previously is measured or measurements are made with an ionization chamber without bottom placed directly on the ground and moved from spot to spot or the radiation from the soil is measured at intervals with a shielded chamber or, finally and most commonly, an ionization

chamber mounted on a tripod is moved from place to place, air from a hole in the surface of the ground is pumped into it and its radioactivity is measured.

This general method lends itself to the detection not alone of radioactive minerals but of faults and dykes as well. Faults show enhanced radioactivity probably because they facilitate the circulation of radioactive air and water, and their location is desirable when dams are projected. In Europe this method has succeeded in finding faults and dykes.

Electric Methods.—For their application these depend on the formations sought having a different electric conductivity from their surroundings. Ore bodies are good conductors especially sulphides. Electric prospecting for ore has proved to be quite successful in Canada, Upper Michigan, Newfoundland, Arizona and New Mexico. It is, however, applied to oil geology also. Salt domes that accompany oil are insulating in character and besides this conclusions as to general structure can be drawn from a study of the distribution of electrical conductivity. The method is not as useful for oil as for ore. There is a great variety of these methods and the literature is considerable. G. F. S.

Ozone in the Northern and Southern Hemispheres. F. E. FOWLE. (*Ter. Mag.*, Sept., 1928.) It is easier to measure the radiation reaching the earth from the sun when the localities of the observations are in the southern hemisphere (Calama and Montezuma, Chile) than when they are in the northern hemisphere (Mt. Wilson, Harqua Hala, Table Mountain). The question is raised whether this may not be due to a difference between the amount of ozone "which spoils the apparently excellent skies of such a station as Table Mountain, California." Ozone is formed from oxygen by the action of a range of short wave-lengths in the solar radiation lying in the ultraviolet. It is decomposed by the action of group of longer wave-lengths likewise in the same region of the spectrum. "Hence, a layer of ozone might exist from this cause high up in the atmosphere where the former effect preponderated. Lower down as the energy of the shorter wave-lengths of the sun's energy became more rapidly weakened than the longer, a place would be reached where the decomposition effect would exceed the generation, and no ozone would be found." From a study, made in the laboratory, of the absorption of solar radiation by ozone one can calculate the amount of this substance needed to produce the absorption due to it that is observed in the solar spectrum, but only about one-twentieth of this amount is found in the air at sea level or at the top of Mt. Blanc. This was confirmed by a study of the trans-

mission through air near the earth of a certain mercury line. It was transmitted through 6.5 kilometers of air. This would have been quite impossible had all the ozone, indicated as present in the entire atmosphere, been equally distributed throughout the whole mass of the air. "The ozone causing the absorption in the solar spectrum should therefore be found at a considerable altitude, and Cabannes and Dufay have estimated this altitude at about 50 kilometers."

From a daily study of an absorption band in the solar spectrum that is caused by ozone the author has derived the absorption by this gas in the atmosphere. This was done for the period from Dec., 1925, to Dec., 1927, and for Table Mountain, Cal., and Montezuma, Chile. "There is a comparatively small and apparently constant amount of ozone above the Chilean solar station. Above Table Mountain it exists in an amount capable of exercising a considerable disturbing effect upon the incoming solar radiation. There is above Table Mountain a distinct seasonal variation for the two years shown, the absorption being greatest in the early months of the year, reaching a minimum along the early part of November for the two years stated. The amount of ozone above Table Mountain shows some relationship between the magnetic character of the day, the total spottedness (of the sun), and a very decided relationship with the number of flocculi over the whole disc."

Attention is directed to the importance of ozone for three branches of science: (1) To meteorology because of its effect upon solar radiation by absorbing this and thus keeping it from reaching the surface of the earth and because of its acting as a blanket to prevent the escape of infra-red radiation coming from the earth itself; (2) To astrophysics, and (3) To terrestrial magnetism and atmospheric electricity, as the ozone in the upper regions of the atmosphere is equal to a conducting layer there that moves about the magnetic earth with the motion of the sun. G. F. S.

Production and Properties of High-frequency Radiation. SIR E. RUTHERFORD. (*Nature*, Dec. 8, 1928.) The expression 'a quantum of radiation' has grown to be quite familiar to readers of modern physics. It indicates a definite amount of radiation-energy that is emitted and absorbed as an entity. It is the modern counterpart of Newton's corpuscle of light. Radiations of different wave-lengths have different quanta. The shorter the wave-length the larger the energy of the quantum, for the energy of a quantum is the product of Planck's constant h and the frequency of the particular radiation. "It is very convenient to express the energy of a quantum not in ergs but in terms of a potential difference in volts, through which an electron must fall to acquire an equal

energy. Expressed in this way, the energy of a quantum of green light corresponds to 2 electron-volts, or 2 volts for brevity. Before the advent of X-rays the highest frequencies examined were confined to the ultraviolet part of the light spectrum, corresponding to less than 10 volts. Following the discovery of X-rays and the application of methods for determining their frequency, we have been enabled to study radiations over a wide range of individual energy, varying from a few hundred volts to 300,000 volts or more." When electrons impinge upon matter X-ray radiation of many frequencies is produced. The faster the electrons travel the greater is the maximum frequency in the resulting group of radiations produced. It seems that there is no limit to the frequency attainable except that set by our ability to impart velocity to the bombarding electrons. "In some recent experiments in the Institute of Technology, Pasadena, about a million volts have been successfully applied for a short time to a suitably designed X-ray tube. It is stated that the X-rays obtained were of such intense and penetrating power that they could easily be observed by the luminosity on a phosphorescent screen 100 feet away." So far the laboratory cannot yield radiation with the penetrating power of the gamma rays spontaneously emitted in radioactive transformations. The energy of their quanta corresponds to 3 or 4 million volts, and the rays coming from weakly radioactive potassium seem to have even greater quanta.

From the equivalence of mass and energy it is possible to calculate the frequency of the energy emitted in some simple atomic changes, but experimental verification is difficult because of the enormous amount of energy associated with a gram of matter. "The atom of uranium of mass about 238, after successive transformations involving the loss of 8 α-particles, changes into an isotope of lead of mass about 206. It is to be anticipated that, if the methods of positive ray analysis could be applied to these elements, the difference between the atomic masses of the uranium and the resulting lead would include not only the mass of 8 helium nuclei in the free state, but also about 0.05 unit of atomic mass, corresponding to the total emission of energy of about 46 million electron-volts per disintegrating atom of uranium. This difference—about 1 in 4000—should be just detectable by the methods employed by Aston in his study of isotopes." The building up of a helium atom from four hydrogen atoms produces a structure, the helium atom, that has less mass than its constituent parts had. "This indicates a loss of energy of 27 million electron-volts in the process of building a helium nucleus from free protons and electrons. The energy emitted per atom is thus very large, and it has been suggested by

Eddington and others that the formation of helium from hydrogen nuclei may be one of the sources of the energy radiated from the stars." Jeans, on the other hand, suggests that protons and electrons both may turn into radiation and thus account for the age-long shining of the stars. If the energy emitted by simultaneous annihilation of a proton and an electron should be given off in a single quantum its energy would be about 940,000,000 volts. Do the cosmic rays of Kolhoerster and Millikan meet the requirements for such radiation? There is some reason to believe that they do but the evidence is by no means conclusive. "The study of these extraordinarily penetrating radiations is not only of great interest in itself, but also for its promise of throwing new light on fundamental processes in our universe connected with the building up and destruction of atoms." G. F. S.

Isaac Newton. ALBERT EINSTEIN. (From the *Manchester Guardian*, March 19, 1927; reprinted in the Annual Report of the Smithsonian Institution for 1927, published in 1928.) This paper appeared a short time before the two-hundredth anniversary of the death of Sir Isaac Newton, "this shining spirit, who pointed out, as none before or after him did, the path of Western thought and research and practical construction. He was not only an inventor of genius in respect of particular guiding methods; he also showed a unique mastery of the empirical material known in his time, and he was marvellously inventive in special mathematical and physical demonstrations. For all these reasons he deserves our deep veneration. He is, however, a yet more significant figure than his own mastery makes him, since he was placed by fate at a turning point in the world's intellectual development. This is brought home vividly to us when we recall that before Newton there was no comprehensive system of physical causality which could in any way render the deeper characters of the world of concrete experience." The Greek materialists had referred all the phenomena of matter to the motions of atoms governed by law; Descartes had revived this conception but it was largely without foundation before Newton. Knowing Kepler's empirical laws of planetary motion Newton set himself to solve the question "Does there exist a simple rule by which the motion of the heavenly bodies of our planetary system can be completely calculated, if the state of motion of all these bodies at a single moment is known?" The affirmative answer and the solution are found in Newton's laws of motion and of attraction. The impressive success of Newton's method in explaining the complicated motions of the solar system led to its dominating physical research until the close of the nineteenth century. Electrical and

magnetic phenomena were attributed to imponderable substances acting at a distance. Newton tried to explain light effects by inert corpuscles. The kinetic theory of matter employed the laws of motion. Even when Faraday and Maxwell's new conceptions in optics and electro-dynamics came in, Kelvin, Boltzmann and Maxwell tried again and again to clothe the new ideas in the garb of mechanical processes. "Newton himself was better aware of the weak sides of his thought-structure than the succeeding generations of students. This fact has always excited my reverent admiration." Three weak points in Newton's system are indicated by Einstein.

1. His use of the conceptions of absolute space and absolute time.
2. The assumption, in contradiction to our usual experiences, of direct, instantaneously acting forces at a distance.
3. "Newton's theory offered no explanation of the very remarkable fact that the weight and inertia of a body are determined by the same magnitude (the mass). The remarkable nature of this fact struck Newton also."

The first serious blow to Newton's theory came when, in accord with Maxwell's theory it was shown that magnetic and electrical attractions and repulsions are not transmitted instantaneously but travel with a finite speed through space. As a competitor to Newton's point-mass and its movements there arose Faraday's conception of a field as a real physical entity. "We owe to H. Hertz the deliberate liberation of the conception of the field from all the scaffolding of the conceptions of mechanics, and to H. A. Lorentz the liberation of the conception of the field from a material bearer."

The Maxwell-Lorentz theory led to the special theory of relativity. According to this mass is not unchangeable in quantity but is identical with energy, and Newton's laws of motion hold only for small velocities and are comprised in a broader law wherein the velocity of light in a vacuum appears as the highest possible velocity. Then followed the general theory of relativity. "Inertia, gravitation and the metrical behavior of bodies and clocks were reduced to the single quality of a field, and this field in turn was made dependent on the bodies. Space and time were so divested, not of their reality, but of their causal absoluteness, which Newton was compelled to attribute to them in order to be able to give expression to the laws then known."

"The whole development of our ideas concerning natural phenomena, which has been described above, may be conceived as an organic development of Newton's thought,—but while the construction of the theory of fields was still actively in progress, the facts of heat radiation, spectra, radioactivity, and so on, revealed a

limit to the employment of the whole system of thought, which, in spite of gigantic successes in detail, seems to us to-day completely insurmountable. Many physicists maintain, not without weighty arguments, that in face of these facts not only the differential law but the law of causality itself—hitherto the ultimate basic postulate of all natural science—fails." G. F. S.

Cosmic Radiation and Radioactive Disintegration. L. R. MAXWELL and W. F. G. SWANN. Bartol Research Foundation of The Franklin Institute. (*Nature*, Dec. 29, 1928.) Perrin has suggested that the absorption of cosmic radiation is the occasion of the transformations of radioactive substances. To test this hypothesis Dr. Maxwell investigated the activity of polonium at the bottom of the New Jersey Zinc Co.'s mine at Franklin. It was thought that here, 1,150 ft. below the surface of the earth, there might be detected a change in the radioactivity of polonium due to the decrease in the intensity of cosmic radiation caused by its absorption by the superincumbent layer of earth. Within the limits of the experiment, however, polonium displayed the same radioactivity at the surface as at the bottom of the mine. This appears to negative Perrin's theory. Swann, however, basing his argument upon the absorption coefficient of the cosmic rays got by Millikan and Gale and upon Dirac's formula, says: "It is therefore clear that we should only expect a cosmic ray to be absorbed by one of these (polonium) atoms once in about 10^{15} secs., i.e., once in twenty million years. This consideration would appear to render very improbable a direct effect of the cosmic radiation even in the most favorable case." G. F. S.

Bakerian Lecture.—The Aurora and Its Spectrum. J. C. MCLENNAN. (*Proc. Roy. Soc.*, A 785.) Whether in the northern hemisphere, the Aurora Borealis, or in the southern hemisphere, the Aurora Australis, the aurora occurs most frequently in zones that make angles of approximately 20° with the earth's magnetic axis. Magnetic storms generally accompany the aurora and both are attributed to streams of electrons coming from the sun. Such streams, according to Birkeland, would, upon entering the terrestrial magnetic field, follow paths having the shape of the streamers of the aurora. In form and intensity the aurora varies widely. "At times it is seen as a weak, diffuse, formless luminosity; at other times it appears as luminous draperies, bands or arcs, and often only as luminous beams or rays." Sometimes the figures of the aurora are stationary while at other times they move. The brightness may pulsate. Greenish-yellow predominates as the color of

faint, diffuse auroras as well as for those of curtain or arc form with rays and of small elevation. Violet, blue, red and other colors are, however, seen particularly with bright and shifting manifestations. In auroras high above the earth red-violet has been observed and curtains of violet-grey. The spectra of these displays are strong in the short wave-lengths. Størmer since 1907 has made many investigations of the elevation of the aurora. His method depends on photographing the same aurora simultaneously from two stations separated by distances ranging from 4.5 to 258 km. Now only one second is required for the exposure. The stars, photographed at the same time as the aurora, serve as points of reference for the determination of height. Størmer has taken hundreds of photographs of auroras. "His results show that very few displays occur below 80 km., and that above this level the number increases rapidly with the most frequent occurrences, beginning at a height of 106 km. Above this height the number of auroras falls off rapidly. Some, however, have been observed as high as 500 km. and 750 km. One reached even to 1,000 km. The auroras seen at Oslo in general extend higher than those viewed at Bossekop near Finland.

Ångstrom, after whom the unit of wave-length, the Å.U., is named, in 1869 was the first to examine the spectrum of the aurora. He saw a green line and faint lines in the blue. Since that time many have followed him of whom Vegard is the most notable. He has observed 35 or 40 lines or bands. On the other hand it has been known since 1901 that the night sky glows with a faint green light. This holds true for all latitudes, for all parts of the sky, for moonless and cloudless nights and also for moonlit and somewhat cloudy nights. This "non-polar" aurora occurs without dependence on polar auroras or magnetic storms. In the spectrum of the night sky the chief feature is a line in the yellow-green of wave-length 5,577 Å.U., also appearing as a strong line in the auroral spectrum. The polar and the non-polar aurora do not of necessity have the same origin.

Most of the spectrum of the polar aurora is due to the band spectrum of nitrøgen. In the auroral spectrum one prominent feature is a group of four bands ranging toward the violet end from 5,227 Å.U. onward. These belong to the first negative band spectrum of gaseous nitrogen. The auroral spectrum contains also three additional but fainter bands of the same system. Further there are about sixteen faint bands in the spectrum of the aurora. These are now identified with members of the second positive band system of nitrogen. There are two bands in the auroral spectrum that are awaiting identification, but one of them seems likely to belong to the band spectrum of nitrogen. In addition the spectrum

has ten wave-lengths ranging from 3,000 to 5,000 Å.U. that are not in the spectrum of nitrogen nor in that of atomic oxygen with perhaps one exception.

The famous green line, 5,577 Å.U., long defied identification. At last in 1925 McLennan and Shrum showed it to be emitted by atomic oxygen. A study was made of the intensity of this line in the spectrum of the night sky. It was found that after sunset the intensity of the green light from the zenith increased until a maximum was attained about one and a half hours after midnight. From this time onwards until sunrise there was a decrease.

G. F. S.

The Velocity of Electrical Waves on Thin Wires of Different Specific Conductivities. L. BERGMANN and G. HOLZLOEHNER. (*Annalen der Physik*, No. 21, 1928.) On the basis of the mathematical investigations of Sommerfeld, Mie and Mercier a formula is derived for the difference between the wave-length of an electrical oscillation in air and the wave-length of the same vibration on a wire. The wave-length in air is the greater by a quantity that is greater as the wave-length increases. For a wire of zero resistance the difference disappears, that is, the velocity along the wire equals the velocity of light. According to the formula the difference in velocities in air and along the wire is inversely proportional to the square root of the specific conductivity and roughly inversely proportional to the diameter. The frequency has the main effect upon this difference.

The measurements were carried out with two frequencies corresponding to wave-lengths of 400 and 172 cm. respectively. Two Lecher systems were employed, the wave-lengths obtained on one system with the different thin wires being compared with the wave-lengths got with a copper wire 1.5 mm. in diameter on the other system with the same oscillation. The wave-length in air is not measured but in its stead is substituted the wave-length on the thick copper wire. The error made thereby lies well within the range of error in the experiment. Measurements were made on wires of copper, aluminium, brass, German silver, manganin and constantin, the diameters varying from .04 to .3 mm. With the higher frequency (wave-length of 400 cm.) the difference between the wave-length in air and the wave-length on the wire is in general rather more than three times the difference for the lower frequency (wave-length of 172 cm.). This holds true for all materials investigated and for all diameters. For example with copper .04 mm. in diameter the difference for high frequency is 4.50 cm. and for low frequency 1.33 cm. The thicker the wire the smaller the

difference as should be the case according to the theoretical formula. For copper wires of diameters .04, .06, .10, .15, .20 and .25 mm. the differences for the higher frequencies were 4.50, 3.20, 1.90, 1.50, 1.05 and .90 cm. respectively.

Constantin has a much lower specific conductivity than copper. Hence it is not surprising to note for the same frequency and the same diameter that constantin shows a difference of velocity of 6.50 and copper only 1.05 cm. When the calculated differences are compared with those observed it appears that the agreement is quite good, especially for wires of larger diameters and high specific conductivity.

With the higher frequency, 7.5×10^7 cycles per sec. (wave-length of 400 cm.) the smallest difference between the velocity in air and the velocity on the wire was observed to be .9 cm. This means that the electric wave in air outruns the wave along the wire at the rate of 6,750 km. per sec. G. F. S.

Absolute X-Ray Wave-Length Measurements. A. P. R. WADLUND. (*Phys. Rev.*, Dec., 1928.) Hitherto most of the measurements of the wave-length of X-rays have been made by means of reflection from a crystal. Uncertainty in the exact value of the crystal grating space introduced a corresponding uncertainty into the calculated wave-length. Mr. Wadlund, working in the Ryerson Physical Laboratory, has sought with success to obtain accurate values of the wave-lengths by using a diffraction grating ruled on speculum metal. The rulings were 2 cm. long and the group covered a width of 5 mm. The grating space was .02 mm., i.e., 50 lines to the mm. The grating was ruled on one of Michelson's engines and was especially free from ghosts.

For the K_{α_1} lines of copper, iron and molybdenum the wave-lengths are calculated to be in Ångstrom units 1.5373, 1.937 and .708 respectively. The crystal grating space of calcite is computed to be 3.029 Å. This agrees well with the value of 3.033 Å. derived by Baecklin from measurements made on the K_α line of aluminium.

From the wave-lengths determined in this paper Avogadro's Number and the electrostatic charge on an electron are calculated to be 6.061×10^{23} and 4.774×10^{-10} e.s.u. respectively. Millikan's values from his oil-drop measurements are just the same, the only difference being that his probable errors are somewhat smaller.

G. F. S.

The Influence of a High Potential Direct Current on the Conductivity of an Electrolyte. J. A. H. LEECH-PORTER and T. ALTY. (*Phil. Mag.*, Jan., 1929.) "A cylindrical cell 7 cm. long and 2.5 cm.

in diameter and having platinum end-pieces was filled with distilled water and the resistance between the end-pieces was measured. A potential of 240 volts was then applied between the platinum plates for some minutes, after which the resistance was again measured. Usually the application of the potential increased the conductivity by about 100 per cent. On stirring the water the original resistance was restored." The resistance was found by the use of an alternating current. Another form of the experiment was this. The cell was filled with water and 40 volts were applied for 10 minutes. Then the resistance was measured. A potential of 80 volts was then applied and the resistance measured after the lapse of 10 minutes. This was continued up to 600 volts. The cell was refilled and the series repeated with voltages decreasing from 600 to 40. With both increasing and decreasing voltages the change of resistance was greatest at 80 volts. The number of ohms of change in the resistance was likewise independent of the ascending or descending sequence.

Since the alternating voltage for measurement of resistance and the direct voltage were both applied at the electrodes the effect observed might be attributed to some change in the electrodes. To clear this up a modification was introduced. Two large electrodes in dilute HCl were used for the direct voltage only, and the resistance was measured by exploring electrodes placed in the liquid between the main electrodes. Thus any change of the latter would be without effect on the resistance between the auxiliary electrodes. After the D.C. current had flowed for a while the resistance midway between the large electrodes was always greater than before the passage of the current. Near either electrode, however, the resistance was much reduced. The normal resistance was restored by stirring.

Again, two pairs of exploring electrodes were kept in the liquid, the line joining the two members of each pair being parallel to the faces of the large electrodes. One pair was as far from the positive electrode as the other pair was from the negative one. Minute by minute the resistance of the liquid between the electrodes of each pair was measured, while the D.C. current was flowing. "The resistance across one pair of electrodes increased while that across the other decreased. When the current was reversed, the resistances (at the two pairs of exploring electrodes) were interchanged. Also, on reversal, it will be noted that the conductivity at both pairs of electrodes first rises to a maximum and then falls to an equilibrium value. If the current be again reversed, this same effect is repeated." The following seems to be the explanation. When the D.C. voltage is applied ions are swept out of the liquid

and collect at the electrodes. The liquid midway between the electrodes has a high resistance because it is poor in ions. It is the resistance of the pair of electrodes near to the positive electrode that diminishes while the resistance near to the negative plate increases. This may indicate that the cathode holds its ions closer than does the anode. It may be that the difference depends on different rates of diffusion for the positive and the negative ions since at an electrode the ions will be in a state of temporary equilibrium resulting from the interaction of the current effect and of diffusion. A study of the change of resistance at the two pairs of auxiliary electrodes taking place when the D.C. voltage is reversed indicates that the anions and the cations move across to the opposite electrode in a group and not in a straggling procession. Observations of the negative Cl ion show that it moves with a velocity of about 74×10^{-5} cm./sec. when the difference of potential in the liquid is one volt per cm. G. F. S.

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Journal
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Devoted to Science and the Mechanic Arts

Vol. 207 JUNE, 1929 No. 6

INFLUENCES OF METEORIC ASTRONOMY ON EVOLUTION.*

BY

CHARLES P. OLIVIER, PH.D.,

Professor of Astronomy and Director of the Flower Observatory, University of Pennsylvania.

Mr. President, Members of The Franklin Institute, Ladies and Gentlemen: It is a real pleasure to have the honor of talking to you tonight on a comparatively neglected branch of astronomy, here in The Franklin Institute, which has seen science in America grow from very humble beginnings to its present commanding position. But the important part played in this development by men connected with The Franklin Institute is too well known to all of you to need further mention by me. From a personal standpoint I am glad to speak here, so soon after my arrival in Philadelphia to take up my residence in your city. Because it was before the American Philosophical Society down by Independence Hall that, in 1911, I made what might be called my scientific début, being permitted to read a résumé of my thesis which was afterwards published in full by that Society. It may interest you to know that it was my first extensive publication on meteoric astronomy, the very subject of our talk this evening.

Before it is possible to come to an intelligent conclusion regarding any scientific hypothesis, the history of the subject must be briefly sketched for the benefit of those who perforce have paid only casual attention to it. Further the limits

* Presented at the annual meeting of the Institute held Wednesday, January 16, 1929.

(Note.—The Franklin Institute is not responsible for the statements and opinions advanced by contributors to the JOURNAL.)

where apparent certainty ends and conjecture begins should be made very clear. To do this properly, enough must be said about the data, which are available, so that others may readily weigh their relative accuracy. It is scarcely necessary before an audience of scientists to add that any conclusions reached by me—or in general by any individual—are perforce based also upon the results of many others.

Great Bolide of September 12, 1923, by Josef Klepesta.
From "Meteors" by C. P. Olivier, Williams & Wilkins Co., Baltimore, Md.

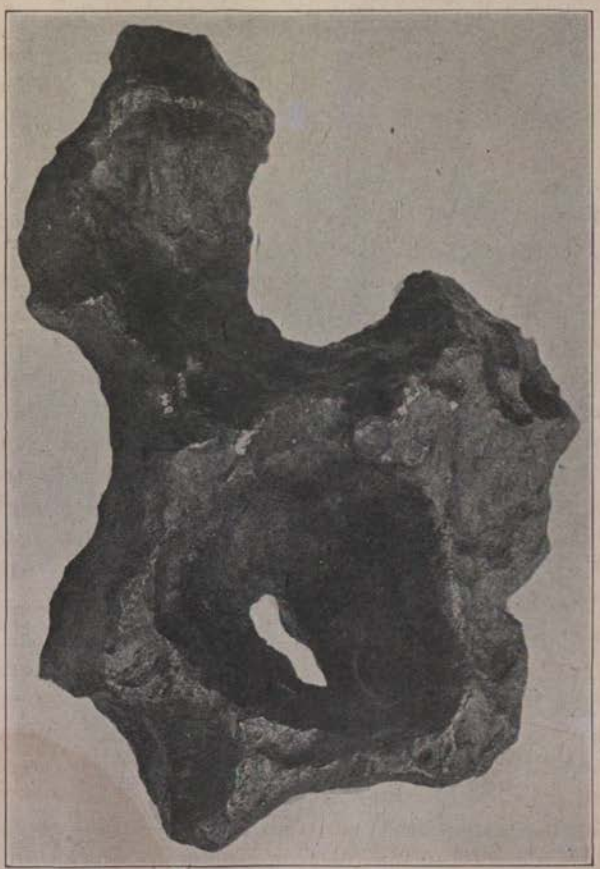
Before giving any description of phenomena connected with meteoric astronomy it should be stated that the general term "meteor" is used to cover three classes of bodies. First and most common the ordinary shooting star or, technically, meteor; secondly the fireball or super-shooting star, sometimes called bolide; and thirdly the meteorite,

which is merely a fireball large enough to survive its fiery passage through our atmosphere and reach the ground in one or more solid pieces. Obviously the definitions are loose: for what one man might call a large meteor, another would call a small fireball. Nevertheless, when interpreted with common sense, the terms are valuable.

The Sharps, Virginia, Meteorite, of April 1, 1921, by Thomas L. Watson.
From "Meteors" by C. P. Olivier, Williams & Wilkins Co., Baltimore, Md.

Again it is certain that objects of the same material and equal mass will be merely fireballs or survive to be meteorites according to the velocity with which they meet our atmosphere. If they move slowly, they survive; if fast, they are destroyed; as kinetic energy varies as the square of the velocity. For example, the Earth moves 18 miles per second, the average cometary meteor 26 miles per second, as it passes our orbit. Then the limits of the relative velocity with which such bodies meet us are 26 ± 18, or 44 m/s and 8 m/s respectively. The ratio of the squares of these numbers

is 30.25 to 1. When allowance is made for the Earth's attraction, this ratio actually falls to about 20 to 1, which perhaps represents inversely their respective chances for survival as meteorites.

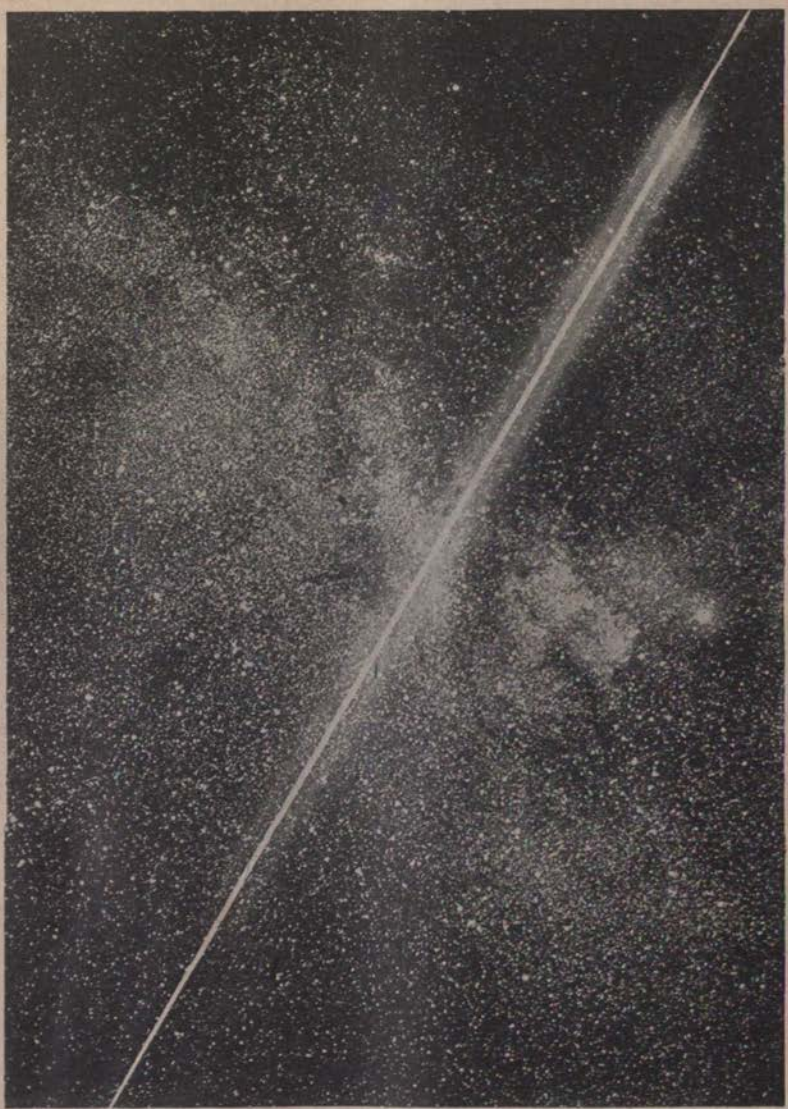


Iron Meteorite from Meteor Crater, Arizona, Photograph loaned by Geo. P. Merrill.
From "Meteors" by C. P. Olivier, Williams & Wilkins Co., Baltimore, Md.

Astronomical observations were made at least 4,000 years ago, for records stretch back that far. These refer to a variety of celestial objects. We may be quite sure that the appearance of a meteoric shower, a brilliant fireball, or the fall of a meteorite, always unexpected as they must be,

caused terror to our ancestors and were considered miraculous. We find records that meteoric stones were worshipped, during the ten centuries preceding our era, in western Asia and southern Europe. Omitting the controversial case mentioned in the Book of Joshua in the Bible, of about 1400 B.C., our earliest records of falls come from the Chinese. The first two dates are 687 and 644 B.C., with 14 others mentioned in the next six centuries. What disposition was made of these stones or exactly how they were regarded we do not know further than that the Chinese record them as coming from heaven or the sky. Fewer accounts are given by classical writers, though they say the stones were generally held to be sacred. In recent centuries the same opinion was held by various nations in the two Americas, in India, and in Japan. But the oldest preserved meteorite, whose fall is recorded, dates only from 1492. At present over 1,000 meteorites are represented in various museums, but a large percentage of the nickel-irons were not actually seen to fall. Nearly all the stone meteorites were seen to strike the ground or they would never have been found. This is because they look not unlike common rocks, and also soon disintegrate when exposed to our damp atmosphere. As for meteor showers, or the appearances of great numbers of shooting stars, we have several dozen accounts in ancient or mediæval chronicles.

Our very term "meteorology" proves that meteors were for a long time considered to be merely atmospheric phenomena like the aurora, lightning, etc. As for meteorites, scientists during the 18th century ridiculed the idea that a stone could fall from the sky, and branded as ignorant dupes or deliberate liars everyone who reported such occurrences. Their argument was simplicity itself: stones could not fall from the sky, therefore they did not. Who could ask for more? But despite the dictum of the greatest scientists (and this included every branch of natural science as well as astronomers) stones kept on falling, so in 1803 Biot was finally delegated by the French Academy to investigate the fall at l'Aigle in France, and his report carried conviction to the most opinionated of his colleagues. From that day on, meteorites took their place as respectable, if minor, members of the Solar System.



Great Bolide of August 13, 1928, by M. de Kerolyr.

However, despite the fact that as early as 1798, in Germany, simultaneous observations had proved that meteors were from 50 to 100 miles high, when they disappeared, and that Humboldt described a wonderful shower in November, 1799, while traveling in South America, while another fine shower was seen in the United States in April, 1803, still meteors were wholly neglected by astronomers in general. Only casual mention can be found here and there in the scientific literature of the early nineteenth century. It remained for the great Leonid shower of November 13, 1833, to finally force the attention of the scientific world to the fact that here was a wholly neglected field of astronomy waiting for investigation.

This great shower of 1833 was seen as such only in the Western Hemisphere, but luckily by this time there were able men in our country who made valuable observations during its progress. We may consider this as the real birthday of meteoric astronomy. Olmsted of Yale, along with a half a dozen others elsewhere, discovered the existence of a radiant point for the Leonid meteors on this night. He however followed up his discovery by notable work and was therefore the first American to deserve a permanent place in the annals of meteoric astronomy.

Serious research work done by many culminated in the 1860's in the brilliant results of Newton and Schiaparelli. The first was able to predict successfully the return of the Leonid shower in 1866, the second proved the connection between Tuttle's Comet and the Perseid meteors which appear every August, coming to a maximum about the eleventh. The Leonids soon after were proved to follow the same orbit as Tempel's Comet, and similar relations were found for the Bielids and Lyrids. The next important event was the splendid shower in November, 1872, which at once was proved to follow the orbit of Biela's Comet. This shower again appeared in 1885, in 1892, and died out in a small one in 1899. This last was seen by the speaker, as a boy, though he did not then know its appearance had been predicted.

We therefore see a close correlation between meteor showers and comets. Let us pause to remind you that a typical comet consists of three parts, nucleus, coma, and tail.

The nucleus is not one solid mass, but a multitude of small, solid masses, of varying size. The coma must be largely gaseous, with a certain amount of the nuclear material scattered through it. The tail is made up of particles repelled from the coma, and is always directed away from the Sun. The size of these particles must be sub-microscopic.

Fireball of Fireball of June 29, 1927, by Issei Yamamoto.

It must be clearly understood that meteors cannot come from debris in comets' tails, but from larger nuclear debris. Comets therefore continually lose part of their material on each visit to the Sun. The famous Biela's Comet, for instance, seems to have been wholly dispersed since 1852.

Questions of evolution may be first properly introduced here. A few personal opinions must be pardoned, because they will clear up later statements. As the way in which the heavenly bodies evolved is, in the purely physical sense, the greatest problem offered to the human mind—the problem of the universe indeed—a complete and final solution is not to be expected. The time scale and the space scale are so immense, that even were we better acquainted with all physical laws, our solutions inevitably would be approximate.

In the present state of science contrary opinions can be offered, without possibility of definite disproof. Each theory presented probably has some truth, and certainly much error. At least this momentary advantage comes to the individual astronomer—he can use his trained imagination, if he has any, and perhaps add a little to the ultimate solution.

Being primarily interested here in our Solar System, the speaker hastens to add that in a broad sense he accepts the Planetesimal Hypothesis of Chamberlin and Moulton as giving the most satisfactory general picture of evolution. However, appearing just in time for me to hurridly read over part of it, the final book on this subject [1] by the late Prof. T. C. Chamberlin came into my hands a few days ago. While one speaks with diffidence about disagreement with such an eminent man, for I believe that the tremendous significance of his work in this line is still largely unappreciated, I cannot agree with his detailed ideas on the origins of comets and meteorites. Yet I believe that the origins of meteorites, and possibly comets, can be fully explained on ideas given earlier in his book, and that we need go to no other hypothesis than the original planetesimal. These ideas will be developed during this lecture. As for comets, he at least has given a coherent theory, a thing no other astronomer in the past has succeeded in doing. For comets are perhaps the hardest things to explain in the universe.

Briefly, his idea was that meteors and the nuclei of comets are simple chondrulites or groups of them respectively. These chondrulites are composed of matter ejected from the Sun; indeed we see eruptive prominences active on that body continually. He believed that such matter, projected into space, cooled rapidly forming what he called precipitates. These in turn aggregated into somewhat larger bodies, the primitive chondrules.

If his view is correct, the argument to be developed here would be largely wrong. But to the speaker it appears impossible that the gaseous matter thrown out from the Sun into space, as we see in the prominences, which would have little mutual gravitation due to its extreme tenuity, could so condense. Would it not rather simply tend to diffuse away

[1] "The Two Solar Families," T. C. Chamberlin, 1928.

Halley's Comet, May 1, 1910, by Heber D. Curtis.

under radiation pressure, the kinetic energy of its separate molecules, release from all external pressure, and tidal actions due to any passing body?

Unfortunately, no one can try the experiment under solar conditions, and I fear that as yet what really happens is a matter of opinion, not proof. In partial support of my own view, however, Jeffreys the distinguished English astronomer, in 1918, concluded that no planet or satellite whose present diameter is less than 1,000 km. could ever have been gaseous. This conclusion, if even approximately correct, bears on our problem.

This discussion could not properly be introduced here except that the nature of comets' nuclei, as postulated by Chamberlin, is I think disproved by the two known cases of collisions with our Earth. These are now taken up as they furnish first-hand information about the nature of comets and meteorites and their present effects on other bodies, which is evolution still in progress.

The first case is the famous Meteor Crater,[2] Arizona, a hole 4,200 feet in diameter and 600 feet in visible depth, dug out in the solid rock by the impact of an immense compact group of meterorites, which must have been the nucleus of a small comet. The proof of the impact origin we owe to the untiring labors of my friend, Mr. Daniel Moreau Barringer, a resident of Philadelphia and well known to you all. The great mass was finally located by a bore-hole 1,400 feet under the south wall, and is now being mined for. Its recovery will be of the vastest value to science and is eagerly being awaited by us all. This fell almost certainly within the past 5,000 years, probably within the past 2,000.

The second case was on June 30, 1908, in Siberia.[3] Due to the inaccessibility of the region, authentic news only reached the scientific world much later, and not until 1927 was an expedition able to reach the spot. This reported an area of 30 to 40 km. in diameter wholly devastated, the trees dead, their trunks seared as by a sudden blast of hot gases, and many lying on the ground, their tops pointing outward from the central region. Here over a large area were hundreds

[2] "Coon Mountain and its Crater," D. M. Barringer, *Proc. Acad. Nat. Sci.*, Philadelphia, 1906.

[3] *Scientific American*, July, 1928.

of "shell-holes" from one to 50 meters in diameter, but as the ground was swampy, water must have been in them. The expedition was too poorly equipped to dig out whatever masses lay in their bottoms. The waves set up by the passage through the air of these celestial visitors were so violent as to knock down men 100 km. from the edge of the devastated area, and affected barographs several hundred kilometers distant. Luckily nobody was within the area,

Microphotograph of Carleton, Texas, Meteorite, by Lewis E. Jewell.

but hundreds of reindeer, sent there to graze, perished. Here we have the example of the effects of collision with the head of another comet, certainly a small one as it was not seen out in space. There is some reason to think this was a fragment connected with Pons-Winnecte's Comet which came to perihelion in 1909. This probability is heightened by the fact that the speaker was able to prove in 1916 that certain meteors seen in May and June of that year followed the same orbit as Pons-Winnecke's Comet,[4] proving that the forces of disintegration were well at work on that body. So it appears that we have here two rather distinct forms of comets, one

[4] *Monthly Notices*, R. A. S., 77, 71, 1916.

with a very compact nucleus, with only relatively small fragments accompanying it for a few miles on either side, the other a nucleus made up of widely scattered single masses, no one of them very large.

From these examples it appears clear that both classes of nuclei must be made up of fragments of considerable size, as well as little fragments, which latter is the composition of such nuclei according to Professor Chamberlin's opinion.

The craters on the Moon are ascribed to two possible agencies: volcanic action and impact of masses of various sizes. While quite open to conviction and despite the increasing favor with which the meteoric theory is viewed, my present opinion is that while impact certainly accounts for a percentage of those visible, still there are almost insuperable difficulties in accepting the impact origin as a general explanation for lunar surface features. I freely admit great difficulties in accepting the volcanic theory also, but not quite so grave, I believe, as with the impact theory. Professor Chamberlin agreed with this viewpoint, which of course is nothing new. I have, however, been able to point out that even today there is on the unprotected lunar surface an action,[5] not unlike erosion, going on due to the continual impact of meteoric bodies of all sizes. This obvious action seems to have been formerly wholly overlooked, or certainly no one realized its consequences.

Next in order comes a brief study of the structure of meteorites. The slides thrown upon the screen illustrate their extreme complexity. It seems fairly certain that these bodies once formed part of the masses of larger ones, and that forces like those acting within the Earth caused the extreme complexity of structure you can see. Both heat and pressure played their part. We see examples of refusion and alteration, and in certain cases the introduction of metal can be proved to be subsequent to the consolidation of the stony material. In the speaker's opinion it appears impossible to assume that such actions could take place merely by the meeting of smaller particles in space, or indeed elsewhere than within a solid body of some size, which was afterwards destroyed or at least disintegrated. The idea advanced is simply this: that while 8 large and at least 1,000 small

[5] "Meteors," C. P. Olivier, 254, 1925.

planets survived the processes of evolution, to say nothing of satellites and comets, many that reached partial growth were destroyed by the very forces that built up the others. The planetary bodies at present in the Solar System merely represent the survival of the fittest.

Again, that there can be no hard and fast line of demarcation between asteroids and comets is proved, for instance, by Asteroid No. 944.[6] Its orbit is quite cometary, and if it showed either a coma or a tail would be so classed. Other asteroids have orbits of high inclination and large eccentricity, not unlike short period comets. It will be objected that we have no asteroids with retrograde motion, while half of the comets have it. But let us point out that all the very short period comets have direct motion and small inclinations. There are a few comets of medium period and retrograde motion. Cannot this be interpreted to mean that comets (or asteroids) of short period, retrograde motion, and small inclinations, had a poor chance for survival? They were, to express it crudely, swimming against the stream, when, according to hypothesis, the whole region within the orbit of Jupiter must have been filled with planetary building material. The possibility is therefore suggested that asteroids and comets had the same kind of origin. In the first case it seems probable that enough material came together under favorable circumstances for its mutual gravitation to draw it all into a compact or comparatively compact body.[7] In

[6] "Meteors," 267–8, 1925.

[7] With regard to both asteroids and satellites, it seems to have been generally tacitly assumed that they were solid, continuous bodies. But on what is this based? So far as asteroids are concerned we have no way to determine the masses of any of them, nor are we liable to do so unless in the cases of the very largest a comet should pass extremely near one of them. With satellites we are in a more favorable position, and very curious results follow in some cases. For instance the fourth satellite of Jupiter—Callisto—turns out to have a density of only 0.58. Also for the satellites of Saturn we find Minas, 0.24; Euceladus, 0.52; and Tethys, 0.54. These values are calculated from data in Table IV of "Astronomy" by Russell, Dugan and Stewart. How can such low densities be explained on the basis of a single, continuous, solid body? For it seems impossible to assume these small bodies are still in the gaseous or liquid form. Is it possible that such satellites furnish us with examples of bodies that are in constitution half way between a typical solid satellite or asteroid, and the nucleus of a typical comet? Is it possible that the variability of some asteroids and satellites could be explained on the basis of their being a fairly compact group of separate masses rather than one continuous body?

the second case, the circumstances were unfavorable and the material less. Remember that practically all asteroids we can see are within the orbit of Jupiter; we do not know about those outside. Again we find meteorites and fireballs with orbits of all inclinations. Does it not seem probable therefore that the action of Jupiter, plus collisions with other masses, did not permit retrograde asteroids of considerable size to survive? Very small ones, with orbits of high inclination, would escape discovery except under most favorable circumstances. But these actions just mentioned might well permit the survival of retrograde comets of long period, which would spend so little of their life within the critical region.

Calling up again the Meteor Crater and Siberian cases, here we have one body, that if it could have been seen at all would have looked like a tiny asteroid, the other like a tiny comet. Yet each are technically comets. Again I find that work at the Lick Observatory,[8] that just came to my notice after most of this paper was written, showed that spectrograms of asteroids are remarkably weak in the ultra-violet region and this in the opinion of the observers connects these bodies physically with the nuclei of comets. Also the presence of small retrograde satellites in the systems of Jupiter and Saturn, without the orbits of the direct moving satellites of these planets, proves that retrograde bodies of small size survive only under special conditions. So while the chain of evidence is far from complete, these various facts are certainly very significant. The probable reason we see so few cases of "stripped nuclei" of comets—to borrow a physical term—such as the Meteor Crater nucleus probably was, is that such a comet, stripped of its coma, assuming indeed it ever had one, would usually be too small to be seen at any distance from the Earth. We can by no means assume that many do not pass us annually undetected.

You will notice that no mention has been made of the great Nebular Hypothesis of LaPlace, nor of Lockyer's Meteoric Hypothesis. The first ignores meteors and one can find no place for them in it. The second starts with meteorites as the building material of the universe. As

[8] *Pop. Astr.*, 37, 31, 1929.

meteorites are very complex bodies, this seems to me to be an insuperable difficulty to its acceptance. It is almost a case of putting the cart before the horse. Also while this theory played an important part in its day, it really added nothing to meteoric astronomy proper except calling attention to these bodies. However, many eminent astronomers now consider certain nebulæ to be meteoric in composition. This may

Microphotograph of Toluca, Mexico, Meteorite, by Lewis E. Jewell.

particularly apply to the dark nebulæ. This phase of the subject is, I think, out of the direct line of our discussion and besides is as yet wholly in the conjectural stage.

A study of the paths of fireballs and meteorites by von Niessl and others has proved that many of these bodies move with a strongly hyperbolic velocity. Of one fact we are sure: a body moving with such a celocity never originated in the

Solar System, no matter what theory we choose to accept as to its origin. Please bear in mind that most important fact, as the rest of the argument depends upon it. Again a study of the structure of meteorites of all kinds shows that while they indeed contain some minerals not found on the Earth, no new element whatever can be detected. So far about half of the familiar ones have been found. This proves the chemical unity of the Universe. Next, in many cases we find meteorites moving with such velocities that they are well within the parabolic limit, in other words they just as surely originated in the Solar System as those with great hyperbolic velocities could not. We also have every reason to believe that meteorites of assorted sizes compose the nuclei of comets, which again originated in our system.

Hence we come to the conclusion: That finding meteorites which certainly originated in our system (irrespective of what theory of their evolution we accept, for that does not enter now into the argument), and finding similar meteorites which certainly came to us from without our system (which means they originated in some other distant stellar system) then a similar evolution took place there and here. This important conclusion destroys definitely (if it can be accepted as true) the uniqueness of the Solar System, and we would expect to find innumerable similar systems among the stars. For if our argument is correct meteorites and comets are the products of the same catastrophic event that gave birth to the planets themselves. We would therefore infer that planets also, not unlike our own, circulate about other stars as well as these meteorites which alone with an occasional comet can escape from their original homes and bring this message to us. That no strongly hyperbolic comet [9] has yet been observed seems purely a matter of chance, for only those which came very near indeed to our Sun could be observed from the Earth. It does not prove that such comets do not visit our system at intervals, much less never.

It is but fair now to examine the reliability of the data

[9] The discussion of comets in many books on astronomy is very misleading. An uninformed reader might infer that most comets which visited the Sun, either periodically or otherwise, came within sight of the Earth's orbit. This assumption is probably quite contrary to fact.

we have used. Obviously the velocities were the most important, and unless warned you would think they were easy enough to obtain. But due to the fact that fireballs and meteorites come unannounced their orbits must be calculated, generally, using data sent in by untrained men. Until such velocities can be gotten by instrumental means, there will always remain some doubt as to their reliability. Yet this fact comes out: those cases with the very best observations usually give higher velocities than those based on poorer material. Again Hoffmeister [10] has proved that small meteors have as a rule strongly hyperbolic velocities, basing his theory on the variation of hourly numbers, and more recently on more involved considerations. So every line of approach, at present available, gives us reason to believe that the larger percentage of all meteoric bodies comes from outer space. But the annual showers, which come from groups following comets with known periods, prove that others move in ellipses and are hence members of the Solar System and originated within it. You see, however, why we await so anxiously the development of faster photographic plates so that we may definitely test the velocities by instrumental means, on a large scale. That done, we will have no longer this weak link in the chain of argument, no matter in which direction we are finally led.

May we add that the studies of the changing speeds of meteors in our upper atmosphere, and the behavior of their trains, give us invaluable information as to its constitution and density, as yet approachable in no other way. Studies of their spectra, when available in some numbers, also will be valuable. The physical theory of meteors in our atmosphere has not yet been solved in a way generally accepted, and indeed there are numerous other problems for physicists and meteorologists as well as astronomers furnished by these little debris of evolution, as we may justly call them.

To review rapidly some of the points raised in this discussion, we will now run through a number of slides.

Finally, just as the brickbats, pieces of stone, scraps of metal, and wooden chips that lie around a newly completed house would give us an idea of how the building was erected,

[10] *Astron. Abhandlungen*, Band 4, Nr. 5.

so meteors and meteorites can give us similar information as to the evolution of the Solar System. Yet in some of the older theories there was no place for them whatever! But in the future no theory of how this wonderful masterpiece of the Creator evolved can be considered even approximately complete that does not take full account of these bodies which are so small in size but so important in the information they bring to us.

As for this talk which is just being concluded, while I have been rather bold in offering a partly new theory, knowing it has as yet so many unproved links, still, even should the theory itself eventually prove wholly unacceptable, some of the data on which it is based and the arguments brought forward in its support should have a more permanent value.

A New Method of Developing Electricity in Quartz Crystals by the Use of Torsion. E. P. TAWIL. (*Comptes Rendus*, Dec. 3, 1928.) Pierre Curie and his brother discovered that certain crystals when subjected to changes of pressure develop the two kinds of electricity at different parts of their surfaces-piezoelectricity. The author has found that torsion applied to quartz crystals produces a similar separation of electricity. A cylinder of quartz is made with its axis parallel to the optical axis of the crystal. The ends of the cylinder are provided with mountings one of which is kept fixed while the other is provided with a lever for torsion. As soon as the cylinder is twisted the surface of the cylinder shows everywhere an electrical charge of the same sign throughout, while the opposite charge manifests itself on the ends perpendicular to the axis. When the crystal is twisted in the opposite direction the charges are reversed. Even relaxing the torsion so that the cylinder resumes its position of equilibrium causes a reversal of the charges. It makes a difference whether the quartz rotates the plane of polarization to the right or to the left. A crystal receives a positive charge on its outer surface when it is twisted in the sense of its optical rotation. Natural crystals of quartz show a similar effect. From the sign of the electricity set free by torsion it is possible to determine in which sense the crystal rotates the plane of polarization.

In applying torsion care must be exercised to avoid bending the quartz, for the author finds that this also brings about a development of electricity. For the effect due to torsion he suggests the name 'strepho-electricity.' Roentgen in 1889 investigated the effect of torsion on crystals but failed to detect the phenomena above described. G. F. S.

Are Characteristic X-Rays Polarized? E. O. WOLLAN. (*Proc. Nat. Acad. Sci.*, Nov., 1928.) Barkla in 1905 showed that the general radiation from an X-ray tube is partially polarized. In 1926 Bishop made the same claim for the characteristic radiation. The importance of the matter justifies the repetition of the investigation. This was carried out but with a method different from that used by Bishop. The conclusion is that "the K lines of molybdenum are not polarized in excess of one per cent." Bearden has recently published the results of his investigation of the same problem. They agree with those of the present paper. G. F. S.

ON THE THEORY OF ELECTRON SCATTERING IN GASES.

BY

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BARTOL RESEARCH FOUNDATION

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A GENERAL wave-mechanical theory of collisional processes has been developed by Born [1] and applied by Wentzel [2] to the scattering of alpha-particles by heavy atoms, and by Elsasser [3] to collisional processes in atomic hydrogen. Recently Sommerfeld [4] has applied the method to the scattering of electrons in argon and has found agreement with the experimental results.

To carry out the calculation it is necessary to know the potential of the atom $v(r)$ as a function of the distance from the nucleus. Wentzel first assumed the potential of an atom of atomic number Z to be given by the Coulomb law $v(r) = Ze/r$, neglecting the effect of the electrons around the nucleus but found that this led to integrals which would not converge. On assuming the potential function to be $v(r) = Ze/re^{-r/R}$, where R is of the order of magnitude of an atomic radius, he was able to obtain a scattering formula which, for alpha-particles, agreed with the Rutherford scattering law. On the other hand Elsasser found the potential of the hydrogen atom in terms of its eigenfunctions, the potential for the hydrogen atom in its normal state being similar to that used by Wentzel, thus justifying his assumption, and showed that a scattering law for atomic hydrogen could be obtained. Finally, Sommerfeld considered scattering from other atoms by treating all the electrons of the atom as united in the K-shell without mutually influencing each other. "The action of these electrons toward the outside would then

[1] M. Born, *ZS. f. Physik*, **37**, 863; **38**, 803 (1926).

[2] G. Wentzel, ibid., **40**, 590 (1927).

[3] W. Elsasser, ibid., **45**, 522 (1927).

[4] A. Sommerfeld, "Atombau und Spektrallinien," Wellenmech. Ergänzungsband, pp. 226 et seq.

be represented wave-mechanically by the Z-fold electron cloud of hydrogen in its normal state, whose charge density is

$$\rho = \frac{eZ^4}{\pi a^3} e^{-2r(Z/a)},$$

where a is the radius of the first Bohr circle." He then showed that the potential energy of an electron in the field of the atom is

$$V(r) = -e^2 Z \left(\frac{1}{r} + \frac{Z}{a}\right) e^{-2r(Z/a)}. \qquad (1)$$

By using this potential energy in the Born collision equations he arrived at a scattering formula, which for high velocity alpha-particles and for not too small scattering angles, agrees with the Rutherford scattering law. The formula also agrees with the experiments of Kirchner [5] on the scattering of electrons in argon (elastic collisions). It will be noted that expression (1) for the potential energy takes into account shielding by the electrons of the atom.

It is the purpose of this paper to carry through the calculations using a somewhat different method of obtaining the potential of the atom at a distance r from the nucleus. Thomas [6] and Fermi [7] have recently shown that if the electrons of an atom of atomic number Z be considered to be a perfectly degenerate gas in the sense of the Fermi statistics,[8] the potential at a distance r from the nucleus is given by

$$v = \frac{Ze}{r} \Phi\left(\frac{r}{\mu}\right), \qquad (2)$$

where

$$\mu = \frac{3^{2/3} h^2}{2^{13/3} \pi^{4/3} m e^2 Z^{1/3}}$$

(m = mass of electron).

This function $\Phi(r/\mu)$ is unity when r is zero and decreases to zero for increasing r, such that for distances a little greater than the atomic radius the potential will be zero. No

[5] F. Kirchner, *Ann. der Phys.*, 83, 969 (1927).
[6] L. H. Thomas, *Proc. Cambridge Phil. Soc.*, 23, 542 (1926).
[7] E. Fermi, *ZS. f. Phys.*, 48, 73 (1928).
[8] E. Fermi, ibid., 36, 902 (1926).

analytical expression is known for the function Φ over its whole range, but a table of values is given by Fermi.[9] He has used this function to calculate the Rydberg correction terms of the alkali atoms,[10] generally thought of as due to the shielding of the nonradiating electrons. In this paper we shall consider this potential function as giving the true potential of an atom at a distance r from the nucleus, inclusive of shielding, and use the potential energy derived therefrom in conjunction with the Born collision theory to calculate the scattering from different atoms.

To calculate the scattering of an electron by an atom situated at O (Fig. 1), we consider a plane wave for an electron

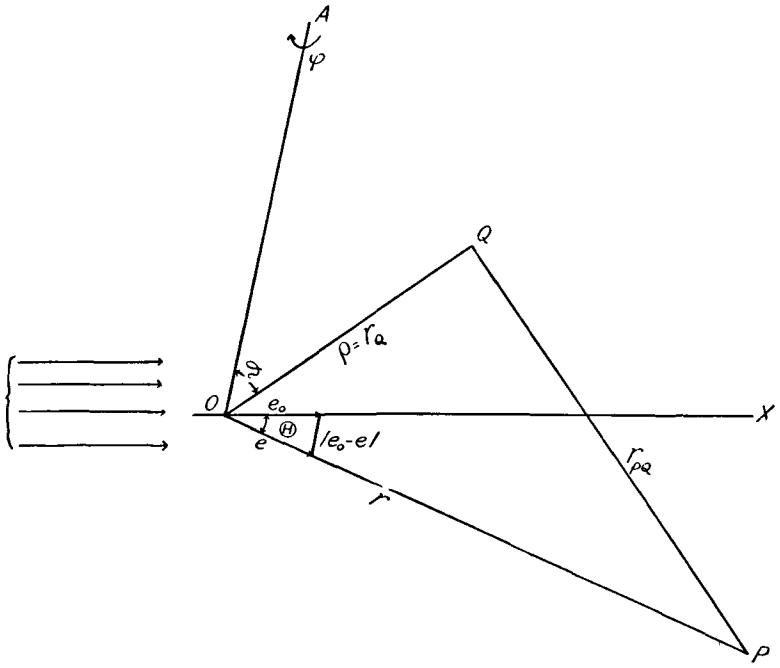


FIG. 1.

approaching the atom from the negative X-direction. The wave equation for the total system, electron plus atom, is

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0. \qquad (3)$$

[9] See ref. 6, p. 75.

[10] E. Fermi, *ZS. f. Phys.*, **49**, 550 (1928).

We consider V, the potential energy of the system, as a perturbation term and write

$$\psi = \psi_0 + \psi_1 + \cdots. \tag{4}$$

Substituting in (3) and writing $k^2 = \frac{8\pi^2 mE}{h^2}$ we have

$$\nabla^2(\psi_0 + \psi_1 + \cdots) + k^2\left(1 - \frac{V}{E}\right)(\psi_0 + \psi_1 + \cdots) = 0, \tag{5}$$

where

$$\nabla^2\psi_0 + k^2\psi_0 = 0, \tag{6}$$

since $V\psi_1$ is to be neglected in comparison with $\nabla^2\psi_0$ and $k^2\psi_0$. From (3) and (4) we have

$$\begin{aligned}
\nabla^2\psi_1 + k^2\psi_1 &= k^2\frac{V}{E}\psi_0, \\
\nabla^2\psi_2 + k^2\psi_2 &= k^2\frac{V}{E}\psi_1, \\
&\cdots\cdots\cdots \\
\nabla^2\psi_n + k^2\psi_n &= k^2\frac{V}{E}\psi_{n-1} = F_{n-1}.
\end{aligned} \tag{7}$$

It will be noted that (6) is the equation for the oncoming electron since the interaction term has been neglected. The solution of (6) for the plane wave of the electron is

$$\psi_0 = e^{ikx}.$$

We must now solve the inhomogeneous equation (7), which may be accomplished by Green's theorem in the following manner. If ξ and η are functions satisfying the usual conditions, Green's theorem gives

$$\int\int\int (\xi\nabla^2\eta - \eta\nabla^2\xi)d\tau = \int\int\left(\xi\frac{\partial\eta}{\partial n} - \eta\frac{\partial\xi}{\partial n}\right)d\sigma. \tag{8}$$

We shall let

$$\xi = \frac{e^{ikr}}{r}, \tag{9}$$

a function satisfying the homogeneous equation

$$\nabla^2\xi + k^2\xi = 0, \tag{10}$$

and

$$\eta = \psi_n. \tag{11}$$

We take the origin of r as the point P, at which we shall seek the value of ψ_n. We shall take for our boundary of integration two surfaces, one a sphere of radius R about P, which sphere shall eventually be made infinite, and for the other boundary a small sphere of radius r_0, which shall eventually be made zero. Substituting (9) and (10) in (8) we obtain

$$\begin{aligned}\int\int\int \frac{e^{ikr}}{r}(\nabla^2\psi_n + k^2\psi_n)d\tau = &\int\int\left(\xi\frac{\partial\psi_n}{\partial r} - \psi_n\frac{\partial\xi}{\partial r}\right)d\sigma \\ &- \int\int_{r_0}\left(\xi\frac{\partial\psi_n}{\partial r} - \psi_n\frac{\partial\xi}{\partial r}\right)d\sigma,\end{aligned} \tag{12}$$

where the negative sign in the integral over the sphere of radius r_0 is incorporated to permit of the normal being drawn outwards from P.

Since ψ_n, by the fundamental conditions of the Schroedinger theory must vanish to the order of $\frac{1}{r\epsilon}$, where $\epsilon > 0$ (a conclusion which will subsequently be seen to be true) the integral over the surface R will vanish to the order $\frac{1}{r\epsilon}$.

Consider now the integral over the surface r_0. Let M be the maximum value of $\partial\psi_n/\partial r$ then

$$\int\int_{r_0}\frac{e^{ikr}}{r}\frac{\partial\psi_n}{\partial r}d\sigma < \frac{e^{ikr_0}}{r_0}M4\pi r_0^2$$

and is zero in the limit. Now

$$\int\int_{r_0}\psi_n\frac{\partial\xi}{\partial r}d\sigma = \int\int_{r_0}\psi_n e^{ikr}\left(\frac{ik}{r} - \frac{1}{r^2}\right)4\pi r^2 dr = -4\pi\psi_n$$

in the limit.

Hence (12) becomes

$$\psi_n = -\frac{1}{4\pi}\int\int\int\frac{e^{ikr}}{r}(\nabla^2\psi_n + k^2\psi_n)d\tau'$$

and remembering (7) we obtain

$$\psi_n = -\frac{1}{4\pi}\int\int\int F_{n-1}(r_Q)\frac{e^{ikr_{PQ}}}{r_{PQ}}d\tau'. \tag{13}$$

Taking the first approximation we have

$$\psi_{1(P)} = -\frac{k^2}{4\pi E}\int\int\int V(r_Q)\frac{e^{ikr_{PQ}}}{r_{PQ}}e^{ikx_Q}d\tau'. \tag{14}$$

From Fig. 1 we have

$$r_{PQ} = |\mathbf{r} - \mathbf{r}_Q| = [\mathbf{r}^2 + 2\mathbf{r}\cdot\mathbf{r}_Q + \mathbf{r}_Q{}^2]^{1/2}.$$

If $r \gg r_Q$ then

$$\begin{aligned} r_{PQ} &= r\left(1 - \frac{1}{r^2}\mathbf{r}\cdot\mathbf{r}_Q\right) \\ &= r - \mathbf{e}\cdot\mathbf{r}_Q, \end{aligned} \tag{15}$$

where $\mathbf{e}$ is a unit vector in the direction r. We also write

$$x_Q = \mathbf{e}_0\cdot\mathbf{r}_Q \tag{16}$$

where $\mathbf{e}_0$ is a unit vector in the direction X. Equation (14) becomes

$$\psi_{1(P)} = -\frac{k^2}{4\pi E}\frac{e^{ikr}}{r}\int\int\int V(r_Q)e^{ik(\mathbf{e}-\mathbf{e}_0)\cdot\mathbf{r}_Q}d\tau'. \tag{17}$$

We now change to polar coördinates taking OA, parallel to $|\mathbf{e}_0 - \mathbf{e}|$, as the polar axis, φ as the longitude and θ as the co-latitude. Then

$$\rho = r_Q \tag{18}$$

$$(\mathbf{e}_0 - \mathbf{e})\cdot\mathbf{r}_Q = {}^{Q}|\mathbf{e}_0 - \mathbf{e}|\rho\cos\theta. \tag{19}$$

If now we introduce the scattering angle Θ, we see from the figure

$$|\mathbf{e}_0 - \mathbf{e}| = 2\sin\frac{\Theta}{2}. \tag{20}$$

Making the above substitutions (17) becomes

$$\psi_{1(P)} = -\frac{k^2}{4\pi E}\frac{e^{ikr}}{r}\int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} V(\rho)e^{i2k\sin\Theta/2\rho\cos\theta}\rho^2 d\rho\sin\theta d\theta d\varphi. \tag{21}$$

We now use the potential energy derived from the Fermi

statistics, which is

$$V(\rho) = -\frac{Ze^2}{\rho}\Phi\left(\frac{\rho}{\mu}\right) \tag{22}$$

for an electron at a distance ρ from the nucleus. Equation (21) then becomes

$$\psi_{1(P)} = \frac{k^2}{4\pi}\frac{Ze^2}{Er}e^{ikr}\int_0^{2\pi}\int_0^{\pi}\int_0^{\infty}\Phi\left(\frac{\rho}{\mu}\right)e^{i2k\rho\sin\Theta/2\cos\theta}\rho d\rho\sin\theta d\theta d\varphi. \tag{23}$$

We first carry out the integration with respect to θ keeping ρ constant. Writing $\delta = 2k\rho\sin\Theta/2$ we have

$$\int_0^{\pi} e^{i\delta\cos\theta}\sin\theta d\theta = \frac{2}{\delta}\sin\delta, \tag{24}$$

and (23) becomes after integrating with respect to φ,

$$\psi_{1(P)} = \frac{k}{2E}\frac{Ze^2}{\sin\frac{\Theta}{2}}\frac{e^{ikr}}{r}\int_0^{\infty}\Phi\left(\frac{\rho}{\mu}\right)\sin\left(2k\rho\sin\frac{\Theta}{2}\right)d\rho. \tag{25}$$

The analytical expression of $\Phi(\rho/\mu)$ is not known, hence the integration of (25) must be carried out graphically. We form the quotient $\left|\frac{\psi_1}{\psi_0}\right|^2$, and remembering that $|\psi_0|^2 = 1$ we obtain

$$P = \left|\frac{\psi_1}{\psi_0}\right|^2 = \left[\frac{kZe^2}{2Er}\right]^2\frac{1}{\sin^2\frac{\Theta}{2}}\left[\int_0^{\infty}\Phi\left(\frac{\rho}{\mu}\right)\sin\left(2k\rho\sin\frac{\Theta}{2}\right)d\rho\right]^2. \tag{26}$$

Equation (26) *may be interpreted as the number of particles per unit solid angle which are scattered through an angle* Θ *in relation to the number which fall on the atom.*

We shall now compare this with the expression obtained by Sommerfeld using the Born method. He obtains

$$P = \left[\frac{e^2Z}{4Er}\right]^2\left[\frac{1}{\sin^2\frac{\Theta}{2}+\alpha^2} + \frac{\alpha^2}{\left(\sin^2\frac{\Theta}{2}+\alpha^2\right)^2}\right]^2, \tag{27}$$

where $\alpha = \frac{Z}{ak}$; a = radius of first Bohr circle. It will be

noted that the above formula gives exactly the Rutherford scattering law for alpha-particles if we neglect α^2 in comparison to $\sin^2 \Theta/2$ and if we introduce the proper factor into the constant to take care of the double charge of the alpha-particle. It has been shown by Wentzel that for alpha-particles and heavy nuclei α^2 may be neglected in comparison to $\sin^2 \Theta/2$ for not too small angles of scattering. On the other hand equation (27) gives for $\Theta = 0$

$$P = \left[\frac{e^2 Z}{4Er}\right]^2 \frac{4}{\alpha^2}, \tag{28}$$

i.e., a finite number of particles scattered at $\Theta = 0$ instead of an infinite number as given by the Rutherford law.

Kirchner has measured the number of electrons of 10 to 40 kv. energy scattered in argon by the Wilson cloud expansion chamber method and the results are in good agreement with expression (27). The number scattered at $\Theta = 0$ is finite and can be calculated from the value of α^2 given above.[11]

We have now to investigate whether our expression (26), derived on the assumption that the potential in the atom is that given by the Fermi statistics, leads to a finite number of particles scattered at $\Theta = 0$. It will be readily seen that on putting $\Theta = 0$ (26) results in an indeterminate form. Differentiation of both numerator and denominator yields a second

[11] What Kirchner actually measured was the number of electrons scattered through angles greater than Θ, which is given by

$$N = C \int_{\Theta}^{\pi} \left[\frac{1}{\sin^2 \frac{\Theta}{2} + \alpha^2} + \frac{\alpha^2}{\left(\sin^2 \frac{\Theta}{2} + \alpha^2\right)^2}\right]^2 \sin \Theta d\Theta.$$

On using the correct value of α^2 (calculated for an atom of atomic number Z and for electrons of a given velocity), the experimental data were found to fit the theoretical curve (see curves in Sommerfeld's book p. 225). Unfortunately, (26) does not readily admit of the integration performed above, owing to incomplete knowledge of the function $\Phi(x)$, hence we cannot expect to get numerical agreement with experiment. Although it is very difficult to perform the integration indicated in Note 11, one can at least get an indirect comparison with experiment by comparing the results obtained from (27) with those obtained from (26), after correct substitution of the constants involved. A comparison has been made, making use of a graphical integration of (26), and the results are to appear shortly in the Proceedings of the National Academy.

indeterminate form

$$P_{\omega=0} = \frac{2kC}{\sin\omega}\left[\int_0^\infty \Phi\left(\frac{\rho}{\mu}\right)\sin(2k\rho\sin\omega)d\rho\right] \times\left[\int_0^\infty \rho\Phi\left(\frac{\rho}{\mu}\right)\cos(2k\rho\sin\omega)d\rho\right]$$

where

$$C = \left[\frac{e^2Zk}{2Er}\right]^2; \qquad \omega = \frac{\Theta}{2}.$$

Finally on differentiating both numerator and denominator a second time and placing $\omega = 0$ we obtain

$$P_{\omega=0} = 4k^2C\left[\int_0^\infty \rho\Phi\left(\frac{\rho}{\mu}\right)d\rho\right]^2. \tag{29}$$

We now make the substitution $x = \rho/\mu$, then

$$P_{\omega=0} = 4k^2\mu^4C\left[\int_0^\infty x\Phi(x)dx\right]^2. \tag{30}$$

Fermi [9] gives a table for $\Phi(x)$ for values of x from 0.00 to 20.0. $\Phi(x)$ is given by the approximate formula [6]

$$\Phi(x) = \frac{144}{x^3\left(1+\frac{1}{x^\lambda}\right)^4},$$

where λ (about $\frac{3}{4}$) is an arbitrary constant. For very large values of x the above equation becomes $\Phi = \frac{144}{x^3}$. The integral $\int_0^\infty x\Phi(x)dx$ will converge and hence a finite value of $P_{\omega=0}$ will be obtained from equation (30), in agreement with the results obtained using the other theory.

Finally, we shall show that the general form of the scattering curves is the same on the basis of the two theories, namely that the scattering is a maximum for $\omega = 0$ and a minimum for $\omega = \pi/2$. Consider, first, equation (27). By taking the first and second derivative of P with respect to ω (where $\omega = \Theta/2$), and applying the criteria for maxima and minima, it is easily seen that P has a maximum at $\omega = 0$ and

a minimum at $\omega = \pi/2$. Similarly for equation (26), based on the Fermi potentials, we obtain for the first derivative

$$\frac{dP}{d\omega} = \frac{2C\cos\omega}{\sin^2\omega}\left[\int_0^\infty \Phi \sin(2k\rho\sin\omega)d\rho\right] \cdot \left\{-\frac{1}{\sin\omega}\left[\int_0^\infty \Phi\sin(2k\rho\sin\omega)d\rho\right] + 2k\left[\int_0^\infty \rho\Phi\cos(2k\rho\sin\omega)d\rho\right]\right\}.$$

This will obviously be zero for $\omega = \pi/2$. For $\omega = 0$ the conditions for $\frac{dP}{d\omega} = 0$ are

$$\frac{1}{\sin^2\omega}\left[\int_0^\infty \Phi\sin(2k\rho\sin\omega)d\rho\right] \neq \infty \qquad \text{at} \qquad \omega = 0 \tag{31}$$

and

$$\frac{1}{\sin\omega}\left[\int_0^\infty \Phi\sin(2k\rho\sin\omega)d\rho\right] = 2k\int_0^\infty \rho\Phi\cos(2k\rho\cos\omega)d\rho. \tag{32}$$

The first condition gives an indeterminate form, which on evaluating gives

$$\frac{1}{\sin^2\omega}\left[\int_0^\infty \Phi\sin(2k\rho\sin\omega)d\rho\right] = 2k^2\left[\int_0^\infty \rho^2\Phi\sin(2k\rho\sin\omega)d\rho\right].$$

$$= 0 \qquad \text{for} \qquad \omega = 0.$$

The second condition likewise is fulfilled for $\omega = 0$, since on evaluating the indeterminate form on the left hand side of (32) we obtain the right hand side of (32) for $\omega = 0$. P is obviously a maximum for $\omega = 0$. To show that it is a minimum for $\omega = \pi/2$ we take the second derivative with respect to ω

$$\frac{d^2P}{d\omega^2} = -\frac{2C}{\sin\omega}\left\{\left[\int_0^\infty \Phi\sin(2k\rho\sin\omega)d\rho\right]\right\} \times \left\{-\frac{1}{\sin\omega}\left[\int_0^\infty \Phi\sin(2k\rho\sin\omega)d\rho\right]\right.$$

$$+ 2k\left[\int_0^\infty \rho\Phi \cos(2k\rho \sin\omega)d\rho\right]\Big\} + A\cos\omega(\cdots).$$

The last term indicates a number of factors which are multiplied by $\cos\omega$ and which vanish for $\omega = \pi/2$. The condition that P should have a minimum at $\omega = \pi/2$ is

$$-\frac{1}{\sin\omega}\left[\int_0^\infty \Phi \sin(2k\rho \sin\omega)d\rho\right] + 2k\left[\int_0^\infty \rho\Phi \cos(2k\rho \sin\omega)d\rho\right] > 0.$$

A numerical integration of the integrals involved shows that this condition is fulfilled.

In conclusion we may summarize the results of this investigation as follows. By using the Fermi statistical potentials in conjunction with the Born collision theory, we have derived a formula for the scattering of electrons in gases. The scattering formula here developed gives a finite number of particles scattered at small angles, $\Theta = 0$; the maximum scattering occurs at $\Theta = 0$, and the minimum at $\Theta = \pi$. This is in qualitative agreement with the results obtained by Sommerfeld, by considering all the electrons to be gathered together in the K-shell and treating the result as a hydrogen-like atom. The method of Sommerfeld, although it gives results in agreement with experiment, seems somewhat illogical in the light of our knowledge of the periodic table. On the other hand, the argument presented in this paper appears to be logical in so far as the calculation of atomic fields by the method of Fermi is logical. The latter, however, seems to be in agreement with the laws of the periodic system, since Fermi has used it to calculate in which atoms the s, p, and d terms appear and has found agreement with the facts. The method of Fermi has the disadvantage of mathematical complexity which does not allow us to readily calculate numerical values of scattering which can be compared with experiment.

The writer wishes to acknowledge his indebtedness to his colleagues of the Bartol Research Foundation, in particular to Prof. W. F. G. Swann and Dr. A. Bramley for many helpful suggestions in connection with this work.

The Ellipticity of the Earth is not a Proof of a Former Liquid State. JOHN F. HAYFORD. (*Am. J. Science*, Aug., 1928.) "The idea is often expressed, even by physicists in good standing, that the observed ellipticity of the earth is a proof of a former liquid state. I am here to challenge this idea. The challenge is not new, nor are the ideas I shall express new. But the frequency with which this mistaken idea is repeated shows that it should be challenged again, and in a direct and positive manner." The error seems to rest on an underestimate of the stress-differences that would be called into action by a departure of the actual ellipticity of the earth from the ellipticity that corresponds to its present rate of rotation. Sir George Darwin has subjected this matter to computation and "finds that if the present ellipticity differs from the ellipticity due to present rotation by one-eighth part, the stress-differences due to this cause which necessarily exist in the earth are one-half ton per square inch over the whole surface of the earth, are as a rule greater the greater is the depth below the surface, and are four tons per square inch at the center of the earth." The author now proceeds to consider the physical properties of granite and the safe stresses to which it may be subjected. The conclusion is that, were the earth composed of good granite throughout, it could not withstand the stress-differences cited above but would yield and assume a changed shape. Of all materials in the earth's crust granite is one of the strongest. A heterogeneous mass made up of granite and weaker constituents would yield more readily than granite alone. Moreover the presence of high temperature at the center of the earth and the persistance of the deforming forces through long periods assist in bringing about the same result. The author holds that "the departure of the actual ellipticity from that due to rotation must always have been and is now less than 1/160 part, even if the earth has always been and is now solid throughout." This is so small a part of the small ellipticity of the earth that it may well be masked by other factors and cannot be trusted as a basis for conclusions on the strength of the earth. Hence "the evidence afforded by the observed ellipticity of the earth is not sufficient to prove that the strength of the material in the interior of the earth, available to resist stress-differences, is now, or ever has been, so small as to justify the statement that said material is now or ever has been, a liquid."

G. F. S.

THE HERSCHEL EFFECT.*

BY

A. P. H. TRIVELLI.

HISTORY.

In 1840 Sir John F. W. Herschel published in the *Philosophical Transactions* an observation he had made on August 27, 1839:[2]

"A highly concentrated spectrum having been formed on muriated paper, and kept for some time on the same spot, a coloured picture was formed. On examining this picture by dispersed daylight, I now for the first time noticed, that in all that part of the paper on which the full red of the spectrum had fallen, there was an appearance of whiteness, a sort of white prolongation of, or appendage to, the dark photographic impression, contrasting itself with the general ground of the paper which, under the influence of dispersed light from passing clouds, sky, *etc.*, had acquired a very sensible discolouration. This discolouration had, therefore, as would appear, been counteracted by the red rays of the spectrum, which are, therefore, by no means to be regarded as inactive, but rather, at least in this instance, as exciting an action of an opposite nature to that of the blue, violet, and lavender rays." On the next page he wrote about the conservative whitening or oxidating action of the red rays.

It is evident that what we now term the Herschel effect is not the effect observed by Herschel. In recent literature, most authors regard the Herschel effect as a bleaching-out of the latent image by red or infra-red radiation. Originally, it was an effect on the direct visible photochemical image, observed on silver chloride. Since the visible photochemical decomposition of the silver halides shows so many analogies to the action of the latent image, there is no objection to acceptance of the same name for the corresponding effect

* Communication No. 383 from the Kodak Research Laboratories.

[1] *On the Chemical Action of the Rays of the Solar Spectrum, Etc.*, London 22 (1840). (Reprint.)

with the latent image. As we distinguish between the visible decomposition of the silver halides and the latent image, we shall have to distinguish between the visible Herschel effect and the latent Herschel effect. This paper deals with the latent Herschel effect only.

Herschel's explanation of the effect as an oxidation of the photochemical decomposition product of silver chloride is in agreement with the regression concept held by many of the present investigators, who mention Claudet as the first to explain the effect in this way.

It is unnecessary to give further history of the Herschel effect, since Lüppo-Cramer [2] has recently given a thorough account of it with a complete bibliography.

INTRODUCTION.

The investigation of the Herschel effect was begun, in these Laboratories, for several reasons: (1) For practical application to infra-red photography; (2) to obtain direct positives and duplicate negatives; (3) for collection of data on the latent image.

Following the publication by R. Terenin,[3] A. L. Schoen and R. M. Briggs [4] of the Department of Physics of these Laboratories investigated the practical application of the Herschel effect to infra-red photography. Working with various materials, they investigated spectroscopically the application of a suitable red filter for a simplified study of the Herschel effect. It was found that the Wratten No. 87 filter transmitted the most effective wave-lengths for plates treated with malachite green. Filters Nos. 89 and 88 cut too far into the visible spectrum and the radiation which they transmitted did not reverse a plate treated with malachite green but increased the density. Filters Nos. 740 and 724 produced a much weaker reversal than the No. 87 filter, indicating that they, also, cut too far into the red. From this it appears that the maximum reversal for the plate treated with malachite

[2] "Grundlagen der photographischen Negativverfahren," Eder's Handbuch, 2: 1927; "The Herschel Effect as a Regression Phenomenon," Seventh International Congress of Photography, London, 1928.

[3] *Z. Physik*, 23: 294 (1924).

[4] "Infra-red Photography by Means of the Herschel Effect, Report No. 2484/3a.

green is between 725 and 850 $m\mu$; beyond 850 $m\mu$ reversal becomes rapidly weaker, until at 1000 $m\mu$ very little reversal is obtained.

In their investigations A. L. Schoen and R. M. Briggs succeeded in producing results which were much better than those produced in the infra-red with plates bathed in dicyanine. Nevertheless, this work was discontinued after the discovery in these Laboratories of several very powerful optical sensitizers for the extreme red and the infra-red, which gave much better results than those obtained previously in the infra-red. The investigations showed that the best results for a Herschel effect were obtainable with filter No. 87, Preliminary experiments showed that to obtain a pronounced Herschel effect the intensity of the long wave-length exposure is of the greatest importance. Photographic films and plates, which often showed no trace of a Herschel effect in the spectrograph, showed this effect immediately upon increasing the intensity of the light source. Lüppo-Cramer [5] mentions: "Draper was aware of the importance of the intensity of the light in its bleaching action." It is clear that we have to deal with the failure of the reciprocity law. It is much easier to get high intensities of radiation with a light source and a filter, than with a spectroscope, as long as strictly monochromatic light is not required.

THE SIGNIFICANCE OF THE HERSCHEL EFFECT FOR THE LATENT IMAGE.

To some extent one is able to draw a parallel between the light action on photographic plates and certain chemical reactions. A well known example is the action of hydrogen peroxide on highly sensitive plates. The curves obtained are of the same shape as those obtained with blue light exposure.[6]

Dombrowsky and Lüppo-Cramer were able to obtain reversal with excess of hydrogen peroxide, just as solarization is obtained with overexposure. The latent image can be

[5] *Loc. cit.*

[6] Dombrowsky, Leipzig, 1911 (*Dissertation*). Lüppo-Cramer, *Z. wiss. Phot.*, **25**: 121 (1928); **25**: 129 (1928). *Phot. Korr.*, **64**: 49 (1928). S. E. Sheppard and E. P. Wightman, J. FRANK. INST., **195**: 337 (1923). E. P. Wightman and R. F. Quirk, J. FRANK. INST., **199**: 261, 731 (1927). E. P. Wightman, *J. Amer. Chem. Soc.*, **50**: 2923 (1928). E. P. Wightman and R. F. Quirk (to be published in *Proc. Seventh International Congress of Phot.*, Abstract in *Brit. J. Phot.*, **75**: 420 (1928).

intensified with small amounts of hydrogen peroxide or, as shown by M. L. Dundon, by means of a short after-exposure. Of all the substances which have, from time to time, been examined because of their activity on the photographic plate, hydrogen peroxide has shown itself the most suitable for quantitative investigation. Allylthiourea in a developer causes the plate to fog. If allylthiourea is added to a slow developer during the process of development, a reversal occurs which corresponds to the Sabatier reversal,[7] produced by active light falling upon an exposed photographic plate during the process of slow development. In all probability, therefore, the Waterhouse reversal [8] belongs to the same class of phenomena as the Sabatier reversal; the light action is replaced only by an equivalent chemical fogging action. If this is true, the sulfite reversal [9] by sulfite fog belongs in this same class of reversal phenomena.

Investigators of about 30 years ago wrote of the possibility of increasing the sensitivity of a photographic plate by pre-exposure. J. M. Eder [10] states that by exposing a silver bromide emulsion for 2 minutes to the red light of his dark-room lamp, he was able to increase the threshold speed from 15°–19° Warnerke; 6 minutes' pre-exposure gave an increase of 19°–20° Warnerke. Longer pre-exposure produced fog. Similar experiments were published by J. Gaedicke.[11] Experiments of recent date show very little or no increase of sensitivity by pre-exposure of photographic emulsions. During the last 30 years the limits of sensitivity of various photographic emulsions have been approached.

A parallel effect was noticed by sensitizing silver halide with silver sulfide. S. E. Sheppard [12] discovered that allylthiourea in slightly alkaline gelatin solutions forms small specks of silver sulfide on silver halide crystals. These specks sensitize the silver halide very powerfully. After this discovery of Sheppard, other investigators tried to sensitize with allylthiourea already highly sensitive photographic

[7] Eder's Handbuch, **2**: 82 (1898).
[8] *Brit. J. Phot.*, **37**: 601 and 613 (1890).
[9] C. E. K. Mees and C. W. Piper: *Brit. J. Phot.*, **58**: 491 (1911).
[10] Eder's Handbuch, **3**: 85 (1903).
[11] *Eder's Jahrbuch*, 122 (1898).
[12] *Phot. J.*, **65**: 380 (1925).

emulsions and found that the sensitizing effect was very small; in some cases the only effect was fog. Silver sulfide is a sensitizer only for silver halide that has not yet reached its maximum sensitivity. This limit is determined by the appearance of fog. Before a grain in an insensitive photographic emulsion becomes developable it must be affected by an exposure to light. This final exposure to light may be replaced in part by a pre-exposure or by a chemical reaction. This use of pre-exposure or chemical reaction to replace the pre-exposure is known as increasing sensitivity. Too much pre-exposure gives fog. Too much chemical action also gives fog.

Several fogging agents are able to give effects similar to those of silver sulfide, even if they are not of the same degree. Recently B. H. Carroll and D. Hubbard [13] were able to increase the sensitivity of silver halide with colloidal silver and colloidal gold.

On the other hand there are desensitizers which have the same reaction on the latent image as the extreme red radiation. Phenosafranine, well known through Lüppo-Cramer's [14] investigations on desensitizers, reduces, in higher concentrations, just as do Fe'''—and Cu''—salts. These salts are desensitizers of silver halide for ordinary exposure, but at the same time are sensitizers for the Herschel effect.[15]

There are certain dyes, which desensitize and at the same time produce fog. Methylene blue is a well known example. This double action, which seems contradictory, is not exceptional. Sensitizing of silver halide can be produced by various substances. It is possible that methylene blue forms with silver halide a complex compound in the form of developable centers and at the same time desensitizes already existing sensitivity centers of another chemical nature. Fog centers and sensitivity centers are not always of the same chemical composition. The most important fact regarding desensitization, however, is the discovery of S. E. Sheppard and E. P. Wightman [16] that potassium bromide is a desensitizer. This

[13] *Research Paper No. 20, Bur. of Standards J. Research*, 1: Oct., 1928.

[14] *Negativ entwicklung bei hellem Lichte.*

[15] B. H. Carroll, *J. Phys. Chem.*, **29**: 693 (1925).

[16] "Effect of Environment on Photographic Sensitivity: II. Effect of Certain Salts on Sensitivity, *Phot. J.*, **69**: 134 (1929).

was to be expected, since H. Arens [17] had already shown that potassium bromide is a sensitizer for the Herschel effect. By carefully washing with distilled water all of the potassium bromide from a photographic emulsion, the Herschel effect disappeared. Arens concludes that the Herschel effect is a desensitizing effect of potassium bromide in red light. As a matter of fact the Herschel effect is not a desensitizing phenomenon; it is a real destruction of the developability of the latent image already formed by some other exposure. Also, it seems better to change Arens' statement that desensitizing is a sensitized Herschel effect to the statement that desensitizers are, in general, sensitizers for the Herschel effect. It seems that the reverse is also true, sensitizers for the latent image are desensitizers for the Herschel effect. Most photographic emulsions contain more or less small amounts of free potassium bromide. Since little is known of desensitizers, an investigation of the Herschel effect is of importance in their study. Until now the quantitative study of the Herschel effect has been made on a limited scale. The purpose of this investigation is to extend this quantitative study and obtain more information relating to the action of desensitizers.

PHOTOGRAPHIC MATERIAL AND DEVELOPERS.

Preliminary investigations with various kinds of photographic emulsions showed that pure silver bromide emulsion gives a more pronounced Herschel effect than a silver bromoiodide emulsion. The silver bromide was precipitated using potassium bromide of which the iodide content was less than 0.002 per cent. The free potassium bromide content of the coated plates was found by A. E. Ballard of the Department of Analytical Chemistry of these Laboratories to be about 0.03 per cent. of the amount of silver bromide. The emulsion has a wide range of grain sizes as is shown by the size-frequency curves and photomicrographs in Fig. 1. For 4 minutes' development with pyro-soda at 68° F. (exposure made to acetylene light, screened to daylight with No. 79 Wratten filter) the sensitometric test showed a speed of 63 H. and D.; gamma, 1.12; with fog, 0.07. To investigate the Herschel

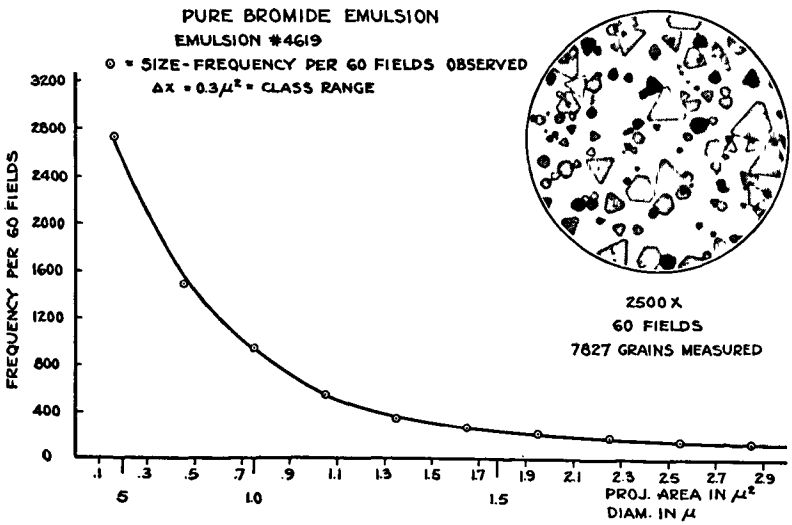
[17] *Z. physik. Chem.*, 114: 379 (1925).

effect, development was effected at 17° C., controlled by a thermostat, with brush [18] in pyro-soda:

Pyro (M/20)	6.304 grams
Sodium sulfite, anhydrous	25 "
Sodium carbonate	21.2 "
Potassium bromide (M/100)	1.19 "
Water to make	1 liter

The sulfur dioxide content of the sodium sulfite used was determined and the actual amount calculated for an equiva-

FIG. 1.



lent of 100 per cent. pure salt. Development was stopped at the required time with dilute acetic acid (1%); the plate was fixed in an acid fixing bath. Densities were read on a Jones' densitometer.[19]

THE SENSITOMETER.

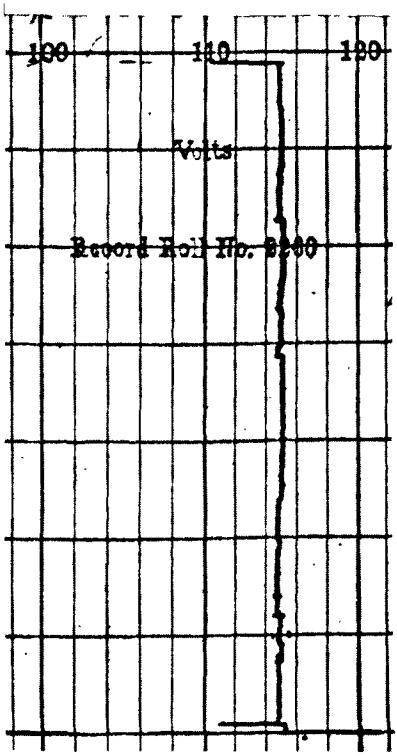
The exposures were made with a non-intermittent sensitometer of 26 steps of $\sqrt{2}$ ratio. The light intensity was kept constant while the time of exposure was varied; the maximum time of exposure was 579 seconds. The light source was a 500-candle power standard monoplane tungsten filament lamp operated at 104.8 volts, with a color temperature of 2700° K.

[18] *Cf.* W. Clark, *Phot. J.*, **65**: 76 (1925).

[19] *J. Opt. Soc. Amer.*, **7**: 230 (1923).

On account of the many and long exposures it was necessary to keep the light intensity as constant as possible. For this purpose a voltage regulator which maintained voltage constant to within one volt, was built by A. L. Schoen. A record is shown in Fig. 2 in which vertical lines represent

FIG. 2.



time, and every step, 10 minutes. The horizontal lines represent potential differences; the distance between two vertical lines representing 2 volts. Several tests were made. The deviations in potential correspond with less than 6 per cent. intensity difference in the blue light and with less than 3 per cent. intensity difference in the extreme red.

Both exposures were given without any change of the light source at one-meter distance from the photographic plate;

only the filter, the diaphragm, and the diffusion screen between light source and photographic plate were changed. The changes are given in the following table:

Exposure	Blue Light	Infra-red radiation
Filter	No. 49	No. 87
Total transmission	0.5 per cent.	Not determined
Wave-length transmission	505–535 $m\mu$	Beyond 700 $m\mu$
Diffusion medium	Opal glass	Ground glass
Diaphragm (diffusing media and diaphragm)	1.119 c.p.	444.66 c.p.

The blue filter transmits some infra-red light. Even when this transmission is of the same order as that of filter No. 87, the total intensity on the photographic plate would be 400 times less than with the infra-red radiation exposure. To absorb all infra-red radiation, filter No. 49 was combined with a cell, 2.5 cm. thick, containing a 5 per cent. solution of copper sulfate in water. No difference was noticeable when filter No. 49 was used alone or with the copper sulfate cell. Therefore, this effect can be neglected.

EXPOSURE AND DEVELOPMENT.

J. Plotnikow [20] suggested an investigation of the Herschel effect with cross exposures. In this way it is possible to investigate the Herschel effect with constant infra-red radiation on different blue light exposures, and different infra-red radiations on a constant blue light exposure.[21] The blue light exposure was made in 26 steps and the infra-red light exposure was then made perpendicularly upon it, also in 26 steps. Only a part of these exposed areas appeared in the developer. From the blue light exposure only 22 steps, and from the infra-red radiation only 12 steps were readable. Until now no investigations have been published on the action of the Herschel effect with time of development, so that many data had to be collected to determine the density-time of development curves. The plates were developed for 2, 3, 4, 5, 6, 7, 8, 10, 12, 16, 22, and 32 minutes. The same procedure was repeated 5 times, so that 60 plates were exposed and developed, each containing 22 by 12 densities. Not all these densities

[20] "Photochemie," 650 (1920).

[21] *Cf.* Lüppo-Cramer, "Die Grundlagen der photographischen Negativverfahren," 716 (1927).

were read but only as many as were needed for obtaining the required results.

TREATMENT OF DATA.

The results of averaging five sensitometric data are not sufficient for the determination of the final data. Compared with the development errors, the errors in exposure and density reading are negligible. Five sensitometric determinations are sufficiently reliable to determine the shape of the curve. The gamma of the curve for time of development was corrected by a method of smoothing suggested by L. A. Jones. Values from the H. and D. curves were plotted as functions of development time; the smoothed values from the density-time errors were then replotted as H. and D. curves. This may be repeated. The results obtained show that up to about 10 minutes' development, the curves are very reliable and represent the real data as nearly as it is possible under these circumstances to determine them. Beyond 10 minutes' development, differences in thickness of the emulsion influence the densities so greatly that the deviations in densities increase and the averaging becomes less accurate. No fog correction was applied to the data.

DISCUSSION OF THE DATA.

In Fig. 3 are given the H. and D. curves and in Fig. 4 are given the density-time of development curves for blue light exposures alone. For these and the next group of curves the density measurements have been carefully extended from 22 to 25 steps, but they were not drawn in the sets of H. and D. curves. Step 25 represents the increase of fog with time of development. In all cases the strongest exposure was represented by step 1.

In Fig. 5 are given the H. and D. curves and in Fig. 6 the density-time of development curves for blue light exposures, which have received an additional infra-red exposure for 579 seconds (log $E_r = 3.60$), the maximum infra-red exposure given in this series of experiments. In Fig. 6, step 25 represents the increase of density with time of development for infra-red exposure alone.

FIG. 3.

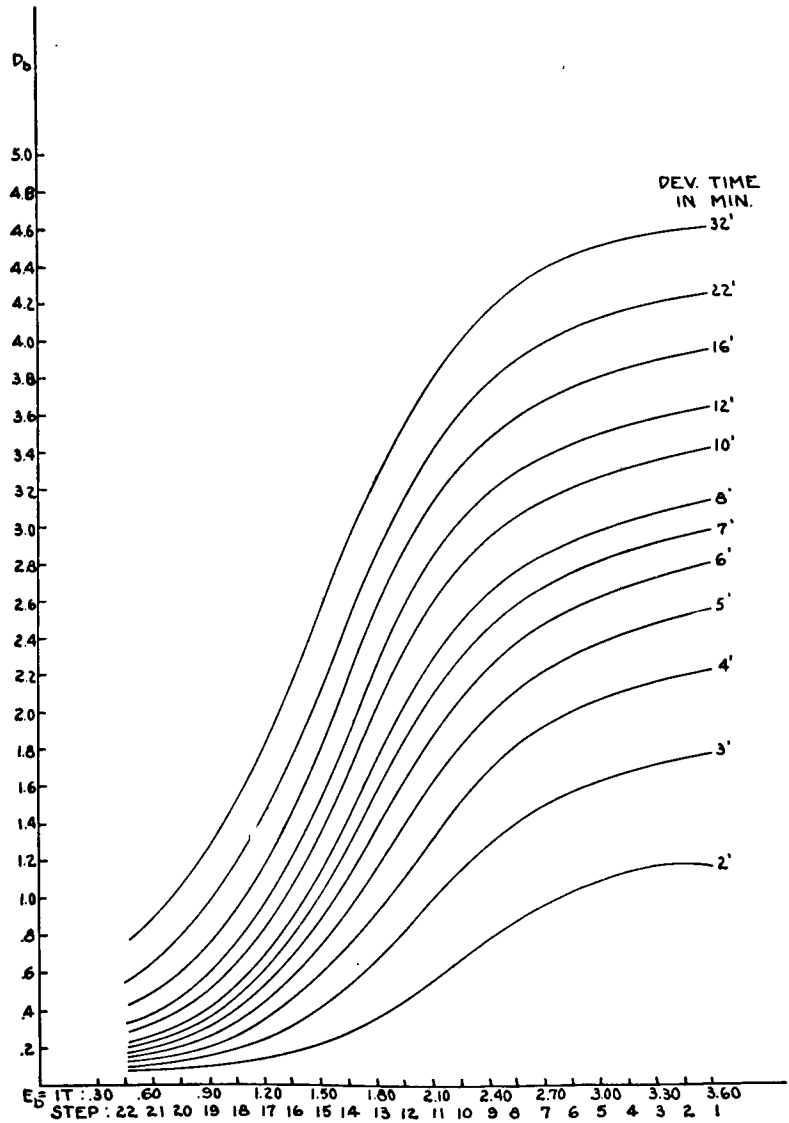

D_b
5.0
4.8
4.6
4.4
4.2
4.0
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
DEV. TIME IN MIN.
32'
22'
16'
12'
10'
8'
7'
6'
5'
4'
3'
2'
E_b = IT : .30 .60 .90 1.20 1.50 1.80 2.10 2.40 2.70 3.00 3.30 3.60
STEP : 22 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

FIG. 4.

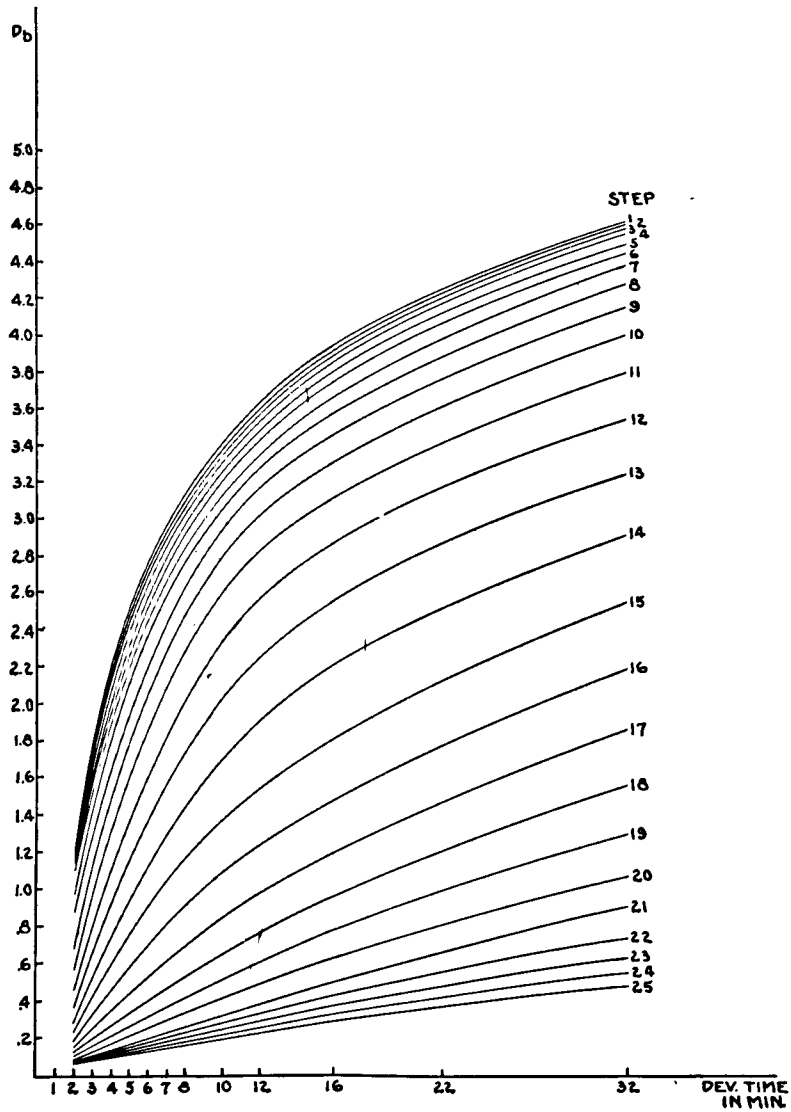

Db
5.0
4.8
4.6
4.4
4.2
4.0
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
STEP
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
1 2 3 4 5 6 7 8 10 12 16 22 32
DEV. TIME IN MIN.

FIG. 5.

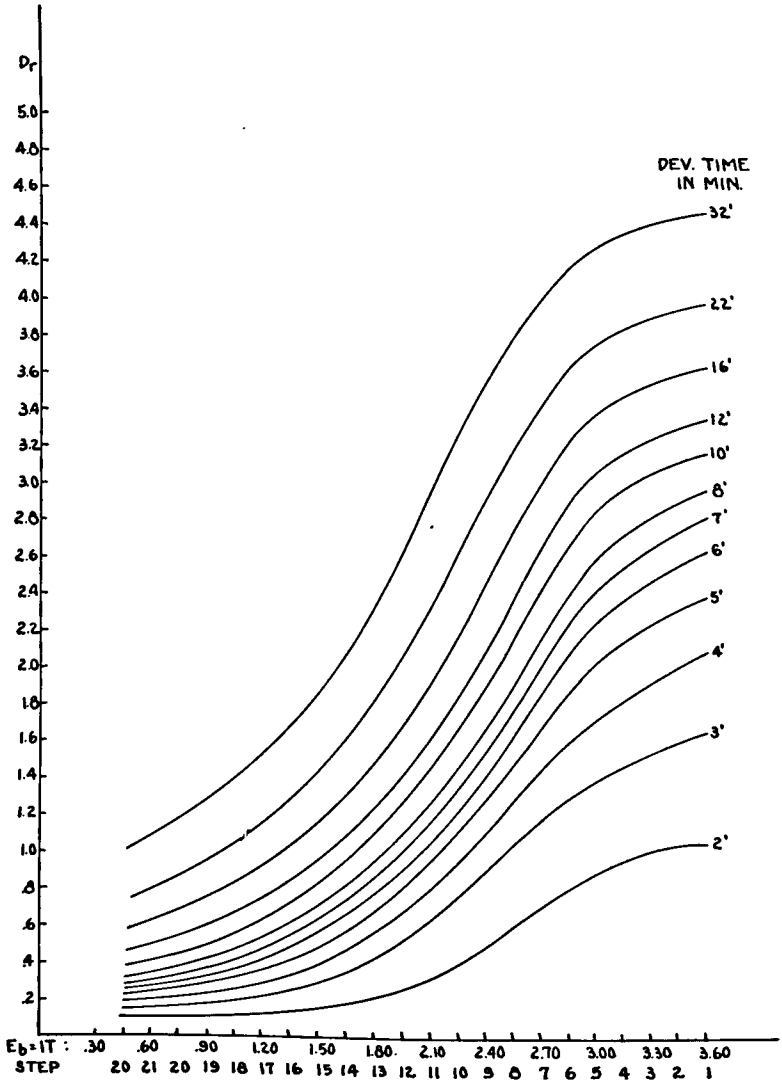
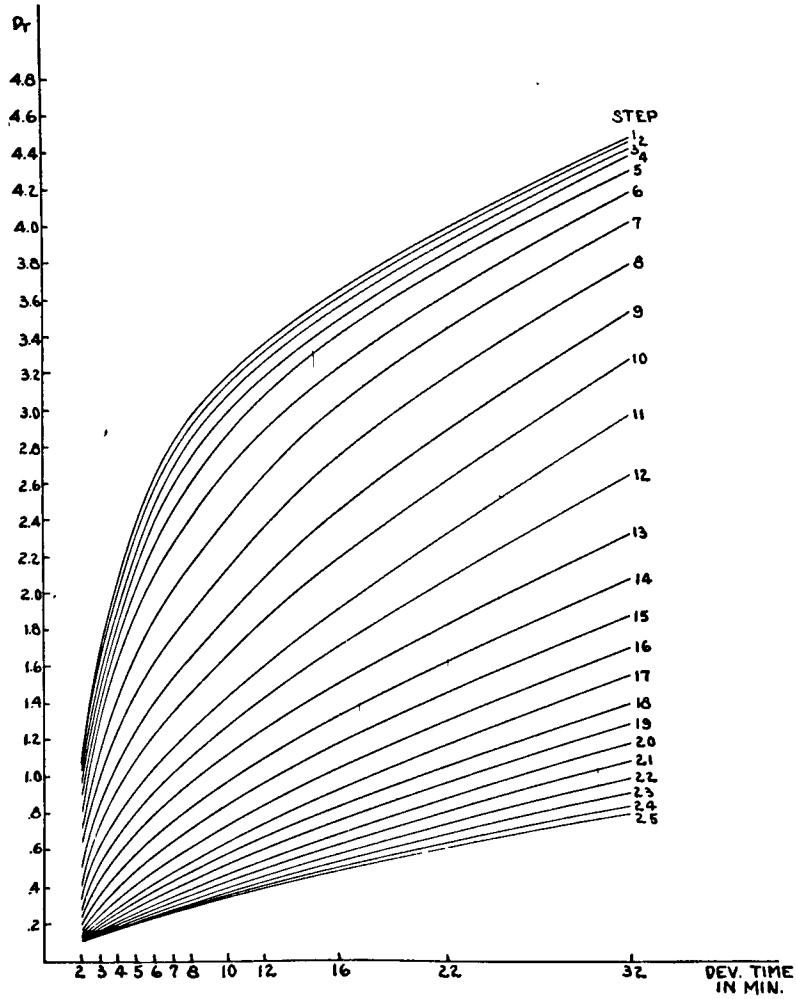

D_r
5.0
4.8
4.6
4.4
4.2
4.0
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
DEV. TIME IN MIN.
32'
22'
16'
12'
10'
8'
7'
6'
5'
4'
3'
2'
E_b=IT : .30 .60 .90 1.20 1.50 1.80 2.10 2.40 2.70 3.00 3.30 3.60
STEP 20 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

FIG. 6.


Pr
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STEP
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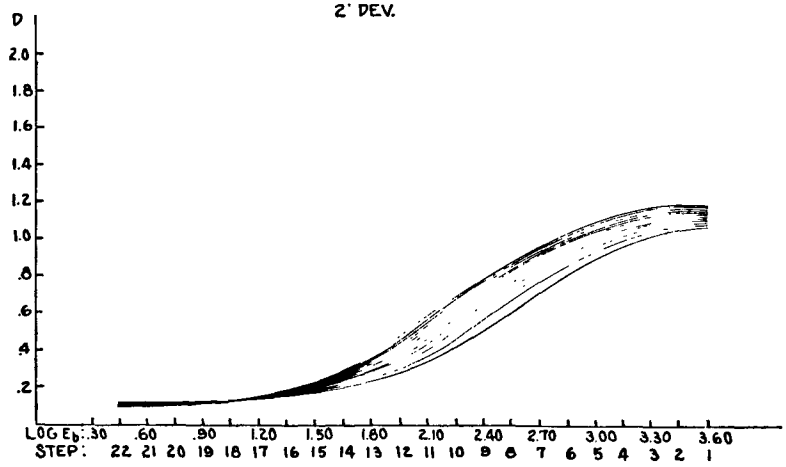
2 3 4 5 6 7 8 10 12 16 22 32
DEV. TIME IN MIN.

In comparing these curves, it is noticeable that their shapes change with a change in gamma:

Time of Development in Min.	Gamma (Blue Light Exposure)	Gamma (Blue and Infra-red Exposure)
2	0.72	0.70
3	1.07	0.97
4	1.36	1.18
5	1.58	1.34
6	1.76	1.47
7	1.91	1.58
8	2.02	1.66
10	2.19	1.75
12	2.29	1.82
16	2.40	1.91
22	2.49	1.98
32	2.52	2.00

From the complete data of the cross exposure only the 2, 4, 8, 16, and 32-minute developments are given in Tables I, II, III, IV, and V. These data can be plotted in two ways: Density against blue light exposure (E_b) and density against infra-red radiation exposure (E_r). Figs. 7, 8, 9, 10, and 11 show the

FIG. 7.



gradual decrease in densities with increasing infra-red exposure. The toe of the curve is of special interest since we see here that the infra-red exposure is too strong, in relation to the weak blue light exposures, to show any Herschel effect. The density increases with the secondary infra-red exposure, but always to a less degree until a certain neutral point is reached, after which the Herschel effect appears.

TABLE I.

2 MINUTES DEVELOPMENT.

Step	Infra-red →	1	2	3	4	5	6	7	8	9	10	11	12
Blue ↓	Log E	3.60	3.45	3.30	3.15	3.00	2.85	2.70	2.55	2.40	2.25	2.10	1.95
1	3.60	1.06	1.08	1.10	1.11	1.12	1.13	1.14	1.15	1.16	1.17	1.18	1.18
2	3.45	1.05	1.07	1.09	1.10	1.11	1.12	1.13	1.14	1.15	1.16	1.17	1.17
3	3.30	1.01	1.03	1.06	1.08	1.09	1.11	1.12	1.13	1.14	1.15	1.16	1.16
4	3.15	.96	.99	1.02	1.04	1.06	1.08	1.10	1.11	1.12	1.13	1.14	1.14
5	3.00	.89	.92	.96	.98	1.01	1.03	1.04	1.06	1.07	1.08	1.09	1.10
6	2.85	.80	.84	.88	.91	.94	.96	.98	1.00	1.01	1.02	1.03	1.04
7	2.70	.71	.76	.80	.83	.86	.88	.90	.92	.93	.94	.95	.96
8	2.55	.60	.65	.70	.73	.76	.79	.81	.83	.84	.85	.86	.87
9	2.40	.50	.56	.60	.64	.67	.70	.72	.74	.75	.76	.77	.78
10	2.25	.40	.45	.49	.52	.56	.58	.60	.62	.63	.64	.66	.67
11	2.10	.32	.36	.40	.42	.45	.47	.49	.51	.52	.54	.55	.56
12	1.95	.27	.29	.32	.34	.36	.38	.40	.41	.42	.43	.44	.45
13	1.80	.22	.24	.26	.28	.29	.30	.31	.32	.33	.34	.35	.36
14	1.65	.19	.20	.21	.22	.23	.24	.25	.26	.27	.28	.29	.29
15	1.50	.17	.17	.18	.19	.20	.21	.22	.22	.23	.23	.24	.24
16	1.35	.15	.16	.16	.17	.17	.18	.18	.19	.19	.20	.20	.20
17	1.20	.14	.14	.14	.15	.15	.15	.15	.16	.16	.16	.16	.16
18	1.05	.13	.13	.13	.13	.13	.13	.13	.13	.13	.13	.13	.13
19	.90	.12	.12	.12	.12	.11	.11	.11	.11	.11	.11	.11	.11
20	.75	.11	.11	.11	.10	.10	.10	.10	.09	.09	.09	.09	.09
21	.60	.10	.10	.10	.09	.09	.09	.09	.08	.08	.08	.08	.08
22	.45	.10	.10	.09	.09	.09	.08	.08	.08	.07	.07	.07	.07

TABLE II.

4 MINUTES DEVELOPMENT.

Step	Infra-red →	1	2	3	4	5	6	7	8	9	10	11	12
Blue ↓	Log E	3.60	3.45	3.30	3.15	3.00	2.85	2.70	2.55	2.40	2.25	2.10	1.95
1	3.60	2.09	2.11	2.12	2.14	2.16	2.17	2.18	2.19	2.20	2.21	2.22	2.22
2	3.45	2.01	2.04	2.07	2.09	2.11	2.13	2.14	2.16	2.17	2.18	2.19	2.20
3	3.30	1.92	1.97	2.01	2.04	2.06	2.08	2.10	2.12	2.14	2.15	2.16	2.16
4	3.15	1.83	1.89	1.94	1.98	2.01	2.04	2.06	2.08	2.10	2.11	2.12	2.12
5	3.00	1.72	1.80	1.86	1.91	1.95	1.98	2.00	2.02	2.04	2.05	2.06	2.07
6	2.85	1.59	1.69	1.76	1.81	1.86	1.90	1.93	1.96	1.98	1.99	2.00	2.01
7	2.70	1.44	1.56	1.65	1.71	1.76	1.80	1.84	1.86	1.89	1.91	1.92	1.93
8	2.55	1.27	1.40	1.50	1.57	1.62	1.67	1.71	1.74	1.76	1.78	1.80	1.82
9	2.40	1.10	1.23	1.32	1.40	1.46	1.50	1.55	1.59	1.62	1.64	1.66	1.68
10	2.25	.95	1.06	1.15	1.21	1.27	1.32	1.36	1.40	1.43	1.46	1.49	1.51
11	2.10	.80	.91	.98	1.04	1.10	1.14	1.18	1.21	1.24	1.27	1.29	1.31
12	1.95	.68	.76	.82	.87	.92	.96	.99	1.02	1.04	1.06	1.09	1.11
13	1.80	.57	.63	.68	.72	.76	.80	.82	.85	.87	.89	.91	.93
14	1.65	.47	.52	.56	.60	.63	.65	.68	.70	.72	.73	.75	.76
15	1.50	.40	.43	.47	.50	.52	.54	.55	.57	.58	.59	.60	.60
16	1.35	.33	.35	.37	.39	.40	.42	.43	.44	.44	.45	.46	.46
17	1.20	.28	.29	.30	.31	.32	.33	.33	.34	.34	.34	.34	.34
18	1.05	.24	.24	.25	.25	.25	.26	.26	.26	.26	.26	.26	.26
19	.90	.22	.21	.21	.21	.20	.20	.20	.20	.20	.20	.20	.20
20	.75	.20	.19	.19	.18	.18	.17	.17	.16	.16	.16	.16	.16
21	.60	.19	.18	.17	.16	.16	.15	.15	.14	.14	.13	.13	.13
22	.45	.18	.17	.16	.15	.14	.13	.13	.12	.12	.11	.11	.11

TABLE III.

8 MINUTES DEVELOPMENT.

Step	Infra-red →	1	2	3	4	5	6	7	8	9	10	11	12
Blue ↓	Log E	3.60	3.45	3.30	3.15	3.00	2.85	2.70	2.55	2.40	2.25	2.10	1.95
1	3.60	2.96	2.99	3.02	3.04	3.06	3.07	3.09	3.10	3.11	3.12	3.12	3.13
2	3.45	2.90	2.93	2.96	2.99	3.02	3.03	3.05	3.06	3.08	3.09	3.10	3.10
3	3.30	2.82	2.86	2.90	2.93	2.96	2.98	3.00	3.02	3.03	3.04	3.05	3.06
4	3.15	2.72	2.78	2.82	2.86	2.90	2.93	2.95	2.97	2.99	3.00	3.01	3.02
5	3.00	2.57	2.65	2.72	2.76	2.81	2.84	2.88	2.90	2.92	2.94	2.96	2.97
6	2.85	2.39	2.50	2.59	2.65	2.70	2.75	2.79	2.82	2.84	2.87	2.89	2.91
7	2.70	2.14	2.31	2.42	2.51	2.58	2.63	2.68	2.72	2.76	2.79	2.82	2.84
8	2.55	1.89	2.10	2.24	2.35	2.44	2.51	2.58	2.63	2.67	2.70	2.73	2.75
9	2.40	1.66	1.90	2.06	2.18	2.28	2.38	2.45	2.51	2.55	2.58	2.61	2.63
10	2.25	1.45	1.65	1.80	1.94	2.05	2.16	2.25	2.32	2.38	2.42	2.46	2.48
11	2.10	1.26	1.41	1.55	1.66	1.78	1.88	1.97	2.05	2.12	2.19	2.22	2.27
12	1.95	1.10	1.21	1.33	1.42	1.52	1.61	1.70	1.78	1.86	1.93	1.98	2.02
13	1.80	.95	1.04	1.13	1.22	1.30	1.38	1.45	1.52	1.58	1.64	1.69	1.73
14	1.65	.82	.89	.96	1.02	1.08	1.15	1.20	1.26	1.31	1.36	1.40	1.44
15	1.50	.71	.76	.80	.85	.90	.94	.98	1.02	1.06	1.09	1.12	1.15
16	1.35	.61	.64	.67	.70	.74	.76	.79	.82	.84	.86	.88	.90
17	1.20	.54	.56	.58	.60	.61	.63	.74	.76	.77	.78	.79	.80
18	1.05	.48	.48	.49	.50	.50	.51	.52	.52	.53	.54	.54	.54
19	.90	.42	.42	.42	.42	.42	.42	.42	.42	.42	.42	.42	.42
20	.75	.38	.37	.36	.35	.34	.34	.34	.33	.33	.33	.32	.32
21	.60	.34	.33	.32	.31	.30	.29	.28	.28	.27	.27	.26	.26
22	.45	.32	.30	.29	.28	.27	.26	.25	.24	.23	.23	.22	.22

TABLE IV.

16 MINUTES DEVELOPMENT.

Step	Infra-red →	1	2	3	4	5	6	7	8	9	10	11	12
Blue ↓	Log E	3.60	3.45	3.30	3.15	3.00	2.85	2.70	2.55	2.40	2.25	2.10	1.95
1	3.60	3.62	3.67	3.72	3.76	3.79	3.82	3.85	3.87	3.89	3.91	3.93	3.94
2	3.45	3.59	3.64	3.69	3.73	3.76	3.80	3.82	3.85	3.87	3.89	3.90	3.91
3	3.30	3.54	3.60	3.65	3.70	3.73	3.76	3.79	3.81	3.83	3.85	3.87	3.88
4	3.15	3.47	3.53	3.59	3.64	3.68	3.71	3.74	3.77	3.79	3.81	3.83	3.84
5	3.00	3.37	3.45	3.51	3.57	3.61	3.65	3.68	3.71	3.74	3.76	3.78	3.80
6	2.85	3.22	3.32	3.40	3.46	3.51	3.56	3.59	3.63	3.66	3.69	3.72	3.74
7	2.70	3.00	3.14	3.24	3.33	3.39	3.44	3.50	3.53	3.57	3.60	3.63	3.66
8	2.55	2.73	2.92	3.06	3.17	3.25	3.32	3.39	3.44	3.48	3.52	3.54	3.57
9	2.40	2.42	2.62	2.80	2.96	3.08	3.18	3.26	3.32	3.36	3.40	3.42	3.45
10	2.25	2.16	2.36	2.54	2.69	2.84	2.96	3.05	3.14	3.20	3.24	3.28	3.30
11	2.10	1.91	2.10	2.28	2.44	2.59	2.72	2.83	2.92	2.99	3.04	3.08	3.10
12	1.95	1.68	1.83	1.98	2.12	2.24	2.38	2.49	2.60	2.68	2.76	2.81	2.84
13	1.80	1.50	1.63	1.76	1.87	1.98	2.09	2.19	2.28	2.36	2.43	2.50	2.54
14	1.65	1.32	1.42	1.52	1.61	1.71	1.80	1.88	1.95	2.02	2.09	2.15	2.20
15	1.50	1.17	1.24	1.31	1.38	1.44	1.51	1.57	1.62	1.67	1.72	1.77	1.81
16	1.35	1.04	1.08	1.13	1.17	1.22	1.26	1.30	1.34	1.38	1.41	1.44	1.47
17	1.20	.94	.96	.99	1.02	1.04	1.07	1.09	1.11	1.13	1.15	1.17	1.19
18	1.05	.84	.85	.86	.88	.89	.90	.92	.93	.94	.95	.96	.96
19	.90	.76	.76	.76	.76	.76	.76	.76	.76	.76	.76	.76	.76
20	.75	.68	.67	.66	.65	.64	.63	.62	.62	.62	.61	.61	.61
21	.60	.61	.59	.58	.56	.55	.54	.53	.52	.52	.51	.51	.50
22	.45	.56	.54	.51	.49	.48	.46	.45	.44	.43	.42	.42	.41

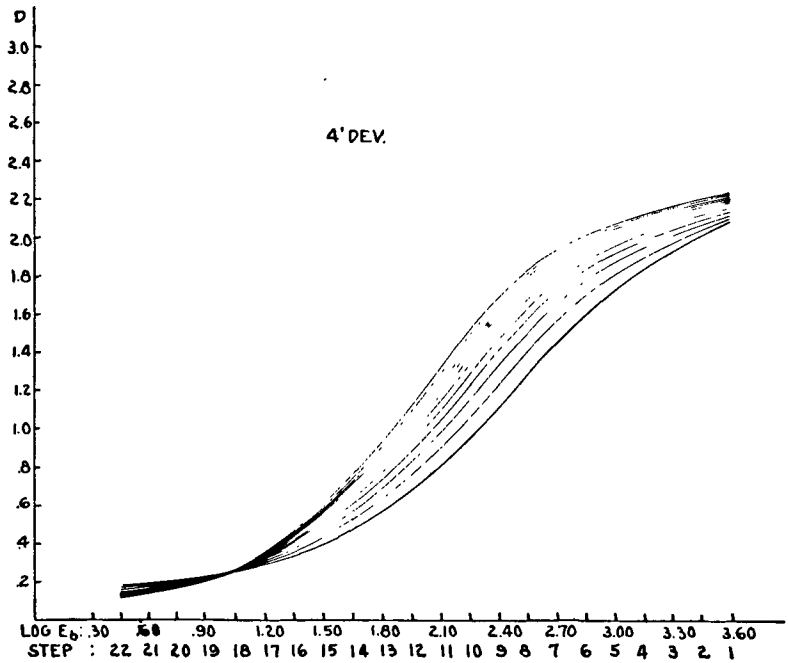
TABLE V.

32 MINUTES DEVELOPMENT.

Step	Infra-red →	1	2	3	4	5	6	7	8	9	10	11	12
Blue ↓	Log E	3.60	3.45	3.30	3.15	3.00	2.85	2.70	2.55	2.40	2.25	2.10	1.95
1	3.60	4.46	4.49	4.51	4.53	4.55	4.56	4.57	4.58	4.59	4.59	4.60	4.60
2	3.45	4.44	4.47	4.49	4.51	4.53	4.54	4.55	4.56	4.57	4.57	4.58	4.58
3	3.30	4.40	4.43	4.45	4.47	4.49	4.50	4.51	4.52	4.53	4.54	4.55	4.56
4	3.15	4.36	4.39	4.41	4.43	4.46	4.48	4.49	4.50	4.51	4.52	4.52	4.53
5	3.00	4.28	4.32	4.35	4.38	4.40	4.42	4.44	4.45	4.46	4.47	4.48	4.49
6	2.85	4.16	4.21	4.25	4.28	4.31	4.34	4.37	4.39	4.40	4.42	4.43	4.44
7	2.70	3.98	4.04	4.10	4.15	4.19	4.22	4.25	4.28	4.31	4.33	4.35	4.36
8	2.55	3.76	3.85	3.92	3.98	4.03	4.08	4.12	4.15	4.19	4.22	4.24	4.27
9	2.40	3.52	3.66	3.76	3.83	3.89	3.94	3.99	4.02	4.06	4.09	4.12	4.14
10	2.25	3.24	3.40	3.52	3.60	3.68	3.74	3.79	3.83	3.88	3.92	3.95	3.98
11	2.10	2.93	3.13	3.26	3.36	3.44	3.50	3.56	3.61	3.66	3.70	3.74	3.78
12	1.95	2.61	2.78	2.91	3.01	3.11	3.19	3.27	3.33	3.39	3.43	3.48	3.51
13	1.80	2.32	2.44	2.56	2.66	2.76	2.86	2.95	3.03	3.10	3.17	3.21	3.24
14	1.65	2.07	2.17	2.27	2.36	2.44	2.52	2.60	2.68	2.75	2.81	2.86	2.90
15	1.50	1.86	1.94	2.01	2.08	2.16	2.22	2.28	2.34	2.40	2.44	2.49	2.53
16	1.35	1.68	1.73	1.78	1.83	1.88	1.93	1.98	2.02	2.06	2.09	2.12	2.15
17	1.20	1.53	1.56	1.59	1.62	1.65	1.68	1.71	1.73	1.76	1.78	1.80	1.82
18	1.05	1.40	1.41	1.43	1.44	1.46	1.47	1.48	1.50	1.51	1.51	1.52	1.52
19	.90	1.29	1.29	1.29	1.29	1.29	1.29	1.29	1.29	1.29	1.29	1.29	1.29
20	.75	1.18	1.16	1.15	1.14	1.12	1.11	1.10	1.09	1.08	1.08	1.07	1.07
21	.60	1.08	1.05	1.02	1.00	.98	.96	.94	.93	.91	.90	.89	.88
22	.45	.99	.95	.91	.87	.85	.82	.80	.79	.78	.77	.76	.75

Figs. 12, 13, 14, 15, and 16 show the influence, on the blue light latent image, of gradually increasing the infra-red exposure. The data are plotted from the lowest infra-red exposure, as log $E_r = 1.95$. These curves show that the infra-red exposure is able to give a number of different curves

FIG. 8.

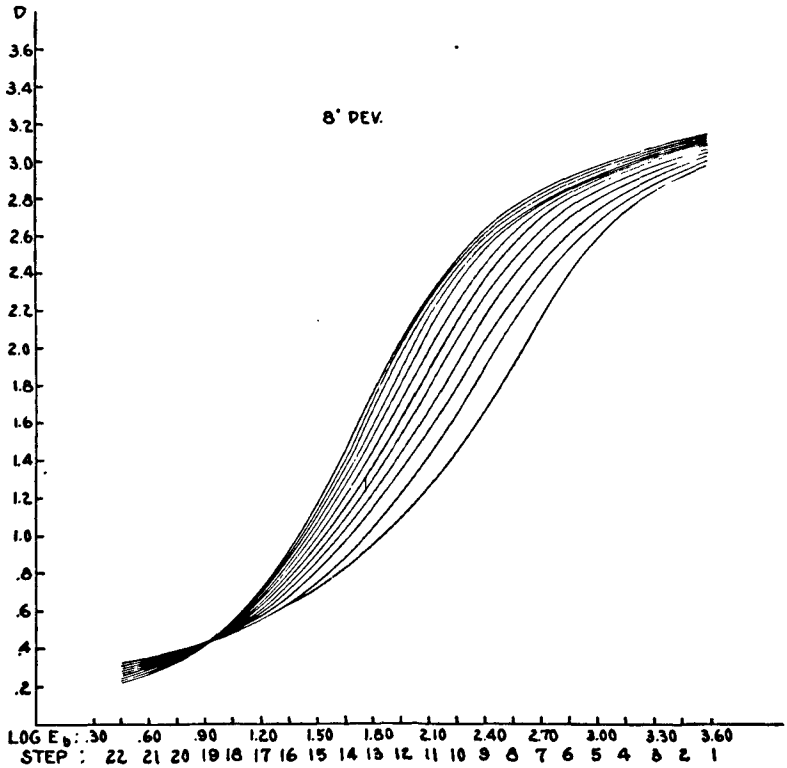


of constant infra-red intensity depending on the amount of the first blue light exposure and the time of development. In spite of the fact that the highest infra-red exposures are unable to form more than the toe of the reversal curve, it is clear that many H. and D. curves varying in form and gamma, are probably possible, if the plates are sensitized with proper sensitizers and more complete Herschel effects are obtained. This makes the sensitized Herschel effect probably suitable for the making of duplicate negatives and direct positives.

If we express the Herschel effect as the difference between the density of the blue light exposure (D_b) and the density of the blue light exposure with the additional infra-red exposure

(D_r) thus: $(D_b - D_r)$ we can collect the data in Table VI, where only the data with the strongest infra-red exposure (log $E_r = 3.60$) are taken for the values of D_r. Table VI shows that with increasing time of development the neutral point changes from step 18 to step 19. There are two maximum Herschel effects,—one with variable first blue light

FIG. 9.

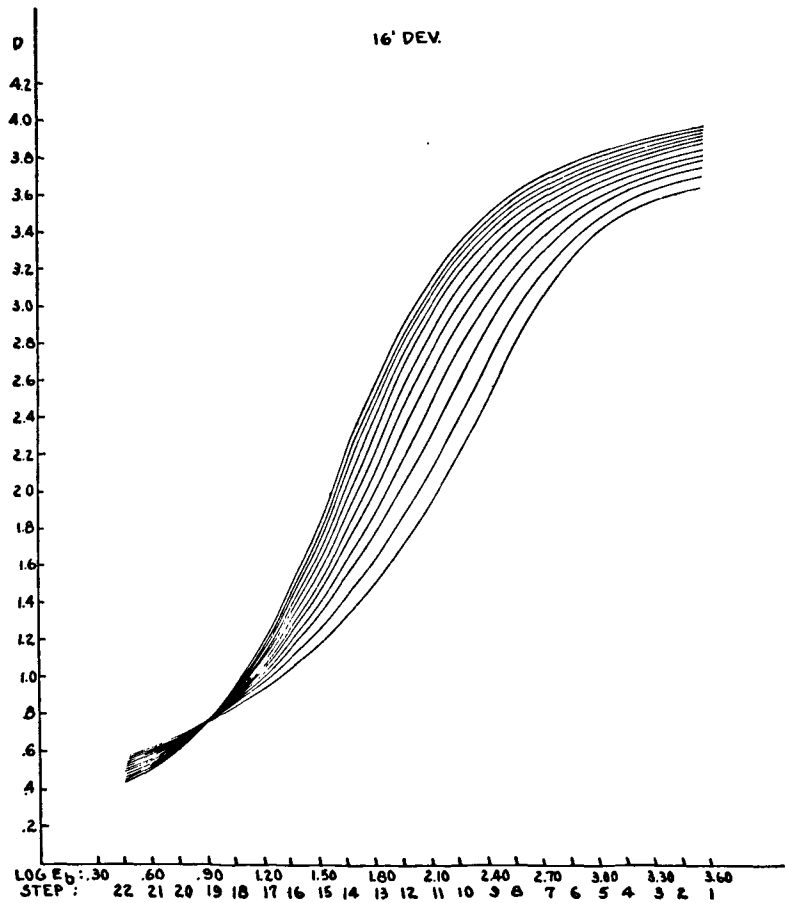


exposure and one with variable time of development. The maximum densities are placed in black squares with all densities $\pm$ 0.03. We see that with development time one maximum moves from step 9 with 2 minutes' development to step 13 with 32 minutes' development. The other maximum moves from 16 minutes' development at step 1 to 10 minutes' development at step 11; and then again back to 22 minutes' and 32 minutes' development at step 18.

The absolute maximum lies at step 11 with 10 minutes' development.

The Herschel effect can also be expressed as the ratio of the two densities D_r/D_b. The data for the strongest infra-red

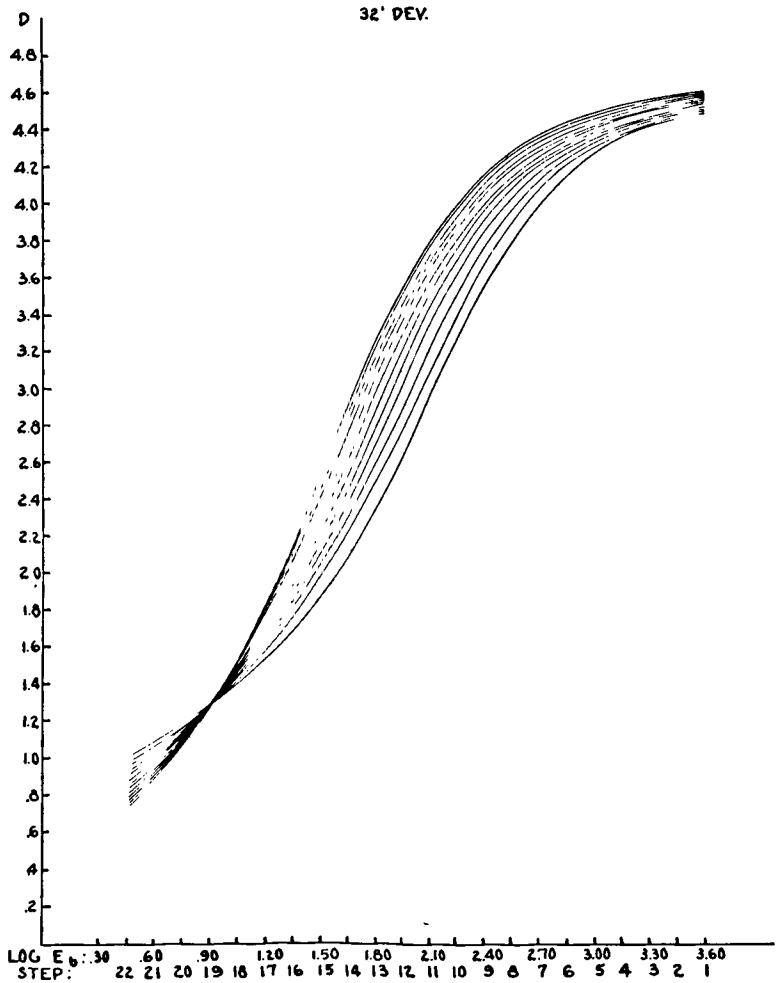
FIG. 10.



exposure, $E_r = 3.60$, are collected in Table VII. The numbers obtained are multiplied by 10^3. The smaller the number obtained therefore, the greater the Herschel effect. Again, two different maxima appear.

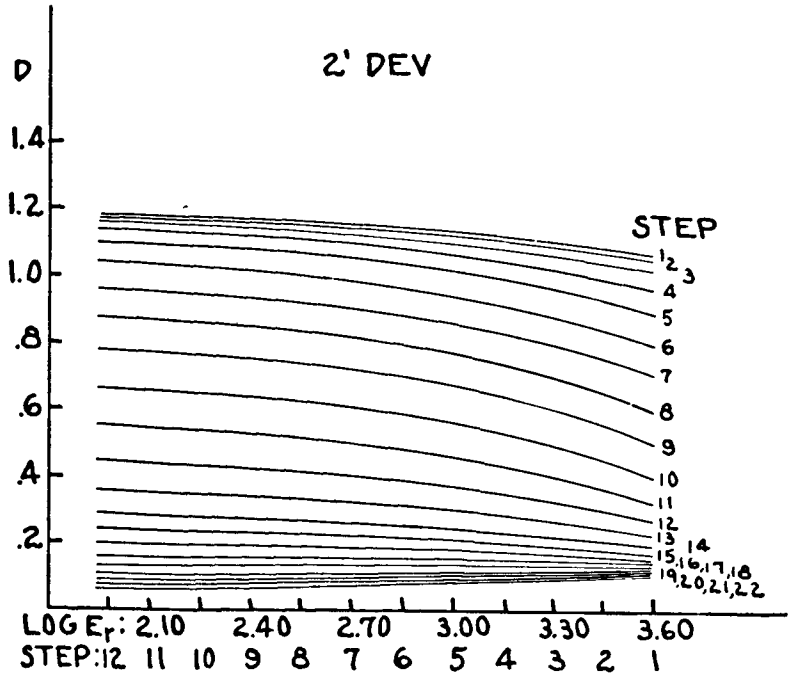
The maximum with different first blue light exposures lies

FIG. 11.


32° DEV.
D
4.8
4.6
4.4
4.2
4.0
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
LOG E .30 .60 .90 1.20 1.50 1.80 2.10 2.40 2.70 3.00 3.30 3.60
STEP: 22 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

at step 11 with 2 minutes' development and moves to step 14 with 32 minutes' development. The maximum with various times of development moves from step 1–7 with 2 minutes' development to step 18 with 16 minutes' development.

FIG. 12.



The absolute maximum lies at step 12 with 10 minutes' development.

The ratio D_r/D_b expresses the Herschel effect in the unity of density of the first blue light exposure. The same thing can be applied to the first expression of the Herschel effect by expressing the effect per unit of density of the first blue light exposure. We get then

$$\frac{D_b - D_r}{D_b}.$$

Multiplying by 10^2, we get the well known percentage formula; the data are collected in Table VIII.

FIG. 13.

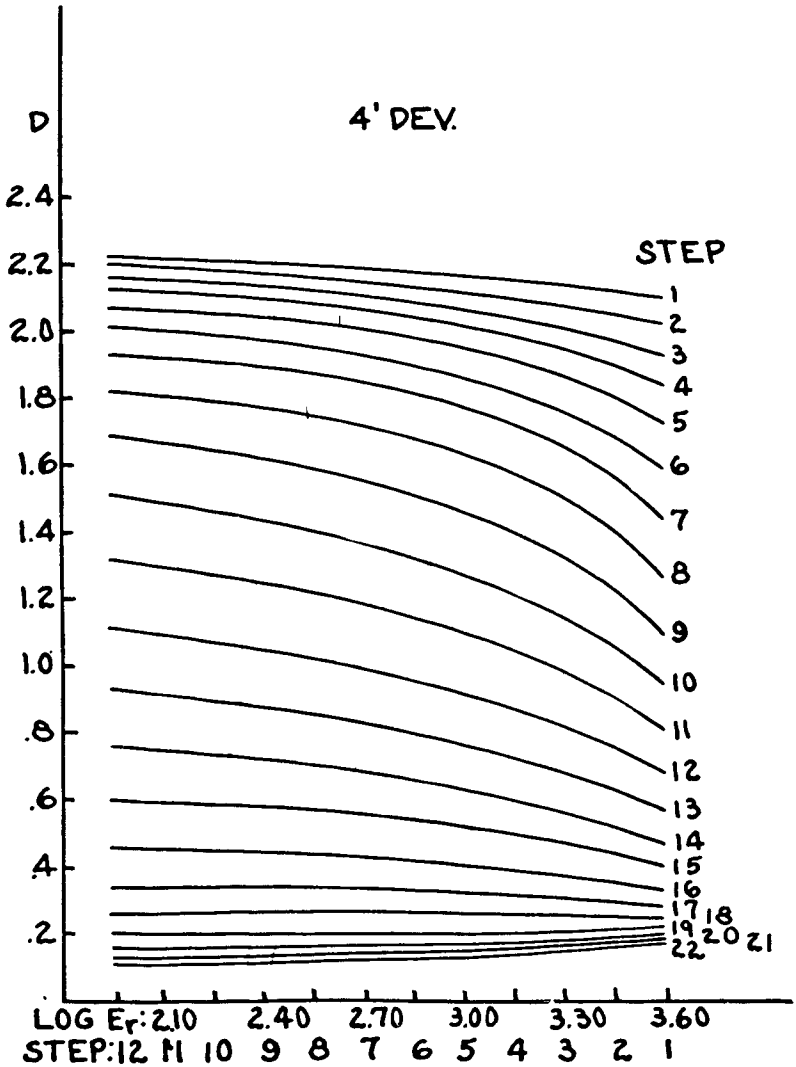

D
4' DEV.
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
STEP
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
LOG Et: 2.10 2.40 2.70 3.00 3.30 3.60
STEP: 12 11 10 9 8 7 6 5 4 3 2 1

FIG. 14.

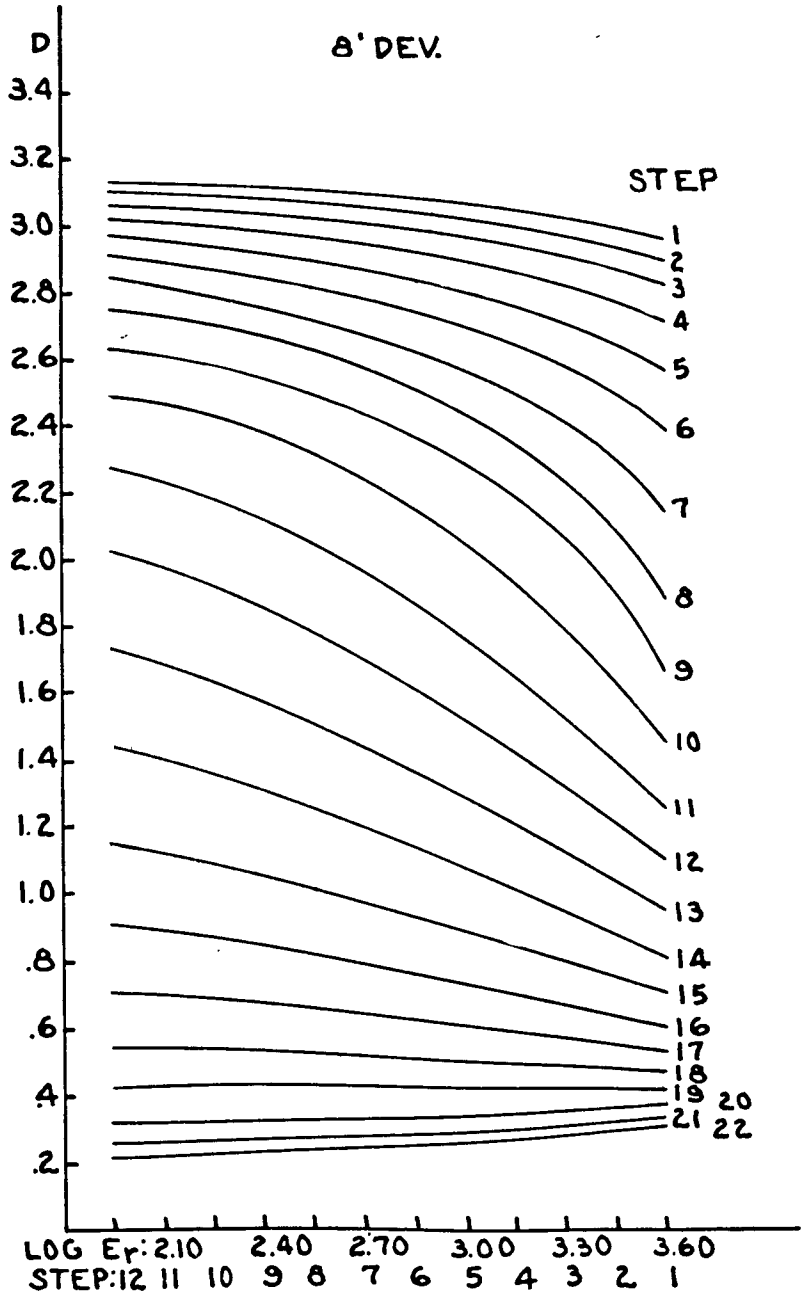

D
8' DEV.
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
STEP
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
LOG Er: 2.10 2.40 2.70 3.00 3.30 3.60
STEP: 12 11 10 9 8 7 6 5 4 3 2 1

FIG. 15.

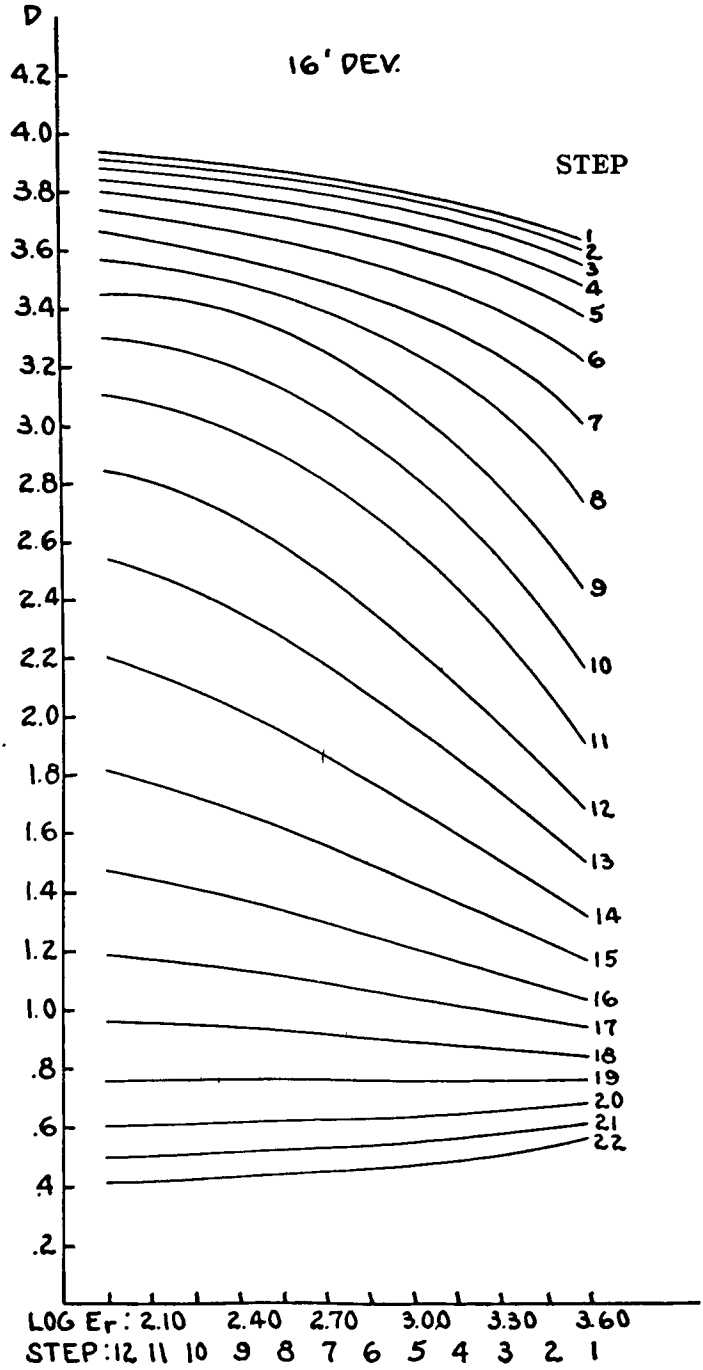
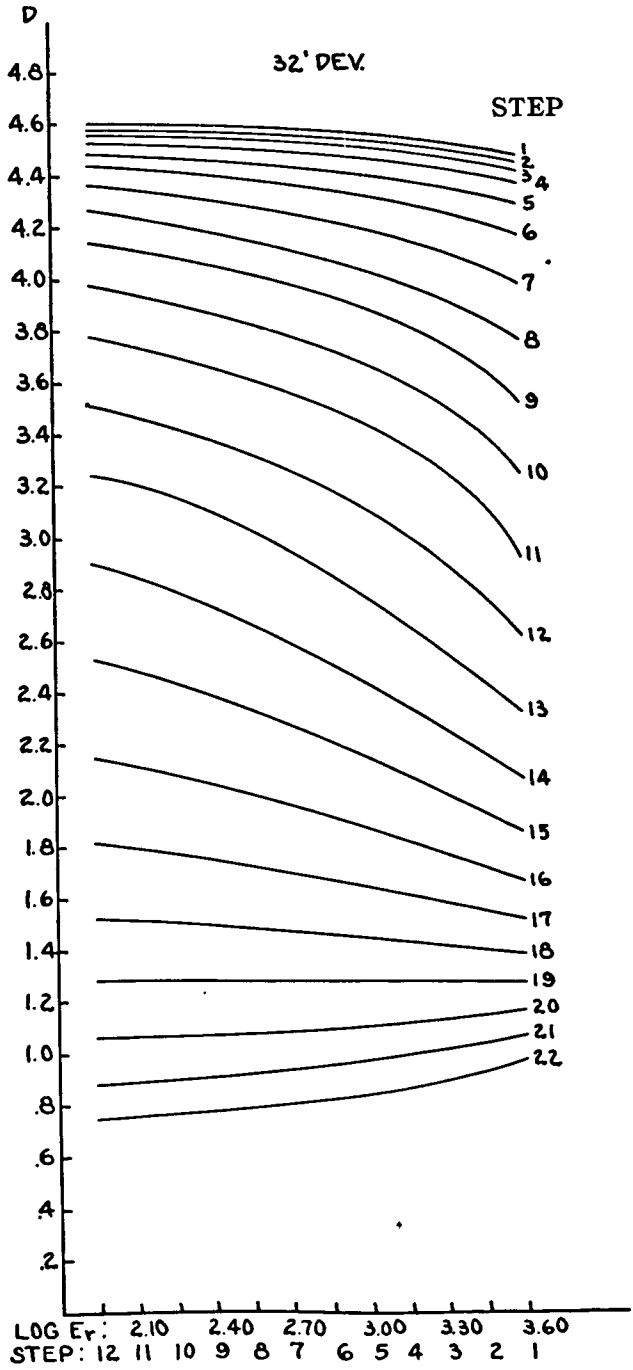

D
16' DEV.
STEP
4.2
4.0
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
LOG E: 2.10 2.40 2.70 3.00 3.30 3.60
STEP: 12 11 10 9 8 7 6 5 4 3 2 1

FIG. 16.


D
32' DEV.
STEP
4.8
4.6
4.4
4.2
4.0
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
LOG Er: 2.10 2.40 2.70 3.00 3.30 3.60
STEP: 12 11 10 9 8 7 6 5 4 3 2 1

Again we find there two series of maxima: A maximum with blue light exposure and a maximum with time of development.

TABLE VI.

$D_b - D_r$.

LOG $E_r = 3.60$.

Step	Log E_b	2′	3′	4′	5′	6′	7′	8′	10′	12′	16′	22′	32′	Dev. Time	
1	3.60	.12	.12	.13	.16	.17	.16	.17	.25	.29	.32	.27	.14		
2	3.45	.12	.15	.19	.20	.20	.19	.20	.27	.30	.32	.28	.14		
3	3.30	.15	.18	.24	.25	.24	.24	.24	.29	.32	.34	.29	.16		
4	3.15	.18	.21	.29	.30	.31	.29	.30	.33	.36	.37	.31	.17		
5	3.00	.21	.25	.35	.34	.38	.40	.40	.40	.43	.43	.36	.21		
6	2.85	.24	.28	.42	.46	.49	.49	.52	.55	.55	.52	.44	.28		
7	2.70	.25	.34	.49	.57	.62	.66	.70	.72	.74	.66	.58	.38		
8	2.55	.27	.39	.55	.66	.75	.80	.86	.92	.93	.74	.72	.51		
9	2.40	.28	.41	.58	.71	.83	.90	.97	1.06	1.08	1.03	.88	.62		
10	2.25	.27	.40	.56	.71	.85	.94	1.03	1.15	1.18	1.14	1.00	.74		
11	2.10	.24	.35	.51	.66	.80	.91	1.01	1.26	1.21	1.19	1.09	.85		←max.
12	1.95	.18	.27	.43	.58	.71	.82	.92	1.07	1.13	1.16	1.09	.90		
13	1.80	.14	.21	.36	.50	.60	.69	.78	.92	1.00	1.04	1.01	.92		
14	1.65	.10	.17	.29	.39	.48	.54	.62	.70	.79	.88	.88	.83		
15	1.50	.07	.11	.20	.28	.34	.39	.44	.54	.58	.64	.66	.67		
16	1.35	.05	.07	.13	.18	.22	.24	.29	.36	.38	.43	.46	.47		
17	1.20	.02	.03	.06	.08	.12	.14	.16	.19	.22	.25	.31	.29		
18	1.05	.00	.00	.02	.02	.04	.04	.06	.07	.09	.12	.14	.12		
19	.90	−.01	−.01	−.02	−.03	.00	.00	.00	−.01	.00	.00	.00	.00		
20	.75	−.02	−.03	−.04	−.05	−.04	−.05	−.06	−.06	−.07	−.07	−.09	−.11		
21	.60	−.02	−.03	−.06	−.06	−.07	−.07	−.08	−.08	−.11	−.11	−.16	−.20		
22	.45	−.03	−.03	−.07	−.07	−.08	−.08	−.10	−.10	−.13	−.15	−.21	−.24		
									↑ max.						

The maximum with exposure moves from step 11 with 2 minutes' development to step 14 with 32 minutes' development.

The maximum with time of development moves from 2

minutes' development at steps 1–7 to step 18 with 16 minutes' development.

The absolute maximum lies at step 11 with 10 minutes' development.

TABLE VII.

$$\frac{D_r}{D_b} \times 10^3.$$

LOG $E_r = 3.60$.

Step	Log E_b	2′	3′	4′	5′	6′	7′	8′	10′	12′	16′	22′	32′	Dev. Time
1	3.60	898	933	941	937	939	946	946	927	920	919	936	969	
2	3.45	897	914	914	921	927	935	935	920	917	918	934	969	
3	3.30	871	895	889	899	911	917	921	913	910	912	931	965	
4	3.15	842	875	863	877	884	898	901	900	898	904	925	962	
5	3.00	809	848	831	857	854	857	865	876	876	887	912	953	
6	2.85	769	823	791	802	808	814	821	828	839	861	891	931	
7	2.70	740	776	746	744	749	752	753	769	780	820	854	913	
8	2.55	690	725	698	689	682	689	687	698	715	765	814	880	
9	2.40	641	685	655	641	628	631	631	639	658	701	766	850	
10	2.25	597	655	629	608	589	588	585	589	609	654	722	814	
11	2.10	571	646	611	588	570	562	555	554	571	616	679	775	
12	1.95	600	667	613	579	562	554	544	541	557	591	653	743	←max.
13	1.80	611	681	613	573	565	560	549	544	553	590	642	716	
14	1.65	655	685	618	594	579	585	567	568	582	600	646	714	
15	1.50	708	738	666	632	622	621	617	606	623	646	686	735	
16	1.35	750	788	718	695	686	700	677	667	691	707	739	781	
17	1.20	875	880	823	818	773	774	675	771	770	790	789	841	
18	1.05	—	—	923	937	900	913	888	891	880	875	882	921	
									↑ max.					

Three different expressions for the Herschel effect are given, of which the last two express the effect for unity blue light density and which give the same maxima for different blue light exposures and for different times of development. Table VII gives quantitative data, which are less practical than those in Table VIII. It is much more convenient to

express a greater Herschel effect in a higher number than in a smaller number. The percentage formula, therefore, is the most suitable for practical application.

TABLE VIII.

$$\frac{D_b - D_r}{D_b} \times 10^2.$$

LOG E_r = 3.60.

Step	Log E_b	2′	3′	4′	5′	6′	7′	8′	10′	12′	16′	22′	32′	Dev. Time
1	3.60	10.17	6.74	5.86	6.27	6.07	5.39	5.39	7.33	7.99	8.12	6.37	3.04	
2	3.45	10.26	8.57	8.64	7.94	7.25	6.46	6.45	7.99	8.33	8.18	6.63	3.06	
3	3.30	12.93	10.47	11.11	10.08	8.86	8.28	7.84	8.68	8.99	8.76	6.92	3.51	
4	3.15	15.79	12.50	13.68	12.30	11.61	10.18	9.93	10.00	10.20	9.63	7.46	3.75	
5	3.00	19.09	15.24	16.91	14.29	14.56	14.29	13.47	12.34	12.34	11.31	8.76	4.68	
6	2.85	23.08	17.72	20.90	19.83	19.22	17.88	17.87	17.19	16.08	13.90	10.86	6.31	
7	2.70	26.04	22.37	25.39	25.56	25.10	24.81	24.65	23.08	22.02	18.03	14.57	8.71	
8	2.55	31.03	27.46	30.22	31.13	31.78	31.13	31.27	30.16	28.44	20.73	18.56	11.94	
9	2.40	35.90	31.54	34.52	35.86	37.22	36.89	36.88	36.05	34.18	29.85	23.40	14.98	
10	2.25	40.30	34.48	37.09	39.23	41.06	41.23	41.53	41.07	39.07	34.54	27.78	18.59	
11	2.10	42.86	35.35	38.93	41.25	43.01	43.75	44.49	48.46	42.90	38.39	32.06	22.49	←max.
12	1.95	40.00	33.33	38.74	42.03	43.53	44.57	45.54	45.92	44.31	40.84	34.71	25.64	
13	1.80	38.89	31.82	38.71	42.74	43.48	43.95	45.09	31.51	44.64	40.94	35.81	28.39	
14	1.65	34.48	31.48	38.16	40.63	42.11	41.54	43.06	41.42	41.80	40.00	35.34	28.62	
15	1.50	29.17	26.19	33.33	36.84	37.78	37.86	38.26	39.42	37.66	35.36	31.43	26.48	
16	1.35	25.00	21.21	28.26	30.51	31.43	30.00	32.22	33.33	30.89	29.25	26.14	21.86	
17	1.20	12.50	12.00	17.65	18.18	22.64	22.58	20.00	22.89	22.92	21.01	21.09	15.93	
18	1.05	—	—	7.69	6.25	10.00	8.70	11.11	10.94	12.00	12.50	11.77	7.89	
									↑ max.					

The data given above show that the maximum Herschel effect in these experiments lies at step 11 and 12 with 10 minutes' development. This occurs with a blue light exposure, which gives a density from 2.33–2.60.

SUMMARY.

1. To correct some historical statements concerning its origin and theory the discovery of the Herschel effect is republished.

2. The results of the investigations of A. L. Schoen and R. M. Briggs, who showed that Wratten filter No. 87 is the most suitable for the Herschel effect, are given.

3. The significance of the Herschel effect for the study of desensitizers of the latent image is discussed.

4. It is shown that the Herschel effect shows maximum effects determined by the amount of the first blue light exposure and the time of development.

5. A quantitative expression for the Herschel effect is discussed.

My thanks are due to Mr. R. P. Loveland, who started this investigation with me and made several of the sensitometer determinations published in this paper.

The Floating Needle Experiment. F. E. BEACH. (*Am. J. Science*, Nov., 1928.) A needle can be made to float upon the surface of water. It is usual to describe it as resting in a trough in a yielding elastic film, much as a nail would lie on a horizontal sheet of rubber. The author has found this to be untrue. For the floating needle to rest in a trough water must not adhere to the steel, but by touching the needle to a water surface and withdrawing it it is seen that water adheres. It is further said that oiling the needle slightly makes it easier to float. This also the author denies.

When the needle floats, all of it is below the horizontal plane passing through the undistorted surface of the water. On the assumption that the cross-section of the needle is circular, the water surface bends gently downward until it meets the steel surface much above the center of the needle making an angle of nearly 90° with the surface. Most of the needle is in the water below the horizontal plane through the line of contact between steel and water, while a small part projects above this plane into the air.

Owing to the nature of its ends a needle is likely to plunge into the water. A ring 5 cm. in diameter was made from steel wire of a millimeter diameter and could be readily floated when it was quite clean, but it was nearly impossible to make it float after it had been slightly oiled. For quantitative purposes a tube of aluminium 6 mm. in diameter was bent into a closed ring or torus 6 cm. in diameter. It weighed 7.13 grams and the water met it at almost the same angle as it met steel. When clean it floated readily, but it could not be made to float after it was rubbed with oleic acid. The condition that the ring should float is that its weight equal the sum of 3 forces, viz.,

a. The upward pull of the bent water surface within and outside of the torus.

b. The weight of the water displaced, lying below the horizontal plane through the lines of contact between metal and water.

c. The force of buoyancy due to the absence of water from the space over the ring.

Measurements were made and each of these forces was calculated. They were respectively 1.360 gm., 4.720 gm. and 1.196 gm. Their sum was 7.276 gm., which differs by only $2\frac{1}{2}$ per cent. from the weight of the torus 7.13 gm.

When the water surface was dusted with lycopodium powder the torus rested in a trough since the liquid did not wet the metal. Again the sum of the three supporting forces just about equaled the weight of the ring. G. F. S.

AIRCRAFT DETECTORS AND THE ANTI-AIRCRAFT PROBLEM.

BY

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Ordnance Department, U. S. A.

THERE are three things in which most civilians feel a proprietary interest—these are the Army, the Navy and the Marine Corps. It is my purpose tonight, in a brief space of time, to describe one of the many phases of activity in which this Army of yours is engaged so that you will have an idea of our problems and how they are solved. There are those who think that Army officers, in time of peace, spend their efforts figuring out how to start a war. That, of course, is as ridiculous as it would be to accuse our doctors of prescribing medicine to cause ill health so as to have plenty of patients, or of accusing firemen of being anxious to have conflagrations and policemen of inciting burglaries, and the like.

The Army, in time of peace, prepares not for war, but *against* war.

Science today is traveling along in what we might call "second gear." The last century science moved slowly but established many important and basic facts. The twenty-five years just past have experienced a shift from this "low gear" of scientific progress. The advances made in radio, television, automobiles, airplanes, textiles, talking movies, and many other mechanical and electrical devices illustrate this point.

I well remember the thrill which I received one day twenty years ago while still a boy but filled with enthusiasm over scientific problems. My laboratory was a table and some shelves in my bedroom where miscellaneous electrical and wireless equipment were set up. A wireless telegraph set had been developed with which I had no difficulty in tuning in many of the code transmitting stations in the vicinity of New York City. A newly purchased silicon detector was a highly prized part of the set, and I desire to state that equipment was much harder to obtain in those days than it is today.

On this day as I listened in, I thought I heard voices, and with a little careful adjustment and tuning I brought up the signal strength to a point where I knew I was listening to a conversation—one of the first of radio telephone transmissions, which I then suspected, and afterward verified, was taking place between Dr. Lee de Forest's Laboratory in the Metropolitan Tower and another experimental station in Newark, New Jersey. I was about ten miles from these two stations.

The next day I was to meet with reverses, however, for the housemaid in making up my bed upset an electrolytic interrupter filled with a sulphuric acid solution, the result being a large hole in the bed spread and another in the bedroom carpet. I do not remember if I was promptly ejected from my laboratory like Edison after his baggage car explosion, but soon after I established my laboratory in a small shanty where the members of the household would not interfere with scientific progress.

From the day of de Forest's audions to the present, radio has developed with amazing rapidity.

Soon we are to shift over again into a period of high powered scientific development such as the world has never dreamed of. Aircraft, television and like items, now in their infancy, will be brought to perform way beyond our greatest hopes.

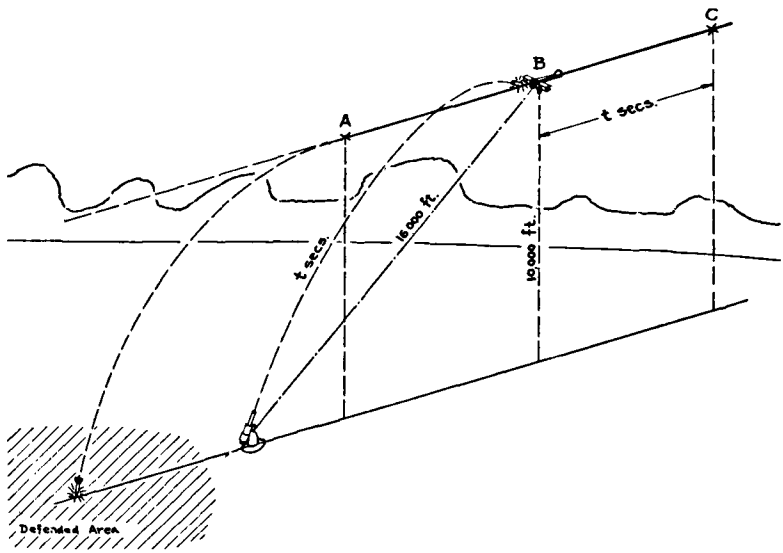
The Army in time of peace must keep up a high grade research program so as to be ready for any emergency that may arise. If the war comes, we must have the best guns, powders, fire control instruments, transportation, aircraft and methods of warfare that it is possible to develop. We are a great research laboratory. Many people think that all one has to do to start a war is to go out, call the men together and go into it. But who is to provide the arms and equipment, the tactical methods, the guns, tanks, optical instruments, sanitary equipment, messing facilities—the myriad of things that go to make up an Army in the field? The foreign armies are keeping up to date, and if we do not, there is a terrific waste of life and money if war comes.

I stood, this spring, on a sand dune at Fort Story on the Virginia coast and watched the firing of a battery of four 16-inch howitzers at a target ten miles away. Each of the rounds

fired weighed 2,000 pounds. The target was about the size of this room and the shots fell close enough to it to have been hits had it been a warship. An error of half a degree in setting the guns would have thrown the projectile off several hundred yards. The mechanism to do this is as complicated as a locomotive and may be even more carefully constructed for the great accuracy required.

By maintaining a modern Army with modern equipment, we may thwart off any desire of a foreign nation to engage us in war. In the last war, it is well known that the Germans discounted at very little the amount that the United States could do. Of course, they were unpleasantly surprised but we might have been saved from the war if we had been known as a prepared nation.

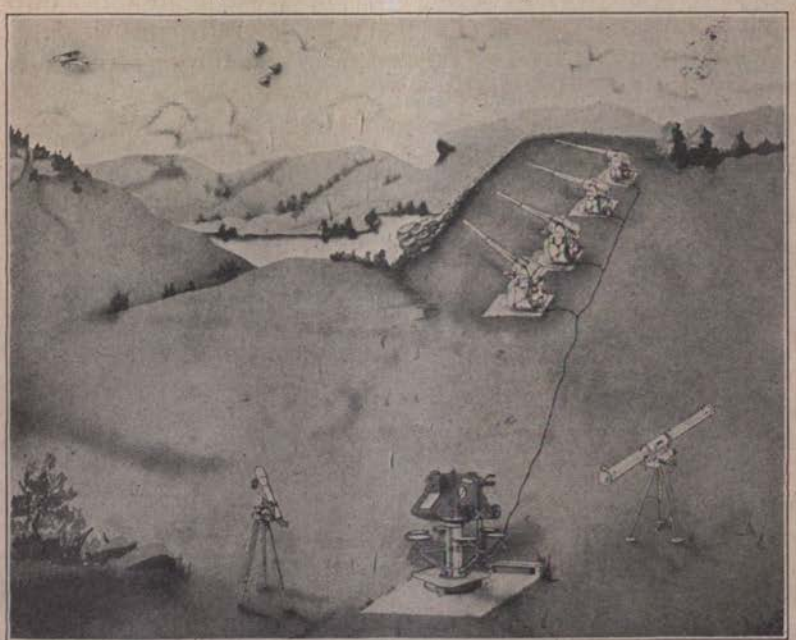
PLATE I.



One of our most urgent problems at present is that of defense against aircraft. The airplane is a small, exceedingly swift target traveling in three dimensional space, and thus has added to previous firing problems the elements of speed and altitude. In three minutes an airplane can come from a distance at which it is a nearly invisible speck to a position overhead from which it can drop its destructive bombs. At

night the plane cannot be seen at all. And here I wish to state that while machine gun fire is used on low flying planes of any class, it is only the bombing plane at high altitudes which it is profitable to attack by ground fire. War must be conducted on a paying basis, and the cost of firing at other than bombing planes, as compared with the worth of the targets and effect, is too great.

PLATE 2.



A battery of anti-aircraft guns with latest fire-control equipment.

Plate I shows a diagram of an area to be defended and an airplane flying toward this area. A bomb released at point *A* would strike within the area. An anti-aircraft gun must fire so as to hit the target at some such point as *B* ahead of *A*. If *B* is 10,000 feet high and three miles away, it will take the high explosive projectile fired 15 seconds to reach *B* so that it must be fired when the target is at *C*, the travel of the plane through *BC* being equal to the time of flight of the projectile.

It is evident that the sighting of the target, computation of data, setting and loading of the gun and firing must take place in a very small period of time. A modern battery of four 3-inch A. A. guns can place a barrage of 100 shells on the line of travel of an airplane within two minutes of the first sighting of the plane.

Plate II shows diagrammatically the set-up of a modern battery of A. A. guns. These guns are automatically set and controlled from a remote mechanism called a director, into which is entered the angular travel of the airplane through telescopes manned by two observers. The target altitude is obtained by a height finder and automatically set into the director. Having the course and height of the target, the director computes automatically and instantly the proper predicted setting for the guns. To this are added corrections for the effect of wind, temperature, velocity of the projectile and other ballistic factors, all of which are automatically set into the guns.

The projectiles are loaded and the pieces are fired with a rapidity of 15 to 28 shots per minute for each gun depending upon the caliber. As one looks in the direction of the tiny target, white puffs of smoke are seen indicating the shell bursts all around the objective.

And here I wish to present the description of a future bit of warfare as I am wont to picture it. In the light of recent demonstrations, at the Army Proving Ground, of the rapid strides made in anti-aircraft material, I do not feel that I have over-estimated the possibilities of near future development.

The faint purr of airplanes is heard in the distance, the sound of several planes mingling into one—doubtless an attack squadron. But almost instantly after we have heard the sound, a sharp crack is heard to the flank and this followed in rapid succession by others and still others, and up in the clouds a tiny flash and then a series of flashes are seen. The purring of the airplanes is drowned out by the crack and boom of anti-aircraft artillery. A tiny flaming meteor is seen to drop from where the flashes were seen; another is seen to start at a lower point. The guns cease firing. Silence reigns in the darkness of a slightly cloudy night. No sound of air-

plane is heard nor other sound except those incident to any night in the country.

We have pictured the successful repulse of an air raid of the future by a battery of anti-aircraft artillery, the surprise of a group of aviators who, traveling without light or other visible manifestation of their presence, have found themselves without warning in an inferno of bursting shells with no knowledge that they were even under observation. Some fortunate ones have maneuvered, dropped from the inferno and turned about, their mission unaccomplished. Others have gone the way.

But how were these bursts placed with such accuracy in utter darkness and no visible or audible indication of preparation?

A large amount of our daily activity depends upon sight for its execution; much of our daily pleasure and enjoyment is obtained through seeing things. The artilleryman in war uses his various sights to set his guns upon the target, just as a surveyor sights through a transit or level. The principal activities of mankind are carried on during daylight hours when objects can be seen. But what happens when light fails with the falling of night, or even during light hours when objects cannot be seen by virtue of screening or other diversion of the light rays? The auditory system comes into play to guide us; to warn of the automobile bearing upon us from behind, to warn of the approach of trains, to guide boats in foggy weather, to assist the blind man as he taps along the street with his cane, to tell one from whence this or that sound comes and to warn us on many occasions.

Some people are born with big ears. Science has provided a means of augmenting the auditory powers of others who would hear into the distance. The second in importance of our senses of perception is that of hearing.

The enemy airplane in future wars, approaching under cover of darkness, in fog or in smoke screen, will be heard and located at distances perhaps greater than the ranges of visual instruments in daylight. The sound locator is one of the newest pieces of ordnance and is part of the fire control equipment of a modern anti-aircraft battery. It came into being during the late war in Europe like many other instruments of

war. The old story of the appearance of a new offensive weapon calling for a new means of defense was repeated. The sound locator is a part of the defense used against the airplane, a new and powerful weapon which sprang into existence as a war machine during the last great war and has steadily gained in importance in this capacity. A large part of the mission of the airplane is accomplished at night under the cover of darkness when visual means for detection cease to function. It is then that we turn to our second great sense of perception, which is only sharpened by darkness and the stillness of night, and seek out the thrum of airplane motors and the whir of the propellers by the aid of acoustics. Just as we use telescopes or field glasses to aid the vision in seeing more clearly or more distantly and to determine direction, so we use sound locators to amplify sounds so that we can hear further and to give us the directions of these sounds.

In conjunction with the ears, the sound locator depends for its action on what is called the binaural phase effect, briefly explained as follows:

Sound is a vibratory disturbance in the atmosphere, or other medium, wherein a pressure wave is set up traveling from the sound source in all directions at the rate of 1,100 feet per second. This sound wave arrives at a listener's head and unless he is looking in the direction of emission, it will impinge upon the diaphragm of one ear a fraction of a second sooner than upon that of the other ear. The resulting sensations transmitted to the brain are there analyzed by point of time so that instinct tells the listener that the sound source is at a certain angle to the right or left. If now the listener turns his head the proper amount, the sound wave strikes both ears simultaneously and is analyzed as coming from the front. This ability of the ears and brain to determine direction acoustically is called the binaural sense; the effect of the sound wave on the ears is called the binaural phase effect.

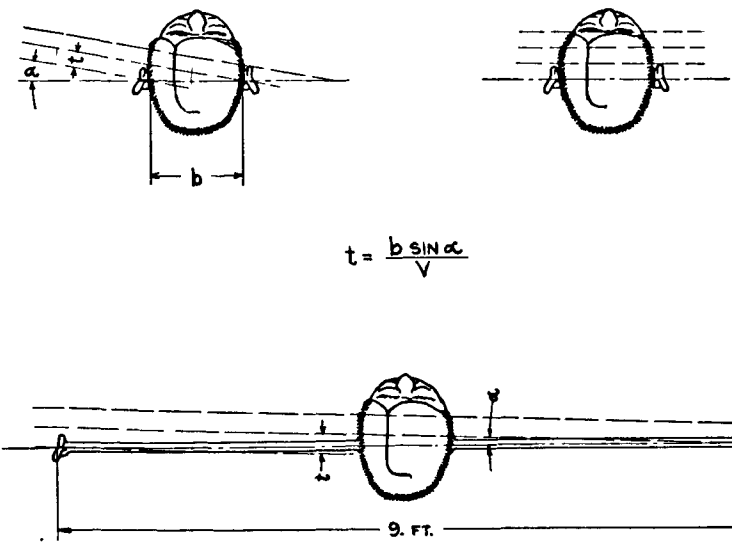
It is evident that if the ears were separated by a greater distance, the difference in time of a sound wave striking one diaphragm over that of the other would be increased and the binaural sense would be sharpened.

Referring to Plate III, if α is the angle of approach of a sound wave and the least value of this which the human ears

can determine is 5° then $t = b \sin \alpha / v$. t equals 40 micro seconds for this value. If we retain this value of t' but lengthen the base line between receivers to 9 feet

α equals 20′ of arc or 1/3°.

PLATE 3.



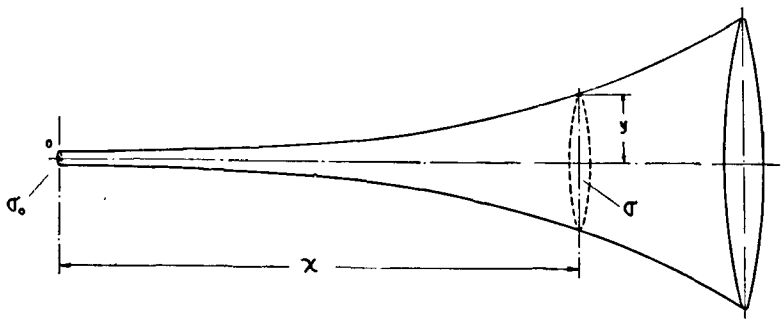
That is exactly what is accomplished by using a sound locator; two large horns separated by a distance, perhaps twenty times that of the ears, are supported always parallel to each other with their small ends terminating at the ears. The sound wave entering both horns travels through each independently until it strikes the ear diaphragms at different instants. The binaural sense immediately tells which way the pair of horns must be swung in order to compensate for the time difference or phase difference. The listener aims to reduce the phase difference to zero at which time his ears will receive the sound impulse "in phase" and the horns will be pointed at the source of sound. The direction can then be read from scales graduated in degrees.

Besides obtaining a greater separation of the receivers by extending the ears through the use of horns, amplification of the sound is obtained by properly designed horns. A horn

upon receiving a continuous series of sound impulses, acts upon these in such a way as to combine their energy and produce a louder sound. There will be losses of energy through reflection and friction but the proper horn design will cut these to a minimum and give the maximum concentration of energy at the small end.

The most efficient type of horn known at present is the exponential horn. The profile of this horn, or its cross-sectional area follows the simple exponential law expressed

PLATE 5.



$\sigma = \sigma_0 e^{mx}$

$e = 2.718$

Exponential horn.

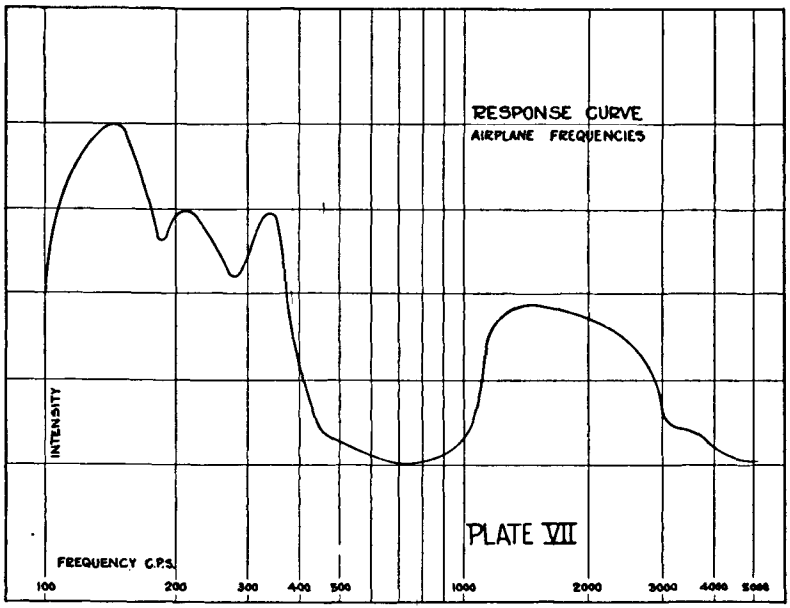
by the mathematical equation $\sigma = \sigma_0 e^{mx}$ where σ is the cross-sectional area and m is a constant whose selection determines the rate of flare of the horn. e is the Napierian base 2.718.

This is the type of horn used commercially in a great many sound radiating devices. The latest phonographs use exponential horns, whose lengths are approximately six or eight feet, depending upon the model. These horns are folded up in an ingenious manner in order to place them in suitable cabinets. The talking movies use large exponential horns to reproduce the sounds made by the characters projected on the screen behind which the horns are placed. Many of the best radio loud speakers depend for their faithful reproduction upon the use of exponential horns. Public address systems use these horns.

In the reproduction or collection of sound, it must first be

determined what frequencies, or tones, are to be reproduced, and then design the horn and other equipment to suit these frequencies. If a known band of frequencies is to be dealt with, as in the case of speech or music, then generally the

PLATE 7.

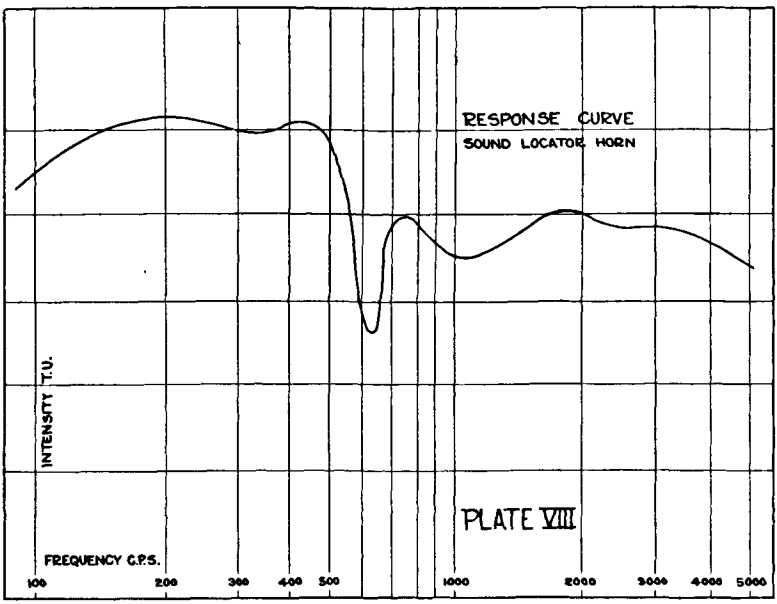


upper and lower frequencies are the determining ones. Formulæ are available for the design of exponential horns so that very faithful, or undistorted transmission through the horn is possible.

In general, the horn becomes larger as we seek to reproduce lower frequencies. A good phonograph horn has a cut-off at about 110 cycles. The sound from an airplane is complex, but much of the energy has a frequency from 100 to 400 cycles. The horns used for its reception have a cut-off around 70 cycles. In this connection, it should be noted that fully half of the sound energy from an airplane does not come from the engine but from the propeller, struts and other unsilenceable parts. The silencing of the engine would not be a handicap in the problem.

The sound locator consists of four large exponential horns mounted on alti-azimuth axes swinging from a seven-foot pedestal. The carriage bearing the pedestal consists of a turntable attached to a pneumatic tired trailer.

PLATE 8.



The four horns are mounted so as to have their axes always parallel. Two of the horns are used for azimuth determination and two for elevation. Two pairs of horns are required because a sound wave in the region of the sound locator travels approximately as a plane wave, hence is two dimensional. It is then necessary to have a pair of horns for the determination of direction in each of two dimensions at right angles. By means of flexible tubing, the small ends of the horns by pairs are connected with the ears of the operators who control their horns by large handwheels. Due to the concentration required by the listeners, the reading of the scales must be done by additional members of the crew of five or six men found to be sufficient for handling the instrument.

Atmospheric corrections are applied to the readings thus made and the direction angles are then automatically transmitted to gun batteries or other material to be set on the airplane. These automatic data transmitters are among the

PLATE 14a.

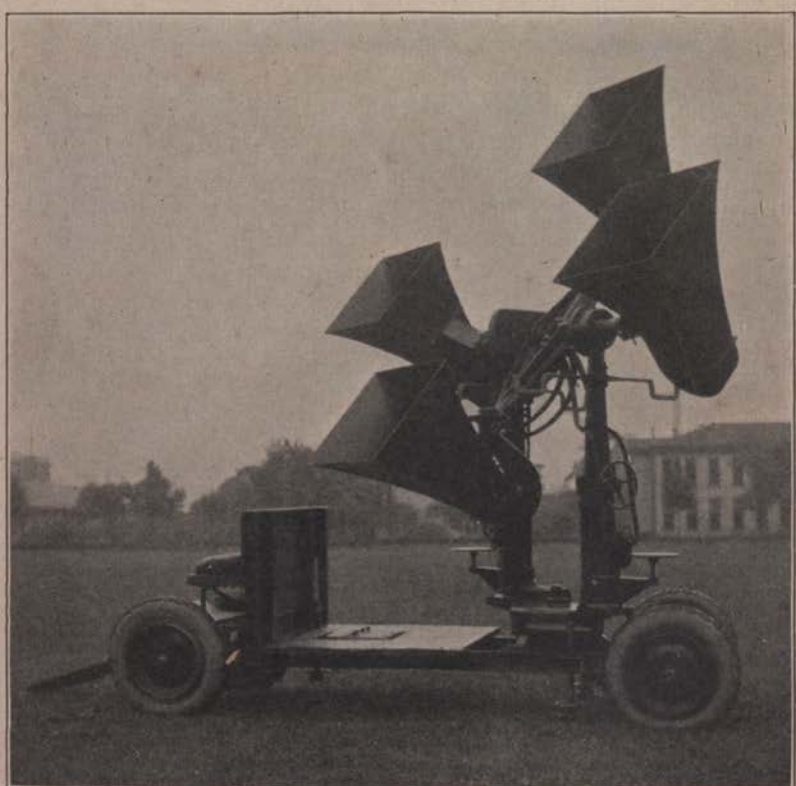
Sound locator with horns in traveling position.

latest and most important additions to the fire control equipment of anti-aircraft batteries, which have to contend with such rapidly moving targets that the making of observations, calculations and setting must be accomplished in the least possible time.

At present the sound locator is used in conjunction with a 60″ searchlight as a finder for the latter. The sound locator and light, separated by a distance of several hundred feet, are connected electrically through a control box called

the comparator. Small synchronous motors transmit to the comparator the angular settings of both sound locator and light. At the comparator also is the remote controller for the light. By using his controller the operator simply keeps two pointers matched whence the searchlight will follow the direction indicated by the sound locator.

PLATE 15.



Sound locator in operating position.

The sound locator has a range of ten to fifteen miles, while the light has a range of five to seven miles. Accordingly, ample time is afforded the listeners to get the plane's course and set the light before the plane comes within range of the

latter's beam. At a given signal the light is turned on and illuminates the plane; in this it is aided by auxiliary lights, the present practice being to have one pilot light and three illuminating lights to a platoon, or two pilots and two illuminating lights. The instant the lights are turned on, the gun battery commences using its visual observation as in the day time. It takes approximately thirty seconds for the gun battery to commence firing after the first observation.

The great disadvantage of this method of procedure lies in the discovery of your position by the enemy airplane when the searchlights are turned on. Assume that the lights strike the airplane the instant they are turned on; the pilot will naturally attempt to maneuver and get out of the beam. It has been quite conclusively demonstrated that once a plane is in the beam of a modern searchlight, it is well nigh impossible for the aviator to get away from the beam. This is particularly so in the case of the bombing plane with which we are chiefly concerned. It is heavy, slow moving and not easily maneuvered from a regular course. However, suppose the aviator, having been illuminated, decides to change his objective and drop his bombs on the searchlight, his target stands out beautifully and might easily be destroyed. The time is not far distant when firing by sound alone can be realized without the give-away of intermediate searchlights. It remains to perfect our aerial sound locating instruments so that accurate enough acoustic data can be obtained for setting guns. Acoustic data will never be as accurate as visual data on account of the differences in the natural characteristics of sound waves and light waves. But acoustic instruments can be perfected to the point where very accurate data are obtainable and the degree of accuracy of these data determined. It then remains for the guns to place a barrage of great enough dispersion to cover the probable error. This is a simple artillery problem.

THE PROPERTIES OF DIELECTRICS. I. ELECTRIC MOMENT AND MOLECULAR STRUCTURE.

BY

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At the request of the Committee on Insulation under the Chairmanship of Dr. J. B. Whitehead, Division of Engineering, National Research Council, a summary of the theoretical aspects of certain of the properties of dielectrics has been prepared. The present contribution aims to present a brief survey of the relations between the electric moments, polarizations, and molecular structures of substances.

Faraday [1] concluded that "the particles of an insulating dielectric whilst under induction may be compared . . . to a series of small insulated conductors" and further concluded that each of these conductors consisted of a single molecule which could be polarized as a whole. When this polarization occurs, one side of the molecule acquires a positive charge and the opposite side a negative charge of equal size, so that the molecule as a whole is neutral. This pair of charges, being at only a small distance apart, may be regarded as forming a small doublet or dipole, the size of which is measured by its moment, the product of either charge by the distance between the two charges. Mossotti [2] treated the idea of Faraday mathematically and Clausius [3] later developed a relation similar to that of Mossotti. Their conclusions are embodied in the so-called Clausius-Mossotti equation,

$$\frac{\epsilon - 1}{\epsilon + 2}\frac{1}{d} = \text{const.},$$

in which ϵ is the dielectric constant and d the density. The approximate constancy of the expression $\frac{\epsilon - 1}{\epsilon + 2}\frac{1}{d}$, called the polarization, was established for many substances of low dielectric constant, but the polarizations of many substances,

particularly those of high dielectric constant, were found to vary greatly with temperature.

The Lorenz[4]- Lorentz[5] expression for refraction, $\frac{n^2 - 1}{n^2 + 2}\frac{1}{d}$, in which n is the refractive index, is found approximately equal to the polarization in many cases, as would be expected from the well-known Maxwellian[6] relation $\epsilon = n^2$. In many cases, however, when n, the index of refraction measured for light in the visible region, is extrapolated to the region of long electric waves in which the dielectric constant is measured, ϵ is found to be much larger than n^2.

Debye[7, 8] has applied to dielectrics a treatment analogous to that used by Langevin[9] in discussing paramagnetism, assuming that a molecule may contain a permanent doublet of moment, μ, which causes the molecule to become oriented in an electric field. The effect of the doublet induced by an applied field is then superimposed upon that of the permanent doublet. Orientation of the molecule containing the doublet is opposed by the thermal agitation of the molecules, which, of course, varies with the temperature, so that the polarization becomes a function of the temperature. Debye has expressed this relation for the polarization of a gram-molecule of a substance, that is, the molar polarization, as

$$P = \frac{\epsilon - 1}{\epsilon + 2}\frac{M}{d} = \frac{4\pi N}{3}\gamma + \frac{4\pi N}{3}\frac{\mu^2}{3kT},$$

in which M = the molecular weight, N = the number of molecules in a gram-molecule = 6.061×10^{23}, γ = the molecular polarizability or the moment of the doublet induced on a molecule by a field of unit strength, k = the molecular gas constant = 1.372×10^{-16}, and T = the absolute temperature. $\mu^2/3kT$ is the mean component of the moment per molecule in the direction of an internal field of unit strength. As T is the only variable on the right-hand side of the equation, it is evident that, when the molecule contains no permanent doublet ($\mu = 0$), the second term on the right drops out and the equation becomes equivalent to the Clausius-Mossotti expression, which takes account only of doublets induced in the molecules. However, when the molecule is unsymmetrical electrically, containing a perma-

nent doublet, the polarization is a linear function of $1/T$ and the second term on the right may be much larger than the first.

The equation may be written in the form:

$$P = a + b/T,$$

in which $a = (4\pi N/3)\gamma$ and $b = 4\pi N\mu^2/9k$. When ϵ and d are measured over a range of temperature, the values of the constants a and b may be calculated and the value of μ obtained from that of b. a may also be calculated approximately as the molar refraction for infinite wave-length obtained by extrapolation of the refraction in the visible region. b/T is then given as the difference between a and P. This method of obtaining the moment is, at best, less accurate and is misleading when any molecular association occurs.

In the derivation of this equation on the basis of the classical mechanics, it is necessary to suppose that, in the absence of an external field, the molecules are oriented at random. The application of the principle of quantized orientation to the problem led to an equation of the same general form but with a different coefficient for the second term.[10, 11] However, applications of the new quantum mechanics to simple poly-atomic molecules have given equations from which results practically identical with those of the Debye equation may be obtained [12, 13, 14] and Van Vleck [15] has recently given a general proof of the equation on the basis of the new mechanics.

Obviously, when the molecules are not entirely free to orient in an applied field, the equation cannot be expected to apply. For gases, the dielectric constants of which are very close to 1, simplification may be effected by setting $\epsilon + 2$ equal to 3. The validity of this simplified equation has been established experimentally by Jona,[16] Zahn,[17] Sänger,[18] and others.[19, 20] In liquids, the molecules are so close together that large doublets attract one another strongly, forming complex molecules or restricting one another's freedom of motion. The first term of the Debye equation is little affected by the mutual action of the molecules as it is dependent upon electronic shifts within the molecules, but the second term is commonly decreased by lack of freedom of the molecules. However, if the single molecules are

associated to form complex molecules in which the doublets point in the same direction, the second term may be greatly increased. The inapplicability of the equation to certain dipole liquids was shown by Smyth,[21] and Debye [8] and Lange [22] have discussed this effect of molecular association upon the polarization. As molecular association, which usually decreases the polarization, is opposed by thermal agitation, which also decreases the polarization, the polarization of a liquid containing large doublets in its molecules may either increase or decrease with temperature, although the latter behavior is much more common. For example, the polarization of pure ethyl bromide, which has a large electric moment, has been found by Smyth and Morgan [23] to increase slightly with rising temperature, while its solution containing about 5 per cent. hexane has a polarization almost independent of temperature. Solutions richer in hexane, which has no electric moment, have polarizations which decrease with rising temperature, because the ethyl bromide molecules are more separated from one another and, therefore, have greater freedom to orient in an applied field. The molar polarization of the ethyl bromide dissolved in the hexane may be calculated from the total polarization of the mixture and that of pure hexane, and the polarization for an infinitely dilute solution may then be obtained by extrapolation of the values at different concentrations. The polarizations at infinite dilution obtained in this way for ethyl bromide, chloroform, and chlorobenzene have been found [23] to conform to the Debye equation over the entire range of temperature within which the solutions are liquid. Lange's [22] results for nitrobenzene in solution in benzene and in toluene over a shorter temperature range also show conformity to the equation.

The equation of Debye [8, 24] has also been developed in a more general form:

$$\frac{\epsilon - 1}{\epsilon + 2}\frac{M}{d} = \frac{4\pi N}{3}\gamma + \frac{4\pi N}{3}\frac{1}{1 + i\frac{\omega}{\omega_0}}\frac{\mu^2}{3kT}$$

in which ω is the frequency of the applied field, and ω_0, a

characteristic frequency $= kT/4\pi a^3\eta$, a being the molecular radius and η, the viscosity. Under ordinary conditions of measurement ω is so small in comparison to ω_0 that ω/ω_0 is negligible and the equation becomes identical with the simpler one already discussed, so that the dielectric constants of gases and of liquids of normal viscosity are independent of frequency down to very short wave-length. As the frequency is increased until the wave-length is but a few centimeters, the value of ω/ω_0 becomes appreciable and rapidly increases with decreasing wave-length. In this region of the electric spectrum the polarization falls off rapidly until it becomes as little dependent on wave-length as the molar refraction with which it is now identical. Mizushima,[25] measuring the dielectric constants of a number of alcohols at wave-lengths of 3.08, 9.5, and 50 m., has found that, for a given temperature, the dielectric constant begins to fall off at a longer wave-length the larger the molecule, while, for a given wave-length, the temperature at which the decrease begins is higher the larger the molecule, and, for a given molecule, the temperature at which the falling off begins is higher the shorter the wave-length. The results are in quantitative agreement with the Debye theory. Similar investigations have been made upon glycerin by Mizushima,[25] Graffunder,[26] and others.

Obviously, when a substance is solid and the molecules are unable to orient in an applied field, the second term should drop out. Substances which possess dipoles in their molecules normally show a great drop in polarization on solidification, after which the polarization shows little variation with temperature.[27, 28] However, Errera[29] reports that the dielectric constants of a number of substances with moments, when measured at wave-lengths of some hundreds of kilometers, show little decrease on solidification and it is only when the wave-length is reduced to about a kilometer or the temperature of the solid lowered far below the freezing-point that the dielectric constant decreases to a value which would make the second term of the equation negligibly small. This seems to indicate that the molecules of a solid near its melting-point possess sufficient mobility to turn slowly in an alternating field.

Attempts[30] have been made to correlate dielectric constant

and chemical composition by treating the constant as an additive property like molar refraction. Thwing [31] assigned values to certain radicals and attempted to calculate the dielectric constant of a substance from the values for the radicals composing its molecule. Walden [32] recognized that certain radicals unsymmetrically distributed in the molecule gave rise to a large dielectric constant, but showed the incorrectness of Thwing's view. As an illustration, he cited the fact that nitromethane, CH_3NO_2, had a dielectric constant of 56, from which by Thwing's method the dielectric constant of tetranitromethane, $C(NO_2)_4$, would be calculated to be 104 as compared to the observed value 2.1.

The problem of the relation between dielectric constant and chemical constitution may be more advantageously attacked by applying the modern views of atomic and molecular structure. Faraday's [1] conception of the molecule as a conducting sphere, which led to the derivation of the Clausius-Mossotti expression,[2, 3] must be replaced by the view of the molecule as an arrangement of electrons and positive charges which may or may not be unsymmetrical so as to be approximately equivalent to an electric doublet and which may, when a field is applied, be slightly distorted so as to form a doublet. The theory may be best illustrated by the description of specific cases. The argon atom is believed to consist of a positive nucleus with 18 electrons distributed about it, the total charge of the atom being zero. The symmetry of the distribution is such that the mean position of the center of gravity of the electrons is coincident with that of the positive nucleus, the atom thus having no permanent moment. When the atom is placed in an electric field the electrons shift so that their center of gravity is no longer coincident with the positive nucleus, the system thus forming an electric doublet. Since the molecule contains no permanent doublet, the second term in the Debye equation is zero and the polarization should, therefore, be constant.

The chloride ion, Cl^-, is supposed to consist of a symmetrical arrangement of 18 electrons around a nucleus which has 17 positive charges. Since the center of gravity of the electrons coincides with the positive nucleus, the system of charges may, as an approximation, be resolved into a single

negative charge located at the nucleus. If, now, a hydrogen nucleus could, without distortion of the system, be attached to form a neutral hydrogen chloride molecule, a permanent electric doublet should result. The moment of the doublet should be the product of the single electronic charge, 4.774×10^{-10} e.s.u., by the distance between the nuclei, which has been calculated from infra-red absorption spectra as 1.265×10^{-8} cm. The moment 6.04×10^{-18} calculated in this way is far larger than the value 1.03×10^{-18} obtained by application of the Debye equation to measurements of the dielectric constant of gaseous HCl, because the positive hydrogen nucleus draws the electrons of the chloride ion toward it distorting the system of charges and greatly diminishing the distance between their centers of gravity. The extent of the distortion has been calculated to be of approximately the right amount to cause the difference between the values of the moments calculated from the molecular structure and those obtained from experimental data.[33]

Although it is impossible, because of the distortion of the electronic systems, to obtain accurate values of the electric moments of molecules from their hypothetical structures, the relative sizes of the moments may often be predicted. Moments have been calculated for a number of substances by applying the Debye equation to measurements of the dielectric constants of gases over a range of temperature.[16, 17, 18, 19, 20] Approximate values[34] have been calculated for the moments of a large number of liquids by the use in a modified form of an equation developed by Gans,[35] who attempted to take account of the effect of molecules upon those immediately surrounding them. Since the equation can now be regarded as, at best, approximate, it need only be stated that its use in modified form provides a means of calculating an approximate value of the moment of a substance from the indices of refraction at two different wavelengths, the density of the substance under the conditions under which these indices are measured, the dielectric constant at one temperature, the density at this temperature, the value of this temperature itself, the molecular diameter, and the universal constants employed in the Debye equation.

Recently, the polarization of a substance at infinite dilution in a non-polar liquid, obtained in the manner already described, has been used with the value of a, calculated from optical data, to obtain the moment.[8, 22, 36, 38, 39, 40] The polarizations at infinite dilution which were obtained by Smyth and Morgan[23] in order to test the Debye equation were also used to calculate a and b and obtain accurate values of the moments.

Since most inorganic substances are solids which are insoluble in non-polar liquids, organic substances lend themselves more readily to investigation. Groups which, in inorganic substances, tend to form ions are generally found, on the basis of our hypotheses of molecular structure, to give rise to unsymmetrical arrangements of charges in organic molecules and it is found experimentally that molecules containing radicals such as Cl, Br, I, NO_2, OH, etc., contain electric doublets. On the other hand, it has been calculated that a hydrogen atom attached to a carbon may be replaced by a methyl radical without causing any direct change in the electric symmetry of the molecule[34] and this has been confirmed by the experimental fact that all of the saturated aliphatic hydrocarbons investigated, including the nine isomers of heptane, have been found to have no electric moment.[37] Moreover, the aliphatic monohydric alcohols and other homologous series are found to have moments which vary but little with the length of the hydrocarbon chain.[34] The effect of two doublets in the molecule is illustrated by the results of Errera[38] for the geometrical isomers of the compounds represented by the formula CHX = CHX, where X represents a halogen atom. In the cis-form of the compound, in which the two halogen atoms are on the same side of the molecule and the two doublets due to the halogens, therefore, point in the same direction, the electric moment is large, but, in the trans-form, in which the halogens are on opposite sides of the molecule and the two doublets point in opposite directions, the moment is zero. When two doublets are attached to a benzene ring, which is symmetrical and has no moment, they should make an angle of somewhat more than 60° when located in the ortho-position on adjacent carbons, an angle of 120° when in the meta-position separated

by one carbon, and should point in opposite directions when in the para-position. The nitro- and halogen disubstituted benzenes are found to have moments predictable, in many cases quantitatively, on this assumption.[38, 39, 40] Many other relations have been established by Williams and his co-workers,[36] Smyth,[34] and Ebert,[41] who avoided the calculation of the moment and dealt merely with the relation between polarization and molecular structure.

It is evident that the electric moment of a molecule is not a mere mathematical quantity arbitrarily adjusted to make an equation fit experimental data, but a definite physical entity dependent upon molecular structure. Knowledge of the electric moment is helpful in formulating the structure of a molecule, but, what is more important from the practical standpoint, knowledge of molecular structure should make it possible to estimate the magnitude of the moment and obtain dielectrics of desired properties. The dependence of the dielectric constant upon the other properties of the material may be summarized in the statement that the dielectric constant depends upon the number and the mobility of the electrons per cm.^{3} of material and, when the molecules contain electric doublets, to a much greater extent upon the size and arrangement of the doublets in the molecules, upon the freedom of the molecules to orient, and upon the number of molecules per cm.^{3}.

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THE INVENTION OF THE MOTION PICTURE.

BY

HENRY LEFFMANN,

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WHAT constitutes priority in invention or discovery? To what extent must foreshadowings be considered as entitling their utterers to the honors of the pioneer? Aviation is now a well-developed mechanical art: may we date its beginning from Daedalus or find its first step in the *suggestions* of Leonardo da Vinci? Aristotle and Augustin make allusions that may be taken as suggesting evolution: are Darwin and Wallace, therefore, mere followers? A search of Greek literature will bear out Sydney Smith's complaint of "Those confounded ancients who anticipated everybody." Perhaps some enthusiast might regard Paul's "Star differs from star in glory" as an adumbration of modern cosmic spectroscopy.

These musings have been provoked by an article by Mr. George E. Nitzsche, Recorder of the University of Pennsylvania, appearing in the April issue of the quarterly magazine of the General Alumni Society, in which it is frankly claimed that the invention of the motion picture is due to Edward James Muggeridge (familiar in the history of photography as Eadweard Muybridge) working under the auspices of the University and with substantial financial aid from it. Mr. Nitzsche's essay is devoted to demonstrating the validity of this claim and not unnaturally shows much earnestness, which has led unconsciously to a somewhat partisan presentation of the early history of the motion picture. No one person can be credited with the whole or even a vital part of the development of this striking and popular entertainment. As with almost all other great developments in science and art, it is the product of many men in many lands, a slow progression, here a little and there a little, often gaining ground through invention and discovery in apparently unrelated fields. Thus the vogue of the motion picture to-day depends on flexible film, high

speed emulsions capable of retaining sensitiveness for a long time and also of retaining undeveloped the latent image, a powerful light, special projection apparatus, and in the latest phases, high dramatic art. The first bridge built by the Pennsylvania Railroad over the Susquehanna at Havre-de-Grace, bore a tablet stating that it was the work of a thousand men for four years. It may be truly said that bearing in view Heinrich Schulze's experiment in 1727, the motion picture is the product of the labors of hundreds of men for two centuries. Mr. Nitzsche gives a brief statement of the early inventions and methods that were leading up to the present state of the art. It is not necessary to review these statements, except to say that at least two men who did important work are unnoticed. Jules Janssen in 1873 constructed a machine to secure photographs during the transit of Venus in 1874 by which could be recorded the phenomena exhibited at the moment of contact of the planet with the sun's limb. This apparatus, though simple, involved some of the fundamental principles of the modern motion-picture camera, and is described in detail in *Compt. rend.*, 1873, *76*, 678. Another well-known demonstration, not noticed, is that of Friese Greene and Evans, who in the late 80's, exhibited in the window of the shop of the former, a commercial photographer, motion pictures visible to passers-by and created such an attraction that the police authorities requested the discontinuance of the exhibition.

There can be no question of Muybridge's service in applications of photography to the study of the motion of living beings. His pictures startled artists, physiologists, athletes and many others, for they showed that the conventional representations of motion such as a horse running, a man walking or an athlete vaulting, were composites on the brain of the observer, due to the persistence of vision, one image on the retina not being discharged before another is produced. The broad and exclusive claim made in the following paragraph of Mr. Nitzsche's paper cannot be accepted: "After careful study, the credit (of the invention of motion picture) unquestionably would seem to belong to Eadweard Muybridge, whose final researches and experiments under the auspices of the University of Pennsylvania made modern motion pictures possible."

On careful examination of the history of efforts at projection of pictures on a screen so as to give the *illusion* of motion—for that is all that has yet been accomplished in the "movie"—the work of Sellers and Heyl in mid-Victorian years, set forth fully in the literature of this subject is seen to be more closely connected with the line of development than was that of Muybridge. The important point of difference is that Muybridge devised and carried out with great ingenuity and success the *analysis* of motion, but the present day screen-play is a *synthesis*. Muybridge used in his pioneer work a battery of twenty-four cameras so arranged that each was released in rapid succession, thus recording the motion of a living animal in twenty-four phases at very short intervals, but each was an independent picture from a separate point of view, while the series to be commercially applicable must be made, as in the modern work, from one camera, that is from one point of view. A foot note in the University article referring to enormous revenue that would have accrued if the basic patents on the motion picture industry should have been taken out by the Institution is misleading in two ways. The Muybridge method would not have yielded pictures that could have had any widespread vogue, and moreover "if" looms large in the significance of such assertions. The pictures made by Sellers and Heyl, though crude in execution were records of steps in motion, and were taken in such a way that they could be composed and thus give to the viewer the illusion of moving objects which is all that the present day frequenter of motion picture theaters expects. The public at large cares nothing for the analytic data revealed by the camera. In fact the educational applications of the motion picture are entirely secondary to its entertainment features. It is true that in some departments of science, especially in medicine much information has been obtained as to the details of physiologic and pathologic phenomena, but these data remain almost entirely within the professions concerned, or if shown publicly find vogue because they gratify a morbid curiosity.

Sellers' and Heyl's cameras operated interruptedly with considerable intervals between exposures, owing to the slowness of the emulsion then available. They were also

handicapped by the inflexible glass support. The composition of the views in projection seems from reports of those who saw them to have been fairly satisfactory. The most advanced form of motion-picture camera to-day, operates also with intervals, much shorter than those of older days, but this is a difference in degree not in kind. The modern film-maker has the benefit of a dry, speedy, well-keeping, sensitized surface and a flexible film, but in these inventions Muybridge had no share, although he utilized them. For the dry-plate upon which the enormous vogue of photography depends, the world is largely indebted to Doctor Maddox of England and for flexible film largely to George Eastman.

In summing up this study of motion-picture history, it surely appears that, as said above, no exclusive claim can be made for any one, nor is such claim necessary. As Schley said at Santiago, "There is glory enough for all." A long line of workers, only a few of whom sought or secured pecuniary reward, constitutes the group from Schulze down through two centuries that must be honored for their labors. Muybridge is entitled to conspicuous recognition in this series, but in any summary of motion-picture history the work of Coleman Sellers and Henry Heyl must be prominently set forth. There is no justification for declaring Eadweard Muybridge to be the "inventor" of motion pictures.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

NEW INSTRUMENT FOR TESTING PIPETTES.

A NEW instrument, employing a volumetric method, for testing dilution pipettes, has been designed by E. L. Peffer in charge of the bureau's capacity and density section, and constructed in the bureau's shops. The time required to test pipettes by the new method is from one-tenth to one-fifth that required by the weighing method, and the results are of equal accuracy. When it is remembered that 4,200 of these pipettes were tested during the last fiscal year and that for the present year the number will run over 6,000, this constitutes an important economy.

The pipettes in question have a ratio of stem to bulb of one to ten or one to one hundred.

The instrument consists of a small horizontal glass cylinder with double walls. The space between the walls is evacuated to reduce temperature effects and the inner cylinder is filled with a colored non-corrosive solution. Into the two ends of the cylinder project small steel plungers which may be forced into or withdrawn from the cylinder by the rotation of micrometer screws. These plungers are lapped to size and the area of one is ten times that of the other. A dome on the upper portion of the cylinder has an opening with a rubber stopper into which the top of the pipette is inserted.

In making a test, the liquid is driven up the tube of the inverted pipette by the large plunger to the first division, and a reading is then taken on the micrometer. The liquid is then driven through the bulb and to the first ruling on the stem, where a second reading is taken. The small plunger is then used to drive the liquid up the stem, readings being taken at the required points. The volume of liquid displaced by the large plunger being ten times that displaced by the small one, the number of turns and hence the readings of the two micrometers will be equal if the ratio of bulb to stem of the pipette is truly ten to one.

* Communicated by the Director.

The micrometer heads used were taken from ordinary depth micrometers, and are sealed into the glass with Kotinsky's cement. Even with the vacuum jacket there is a slight temperature effect, but this could be reduced by silvering the walls.

SOME OLD WEIGHTS STILL VARY WITH CHANGES IN HUMIDITY.

CERTAIN gold-plated "analytical" weights, tested during the past month, showed such variations with changes in atmospheric humidity, that the desired certificate could not be granted. These weights were at least seven years old. It thus appears to be definitely established that variability of gold-plated screw-knob weights does not necessarily disappear with age. Moreover, the smaller weights in the set showed a slight gain averaging about 0.1 mg. These changes are quite consistent with the idea that the electroplating salts that cause this variability are likely to cause permanent gain also. The necessity for exercising great care in the washing that is necessary after weights are plated, is again emphasized.

VISIT OF THE CERAMIC SOCIETY.

ON the morning of May 4 about 60 delegates from the Ceramic Society of England inspected the bureau's work in the clay products and allied fields.

This visit was part of a tour for the British ceramists arranged by the American Ceramic Society, and which includes inspections of the leading manufacturing plants and research laboratories in the United States and Canada.

The trip through the bureau gave the visitors an opportunity to see the work in progress on clay products, Portland cement, lime and gypsum, stone, and optical glass, as well as related investigations covering the strength and fire resistance of structural materials.

American ceramists have learned much from the long-established ceramic industry of England, where some of the world's most famous chinaware has been manufactured for years. It was therefore particularly gratifying to the bureau to receive this visit and to exhibit some of the apparatus and methods which this country has developed.

HYDROGEN AND CARBON MONOXIDE IN FLAMES.

AN investigation is being made of the relative amounts of hydrogen and carbon monoxide which escape from gas flames in which combustion is nearly complete. Information on this subject, which is of importance in connection with the determination of carbon monoxide and also in connection with efficiency of utilization, has heretofore been based upon very meager data and a somewhat uncertain theory. The determination of the two gases when present in concentrations of a few hundredths of one per cent. is being made by first removing water and carbon dioxide by means of liquid air, burning the reducing gases with the aid of a catalyst, collecting the water and carbon dioxide formed by means of liquid air, and transferring to weighing tubes with a relatively small volume of air.

MEETING OF AMERICAN PHYSICAL SOCIETY.

THE 157th regular meeting of the American Physical Society was held in Washington, April 18 to 20, inclusive. The meetings on the 18 and 19 were at the Bureau of Standards while that on the 20 took place at the National Academy of Sciences. Because of the number of papers on the program (131) and the limited capacity of the assembly rooms, two sessions were held at the same time in the East and Industrial Buildings of the bureau.

The program covered a vast range of subjects from electrons to interstellar space with a fair number of papers on the application of physical measurements to important technical problems.

The meetings were well attended, the total registration being 321.

The following papers were presented by members of the bureau's staff, and will be published later in the *Physical Review* (University of Minnesota, Minneapolis, Minn.):

Effect of temperature, pressure, and frequency on the electrical properties of rubber. H. L. Curtis, A. T. MacPherson, and A. H. Scott.

The infra-red absorption spectrum of carbon tetrachloride as related to the Raman spectrum of scattered radiation. W. W. Coblentz and R. Stair.

The infra-red absorption spectrum of chlorophyl and xanthophyl. R. Stair and W. W. Coblentz.

Quantitative measurements of wind tunnel turbulence. H. L. Dryden and A. M. Kuethe.

The first spectrum of krypton. W. F. Meggers, T. L. de Bruin, and C. J. Humphries.

Incoherent scattering in Rochelle salt. R. M. Langer.

ELECTROPLATING RESEARCHES AT THE BUREAU OF STANDARDS.

DURING the past few years conferences have been held at the bureau to discuss the progress of researches on electroplating and appropriate subjects for future research by the bureau's staff and by the research associates of the American Electroplaters' Society. This year, at the invitation of the Newark branch and the research committee of the Electroplaters' Society the conference was held in Newark on April 6. O. J. Sizelove of the Newark branch presided at the morning session and R. J. O'Connor, chairman of the Research Committee, at the afternoon session. About 250 persons attended the meetings, at which the following subjects were presented and discussed:

1. *Spotting Out.*—W. P. Barrows, research associate of the American Electroplaters' Society, presented a summary of his investigation of spotting out. The detailed results of this study are now in press, as a Research Paper of the Bureau of Standards, which will appear about June 1. Copies of this paper will be distributed by the Electroplaters' Society to all of its members, and to all subscribers to the research fund.

2. *Chromium Plating.*—A summary of the researches of the bureau on chromium plating was presented by W. Blum, chief of the electrochemistry section of the Bureau of Standards.

3. *Analysis of Cyanide Solutions.*—M. R. Thompson of the Bureau of Standards reported that after many trials very nearly pure sodium and potassium cyanide have been prepared for research purposes.

4. *Measurement of pH in Nickel-plating Solutions.*—Methods of measuring pH in nickel plating solutions were discussed. It is hoped that at the convention of the American Electroplaters' Society in Detroit in July, some definite recommendations may be presented.

5. *Addition Agents in Copper Electrotyping Solutions.*—R. O. Hull, research associate of the International Association of Electrotypers reported on addition agents in copper electrotyping solutions. He explained that when certain addition agents are used, the deposits are smoother and harder than those from solutions with no addition agent. This solution is now being tried on a commercial scale in several electrotyping plants.

6. *Iron Deposition.*—C. E. Thomas, Bureau of Engraving and Printing, reported on a process for the electrode position of iron. This iron has a tensile strength of 4,000 kg./cm.^{2} (56,000 lbs./in.^{2}).

7. *Future Plans.*—The discussion of future plans emphasized the need for more information regarding the protective value of electroplated coatings against corrosion, as a basis for specifications of quality. It was pointed out that the various tests and specifications for zinc and cadmium coatings are not adequate. It was also stated that although chromium plating has been widely applied in the automobile and other industries, the present methods and specifications do not yield entirely satisfactory products.

It was suggested that in any study of the protective value of electroplated coatings, the Electroplaters' Society and the Bureau of Standards should coöperate closely with committees of the American Society for Testing Materials and similar organizations. The hope was expressed that the automobile industry as well as other metal industries would contribute to the support of such an investigation.

At a subsequent meeting of the executive committee and the research committee of the American Electroplaters' Society, it was decided to have their two research associates undertake a comprehensive study of "Protection against corrosion by means of electroplated finishes." Such a study will probably require about three years. The first subject to be investigated will be the protective value of chromium plating. As soon as possible the plans for the study will be prepared and discussed with interested firms and organizations.

PROTECTIVE COATINGS FOR PIPE LINES.

THE American Gas Association in coöperation with the Bureau of Standards is starting an investigation of protective coatings for pipe lines. A general study of the nature and method of application of the more common types of coatings is now under way. As soon as this is completed a number of manufacturers will be asked to apply their products to pipe samples, and these samples will be buried in a few locations where the corrosion is known to be severe. Some of the samples will be removed at intervals for examination. Each manufacturer will be expected to meet a general specification as to the nature and thickness of the coating.

Manufacturers of coatings should communicate with the bureau in case they wish to have their product considered in the tests which are being planned.

FADING OF DYED TEXTILES IN DAYLIGHT AND IN CARBON ARC LIGHT.

THE American Association of Textile Chemists and Colorists and the Bureau of Standards have been studying the fading of dyed textiles with the primary object of developing standard methods for testing fastness to light. One phase of this work, involving the exposure of over 10,000 individual samples to daylight under various conditions and to carbon arc light, was started in 1926 and has now been completed. A brief outline of the work and some of the results obtained follows. A report by William H. Cady and William D. Appel will be published soon in the American Dyestuff Reporter.

A total of 1,252 specially prepared dyeings on cotton, wool, silk, and weighted silk, representing some 381 different coloring matters, were exposed to daylight in several different ways, and to the light from a glass-enclosed carbon arc. Each sample was exposed for four different periods of time. In general, slight fading occurred in the first period, decided fading in the fourth, and intermediate amounts of fading in the other two. Seven sets of samples, all cut from the original dyeings, were exposed in the following ways: Sets *A* and *K* in the standard sun test of the American Association of Textile Chemists and Colorists, i.e., on clear days between 9 A.M. and

3 P.M., under glass, at an angle of 45 degrees from the horizontal, facing south; set *B*, simultaneously with sets *A* and *K* but with no glass cover; set *D*, continuously, under glass in a vertical position, facing south; set *E*, continuously, under glass, at an angle of 45 degrees from the horizontal, facing south; set *F*, continuously, under glass in a vertical position, facing north; set *G–H*, in the "Fade-ometer," a type of glass-enclosed carbon arc. The results of a sample by sample comparison of the dyeings in the several sets with the corresponding dyeings in set *A* taken as the standard are given in the paper. It is shown that:

1. The "standard sun test" (Set *A* above) of the American Association of Textile Chemists and Colorists gives essentially reproducible results with reference to the relative fading of the dyeings in the series.

2. The presence or absence of a window glass cover over the samples during exposure in the standard test has practically no effect on the fading of 74 per cent. of the samples and only a slight effect on 22 per cent. of them. It has a marked effect on the fading of the remaining 4 per cent. of the samples.

3. Marked divergences in the relative fading of the dyeings in continuous exposures compared with the fading in the standard exposure are noted. These divergences are most marked in the samples exposed continuously, in a vertical position, facing north. A considerable number of the cotton dyeings faded relatively faster than the wool and silk dyeings in this test. In general, the basic dye types, the indigoids, and the ice colors on cotton, fade much faster relative to some other cotton, wool, and silk dyeings in the vertical north exposure.

4. Although the majority of the dyeings fade to about the same extent, relative to one another, in the arc exposures compared with the standard sun exposures (1 arc hour = 1.3 hours in the "standard sun test"), fading in the arc light is different in quality from that in sunlight in many instances. Some samples fade entirely too much, others entirely too little, in the arc exposures, but the agreement between standard sun test and arc test is much better than between the former and the continuous exposure in a vertical position facing north.

Miscellaneous observations on the peculiarities in a behavior of individual dyeings are recorded.

A classification of the dyeings for fastness to light and a selection of standards of fastness will be given in another report.

STANDARD SAMPLES.

THE renewal No. 30c of the bureau's standard sample No. 30b, chrome-vanadium steel, has been prepared and is ready for distribution. The new sample has the following composition: carbon .490, manganese .71, phosphorus .019, sulphur .013, silicon .236, chromium .971, and vanadium .24 per cent. The price of this standard is the same as that of No. 30b, $3.00 per 150 grams.

NOTES FROM THE RESEARCH LABORATORY, EASTMAN KODAK COMPANY.*

PHOTO-VOLTAIC CELLS WITH SILVER—SILVER BROMIDE ELECTRODES.[1]

W. Vanselow and S. E. Sheppard.

PHOTO-VOLTAIC cells consisting of silver—silver bromide electrodes in solutions of potassium bromide have been studied. The potential difference—time curve obtained is the resultant of two effects, viz., an initial negative effect on which is superimposed a positive effect. A hypothesis was developed explaining these in terms of the liberation of electrons and bromine atoms from bromide ions on absorption of light. In agreement with this it is shown that the positive effect can be suppressed, the negative effect made more pronounced, by addition of bromine acceptors. The positive effect is shown to be similar to that produced by allowing bromine to diffuse through the silver bromide layer to the silver.

THE FORMATION OF THIN FILMS OF ORGANIC COLLOIDS BY SPREADING ON MERCURY SURFACES.[2]

R. L. Keenan.

THIN films of cellulose esters, gelatin, rubber, and fatty acids were obtained on mercury when the proper conditions were attained. The method consisted in allowing drops of solutions of varying concentrations to spread on a mercury surface cleaned by the sweeping procedure, the amount of substance being insufficient to cover completely the liquid surface. The film area was developed with talc powder, and

* Communicated by the Director.

[1] Communication No. 362 from the Kodak Research Laboratories and published in *J. Phys. Chem.* 33: 331. 1929.

[2] Communication No. 363 from the Kodak Research Laboratories and published in *J. Phys. Chem.* 33: 371. 1929.

measured by a planimeter. On plotting the thickness of the film, calculated by assuming the density equal to the massive material, against concentration, a point is reached at which this thickness value becomes independent of the concentration (or dilution).

The limit thickness increases on passing from rubber through the carbohydrate derivatives to the protein, that is with increasing complexity of the atom groups

(C, H)	(C, H, O)	(C, H, O, N)
1.5 Å. U.	2.5 – 5.0 Å. U.	7 Å. U.

In the case of the cellulose nitrate there appears to be an inverse relation between the viscosity and the thickness of the limit film.

The method and its control were governed by measuring the length of the molecules of fatty acids and other known molecules which were determined by Langmuir and others.

ILLUMINATION IN MOTION PICTURE PRINTING.[3]

C. Tuttle.

THE selection of a light source for motion picture contact printing has been a matter of experiment for each particular case. Quantitative data concerning the amount of light available from various incandescent sources should aid in this selection. A knowledge of exposure requirements and laws which determine photographic intensity may prove of value to those engaged in motion picture processing.

Experimental evidence shows that a simple exponential relation exists between the electrical input and the photographic intensity of tungsten lamps. This relation is given for six various lamps representative of types commonly used in printing.

[3] Communication No. 371 from the Kodak Research Laboratories and published in *Trans. Soc. Mot. Pict. Eng.* **12**: No. 36. 1040. 1929.

A PHOTOMETER FOR THE MEASUREMENT OF LOW ILLUMINATIONS.[4]

L. A. Jones and E. M. Lowry.

IN the manipulation of photographic materials the provision of satisfactory darkroom illumination is of vital importance. With the advent of high-speed emulsions sensitive to all wave-lengths of the visible spectrum, the problem of providing satisfactory illumination in the darkrooms has assumed greater importance and more and more difficulty has been met in obtaining an illumination which is satisfactory from the standpoint of both photographic safety and visual requirements.

The instrument described in this paper was constructed specifically to meet these special requirements of darkroom photometry and in its design careful consideration was given to those fundamental visual characteristics which determine the precision with which photometric readings can be made. Among the advantages which are claimed for the instrument may be mentioned: Observations are made with unobstructed binocular vision under the same conditions which exist when some task is being performed. The size of the test plate and the test spot are such that the visual angle is that giving maximum contrast sensibility at low illuminations. The intensity adjustment is not accompanied by any change in color of the illumination as of necessity must occur when intensity adjustment is made by control of the lamp current.

A STUDY OF GROUND NOISE IN THE REPRODUCTION OF SOUND BY PHOTOGRAPHIC METHODS.[5]

O. Sandvik.

GROUND noise is the term ordinarily used to designate the extraneous noises present in a film used for photographic sound recording, or more explicitly, on a motion picture film having a

[4] Communication No. 373 from the Kodak Research Laboratories and published in *Trans. Soc. Mot. Pict. Eng.* **12**: No. 36. 1054. 1929; *Phot. Ind.* **26**: 1325. 1928; Kinotechnik **11**: 47. 1929.

[5] Communication No. 374 from the Kodak Research Laboratories and published in *Trans. Soc. Mot. Pict. Eng.* **12**: No. 35. 1929; *Amer. Cinemat.* **9**: 9. 1928; *Phot. Ind.* **27**: 265. 1929.

uniform deposit of silver, the magnitude of which may be zero or finite. The purpose of this investigation is to determine some causes of the ground noise and to measure its growth with the increase in the number of times which the film has run through the projector. The results show that the amount of ground noise present on clean film base is imperceptible when the amplifier is operated on a grain normally necessary for an average sound record. The results show that no appreciable increase in the noise level is introduced at the various stages of manufacture. There is, however, a considerable increase in the noise during processing, that is, during developing, fixing, washing, and drying, even when these operations are carried out with the utmost care. The results on several samples of film repeatedly run through the projector show that after the first few runs the noise increases linearly with the number of times run.

SOME PRELIMINARY EXPERIMENTS IN MEDICAL MOTION PICTURE PHOTOGRAPHY.[6]

C. Tuttle and C. A. Morrison.

AT the instigation of Mr. Hays, President of the Motion Picture Producers and Distributors Association, the Eastman Kodak Company has recently undertaken some experimental work in connection with motion pictures of medical subjects.

While it is too early to state many definite conclusions regarding the progress of this branch of photography, it is felt that a description of our experiments and preliminary conclusions may be of some interest to those engaged in similar work.

The quantity, quality, and direction of lighting for surgical pictures have been considered and some new experimental lighting units have been built.

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[6] Communication No. 375 from the Kodak Research Laboratories and published in *Trans. Soc. Mot. Pict. Eng.* 12: No. 36. 1022. 1929.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, MAY 15, 1929.

The stated monthly meeting of the Institute was called to order at three-forty P.M., by the President of the Institute, Nathan Hayward. Medal Day celebration was held in connection with this meeting.

The Chairman stated that the minutes of the April meeting had been printed in full in the May number of the Journal of the Institute and that unless there were objection they would be approved as printed. No objection was offered and the minutes were declared approved.

The Chairman then called upon the Secretary, who reported concerning certain phases of the activities of the Institute since the Medal Day meeting of 1928. The Secretary stated that much business had to be transacted during the afternoon and that he could not, therefore, speak at length. He felt, however, that the Institute would be remiss if it did not make grateful acknowledgment of the aid which had been received from friends during the past year. He referred to the bequest from the estate of the late Craig D. Ritchie, a long time member of the Institute, which had finally become available for the needs of the Institute. This bequest amounted to $32,049.86. He mentioned especially the bequest of the late Dean Marion Reilly which, while relatively small, had brought an especial measure of encouragement to the authorities of the Institute. Miss Reilly was a woman of ability and of high standing, and was at the same time a strong mathematician. She was familiar with the workings of the Institute and with its plans. The Board of Managers was therefore happy to be able to regard her bequest as an evidence of her approval of the Institute. The Board has already voted to establish, by means of her bequest, $1,000, the "Marion Reilly Memorial Library in Mathematical Physics," the subject in which Dean Reilly was most interested.

The Secretary also made grateful acknowledgment of the receipt from Mrs. James Mapes Dodge, a tested friend of the Institute, of an amount which doubled the endowment which she had already presented for the "James Mapes Dodge Lecture Foundation" for juveniles. The Institute now has a handsome endowment which enables these important lectures for young people to be given by prominent scientists during Christmas Week.

The Secretary referred briefly to the completion of the Bartol Research Foundation at Swarthmore and the prospects of a distinguished record for that Foundation. He made mention also of the favorable outlook for rapid progress in the scheme for the new Franklin Institute Building and Benjamin Franklin Memorial and Museum.

The President then announced that the award of Medals was the next order of business and stated that all the medals which had been recommended during the present Institute year by the Committee on Science and the Arts would be presented at this time. The exercises in connection with the various medals were then held.

PROGRAMME.

	Sponsors
Presentation of Certificate of Merit to	
MR. GEORGE F. MACHLET	Mr. Wm. C. Wetherill
American Gas Furnace Company	
Elizabeth, New Jersey	
Received by Mr. A. A. Machlet	
Presentation of Longstreth Medals to	
MR. EDWARD G. HERBERT	Mr. E. W. Smith
Atlas Works, Manchester, England	
Received by British Consul General	
KONRAD JAGSCHITZ, D. ENG.	Mr. J. M. Rusby
Mainz, Germany	
Received by German Consul General, Consulate General Office, New York City	
MR. A. W. MACHLET	Mr. F. V. Hetzel
American Gas Furnace Company	
Elizabeth, New Jersey	
MR. J. F. PETERS	Mr. C. E. Bonine
Westinghouse Electric and Manufacturing Company	
East Pittsburgh, Pennsylvania	
Presentation of Wetherill Medals to	
MR. GUSTAVE FAST	Mr. C. D. Galloway
The Bartlett Hayward Company	
Baltimore, Maryland	
MR. W. H. MASON	Mr. M. M. Price
Masonite Corporation	
Laurel, Mississippi	
JOHANNES RUTHS, D. ENG.	Mr. C. E. Bonine
Djursholm, Sweden	
Received by Royal Vice-Consul of Sweden	
Presentation of the Levy Medal to	
MR. LODEWYK J. R. HOLST	Dr. T. D. Cope
Brock and Weymouth, Inc.	
Philadelphia	
Presentation of the Clark Medal to	
MR. W. H. GARTLEY	Mr. J. S. Rogers
Philadelphia Gas Works Company	
Philadelphia	
Presentation of Cresson Medals to	
SIR JAMES C. IRVINE, C.B.E.; LL.D.; F.R.S.; F.R.S.E.	Dr. H. J. M. Creighton
St. Andrews, Scotland	
Received by British Consul General	

CHEVALIER JACKSON, M.D.; SC.D.; LL.D.; F.A.C.S. — Dr. Jos. S. Hepburn
Jefferson Medical College
Philadelphia

ELMER A. SPERRY, Sc.D. — Dr. Frederic Palmer, Jr.
Sperry Development Company
Brooklyn, New York

Presentation of the Franklin Medal and Certificate of Honorary Membership to

MR. EMILE BERLINER — Dr. James Barnes
Washington, District of Columbia

Presentation of the Franklin Medal and Certificate of Honorary Membership to

CHARLES THOMSON REES WILSON, LL.D.; F.R.S. — Dr. James Barnes
Cambridge, England
Received by Sir Esme Howard
His British Majesty's Ambassador to the United States

PAPERS.

"Vocal Physics—Historical Notes." by EMILE BERLINER. Read by Dr. McClenahan.

"Some Thundercloud Problems," by PROFESSOR C. T. R. WILSON, Cambridge University. Presented by PROFESSOR JOHN ZELENY, Yale University.

Adjourned.

HOWARD MCCLENAHAN,
Secretary.

(An extended account of the proceedings of this Medal Day meeting will appear in the July number of the JOURNAL.)

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, May 1, 1929.)

HALL OF THE INSTITUTE,
PHILADELPHIA, May 1, 1929.

DOCTOR THOMAS D. COPE *in the Chair.*

The following reports were presented for first reading:

No. 2844: Pulse Relay.

No. 2884: Work of Mr. C. S. Chrisman in the Gas Industry.

No. 2898: Phelps Time Recording Door Lock.

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, May 8, 1929.)

RESIDENT.

MR. JAMES DANIEL EVANS, Attorney, Executive Vice-President and Counsel, Merchant and Evans Company, 2035 Washington Avenue, Philadelphia, Pa. For mailing: Haverford, Pa.

MR. JACQUES L. VAUCLAIN, Vice-President, Baldwin Locomotive Works, Philadelphia, Pa. For mailing: Bryn Mawr, Pa.

NON-RESIDENT.

MR. JOHN J. KNOWLTON, Inspection Electrical Engineer, Western Electric Company, Kearney, N. J.

MR. HOLLAND M. MERRICK, Consulting Engineer, 2028 Land Title Building, Philadelphia, Pa. For mailing: P. O. Box 105, Pottstown, Pa.

MR. VICTOR H. SEVERY, Engineer, Gillette-Severy Laboratories, 1845 South Western Avenue, Los Angeles, California.

ADMIRAL ELLIOT SNOW, C.C., U.S.N., Retired, Apt. E, 2, Bryn Mawr Court, Bryn Mawr, Pa.

STUDENT.

MR. WILLIAM DAVID CHAPMAN, Student and Watchmaker, West Philadelphia High School. For mailing: 5042 Locust Street, Philadelphia, Pa.

CHANGES IN CONTRIBUTING MEMBERSHIP:

NEW MEMBERS.

Leeds and Northrup:
MR. E. SEATON CARNEY,
MR. J. B. WILSON.

Ajax Metal Company:
MR. ROBERT N. BLAKESLEE, JR.

CHANGES OF ADDRESS.

COLONEL GEORGE L. ANDERSON, Delafield, Wisconsin.

DR. DONALD S. ASHBROOK, Bradford Dyeing Association, Bradford, R. I.

MR. HAROLD CALVERT, Crozer Building, 1420 Chestnut Street, Philadelphia, Pa.

MR. BERNARD L. DOUREDOURE, 227 South Front Street, Philadelphia, Pa.

MR. H. LINWOOD GODFREY, Care Emery, Booth, Varney and Whittemore, 149 Broadway, New York City.

MR. HARRY A. GOULD, 6214 Wayne Avenue, Philadelphia, Pa.

MR. NATHANAEL HERRESHOFF, 6 Walley Street, Bristol, R. I.

MR. JOHN B. KLUMPP, 2326 Fidelity-Philadelphia Trust Building, 123 South Broad Street, Philadelphia, Pa.

MR. HOWARD S. LEVY, 1429 North Twenty-first Street, Philadelphia, Pa.

DR. WALTER M. MITCHELL, Care Lukens Steel Company, Coatesville, Pa.

MR. ALBERT W. MORRIS, 827 Linden Avenue, Drexel Hill, Pa.
MR. THOMAS E. MURRAY, 20 Pine Street, New York City.
DR. SAMUEL E. POND, Marine Biological Laboratory, Woods Hole, Mass.
CAPTAIN H. R. STANFORD (C.E.C.), U.S.N., 1735 Eye Street, N.W., Washington, D. C.
DR. PAUL B. TAYLOR, Radio Engineering Department, Westinghouse Elecrtic and Manufacturing Co., East Pittsburgh, Pa.
MR. WILLIAM TIDDY, Servel Solway Engineering Corporation, 40 Rector Street, New York City.

LIBRARY NOTES.

RECENT ADDITIONS.

ALLIEVI, LORENZO. Theory of Water-Hammer. Translated by Eugene E. Halmos. 1925.
American Gas Association. Gas Chemists' Handbook. January 1, 1929.
American Institute of Electrical Engineers. Transactions 1927 to January, 1929, Volumes 46, 47, and 48 part 1. 1927–1929.
American Pharmaceutical Association. Pharmaceutical Recipe Book. 1929.
American Society of Heating and Ventilating Engineers Guide 1929, Volume 7. 1928.
BAILEY, E. H. S., AND HERBERT S. BAILEY. Food Products: Their Source, Chemistry, and Use. Third revised edition. 1928.
BALL, W. W. ROUSE. A Short Account of the History of Mathematics. 1927.
BARBARIN, P. La Géométrie Non Euclidienne. 1928.
BAUER, EDWARD E. Plain Concrete. 1928.
BLACK, ARCHIBALD. Civil Airports and Airways. 1929.
BRAGG, SIR WILLIAM. An Introduction to Crystal Analysis. 1929.
Brassey's Naval and Shipping Annual. 1929.
BRIDGMAN, P. W. The Logic of Modern Physics. 1927.
BRIGGS, MARTIN S. The Architect in History. 1927.
BUTTER, FRANCIS J. Locks and Lockmaking. No date.
CAMERON, JAMES R. Motion Pictures With Sound. 1929.
CARSLAW, R. MCG., C. BURGESS, AND G. LL. ROGERS. Sugar Beet in the Eastern Counties, 1927. University of Cambridge Department of Agriculture, Farm Economics Branch, Report No. 9. 1928.
Chemical Society. Annual Reports on the Progress of Chemistry for 1928. Volume 25. 1929.
COISSAC, G.-MICHEL. Histoire du Cinématographe. 1925.
DIEHL, WALTER S. Engineering Aerodynamics. 1928.
DYKE, A. L., Dyke's Aircraft Engine Instructor. 1928.
FRARY, FRANCIS C., CYRIL S. TAYLOR, AND JUNIUS DAVID EDWARDS. Laboratory Glass Blowing. Second edition, revised and enlarged. 1928.
General Electric Company. Arc-Welding Manual. 1928.
GREGG, WILLIS RAY. Aeronautical Meteorology. 1925.
HARTLEY, LAWRENCE A. Elementary Foundry Technology. 1928.

HATSCHEK, EMIL. The Viscosity of Liquids. 1928.
HAWORTH, WALTER NORMAN. The Constitution of Sugars. 1929.
HUNGERFORD EDWARD. The Story of the Baltimore and Ohio Railroad 1827–1927. Two volumes. 1928.
Institut International de Physique Solvay. Électrons et Photons. 1928.
JAHN, HUGO. The Dictionary of Graphic Arts. 1928.
Jahresbericht über die Leistungen der chemischen Technologie für das Jahr 1928 74 Jahrgang, 1 Abteilung. 1929.
Kaiser-Wilhelm-Institut für Kohlenforschung in Mülheim-Ruhr. Arbeiten. Gesammelte Abhandlungen zur Kenntnis der Kohle. 1924–1927. Bd. 8. 1929.
KLEMIN, ALEXANDER. Airplane Stress Analysis. 1929.
KNOWLTON, A. A. Physics For College Students. 1928.
LASSAR-COHN. Organic Laboratory Methods. Authorized translation from the general part of the fifth revised edition, by R. E. Oesper. 1928.
LYMAN, THEODORE. The Spectroscopy of the Extreme Ultra-Violet. 1928.
MCFARLAND, EARL. Textbook of Ordnance and Gunnery. 1929.
MACGIBBON, W. C. Marine Diesel Engines. No date.
MACINNES, CHARLES RANALD. The Elements of Practical Mechanics. 1929.
MATTHEWS, J. MERRITT. The Textile Fibers. 1924.
Minerva: Jahrbuch der gelehrten Welt. 29 Jahrgang, Bände 2–3. 1928.
MITCHELL, H. H., AND T. S. HAMILTON. The Biochemistry of the Amino Acids. 1929.
MOYER, JAMES A., AND JOHN F. WOSTREL. Radio Receiving Tubes. 1929.
MUNK, MAX M. Fundamentals of Fluid Dynamics for Aircraft Designers. 1929.
NILES, ALFRED S., AND JOSEPH S. NEWELL. Airplane Structures. 1929.
POTONNIÉE, GEORGES. Histoire de la Découverte de la Photographie. 1925.
ROUTSON, H. W. Locksmithing and General Shop Information. 1927.
SEARLE, G. F. C. Experimental Optics: a Manual for the Laboratory. 1925.
SIEMENS-KONZERN. Wissenschaftliche Veröffentlichungen. VII Band, Zweites Heft. 1929.
SINGER, CHARLES. From Magic to Science. 1928.
SMALLWOOD, JULIAN C., AND FRANK W. KOUWENHOVEN. Mechanics for Engineers: Statics and Kinetics. 1928.
STANIAR, WILLIAM. Mechanical Power Transmission. First edition. 1928.
STARRETT, W. A. Skyscrapers and the Men Who Build Them. 1928.
SUTTON, J. R. Diamond: a Descriptive Treatise. 1928.
TEEPLE, JOHN E., AND OTHERS. The Industrial Development of Searles Lake Brines: With Equilibrium Data. 1929.
THOMSON, SIR J. J., AND G. P. THOMSON. Conduction of Electricity Through Gases. Third edition. Volume I. 1928.
WARSHOW, H. T., editor. Representative Industries in the United States. 1928.
WEISER, HARRY BOYER. The Colloidal Salts. First edition. 1928.
WHIPPLE, GEORGE CHANDLER. The Microscopy of Drinking Water. Fourth edition, rewritten and enlarged. 1927.
WHITE, HERBERT JOHN. Oil Tank Steamers. Fourth edition. 1927.

WIEN, W., AND F. HARMS, editors. Handbuch der Experimentalphysik, Bd. 9, I Teil; Bd. 13, I Teil. 1929.

YOE, JOHN H. Photometric Chemical Analysis. Volume 2: Nephelometry. 1929.

BOOK REVIEWS.

ÉLECTRONS ET PHOTONS. Rapports et Discussions du Cinquieme Conseil de Physique tenu a Bruxelles du 24 au 29 Octobre 1927 sous les auspices de L'Institut International de Physique Solvay. vii–289 pages, illustrations, 8vo, paper. Paris, Gauthier-Villars et Cie, 1928. Price, 60 francs.

Under the general designation of "electrons and photons" the papers presented at the fifth convention of the section on physics of the Solvay International Institute of Physics, which was held in Brussels from the 24th to the 29th of October 1927, are collected in this volume with discussions. Preceded by a portrait of the late and most regretted Professor Lorentz with a eulogy by Madame Curie, the six papers by authors of international renown are as follows: W. L. Bragg—Intensity of reflected x-rays, Arthur H. Compton—Discordance between experiment and theory of electromagnetic radiation, Louis de Broglie—The new quanta dynamics, Max Born and Werner Heisenberg—Quanta mechanics, E. Schrodinger—Wave mechanics, N. Bohr—The postulate of quanta and the new atomistic development. All these papers and their discussions are rendered into French.

L. E. P.

PHYSICS FOR COLLEGE STUDENTS, AN INTRODUCTION TO THE STUDY OF THE PHYSICAL SCIENCES. By A. A. Knowlton, Ph.D., Professor of Physics, Reed College. xix–641 pages, illustrations, 8vo, cloth. New York, McGraw-Hill Book Company, Inc., 1928. Price, $3.75.

Students in science and technology have a perfectly definite object for making as thorough a study of physics as their time, inclination and capacity permit: study physics, they must. In a non-technical course of study there is still the need of the precise guidance in a correct estimation of inevitable material background which is afforded by a sound knowledge of physics. Doctor Knowlton has been actuated by two major considerations in the preparation of the present volume. One is to present the subject in an attractive and stimulating form which will enlist the interest of readers whose chief objective is in the academic studies in a course of arts; the other is to provide a text of university grade in which modern theories and accepted developments are incorporated. With regard to the first specification he has adopted the very natural and only too often neglected pedagogical expedient of making use of what he terms the "soaking-in process." "No topic is treated at all fully on its first introduction. So much, and only so much, is done with it as is required for use at that point. At its next introduction, something further is added to material which has become a part of the mental background of the student through use. This requires some sacrifice of the logical arrangement most satisfactory to one already familiar with the subject but has the teaching advantage of keeping the beginner con-

tinually in contact with the fundamental principles in all fields. The laws of motion, for example, appear early and often; so too do the fundamental electrical relations. Perhaps one might describe this as a cyclic arrangement."

The work is discoursive with many historical references and side lights on the circumstances which surrounded the discovery and evolution of important principles and the subject matter is thoroughly modern. Besides the familiar topics of treatises on physics, there are included an explanation of the usual atomic theory and further on an account of the components of atoms with relation to radio-activity and again an explanation of the structure of the dynamic atom. It is a comprehensive and rarely attractive text which will surely and amply fulfil its purpose, perhaps indeed too amply in arousing an excessive degree of interest among "arts" students in physical science at the expense of the purely academic subjects. L. E. P.

MECHANICS FOR ENGINEERS, STATICS AND KINETICS. By Julian C. Smallwood, M.E., A.M., Associate Professor of Mechanical Engineering, Johns Hopkins University, Member of American Society of Mechanical Engineers; and Frank W. Kouwenhoven, M.E., Instructor of Mechanical Engineering, Johns Hopkins University, Associate Member of American Society of Mechanical Engineers. x–185 pages, illustrations, 8vo. cloth. New York, D. Van Nostrand Company, Inc., 1928. Price, $2.50.

Undergraduate university teaching in engineering courses has assumed conditions somewhat akin to mass production in the industries. Much standardization in the selection of material and its arrangement is necessary to conform to a balanced curriculum and to cover essentials in the allotted time without overburdening the student. Particularly is that true of a key-subject like Mechanics which may be elaborated far beyond the particular course of study in which it is included.

The present volume has been prepared to satisfy the requirements of the average student in the principles of Statics and Kinetics over a period of 60 to 80 class-room hours. Being a fundamental subject which is taught not later than the second year, certain topics which can be considered with subjects coming late in the course are properly omitted. The explanatory value of the solution of numerical problems incorporated in the text will be welcomed by the serious student and provide the needed encouragement in attacking the numerous unsolved problems. Modern writers occasionally have been rather insistent on the use of the term "radial acceleration" rather than "centrifugal force" which the authors use. "Radial acceleration" indeed does not get us very far with those who are in closest contact with it. The volume with its logically arranged well illustrated and skilfully presented subject-matter is a most desirable contribution to engineering teaching.

THE ELEMENTS OF PRACTICAL MECHANICS. By Charles Ranald MacInnes, Ph.D., Professor of Mathematics in Princeton University. vi–131 pages, illustrations, 8vo, cloth. New York, D. Van Nostrand Company, Inc., 1929. Price, $2.25.

With substantially the same subject-matter and prepared with a similar objective, Doctor MacInnes's text represents the accumulated material of many

years of class-work. As in the previously noticed work, it is a labor-economizing device for the benefit of the student. A very ample collection of problems as well as illustrative examples in the text is provided. A notable feature is the development of graphic methods along with analytic processes which the author has found very advantageous. Prepared by representatives of institutions of learning of the highest rank, the scholarship of these works needs no other guarantee.

L. E. P.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

Report No. 299. Investigation of Damping Liquids for Aircraft Instruments, by G. H. Keulegan. 24 pages, illustrations, quarto. Washington, Government Printing Office, 1929. Price, ten cents.

This report covers the results of an investigation carried on at the Bureau of Standards with the financial assistance of the Committee.

The choice of a damping liquid for aircraft instruments is difficult owing to the range of temperature at which aircraft operate. Temperature changes affect the viscosity tremendously. The investigation was undertaken with the object of finding liquids of various viscosities otherwise suitable which had a minimum change in viscosity with temperature. The new data relate largely to solutions.

The apparatus consisted of four capillary-tube viscometers, which were immersed in a liquid bath in order to secure temperature control. The method of calibration and the related experimental data are presented in detail.

The viscosity data for the liquids are given in curves in which $\log_{10} \frac{t_\theta}{t_{30}}$ is plotted against temperature, where t_{30} and t_θ are respectively the times of discharge through the viscometer at 30° C. and θ° C. Except for a correction which is usually small, the following relation holds:

$$\log_{10} \frac{t_\theta}{t_{30}} = \log_{10} \frac{\nu_\theta}{\nu_{30}}$$

in which ν_{30} and ν_θ are respectively the kinematic viscosities at 30° C. and θ° C. The density at 30° C., the coefficient of cubical thermal expansion for each solution, and ν_{30} are given, together with other data, so that the absolute viscosity may be computed. The accuracy is within 1 per cent.

Report No. 304. An Investigation of the Aerodynamic Characteristics of an Airplane Equipped with Several Different Sets of Wings, by J. W. Crowley, Jr., and M. W. Green. 14 pages, illustrations, quarto. Washington, Government Printing Office, 1929. Price, ten cents.

This investigation was conducted by the Committee at Langley Field, Virginia, at the request of the Army Air Corps, for the purpose of comparing the full scale lift and drag characteristics of an airplane equipped with several sets of wings of commonly used airfoil sections. A Sperry Messenger airplane with wings of R.A.F.-15, U.S.A.-5, U.S.A.-27, and Göttingen 387 airfoil sections was flown and the lift and drag characteristics of the airplane with each set of wings were determined by means of glide tests.

The results are presented in tabular and curve form.

Report No. 306. Full-Scale Wind-Tunnel Tests of a Series of Metal Propellers on a VE-7 Airplane, by Fred E. Weick. 18 pages, illustrations, quarto. Washington, Government Printing Office, 1929. Price, ten cents.

An adjustable blade metal propeller was tested at five different angle settings, forming a series varying in pitch. The propeller was mounted on a VE-7 airplane in the Twenty-Foot Propeller Research Tunnel of the Committee. The efficiencies were found to be from 4 to 7 per cent higher than those of standard wood propellers operating under the same conditions. The results are given in convenient form for use in selecting propellers for aircraft.

R.

THE ARCHITECT IN HISTORY. By Martin S. Briggs, Fellow of the Royal Institute of British Architects. xii–400 pages, 46 illustrations, 12mo. New York, Oxford University Press, American Branch. Price, $3.75.

Who, after having viewed or studied the Seven Wonders of the Ancient World, the columned structures of Greece and Rome, the rugged grandeur of Gothic cathedrals, has not wondered about the mind which conceived these structural entities of stone and mortar and felt that in the lives of these oldtime architects a touch of the romantic might be disclosed after a patient search? For such persons this book will not be completely satisfying. However, that is not the fault of the author but, as the reader will see, is due to the modesty and retiring nature of these ancient architects, suppression of their identity by jealous and envious employers or in some cases, because the work of building was shared equally by a number of master builders.

A rather astounding fact is that there is so little recorded concerning the architects of earlier times. As a result of this, many writers on architecture have concluded that the architect as such did not exist and gave credit for the design of a structure to the person or persons financing the project. The author of this book is of a different opinion and shows in a very logical manner that the available early records do not deny the existence of architects although at that time they were known by other names. In "The Architect in History" Mr. Briggs has done a good job depicting the evolution of the architect. It is interesting to note how much innate ability and how little part social status played in the attainment of success. In this respect the architect may be classed with the painter and the sculptor. Until the beginning of the nineteenth century royal patronage of either the Church or State was the "Open Sesame" leading to success. Much attention has been paid to the quantity and quality of education these architects received and the gradual changes in the type of early training have been traced up through the ages. Undoubtedly the type of edifices erected depended upon the examples copied and studied by the architect in his youth. Beginning with what information is to be had on the architects of Babylonia, Chaldea and Egypt the author follows the varying but gradually increasing recognition of the architect as an individual through the periods of Greek and Roman supremacy, the Middle Ages, the Renaissance in Italy, France and England and the Nineteenth Century in England. In this way a very comprehensive exposition of the subject has been secured, one which pictures the architect a thoroughly human individual yet does not rob him of any deserved glory.

T. K. CLEVELAND.

AN INTRODUCTION TO CRYSTAL ANALYSIS. By Sir William Bragg, K.B.E., D.Sc., F.R.S. 168 pages, illustrations, 8vo. New York, D. Van Nostrand Company, Inc., 1929. Price, $4.25.

The text of this book is a report of a series of lectures given in 1926 at University College, Aberystwyth. Being intended for a general audience the subjects were treated in a somewhat elementary form. In preparing the material for publication, the treatment has been more extended and more varied. In the welter of publications on colloids it is agreeable to see something about crystals. Study of the intimate molecular structure of crystals dates from a comparatively recent period, Laue's work in 1912 being here given as the starting point. It is well known that Sir William Bragg and his son, W. L. Bragg, have given a large amount of attention to the subject and added a great deal of important and useful information. The principle has been extended to many substances and it appears that, by means of the x-ray, crystal structure may be detected in a large number of organized tissues. The literature of the subject is already very large and increasing rapidly. The author has found that many who are not directly occupied with this line of investigation are yet much interested in its striking and valuable results. He has in this book set forth concisely and clearly the essential features of present day knowledge. This knowledge indicates the wide practical applications that the methods possess, in view of the light thrown on the intimate structure of substances. Special efforts have been made to make clear the phraseology of the subject, as well from the point of view of the old systems of crystallography, as from that of the x-ray methods. It is, indeed informing to compare the modern representations of atomic distribution and association, now so familiar in writings on this subject, with the suggestions as to crystal structure offered in the treatises of the Mid-Victorian era. Yet it would be unfair to deprecate the suggestions then made, for the workers did not have the methods that are now in our hands. Physicists and chemists who desire to keep in touch with the main lines of discovery and practical value of the intimate study of crystal structure will find this book eminently satisfactory. The symbol for the light-wave unit omits the circle over the "A." This is a commendable procedure. The slogan of modern science should be "away with all accented and umlauted letters." They were the product of the perverse ingenuity of the monks of the middle ages.

HENRY LEFFMANN.

RESEARCH PAPER NO. 45. Reprint from Bureau of Standards Journal of Research. Methods for Analysis of Petroleum. By E. W. Washburn, J. H. Bruun and Mildred M. Hicks, Chemistry Division. Pamphlet, 22 pages, illustrations, 8vo. Washington, U. S. Government Printing Office, 1929. Price, ten cents.

In the middle of the last century, the principal causes of world trouble were wheat and cotton. The former, generally called by its British name "corn," was the export product of the North and the latter from the South. "Cotton is King" was the slogan of those who formed the Confederate States, hoping that England's demand for their product would assure early interference on their behalf, but were disappointed. Times change. Oil and rubber are figuring largely in diplomatic complications, and the former is giving rise to both domestic and international trouble.

This paper describes the apparatus and methods which have been developed and are being employed in an investigation on "The separation, identification, and determination of the chemical constituents of commercial petroleum fractions" listed as Project No. 6 of American Petroleum Institute Research. Financial assistance in this work has been received from a research fund of the American Petroleum Institute donated by John D. Rockefeller. This fund is being administered by the institute with the coöperation of the Central Petroleum Committee of the National Research Council.

H. L.

ELECTRIC LINES AND NETS; THEIR THEORY AND ELECTRICAL BEHAVIOR. By A. E. Kennelly, A.M., Sc.D., Professor of Electrical Engineering, Harvard University. Second edition, revised and enlarged. viii–426 pages, 32 tables, 222 figures. New York, McGraw-Hill Book Company, 1928. Price, $5.

If one pauses to consider, it appears remarkable indeed that our knowledge of applied electricity has expanded and ramified to such an extent that a whole volume alone may be written upon the conductance characteristics of transmission lines.

Writing on such a subject, the author has produced a textbook of great value particularly to students of electrical engineering. Looked upon as a revision of the author's earlier book, "Artificial Electric Lines" this more recent text has been enlarged to include the subject of real lines and the corresponding theories which have been developed in the last decade. Great pains have been taken to start right at the beginning. The introductory chapters show how the properties of electrical transmission lines may be expressed by means of trigonometrical functions. Equations for the simplest type of lines are first developed and those of greater complexity are constructed from these simpler elements. The theory originally worked out for continuous currents now has been extended to alternating currents which involves a large number of complex relations. The author shows in great detail just how these relations, characteristic of alternating currents, have been developed for various transmission systems.

The appendix contains a summary of the various formulas and their derivation as well as a number of problems which tend to aid in the practical application of these derived formulas.

T. K. CLEVELAND.

ALLEN'S COMMERCIAL ORGANIC ANALYSIS. By C. Ainsworth Mitchell, M.A., F.I.C., with numerous collaborators. Volume VII, fifth edition, revised and partly rewritten. xi–869 pages, illustrations, 8vo. Philadelphia, P. Blakiston's Son & Company, 1928. Price, $7.50.

C. O. A. as it is commonly known to practicing analysts, marches on like the British Grenadiers at Fontanoy, giving us from time to time a newly revised volume. Except as to one contributor out of eleven it has returned to the land of its birth, and the editorial control is now wholly there. The present editor C. Ainsworth Mitchell is widely and most favorably known in the field to which the work is devoted and chemists may be sure that under his supervision all the text will be trustworthy and up-to-date.

The present volume is devoted to the vegetable alkaloids, thus bringing

together in one volume the data on this important subject. The arrangement of the several groups has been somewhat changed from the previous editions, much of the text having been rewritten. A complete classification according to chemical principles not having been practical, an alphabetical arrangement has been followed. This is entirely satisfactory. The alkaloids intimately associated with the applications and uses of foods and drugs are treated in full in special sections and many other alkaloids of less prominence are duly described. Much space is given to the alkaloids of the familiar beverages. It is to be noted that in the initial paragraph of the article on tea, the product is said to be the leaf of the *Thea Chinensis* of the genus *Camellia*, but the proper form is *Thea sinensis* L. and a plant of the genus *Thea* could not at the same time belong to a genus *Camellia*. The intention was probably to say "family Camelliaceae." It is gratifying to see reference to the work of Howard and Stephenson of Washington, D. C. on the microscopy of the important alkaloids.

The volume now offered is a complete treatise on the available analytic data concerning the vegetable alkaloids. To the chemist who has worked in this field it is not merely a work of reference, it is in many places interesting reading. The reviewer regrets that the sole contributor from the United States is only listed as of the U. S. A. All the others are given their "Local habitation," but Mr. Frank O. Taylor, who has furnished a comprehensive and useful contribution on opium and its alkaloids is merely indicated as an inhabitant of the "great open spaces." In the article on opium, by the way, reference is made to a case reported by Swain Taylor, in which contrary to the usual conditions unsymmetric contraction of the pupils was noted in a patient suffering from opium poisoning. This was cited in the celebrated trial of Carlyle Harris in New York City for poisoning his wife. It was offered by the defense through a distinguished physician from Philadelphia. Mrs. Harris had marked symmetric and strong contraction of the pupils. The prosecuting attorney (referring to Taylor's case) asked the witness "Are you aware that in this case, the man had a glass eye?" The witness answered "no." "Well," said the attorney, "that has been proved in this case and you may go back to Philadelphia."

The volume will be welcome to all chemists occupied with drug and food analysis or dealing with toxicologic problems.

HENRY LEFFMANN.

PUBLICATIONS RECEIVED.

Les Méthodes nouvelles en Analyse Quantique. Mécanique quantique. Mécanique ondulatoire par Julien Pacotte. 139 pages, 8vo. Paris, Librairie Scientifique Albert Blanchard, 1929, price twenty-two francs.

La Structure de la Lumière, par Sir J. J. Thomson, O.M., F.R.S.; traduit par R. Fric, avec une préface de M. A. Cotton. 51 pages, illustrations, plates, 8vo. Paris, Librairie Scientifique Albert Blanchard, 1929, price nine francs.

Réflexions sur la Mecanique Ondulatoire. Une ancienne théorie dynamique ondulatoire par E. de Camas. 98 pages, illustrations, 12mo. Paris, Librairie Scientifique Albert Blanchard, 1929, price fourteen francs.

Introduction a une Connaissance Scientifique des Faits Musicaux par Pins

Servien. 55 pages, 12mo. Paris, Librairie Scientifique Albert Blanchard, 1929, price 7.50 francs.

The Industrial Development of Searles Lake Brines with Equilibrium Data, by John E. Teeple, Ph.D. American Chemical Society Monograph Series. 182 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1929, price $3.00.

Électrons et Photons. Rapports et discussions du Cinquieme Conseil de Physique tenu a Bruxelles du 24 au 29 Octobre 1927 sous les auspices de l'Institut International de Physique Solvay. 292 pages, illustrations, 8vo. Paris, Gauthier-Villars et Cie., 1928, price 60 francs.

Elementary Differential Equations, by Thornton C. Fry, Ph.D. 255 pages, illustrations, 8vo. New York, D. Van Nostrand Company, Inc., 1929, price $2.50.

An Introduction to Crystal Analysis, by Sir William Bragg, K.B.E., D.Sc. 168 pages, illustrations, plates, 8vo. New York, D. Van Nostrand Company, Inc., 1929, price $4.25.

Introduction to College Physics, by Clinton Maury Kilby. 349 pages, illustrations, plates, 8vo. New York, D. Van Nostrand Company, Inc., 1929, price $3.00.

Allen's Commercial Organic Analysis, a treatise on the properties, modes of analysis, and proximate analytical examination of the various organic chemicals and products employed in the arts, manufactures, medicine, etc. Vol. VII. The vegetable alkaloids. Editor, C. Ainsworth Mitchell. Fifth edition, revised and partly rewritten. 869 pages, illustrations, plates, 8vo. Philadelphia, P. Blakiston's Son & Company, 1929, price $7.50.

CURRENT TOPICS.

The Production of New Radiations by Light Scattering. C. V. RAMAN and K. S. KRISHNAN. (*Proc. Roy. Soc.*, A 789.) "In two preliminary papers (*Indian Journal of Physics*) we have recorded the discovery that when monochromatic light is scattered in a transparent medium (be it gas, vapor, liquid, amorphous solid or crystal), the diffused radiation ceases to be monochromatic, and several new lines or sometimes bands (associated in many cases with a continuous spectrum) appear in the spectrograms of the diffused radiation. Further, the new radiations are, in general, strongly polarised. That the phenomenon is entirely distinct from what is usually known as fluorescence is clear from the fact that the effect is observed when both the exciting radiation and the new radiations generated by it are far removed from the characteristic ultra-violet and infra-red frequencies of the medium. Our preliminary studies have proved conclusively that the effect arises in the following way: The incident quantum of radiation is either scattered as a whole, in which case we have the classical scattering, or else is absorbed in part by the molecules of the medium, the remaining part appearing as a scattered quantum. The part absorbed shifts the molecule to a level of energy different from the initial state. The possibility of a process of this kind, in respect of the electronic state of an atom, was first contemplated by Smekal, and figures prominently in the theory of dispersion due to Kramers and Heisenberg, and in the papers of Schroedinger. Our experiments furnish definite proof of the possibility of such processes, and show that they may occur also in such complicated systems as the molecules of a vapour or a liquid or even in a complete crystal."

Further studies of the effect are reported in this paper. By several slow distillations in vacuo the liquids used were made dust-free, the final distillate being in a bulb of 500 or 600 c.c. volume. The radiation from a 3,000 c.p. mercury arc was focused at the center of this bulb. By filters placed in the path of the light the radiation reaching the tube was practically reduced to the 4,358.3 line and its near companions. Rapid plates were used to photograph the spectrum of the scattered light. For benzene and toluene an exposure of 40 hours was required but 25 hours sufficed for carbon tetrachloride, as stronger light was used.

The spectrum of the light scattered by benzene shows 8 modified

lines with wave-lengths longer and 3 with wave-lengths shorter than 4,358.3 Å.U., the length of the most intense wave in the exciting radiation. The longest modified line has a wave-length of 5,026.3 Å.U. and of such waves there are in vacuo 19,877 in the length of one cm. For radiation 4,358.3 there are 22,938 waves to a cm. For these two lines there is a difference of wave-numbers of 22,938 — 19,877 or 3,061. If there were a radiation of wave-number 3,061 its wave-length would be 1/3061 cm. or .000327 cm. and Bell has studied the infra-red absorption of benzene and found a line at .00033 cm. In like manner most of the modified lines of wave-length greater than 4,358.3 for benzene, toluene and carbon tetrachloride are seen to lead to absorption lines in the infra-red that are known already to exist. It is safe to infer the existence of the other lines predicted by this method. Thus we have "a convenient and accurate method in infra-red spectroscopy. In a single spectrogram taken in the visible region we get all in the infra-red frequencies of the molecule simultaneously photographed, and they could be measured much more accurately than with an infra-red spectrometer. The appearance in the scattered spectra of toluene and carbon tetrachloride of modified lines corresponding to several hitherto unknown frequencies of the molecules, in the extreme infra-red region as far as 46 μ, emphasizes the ready applicability of the method especially to regions not accessible to the ordinary infra-red spectrometer."

In the spectra of the light scattered by all three liquids examined there are lines whose frequencies are larger than those of the exciting lines, but they are smaller in number than the scattered lines of smaller frequencies. Such lines of enhanced frequency are particularly conspicuous in the case of carbon tetrachloride. "The appearance of these lines of enhanced frequency proves in the first place the existence in the liquid at ordinary temperatures of some molecules at a level of energy higher than the normal by that corresponding to an infra-red frequency of the molecule, and, secondly, that the incident light induces a return of these molecules to the normal state. That is to say, while most of the molecules taking part in modified scattering are in the normal state and absorb a part of the incident light quantum, thus giving rise to a scattered radiation of smaller frequency, there is also a small number in the liquid, already in the higher level of energy, which under the influence of the incident radiation can be induced to part with their energy by a return to the normal state, thus giving rise to a scattered radiation of correspondingly higher frequency. In the existence of these lines of enhanced frequency we have for the first time a direct experimental proof of *induced emission* of radiation by molecules."

The polarization of both the modified and of the normally scattered lines was studied. All the normal scattered lines are polarized to about the same extent. The modified lines whose frequencies are greater than those of the exciting lines by certain amounts are polarized to the same extent as the modified lines of frequencies less than those of the exciting lines by the same amounts. "The strong modified lines are usually more polarized than the feeble ones."

The intensity of the modified radiation depends upon the wavelength of the exciting radiation. Thus through a direct vision spectroscope the green line from the mercury arc appeared much brighter than the indigo or violet lines, yet the modified lines due to the green line were invisible while those caused by the violet and indigo lines were conspicuous.

Without doubt the discovery of the Raman effect marks a great advance in the study of radiation. G. F. S.

The Duration of Contact between the Pianoforte String and a Hard Hammer. D. BANERJI and R. GANGULI. (*Phil. Mag.*, Feb. 1929.) A string 240 cm. long and weighing 16.4 gm. was stretched horizontally, one end being attached rigidly while the other passed over a fixed pulley and supported a heavy weight. The period of vibration of the string was .0356 sec. A ball hung by fine threads so that, when at rest, it just touched the string. This was pulled aside and coming in contact with the string was the striking hammer. The point of impact was focussed on a slide moved with uniform high speed past the slit of a camera. The wave form of the string was photographed and, the period of the string being known, the speed of the slide was found from the fact that the distance from crest to crest was traversed while the string made a complete vibration. Thus the duration of contact was obtained by dividing the speed into the distance from initial to final contact. With the velocity of the plate equal to 76.33 cm./sec. and the striking ball weighing 46 gr. the duration of contact was determined for different distances of the points of contact from the end of the string. In the table a is the distance in cm. from the nearer end and r the duration in secs.

a (cm.)	r (sec.)	a (cm.)	r (sec.)
15	.0262	40	.0432
20	.0263	50	.0432
25	.0356	60	.0445
32	.0369	80	.0511
		100	.0590

Both Kaufmann and Das have given a theoretical formula for calculating the duration of contact. The experimental results agree better with the Indian than with the German formula. "During the time of contact, while the struck point follows the movement of the hammer, the shorter portion of the string instead of remaining straight as contemplated by Kaufmann executes its fundamental mode of vibration."

G. F. S.

The Difference between the Mechanical and the Optical Lengths of a Steel End-Gauge. F. H. ROLT and H. BARRELL. (*Proc. Roy. Soc.*, A, 789.) Upon the reflection in air of light from transparent surfaces such as glass a change of phase of 180° occurs and the geometrical surface of the glass is coincident with its optical surface at which the reflection takes place. When, however, the reflecting surface is metallic there is an additional change of phase, that varies only slightly with the wave-length of the light. "It has an effect upon optical measurements, where steel gauge surfaces are used as reflectors, which is equivalent to an apparent displacement of the reflecting surfaces from the geometrical surfaces toward the interior of the gauge. In other words light appears to penetrate a metallic surface and is apparently reflected at a plane which is parallel to the geometrical surface." If an end-gauge terminates in lapped steel surfaces that permit its length to be measured by interference methods then its length optically determined is less than its geometrical length by twice the displacement of the reflecting plane, since the displacement occurs at both ends of the gauge.

Let two end-gauges be placed end to end and wrung together. The distance from one end of the combination to the other is the sum of the lengths of the two gauges augmented by the thickness of the wringing film. The authors devised a process of interference measurements by which the optical lengths of six 1/8 inch gauges, single and in combination, were measured, the light of two different neon lines being used. The length of the row of six gauges was measured ten times. "After each measurement the combination was broken down, the gauge surfaces cleaned with the usual mixture of alcohol, ether and ammonia, and then wrung together with a minimum quantity of paraffin oil." With the particular gauges used the optical length of the combination always was less than the 3/4 inch obtained by adding the nominal lengths of the six constituent 1/8 inch gauges. In units of millionths of an inch the largest deficit of the ten determinations was 56.2 and the least 49.0 with light of wave-length 5852.5 A.U. (yellow). With

red light of wave-length 6506.5 A.U. the greatest deficit was 57.7 and the smallest 49.8. The respective means for the two kinds of light were 52.8 and 53.9. The average lengths of single gauges were found by both wave-lengths. In the case of only one gauge did they differ by as much as one millionth inch.

In the National Physical Laboratory the thickness of the wringing film has from experiment been placed at 1/5 millionth inch. By the use of this in combination with the results of the optical measurements cited above the displacement of the reflecting optical surface into the interior of the gauge is calculated to be 3.3 millionths inch whereas Perard and Maudet in France put it at 3.1 millionths. A table is given showing the lengths of separate gauges measured by optical and mechanical methods. In all cases the mechanical length exceeds the optical by about 6 or 8 millionths inch.

G. F. S.

Elastic Constants of Fused Quartz. Change of Young's Modulus with Temperature. H. D. H. DRANE. (*Proc. Roy. Soc.*, A, 789.) This paper is not the outcome of a purely academic investigation but presents results obtained in the laboratory of the The Thermal Syndicate, Ltd. "When fused quartz is heated, its elastic constants for stretch shear and bulk change all increase, a sharp distinction in behaviour from that of most other elastic solids. An elastically-stretched fibre becomes shorter upon heating, and a strained torsion member reduces its twist for a given twisting effort."

In measuring Young's Modulus a little rod of fused quartz was fixed at one end. Weights were applied at the other end and the deflections of an attached pointer were read by a microscope. For low temperature the rod was surrounded by liquid oxygen while an electric furnace was used up to 700° C. beyond which it was not possible to make accurate readings. The author has obtained for Young's Modulus the value 6.7×10^{11} dynes. sq. cm. Taking the modulus at 15° as unity the values at other temperatures are as follows,—183°, .9695; 100°, 1.014; 400°, 1.0525; 700°, 1.079. Thus the value increases from the lowest to the highest observed temperature, though the rate of increase grows less with rising temperature.

"There remains as a problem the specification of the precise thermal and mechanical treatment necessary to yield a specimen of fused quartz with fixed mechanical and optical characteristics."

G. F. S.

The Diffraction of Electrons by Crystalline Powders. M. PONTE. (*Comptes Rendus*, Jan. 14, 1929.) Another similarity between electrons and radiation has been found. Already they have been shown in the United States to be regularly reflected from the faces of crystals and in Scotland to suffer diffraction upon traversing thin films.

A method of crystal analysis depends upon sending a beam of X-rays upon a powdered crystal in which the axis is distributed equally in all directions. The rays are diffracted and symmetrical images are produced on the photographic plate. Similarly now a beam of cathode rays falls on a wire coated with zinc oxide or on an unsupported structure of the oxide. On the plate clear rings manifested themselves. With a voltage of 17,250 applied to the tube their number was about twenty. The dimensions of the crystal calculated from the wave-length to which the electrons were equivalent agree within 2 per cent. with the results furnished by other methods.

The new method makes it possible to study the structure of a crystal by using electrons in place of X-rays and with relatively short exposure and small expenditure of energy. Only a small quantity of the powdered crystal is needed.

G. F. S.

The Limiting Density in White Dwarf Stars. E. C. STONER. (*Phil. Mag.*, Jan., 1929.) Some stars have a mean density that is calculated to be much in excess of any density known on the earth. The companion of Sirius has a mass .85 of that of the sun. Its effective surface temperature is 8,000°. From this fact and from its absolute magnitude being 11.3 its radius can be computed and thus its density determined. The resulting density is 50,000 gm./cu. cm. The density of O_2 Eridani B is even greater, viz., 98,000. "At the temperatures in the interiors of stars atoms will be highly ionized, and if in white dwarfs the last stages of the process are reached, the atoms being broken up into nuclei and free electrons, there is no immediate difficulty in the existence of high densities. On a naive view as to the "sizes" of electrons and nuclei (with radii some 10^{-5} times that of atoms) densities of the order of 10^{14} or 10^{15} would not be impossible." More careful consideration based on Fermi statistics leads to a formula that expresses the limiting density of a uniform sphere made up of fully ionized atoms in terms of the ratio of its mass to the mass of the sun. According to this the limiting density of Sirius B is 2.8×10^6.

G. F. S.

Control of Current in a Discharge-Tube by Means of a Magnetic Field. R. F. EARHART and C. B. GREEN. (*Phil. Mag.*, Jan., 1929.) In his book, "The Conduction of Electricity Through Gases," Sir J. J. Thomson cites an experiment in which the current in a circuit was varied by applying a longitudinal magnetic field to a spark gap, when it was desired to leave the impressed E.M.F. and other electrical properties unchanged. In continuance of this method of regulation this paper presents the results of a study of the effects of both longitudinal and transverse magnetic fields. The discharge tube was placed between the poles of an electromagnet. The electrodes were made from brass rod. Soon it appeared that the magnitude of the current traversing the gap had much to do with the size of the effect of the magnetic field upon the current strength. Two other factors assist in determining the effect of the field, viz., the distance between the electrodes and the pressure of the gas in the discharge chamber. With the field in the direction of the electrical discharge the increase in current due to the application of a magnetic field is greater with currents across the gap previously small than with those previously large. For example when a field of 1,500 gauss is brought to bear on a current of 3 milliamperes it increases it by 400 per cent., whereas applied to a current 8 times as large the field causes an increase of only 75 per cent.

The effects due to transverse fields are more complicated. In many cases the application of the field causes an increase of current, which reaches a maximum and decreases with further increase of the field. Indeed with fields sufficiently strong the current may be smaller than at first. The effect of reversal disappears with adequate reduction of the gas pressure. In the instance of hydrogen a figure "shows an extreme case where the application of a transverse field of from 300 to 500 gauss extinguished the discharge, while slightly higher fields restored it, and an application of 1,000 gauss more than doubled the current. The application of still higher fields reduced the current finally to the point of extinction and here the extinction was final."

G. F. S.

The Quantity of Radium Emanation in the Air according to Measurements made on Aircraft. A. WIGAND and F. WENK. (*Ann. d. Phys.*, No. 13, 1928.) A quantitative determination of the distribution of radium emanation in the air at different elevations will assist in clarifying our knowledge of the causes of ionization in the atmosphere. Up to the present time there have been measurements made in laboratories, in the open air near the earth

and on mountains. In addition samples of air have been brought down for examination and the radioactivity of wires carried by kites and balloons has been studied. In this paper the measurements were carried out by means of airplanes flying horizontally. Thus the mean value for the horizontal layer was effective in producing the readings and the speed of the plane produced a uniform flow of air through the apparatus.

After examination the method of absorbing the emanation in charcoal was rejected in favor of a condensation method. The forward motion of the airplane drove air into a mouthpiece from which it was led into a little cooling tower where it was freed from water vapor and CO_2 that might clog the apparatus. Thence it proceeded to a plate condenser, cooled by liquid air, where the emanation was deposited. About 8 kg. of liquid air had to be taken along in a flight. When the desired height had been attained the pilot was directed to proceed at the same level and air was admitted to the condenser for not more than 7 minutes. After landing the emanation was removed and its amount measured. In a single flight three different experiments could be made. The flights were made near Berlin. Five successful ascensions were made on three of which determinations were made at three elevations. The results show clearly a decrease of radium emanation with elevation. For example in one flight at a height of from 550 to 600 m. the emanation, expressed in 10^{-18} curies per cu. cm., was 168; at 2,200–2,400 m. it was 24, while for 3,800 m. no emanation was found. In no case was any emanation detected at 2,400 m. or over. Of course the derivation and previous history of the mass of air through which the horizontal flight is made have much influence on the emanation content.

The authors claim that their results prove the radium emanation in the air to be derived from the earth, at least on the days of the flights.

Measurements of emanation content were made also in the physical laboratory of the University of Halle where in one room it reached 13,800 of the units named above. This high value may have been due to the long use of radium in the laboratory. On top of the tower values were obtained six times as large as in the adjacent garden. This was attributed to the smoke of the city in which emanation may have condensed.

G. F. S.

The thirteenth number of the Annalen der Physik for 1928 is interesting from the fact that of its five articles three are by women. The two shorter papers by men come first and are followed by

"Experimental Investigations on W. Ritz's Theory of the Transversal Vibrations of Quadrilateral Plates" by Alice Lemke. This is succeeded by Frida Stadie's discussion of "Problems of Brownian Movements." The final contribution is "On the Magnetic Properties of Cobalt" by Margarete Samuel. In this last paper two rods of cobalt were investigated. They came from different sources, Krupp's Works at Essen and the Kaiser Wilhelm Institute for the Investigation of Iron in Düsseldorf. They contained 1.33 and 1.5 per cent. iron respectively. Unfortunately it was impossible to get cobalt free from iron for the experiments. An attempt made at the Institute to prepare a rod of cobalt from Kahlbaum's powdered metal free from both iron and nickel failed because the resulting rod became porous upon cooling. The outstanding feature of the results is that the two rods differed greatly in their magnetic properties and no thermal treatment was found that could make them agree.

G. F. S.

Emulsions Showing Chromatic Effects. R. H. HUMPHREY. (*Proc. Phys. Soc.*, London, Vol. 41, Pt. 1.) Glycerol has a refractive index of 1.467 that lies between the index of benzene (1.502) and that of acetone (1.359). "A stable emulsion of glycerol in acetone may be made by using pyroxylin (1 to 2 per cent.) as the emulsifying agent; such emulsions are milky, the glycerol being the disperse phase." The index of the dispersion medium may be increased continuously by adding benzene which mixes with the acetone. When its index becomes the same as that of the glycerol the emulsion becomes clear. The equality of the indices holds however only for a particular wave-length and when the emulsion is viewed in white light it is this wave-length that gets through. The light transmitted in this case has the color of this one wave-length. The other wave-lengths present in white light are refracted and reflected many times so that the scattered light is of the color complementary to the transmitted light. By the addition of benzene to the emulsion the transmitted color changes from violet through green to orange, with corresponding changes in the scattered light. The addition of acetone makes the transmitted colors return in the reverse order to violet.

"A change of 1° C. causes a perceptible change of the colour, the change being, for a fall of temperature, in the same direction as for increasing proportion of benzene."

G. F. S.

New Superconductors. W. J. DE HAAS. (*Naturwissenschaften*, Feb. 1, 1929.) At the meeting of the British Association for the Advancement of Science held last summer de Haas reported on the work done by himself, van Aubel and Voogt at the Cryogenic Institute, Leyden, upon the subject of superconductivity of metals. They examined for superconductivity not only pure metals but combinations of them. Tin and thallium show this property. Combinations of tin with copper, silver or antimony were tried. Antimony-tin and also thallium-bismuth manifested superconductivity. Ag_3Sn lost resistance continuously from 3.4° abs. to 1.3° without becoming superconducting.

It was further sought to find whether combinations of two metals that separately do not possess the property of superconductivity may show it. The atomic weights of gold and of bismuth lie respectively just above and below the atomic weights of the heavy metals that are superconducting. The eutectic of gold-bismuth was found to be superconducting. "The change of resistance is enormous. While at about 2.2° abs. the resistance has still about .7 of its value at room temperature, at 1.20° lower it disappears completely." At 1.5 abs. neither gold nor bismuth is a superconductor.

G. F. S.

The Production of Electricity by the Torpedo, a Fish. E. HUGUENARD and A. MAGNAN. (*Comptes Rendus*, Vol. 186, No. 26.) A living torpedo was placed between two horizontal tin plates which were connected to the terminals of a voltmeter that registered very promptly. If then a quick downward push was given to the plate just above the electric organ, the voltmeter indicated perhaps 20 volts, came back to 8 or 10 volts and then went up suddenly to 50 volts or more. The interval between the two maxima was about 1/10 sec. If on the other hand the compression was exerted alongside of the organ there was only a single throw of the needle. With a dead torpedo it is possible to get a single electric discharge though it is less intense than that furnished by the living organism. "All this leads to the belief that in the torpedo there is a combination of an electric apparatus developing an electromotive force under pressure and of an arrangement capable of producing compression at the will of the animal. This seems to be confirmed by the fact that when one provokes the discharge by pinching the torpedo's tail its body appears to grow thicker."

G. F. S.

Detection of the Isotopes of Lead by the Band Spectrum Method. S. BLOOMENTHAL. (*Science*, Feb. 22, 1929.) "A few years ago Grebe and Konen found that the lines near 4,250 Ångstroms in the band spectrum emitted by an arc containing uranium lead were sharper than those from an ordinary lead arc, and were displaced .055 Å. units toward the shorter wave-lengths." The author in the Ryerson Laboratory undertook to apply higher resolving power to the same observations. He made photographs of the spectra in the second order of a 21-ft. Rowland grating. The arc of ordinary lead was used, atomic weight 207.2, and again globules of uranium lead of atomic weight 206.1 were attached to copper rods and the arc was struck between them. Three amperes at 220 volts were used and the exposure lasted 15 hours.

"A line for line comparison of our spectrograms, for instance, near the head of the strong 5678.3 Å. band where there is comparatively little overlapping of neighboring series, shows that each line in the band spectrum secured using uranium lead (Pb_{206}) corresponds to the long wave-length member of a group of three lines in the spectrum secured using ordinary lead." There is reason for believing these bands to be due to PbO. If this be so, the separation between the lines of $Pb_{208}O$ and those of $Pb_{207}O$ is given by theory to be .37 wave-number whereas by measurement it is .43.

This spectroscopic method enables lead to be resolved quantitatively into its isotopes, and furnishes results in accord with Aston's analysis.

G. F. S.

The Foucault Pendulum of the Peabody Museum of Yale University. C. S. HASTINGS. (*Am. J. Science*, August, 1928.) Since Foucault in 1858 by the use of his pendulum demonstrated the rotation of the earth without reference to any body outside of the earth's atmosphere, his apparatus has come to be an attractive feature of museums. It has been believed that only exceeding long pendulums can be used. This paper is to prove that a moderate length suffices and that the Foucault pendulum can be made an instrument of precision.

A pendulum started to swing in a plane continues to swing in this plane without regard to the rotation of the point of support. If the pendulum were started to swing from a support vertically over the North Pole, the earth would rotate while the plane of vibration remained as at first, so that the plane would appear to rotate above the earth though in fact it would be the earth that rotates with respect to the unchanging plane. Since the earth

rotates through 360° in 24 hours, the plane would apparently turn through 15° per hour. For other places the hourly angle of apparent rotation is 15° multiplied by the sine of the latitude of the place. In the Peabody Museum the length of the pendulum was limited to 35 ft. by the construction of the building. The two usual methods of supporting the pendulum, by gimbals and by a hook whose point rests on a hard steel plate, were rejected in favor of clamping an elastic wire firmly at the top. The bob consisted of a lenticular mass of 28 lbs. of lead. The form was chosen to reduce air resistance and the wire was as fine as it could be with safety. Twice the weight of the bob would break it. The amplitude of the swing from end to end was 22 inches. As time passed the apparent path of the pendulum changed from a straight line to an elongated ellipse. This was remedied by adjustments made at the suspension at the top, so that for eight hours the ellipse failed to interfere. "The time can be read from the dial (over which the pendulum swings) with greater precision than from a sun dial." Measurements of the position of the plane of oscillation were made every hour from 9 A.M. till 4 P.M. on four days. From these it was calculated that in one hour of sidereal time the plane turned through $9.903° \pm .002$. From this the latitude of the Museum was computed to be $41.312° \pm .016$. "The true latitude, as measured directly from the Yale Observatory is 41.311°. Thus the final conclusion is that the pendulum has determined the latitude with an error of only 0.4 of a statute mile with an expectation of error that is not more than twice as great. This is far more accurate than ordinary sextant determinations at sea." The course of the pendulum brings out clearly the difference between solar and sidereal time. This suggests to the author to ask to what extent physical measurements could be used to replace astronomical observations. If no heavenly bodies could be seen on account of such clouds as surround other planets we could still prove the rotation of the earth by the Foucault pendulum, locate the equator and find the latitude of a place. From a study of latitudes the spheroidal shape of the earth would be shown.

G. F. S.

Scientific Notes from the Books and Letters of John Winthrop, Jr. (1606–1676). First Governor of Connecticut. C. A. BROWNE. (*Isis*, No. 36.) "The outstanding figure in the history of science in America during the seventeenth century is John Winthrop, Jr., obtainer of the Royal Charter of Connecticut and first Governor of this colony from 1659 to 1676, founder of Ipswich, Saybrook, New London and other New England towns, member of the Royal

Society, promoter of colonial industries, soldier, pioneer, lawyer, statesman, physician, philanthropist, man of business and lover of science." No less than 270 books from his library are preserved in the Society Library, New York City, while his papers and correspondence are in the Massachusetts Historical Society. From these sources the author derives the justification for according so high a place to the subject of his memoir. John Winthrop, Jr., was graduated from Trinity College, Dublin and in 1625, was admitted to the bar in London. He then travelled widely on the Continent where he made the acquaintance of men of science with whom he later corresponded in Latin. In 1631 he sailed for Boston which had been established by his father a year earlier. His library of 1,000 volumes came to him in America and grew in size until his death in 1676. The nature of his books indicates the range of the owner's interests. Of his scientific library 52 books are on chemistry or alchemy, 33 on medicine, 10 on physics, 9 on astronomy, 8 on mathematics, 8 on natural history, 3 on navigation, while a dozen more have to do with geography, metallurgy, military science, agriculture and political economy. Of the 300 of his books still preserved 133 are in Latin, 23 in German, 17 in French, 12 in Dutch, 7 in Italian, 4 in ancient Greek, 1 in Spanish and 71 in English. He had Comenius' "Janua Linguarum Reserata" with its parallel columns giving dialogues in Latin and again in the vernacular.

His chief scientific interest seems to have been chemistry. "His scientific friends in England and in Europe kept sending him books and book catalogues. This is shown, for example, in an old bill submitted by his friend, Francis Kirby, in March, 1633 (the first recorded bill for a shipment of chemicals and chemical apparatus to America), at the bottom of which is mentioned the sending of 3 book catalogues." Among the Winthrop books is George Starkey's "Nature's Explication and Helmont's Vindication." Starkey was a graduate of Harvard in the class of 1646 and was the first American college graduate to publish a treatise on chemistry, the book just named. Winthrop had "Das Buch Meteorum" in its medley of German and Latin, Cologne, 1566, and the "Baderbüchlein," Mühlhausen, 1562, that bears the autograph "Johannes Dee 1562." Winthrop himself attests under date of 1640 that this signature and other material added to the book were "written by that famous philosopher and chemist John Dee with his owne hand."

On the practical side Winthrop tried to establish in New England chemical industries in iron, salt and gunpowder. His papers abound in references to tar, potash, indigo and pitch. In his

handwriting there is a proposal for the formation of a joint stock company for the manufacture of saltpeter, the first prospectus of an American industrial company. His chief chemical interest, however, lay in the application of the science to medicine. He seems to have made no charge for his remedies and doctored all from Indians to governors. According to Cotton Mather "wherever he came, the diseased flocked about him as if the healing Angel of Bethesda had appeared in the place." His main chemical remedies were saltpeter, iron, sulphur, antimony, tartar, copperas, white vitriol and calomel. He prescribed in addition red coral, powdered ivory, rosin, saffron, sassafras, aloes, balsam and rhubarb. He used a secret alphabet to conceal the nature of his medicine for ague and small-pox.

Winthrop was much interested in astronomy, having at his service a $3\frac{1}{2}$ foot telescope. With this he saw 5 satellites of Jupiter, according to a letter he wrote to the President of the Royal Society. Later he donated the instrument to Harvard. He had Kepler's treatise "De Stella Nova," Tycho Brahe's "Astronomicae instauratae mechanica," as well as less famous works in the same field of science.

He collected fossils, ores and minerals, "operated mines in New England for the production of ores of iron, blacklead and other minerals." Indeed he appears to have been the precursor of those who unite a knowledge of mining engineering with executive ability in public affairs. Among his correspondents were Kepler, Newton, Cromwell, Helmont, Milton, Wren and Prince Rupert. In his letters from or to members of the Royal Society such subjects are discussed as the tides, the manufacture of pitch and tar, the refining of gold, deep-sea sounding and meteors. Not only was he a member of the recently established Royal Society but his grandson became one of its Fellows. At the time of the Revolution a third John Winthrop, descended from a brother of the subject of this article, was Hollis Professor at Harvard and a Fellow of the Royal Society.

G. F. S.

Stellar Radiation Measurements. E. PETTIT and S. B. NICHOLSON. (*Astrophys. J.*, Vol. 68, No. 4.) The thermocouples used were made of bismuth and of an alloy of bismuth with 5 per cent. of tin. The wires from which the junctions were made had diameters of only .03 mm. "The weight of a compensated thermocouple with two electrically opposed junctions, provided with receivers half a millimeter in diameter, including connecting wires, is about

0.1 mg. The mass of the receiver which is heated by the star is only 0.035 mg. From the electrical properties of the circuit it can be shown that a galvanometer deflection of 1 millimeter corresponds to a rise of temperature of 33-millionths of a degree Centigrade in the receiver. For the giant star Betelgeuse, which is radiometrically the brightest we have measured, the rise in temperature of the receiver is 15-thousandths of a degree; and for the radiometrically faint dwarf, Boss 4342, whose visual magnitude is 7.9, the rise is only 9-millionths of a degree."

A study has been made of 124 stars. The radiation in calories received per minute on a square cm. outside of the earth's atmosphere is from Betelgeuse 132×10^{-12}, from Sirius 145×10^{-12}, from Spica 48.5×10^{-12}, from the sun 1.93.

G. F. S.

The Transmission of Ultra-Violet Light through Tracing Cloth. C. H. YOUNG. (*Nature*, Jan. 12, 1929.) At McGill University the effect of ultra-violet light on different kinds of blue print paper was examined. It was discovered in this connection that the ultra-violet light from a quartz mercury vapor lamp passed to a surprising extent through tracing cloth or linen. Reproductions are given of spectrograms obtained with three secs. exposure. The smallest wave-length transmitted by the cloth varied with its make from 2,535 to 2,482 Ångstrom units. Newspaper transmitted no radiation shorter than 3,984 Å.U., while the limit for writing and for wrapping paper was 3,125 Å.U.

The thickness or number of meshes of the cloth per inch appeared to have but little to do with its transmission. It was further discovered that "the heat from the sun or from a red-hot ball passed through the tracing cloth to a much less extent than through glass or vita glass. Herein may lie the usefulness of this discovery, because, whether in sunlight or in artificial ultra-violet light, it is now possible to screen off much of the heat and yet retain most of the ultra-violet. A single layer of tracing cloth, between wide-meshed wire screens, can now replace curtains or blinds, and with this screen before an open sunny window it is possible to enjoy the advantages of ultra-violet light without undue heat or glare, although the eyes should be protected. Moreover, for country cottages, chicken farms, etc., it is now possible to obtain a cheap and effective substitute for the many glasses which have been manufactured to secure ultra-violet light in the more beneficial regions of the spectrum."

G. F. S.

An Investigation of Pyroelectricity and of Piezoelectricity. A. MEISSNER. (*Naturwissenschaften*, Jan. 11, 1929.) A circular disc of quartz, 15 mm. in diameter was prepared, its faces being perpendicular to an electrical axis. This was brought into vibration by the application of high frequency currents. Lycopodium powder was used to mark the vibrations. From a study of the results joined to a consideration of their implications it was possible to calculate the distances separating the silicon atoms in the quartz. Thus the distance from a silicon atom to its neighbor in the same plane was computed to be 4.9 Å.U., while from it to another atom of the same element above it the distance was 5.44 Å.U. These results agree well with those obtained by the use of X-rays on the quartz. "There comes out the surprising fact that from experiments made on an object of considerable size structural dimensions of the order of 10^{-8} cm. could be determined to an accuracy of a few per cent."

G. F. S.

The Diffraction of Electrons by a Ruled Grating. E. RUPP. (*Zeit. fuer Physik*, Vol. 52, p. 8.) Previous investigations of the diffraction of electrons have used crystal structures as gratings. Naturally the question arose whether artificially ruled gratings would serve also to produce the same effect. The experiments cited below give an affirmative answer to the inquiry. A beam of electrons falls at almost grazing incidence upon a grating of speculum metal with 1,300 rulings per cm. The regularly reflected beam is caught upon a photographic plate which is found to record diffracted beams up to the third order. The electrons came from barium oxide. They passed through three slits in series, fell upon the grating and from it went on to the plate. About 40 photographic records were made, there being three different angles of incidence with the grating and three distinct velocities of the electrons determined by the voltage through which they fell. Reproductions of 6 photographic impressions are presented. The entire apparatus was in a vacuum.

The consistency of the results was carefully examined. de Broglie has given a formula for calculating the equivalent wave-length of electrons from their mass and velocity according to which the wave-length is inversely proportional to the velocity. When the electron beam is diffracted by the grating another formula connects these 5 quantities, viz., order of the diffraction band, equivalent wave-length of the impinging electrons, grating constant, angle between the incident beam and the plane of the grating and, finally, the angle between the regularly reflected beam and the

diffracted beam. The wave-length was varied by changing the difference of potential through which the electrons fell. From the formula, by the insertion of the other observed quantities the angle between the incident beam and the grating plane was calculated. The values computed from different groups of observations agree within a few per cent. Another type of comparison shows again close agreement between de Broglie's formula and the diffraction formula.

This paper forges another link in the chain that is binding electrons and waves together.

G. F. S.

Distilling Water by the Heat of the Sun. J. RICHARD., Director of the Oceanographic Museum at Monaco. (*La Nature*, March 1, 1929.) In regions without fresh water but near the sea it is of great advantage to be able to use the heat of the sun for distilling water from the available salt water. This paper discusses a simple arrangement for doing this.

Inside of a gardener's cold-frame the lower surface of the slanting panes of glass are often covered with drops of water due to the condensation of the water vapor coming from the soil and plants within when it comes in contact with the glass cooled by the outer air. These drops run together and make their way down the inclined glass surface finally dripping away from its lower edge. The government of Tunis instituted a competition for the production of potable water in the south of this country. Maurain then directed attention to the method referred to which seems to have dropped out of sight. It had been used in Salinas near the port of Antofagasta, in the desert of Atacama, Chile. Here the water to be distilled contained 140 gr. of salts per liter, five times as much as in Mediterranean Sea water. 60 wooden frames were installed, each 200 ft. long and 4 ft. wide. The total glass surface was 51,400 sq. ft., the panes being inclined 10° to the horizontal. A gutter for collecting the condensed water ran along the lower edges of the sheets of glass. A windmill pumped the salty water into the frames. Under favorable conditions in summer more than 5,000 gallons or 22 tons of distilled water were thus obtained daily. With the use of the frames a gallon cost less than one cent whereas the cost was four times as much when the water was vaporized by the use of fuel. The result was accomplished with simple materials, wood, glass and putty chiefly.

A similar frame installed in Paris produced 1,083 gr. per sq. m. on a fine day of Sept., 1926, when the sun shone for 7 hours. It was noted that distillation went on even during the night. The

author tried the plan at Monaco on the Mediterranean and in Sept., 1927, got 2,452 gr. sq. m. Ginestous had taken up the observations in Tunis and got on Aug. 23, 1927, 4,043 gr. per sq. m. Inside of his frame the temperature rose to 68° C. Richard has set up at Monaco a small frame with an area of little more than a square m. and with it has made observations of the quantity of water condensed on fine days in every month of the year except January. The greatest production occurred in May and June and the smallest in December, the ratio being about 6 : 1. Days that seem equally fine often produce quantities of water differing by 10 per cent. or more.

When the sun shines more water is obtained with thin glass than with thick. On a certain day a frame with glass 8 mm. thick produced 1,561 gr. while another with glass only 3 mm. thick collected 1,651 gr. On dull days the thicker glass makes the better showing. Reference is made to Ginestous' installing a practical frame in the southern part of Tunis.

G. F. S.

Diffraction of Cathode Rays by Mica. SEISHI KIKUCHI. (*Japanese J. Physics*, Vol. 5, No. 2.) Homogeneous beams of cathode rays with energies ranging from 10 to 25 kilovolts passed through thin sheets of mica and fell upon photographic plates about 15 cm. behind the crystal. Diffraction patterns were produced on the plates the form of which depended on the thickness of the mica sheets. With a sheet having a thickness of the order of 10^{-5} cm., so thin that it showed no interference colors at all, cathode rays of wave-lengths of .047 and .067 Å.U. respectively gave patterns containing many scores of spots. "These figures consist each of an array of spots forming an equilateral triangular network, the dimensions of the meshes depending on the wave-length. The general feature of the pattern is very little affected by the orientation of the crystal. When a magnetic field is applied on the path of the rays between the crystal and the photographic plate, the pattern displaces as a whole and there can be no doubt that it is due to the cathode rays scattered by the crystal." By using the equivalent wave-lengths of the cathode rays employed the distance between two neighboring molecules in the mica was calculated to be 5.18 Å.U., whereas the same distance according to X-ray analysis was 5.17 Å.U. By varying the potential used in the cathode ray tube the equivalent wave-lengths were made to vary from .0432 to .1032 Å.U. yet the distance calculated remained constant within the error of the experiment. The mica

was so thin that it contained in its thickness only about 50 planes and the diffraction was a two-dimensional one. With greater thicknesses of the mica patterns similar to those obtained by Laue with X-rays diffracted by crystals were produced. Some patterns are very complicated. In these cases the diffraction was due to arrays in three dimensions of the diffracting centers.

G. F. S.

The Penetration of Hydrogen into Metal Cathodes and its Effect upon the Tensile Properties of Metals and their Resistance to Repeated Stresses. F. C. LEA. (*Proc. Roy. Soc.*, A. 791.) In many processes of industry pickling is used with resulting brittleness in the metal. This has been attributed to occluded hydrogen. Considerable work has already been done in this field. In this investigation hydrogen is forced into pieces of metal by making them the cathodes in electrolysis. By examination of the specimens thus treated some light on the failure of metals under stress is obtained. For experiments on tensile strength cylinders of metal 2 in. in length and .399 in. in diameter were prepared. These were made the cathodes in either acid or alkali electrolytic baths. Specimens of mild steel were unchanged in tensile strength after the current had deposited hydrogen upon them, but the flow of .25 ampere for 10 min. causes the elongation in per cent. to drop from 38 to 25. The same type of effect was found also for commercially pure nickel and for rustless steel with 26 per cent. of chromium and nickel. In the case of the two kinds of steel electrolysis made a large difference in the area at the time of fracture. Before the deposition of hydrogen the "Reduced in area per cent." was roughly twice as great as afterwards. The type of fracture also was modified. It was discovered that the finish of the surface greatly affected the results. "The tensile strengths of the specimens were the same as the turned specimen; the elongations per cent. and the reductions in area were almost exactly the same as the specimen tested in air." The types of fracture were, however, widely different. Nickel plating mild steel seemed to do away with the effect of electrolysis on tensile strength and other static properties though it lowered the resistance to repeated stresses. A very interesting result was got by subjecting an unplated, unpolished specimen to electrolysis and by subsequently putting it in boiling water for ten minutes. The specimen thus treated reverted in all respects to the characteristics of steel on which hydrogen had not been set free.

"Hydrogen in the cathode does not diminish the resistance to

impact of mild steel and its effect on repeated stress seems of itself unimportant. The fractures under repeated stresses, however, show that the penetration of hydrogen has an important bearing upon the nature and form of fracture of a plastic material, which under torsional repeated stress, fails in a manner similar to quenched and tempered materials.

"Nickel plating mild steel lowers the range of repeated stress but this is not apparently due to occluded gases. As drawn nickel bar is as resistant as mild steel under repeated stresses, it would appear that either initial stresses set up by the deposition or the discontinuity between the metals or, more likely, a crack which early forms in the deposited nickel, is the cause of the lowering of the repeated stress range."

G. F. S.

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